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All the fundamental massless fields in bosonic string theory

E. B. Manoukian*

The Institute for Fundamental Study, Naresuan University, Phitsanulok 65000, Thailand

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A systematic analysis of the massless fields in the mass spectra of bosonic strings is carried in arbitrary spacetime dimension $D > 2$. The emphasis is put on the derivations of their propagators, their polarization aspects and the underlying constraints involved. The treatment is given, in the presence of external sources, in the celebrated Coulomb gauge for the second rank tensors and vector fields, which ensures positivity - a result which is also established in the process. No constraints are imposed on the external sources so that their components may be varied independently generating complete expressions for the propagators. This latter condition is an important one in the generation of dynamical theories with constraints involving such modifications as Faddeev-Popov factors.

1 Introduction

What is remarkable about string theory is that the fundamental fields that are required to describe the dynamics of elementary particles arise naturally in the mass spectra of oscillating strings and are not, a priori, assumed to exist or put in by hand in the the underlying theories. At present only the massless fields string modes are really physically relevant because of the enormous masses of the massive fields string-excitation modes. A massless vector field, for example, may be thought to acquire mass by some mechanism such as, for example, from open strings whose end points are attached to different branes or by some other means such as the Higgs mechanism. In this paper, we are interested in all the massless fields excitations of bosonic strings. The massless fields in question are: the second rank symmetric tensor field, the second-rank anti-symmetric tensor field, the vector field and the scalar one (c.f. [1–3]) which arise in the mass spectra of oscillating bosonic strings in $D = 26$ dimensional spacetime. The second rank symmetric tensor field is associated with the graviton and the scalar one with the dilaton which is thought to provide a coupling strength in string theory through its vacuum expectation value. We are interested in dynamical aspects of these fields. To this end, we are concerned about underlying constraints and the explicit expressions of their propagators. We set the dimension of spacetime arbitrarily to $D > 2$. We work in the celebrated Coulomb gauge, as applied to the second rank tensors and vector fields, which ensures positivity of the formalism as will be established in each case. For earlier four-dimensional studies of the massless symmetric second rank tensor, c.f [4], and for the anti-symmetric one, c.f. [5–7]. Our analysis, however, is involved with dynamical aspects by deriving expressions of the underlying propagators, as just mentioned, and obtain the basic polarization aspects of the fields and the inherit degrees of freedom in arbitrary dimensional spacetime $D > 2$. Quite importantly, the fields are coupled to *non*-constrained external sources so that each of their components may be varied independently and thus the complete expressions of the propagators are obtained. Also the variations of each of the components of external sources is necessary in describing full field theory interactions, such as deriving Faddeev-Popov factors. A fairly detailed

* E-mail: manoukian_eb@hotmail.com

demonstration of the importance of varying the components of the external sources independently with applications to gauge theories is given in Sect. 3 of [4] for the convenience of the reader. In the present work, the scalar field presents no challenge and nothing new emerges about it here. The vector field is relatively simpler to analyze than the second rank tensor cases, but we provide a detailed analysis for it to set up the formalism involved in our analysis, and we also need aspects of its underlying polarization vectors for the other cases. This is given in Sect. 2. Sections 3 and 4 constitute the main contribution of this paper dealing, in turn, with the symmetric and anti-symmetric tensor field cases, respectively. The Minkowski metric is defined by $[\eta_{\mu\nu}] = \mathrm{diag}[-1, 1, \ldots, 1]$, the Greek indices $\mu, \nu, \ldots$ go over $0, 1, \ldots, D-1$, while the Latin ones $i, j, \ldots$ go over $1, \ldots, D-1$.

2 Vector field

The Lagrangian density of a massless vector field A^μ, may be defined by

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \qquad F^{\mu\nu}(x) = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x). \tag{1}$$

It is invariant under the gauge transformation $A^\mu \to A^\mu + \partial^\mu \Lambda$. We work in the Coulomb gauge

$$\partial^i A^i = 0, \tag{2}$$

with a summation over i is understood as a repeated index.

We add an external source contribution to (1), obtaining

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + J^\mu A_\mu. \tag{3}$$

Quite generally, we may write

$$A^\mu = -A^0\, \eta^{\mu 0} + \eta^{\mu i} A^j. \tag{4}$$

Due to the constraint in (2), one cannot vary the components of A^j independently. We may, however, introduce a field with components $\mathcal{A}^i$ that may be varied independently and set

$$A^i(x) = \pi^{ij} \mathcal{A}^j(x), \tag{5}$$

where

$$\pi^{ij} = \Big(\delta^{ij} - \frac{\partial^i \partial^j}{\partial^k \partial^k}\Big), \tag{6}$$

satisfying the identities

$$\pi^{ij} = \pi^{ji}, \quad \partial_i \pi^{ij} = 0, \quad \pi^{ij} \partial_j = 0, \quad \pi^{ij} \pi^{jk} = \pi^{ik}, \tag{7}$$

operating as a projection operator.

Upon varying A^0, $\mathcal{A}^j$ the Lagrangian density (3) leads to

$$-\partial^\mu \partial_\mu A^i = \Big(\delta^{ij} - \frac{\partial^i \partial^j}{\partial^k \partial^k}\Big) J^j, \tag{8}$$

$$-\partial^k \partial_k A^0 = J^0. \tag{9}$$

Clearly only the A^i may propagate. Since no constraints were imposed on $J^\mu(x)$, we may vary its components independently. Upon taking the vacuum expectation values $\langle 0_- | \,.\, | 0_+ \rangle$ of (8), (9), setting

$\langle 0_+|A^\mu(x)|0_-\rangle = (-\mathrm{i}\delta/\delta J_\mu(x))\langle 0_+|0_-\rangle$, and integrating with respect to the source components, we obtain

$$\langle 0_+|0_-\rangle = \exp[\frac{\mathrm{i}}{2}\int (dx)(dx')\, J_\mu(x)\Delta_+^{\mu\nu}(x-x')J_\nu(x')], \tag{10}$$

where

$$\Delta_+^{\mu\nu}(x-x') = \int \frac{(dp)}{(2\pi)^D}\, \mathrm{e}^{\mathrm{i}p(x-x')}\, \Delta_+^{\mu\nu}(p), \tag{11}$$

and $\Delta_+^{0i} = \Delta_+^{i0} = 0$,

$$\Delta_+^{00}(p) = \frac{1}{|\mathbf{p}|^2}, \tag{12}$$

$$\Delta_+^{ij} = \frac{1}{p^2 - \mathrm{i}\epsilon}\, \pi^{ij}, \qquad \epsilon \to +0, \tag{13}$$

where now

$$\pi^{ij} = \delta^{ij} - \frac{p^i p^j}{|\mathbf{p}|^2}. \tag{14}$$

Let $\mathbf{e}_1$, ..., $\mathbf{e}_{D-2}$, $\mathbf{p}/|\mathbf{p}|$, be $D-1$ mutually pairwise orthonormal vectors spanning a $(D-1)$ Euclidean space, i.e.,

$$\delta^{ij} = \frac{p^i}{|\mathbf{p}|}\,\frac{p^i}{|\mathbf{p}|} + \sum_{\lambda=1}^{D-2} e_\lambda^i\, e_\lambda^j, \tag{15}$$

from which

$$\pi^{ij} = \sum_{\lambda=1}^{D-2} e_\lambda^i\, e_\lambda^j. \tag{16}$$

Clearly, $\Delta_+^{00}(p)$ gives rise to a phase to $\langle 0_+|0_-\rangle$. Upon using the identity

$$\mathrm{i}\Big[\,\frac{1}{p^2-\mathrm{i}\epsilon} - \frac{1}{p^2+\mathrm{i}\epsilon}\,\Big] = -\frac{\pi}{|\mathbf{p}|}[\,\delta(p^0-|\mathbf{p}|) + \delta(p^0+|\mathbf{p}|)\,], \tag{17}$$

and (12)-(16), we obtain for the vacuum persistence probability

$$|\langle 0_+|0_-\rangle|^2 = \exp[-\int \sum_{\lambda=1}^{D-2} \frac{d^{D-1}\mathbf{p}}{2|\mathbf{p}|(2\pi)^{D-1}}\, |J(\lambda,p)|^2] < 1, \tag{18}$$

establishing the positivity of the formalism, where

$$J(\lambda,p) = J^i(p)\,\epsilon_\lambda^i, \quad p^0 = +|\mathbf{p}|. \tag{19}$$

The number of independent polarization states may be obtained from (16) to be

$$\sum_{\lambda=1}^{D-2} e_\lambda^i\, e_\lambda^i = \pi^{ii} = \delta^{ii} - 1 = D-2. \tag{20}$$

3 Symmetric traceless second rank tensor field

The Lagrangian density of a massless symmetric tensor field is given by (c.f. [8])

$$\mathcal{L} = -\frac{1}{2}\,\partial^\sigma h_{\mu\nu}\,\partial_\sigma h^{\mu\nu} + \partial_\mu h^{\mu\nu}\partial_\sigma h^\sigma{}_\nu - \partial_\sigma h^{\sigma\mu}\partial_\mu h + \frac{1}{2}\,\partial^\mu h\,\partial_\mu h, \tag{21}$$

where $h = h^\mu{}_\mu$, $h^{\mu\nu} = h^{\nu\mu}$. It is invariant under the gauge transformation

$$h^{\mu\nu} \to h^{\mu\nu} + \partial^\mu \Lambda^\nu + \partial^\nu \Lambda^\mu, \tag{22}$$

up to a total derivative. We work in the Coulomb-like gauge

$$\partial^i h^{i\nu} \,=\, 0, \tag{23}$$

with a sum over i.

We add an external source contribution to the Lagrangian density in (21) to obtain

$$\mathcal{L} = -\frac{1}{2}\,\partial^\sigma h_{\mu\nu}\,\partial_\sigma h^{\mu\nu} + \partial_\mu h^{\mu\nu}\partial_\sigma h^\sigma{}_\nu - \partial_\sigma h^{\sigma\mu}\partial_\mu h + \frac{1}{2}\,\partial^\mu h\,\partial_\mu h + T^{\mu\nu}h_{\mu\nu}. \tag{24}$$

We may write

$$h^{\mu\nu} = \eta^{\mu i}\eta^{\nu j}h^{ij} - (\eta^{\mu 0}\eta^{\nu i} + \eta^{\mu i}\eta^{\nu 0})h^{0i} + \eta^{\mu 0}\eta^{\nu 0}h^{00}. \tag{25}$$

Due to the constraint in (23), the components h^{ij}, h^{0i}, cannot be varied independently. We can, however, introduce fields $H^{k\ell}$, ϕ^j, whose components may be varied independently, and set

$$h^{ij} = \frac{1}{2}(\pi^{ik}\pi^{j\ell} + \pi^{i\ell}\pi^{jk})H^{h\ell}, \tag{26}$$

$$h^{0i} = \pi^{ij}\phi^j, \tag{27}$$

with π^{ij} defined in (6). The variations of the fields ϕ^j, h^{00}, $H^{k\ell}$, lead from the Lagrangian density (24), after some labor, to

$$h^{ii} \,=\, -\,\frac{1}{\partial^k\partial^k}T^{00}, \quad h^{00} = -\frac{1}{(D-2)\,\partial^k\partial^k}\Big[(D-3)\frac{\partial^2}{\partial^\ell\partial^\ell}T^{00} + \pi^{ij}T^{ij}\Big], \tag{28}$$

$$h^{0i} \,=\, -\,\frac{1}{\partial^\ell\partial^\ell}\,\pi^{ik}T^{0k}, \quad \pi^{ij} = \Big(\delta^{ij} - \frac{\partial^i\partial^j}{\partial^k\partial^k}\Big), \tag{29}$$

$$-\,\partial^2 h^{ij} \,=\, \frac{1}{D-2}\Big[\frac{D-2}{2}(\pi^{ik}\pi^{jl} + \pi^{il}\pi^{jk}) - \pi^{ij}\pi^{kl}\Big]T^{kl} + \frac{1}{D-2}\pi^{ij}\,\frac{\partial^2}{\partial^k\partial^k}\,T^{00}, \tag{30}$$

recalling that $\delta^i{}_i \,=\, D-1$, $\partial^2 \,=\, \partial^\mu\partial_\mu$. Clearly, only h^{ij} may propagate. We note that if constraints are imposed on the external sources on the right-hand of these equations, the complete expression of the propagator in question would not follow. Since no constraints were imposed on $T^{\mu\nu}(x)$, we may vary its components independently. Upon taking the vacuum expectation values $\langle 0_-|\,.\,|0_+\rangle$ of (29), (30), setting $\langle 0_+|h^{\mu\nu}(x)|0_-\rangle \,=\, (-\mathrm{i}\delta/\delta T_{\mu\nu}(x))\langle 0_+|0_-\rangle$, and integrating with respect to the source components, we obtain

$$\langle 0_+|0_-\rangle \,=\, \exp\left[\,\frac{\mathrm{i}}{2}\int(dx)(dx')\,T_{\mu\nu}(x)\Delta_+^{\mu\nu,\lambda\sigma}(x-x')T_{\lambda\sigma}(x')\right], \tag{31}$$

where

$$\Delta_+^{\mu\nu,\lambda\sigma}(x-x') = \int\frac{(dp)}{(2\pi)^D}\,\mathrm{e}^{\mathrm{i}p(x-x')}\,\Delta_+^{\mu\nu,\lambda\sigma}(p), \tag{32}$$

$$\Delta_{+}^{00,00}(p) = \frac{D-3}{D-2}\,\frac{(p^2)}{|\mathbf{p}|^4}, \quad p^2 = |\mathbf{p}|^2 - (p^0)^2, \tag{33}$$

$$\Delta_{+}^{00,ij}(p) = \frac{2}{D-2}\,\frac{1}{|\mathbf{p}|^2}\,\pi^{ij}, \quad \Delta_{+}^{00,0i} = \Delta_{+}^{0i,00} = 0, \tag{34}$$

$$\Delta_{+}^{0i,0k}(p) = -\frac{1}{|\mathbf{p}|^2}\,\pi^{ik}, \tag{35}$$

$$\Delta_{+}^{ij,k\ell} = \frac{1}{p^2 - \mathrm{i}\epsilon}\,\frac{1}{D-2}\Big[\frac{D-2}{2}(\pi^{ik}\pi^{j\ell} + \pi^{i\ell}\pi^{jk}) - \pi^{ij}\pi^{k\ell}\Big], \epsilon \to +0, \tag{36}$$

$$\Delta_{+}^{ij,00}(p) = \frac{2}{D-2}\,\pi^{ij}\,\frac{1}{|\mathbf{p}|^2}, \tag{37}$$

and π^{ij} is given in (14).

Clearly, $\Delta_{+}^{00}(p)$, $\Delta_{+}^{0i,0k}(p)$, $\Delta_{+}^{ij,00}(p)$, $\Delta_{+}^{00,ij}(p)$, provide phase factors to $\langle 0_+|0_-\rangle$. We use the completeness relation in (15), the identity in (16), and the following identity

$$\frac{1}{D-2}\Big[\frac{D-2}{2}(\pi^{ik}\pi^{j\ell} + \pi^{i\ell}\pi^{jk}) - \pi^{ij}\pi^{k\ell}\Big] = \sum_{\lambda,\lambda'=1}^{D-2} \epsilon^{ij}(\lambda,\lambda')\,\epsilon^{k\ell}(\lambda,\lambda'), \tag{38}$$

where

$$\epsilon^{ij}(\lambda,\lambda') = \frac{1}{D-2}\Big[\frac{D-2}{2}(e^i_\lambda\, e^j_{\lambda'} + e^i_{\lambda'}\, e^j_\lambda) - \delta_{\lambda,\lambda'}\sum_{\kappa=1}^{D-2} e^i_\kappa\, e^j_\kappa\Big], \tag{39}$$

obtained after a few steps. Finally the identity in (17) gives for the vacuum persistence probability the expression

$$|\langle 0_+|0_-\rangle|^2 = \exp[-\int \sum_{\lambda,\lambda'=1}^{D-2} \frac{d^{D-1}\mathbf{p}}{2|\mathbf{p}|(2\pi)^{D-1}}\,|T(\lambda,\lambda',p)|^2] < 1, \tag{40}$$

establishing the positivity of the formalism, and where

$$T(\lambda,\lambda',p) = T^{ij}(p)\,\epsilon^{ij}(\lambda,\lambda'), \quad p^0 = +|\mathbf{p}|. \tag{41}$$

The number of independent polarization states is obtained from the identity (38) and is worked out as follows:

$$\sum_{\lambda,\lambda'=1}^{D-2} \epsilon^{ij}(\lambda,\lambda')\,\epsilon^{ij}(\lambda,\lambda') = \frac{1}{D-2}\Big[\frac{D-2}{2}(\pi^{ii}\pi^{jj} + \pi^{ij}\pi^{ji}) - \pi^{ij}\pi^{ij}\Big] = \frac{1}{2}D(D-3), \tag{42}$$

where we have used the set of equalities on the right-hand side of (20), and

$$\pi^{ij}\,\pi^{ij} = \pi^{ij}\delta^{ij} = D-2. \tag{43}$$

4 Anti-symmetric second rank tensor field

Consider an anti-symmetric tensor field $C^{\mu\nu} = -C^{\nu\mu}$, and introduce the tensor field

$$F^{\mu\nu\sigma} = \partial^\mu C^{\nu\sigma} + \partial^\sigma C^{\mu\nu} + \partial^\nu C^{\sigma\mu}, \tag{44}$$

defined as a cyclic permutation of the indices. We define the Lagrangian density

$$\mathcal{L} = -\frac{1}{6} F^{\mu\nu\sigma} F_{\mu\nu\sigma}. \tag{45}$$

The Lagrangian density is invariant under the gauge transformation

$$C^{\mu\nu} \to C^{\mu\nu} + \partial^\mu \Lambda^\nu - \partial^\nu \Lambda^\mu. \tag{46}$$

We work in the Coulomb-like gauge

$$\partial_i C^{i\nu} = 0, \tag{47}$$

with a sum over i.

We add an external source contribution to the Lagrangian density in (45) to obtain

$$\mathcal{L} = -\frac{1}{6} F^{\mu\nu\sigma} F_{\mu\nu\sigma} + F^{\mu\nu\sigma} J_{\mu\nu\sigma}. \tag{48}$$

up to an overall multiplicative dimensional constant. We may write

$$C^{\mu\nu} = \eta^{\mu i}\eta^{\nu j} C^{ij} - (\eta^{\mu 0}\eta^{\nu i} - \eta^{\mu i}\eta^{\nu 0})\, C^{0i}. \tag{49}$$

Due to the constraint in (47), one cannot vary all the components of $C^{i\nu}$ independently, we can, however, introduce fields with component χ^{kl}, φ^j, that may be varied independently, and set

$$C^{ij} = \frac{1}{2}(\pi^{ik}\pi^{jl} - \pi^{il}\pi^{jk})\,\chi^{kl}, \quad C^{0i} = \pi^{ij}\varphi^j, \quad \pi^{ij} = \delta^{ij} - \frac{\partial^i\partial^j}{\partial^k\partial^k}. \tag{50}$$

The variations of the components χ^{kl}, φ^j, lead from the Lagrangian density in (48) to

$$-\partial^k\partial^k\, C^{0i} = \pi^{ij}\, J^{0j}, \tag{51}$$

$$-\partial^2\, C^{ij} = \frac{1}{2}(\pi^{ik}\pi^{jl} - \pi^{il}\pi^{jk})\, J^{kl}. \tag{52}$$

Clearly, only C^{ij} may propagate. Again if a constraint is imposed on the external source on the right-hand sides of these equations, such as a transversality condition $\partial_i J^{ij} = 0$, the complete expression of the propagator in question would not follow. Upon taking the vacuum expectation values $\langle 0_- | \,.\, | 0_+\rangle$ of (51), (52), setting $\langle 0_+ | C^{\mu\nu}(x) | 0_-\rangle = (-\mathrm{i}\delta/\delta J_{\mu\nu}(x))\langle 0_+ | 0_-\rangle$, and integrating with respect to the source components, we obtain

$$\langle 0_+ | 0_-\rangle = \exp[\frac{\mathrm{i}}{2}\int (dx)(dx')\, J_{\mu\nu}(x)\tilde{\Delta}_+^{\mu\nu,\lambda\sigma}(x-x')J_{\lambda\sigma}(x')] \tag{53}$$

where

$$\tilde{\Delta}_+^{\mu\nu,\sigma\lambda}(x-x') = \int \frac{(dp)}{(2\pi)^D}\, \mathrm{e}^{\mathrm{i}p(x-x')}\, \tilde{\Delta}_+^{\mu\nu,\sigma\lambda}(p), \tag{54}$$

$\tilde{\Delta}_{+}^{00,00} = 0 = \tilde{\Delta}_{+}^{00,0i} = \tilde{\Delta}_{+}^{0i,00} = \tilde{\Delta}_{+}^{ij,00},$

$$\tilde{\Delta}_{+}^{0i,0j}(p) = -\frac{1}{|\mathbf{p}|^2}\,\pi^{ij}, \tag{55}$$

$$\tilde{\Delta}_{+}^{ij,k\ell}(p) = \frac{1}{p^2 - \mathrm{i}\epsilon}\,\frac{(\pi^{ik}\pi^{j\ell} - \pi^{i\ell}\pi^{jk})}{2}, \qquad \epsilon \to +0. \tag{56}$$

Clearly, $\tilde{\Delta}_{+}^{0i,0j}(p)$ gives rise to a phase factor to $\langle 0_+|0_-\rangle$.

Upon using the identity

$$\frac{(\pi^{ik}\pi^{j\ell} - \pi^{i\ell}\pi^{jk})}{2} = \sum_{\lambda,\lambda'=1}^{D-2} \varepsilon^{ij}(\lambda,\lambda')\varepsilon^{k\ell}(\lambda,\lambda'), \tag{57}$$

where now

$$\varepsilon^{ij}(\lambda,\lambda') = \frac{1}{2}(e_\lambda^i e_{\lambda'}^j - e_{\lambda'}^i e_\lambda^j), \tag{58}$$

the vacuum persistence probability emerges as

$$|\langle 0_+|0_-\rangle|^2 = \exp[-\int \sum_{\lambda,\lambda'=1}^{D-2} \frac{d^{D-1}\mathbf{p}}{2|\mathbf{p}|(2\pi)^{D-1}}\,|J(\lambda,\lambda',p)|^2] < 1, \tag{59}$$

with

$$J(\lambda,\lambda',p) = J^{ij}(p)\,\varepsilon^{ij}(\lambda,\lambda'), \qquad p^0 = |\mathbf{p}|, \tag{60}$$

establishing the positivity of the formalism, except now for the number of independent polarization states, we obtain

$$\sum_{\lambda,\lambda'=1}^{D-2} \varepsilon^{ij}(\lambda,\lambda')\,\varepsilon^{ij}(\lambda,\lambda') = \frac{1}{2}\Big[\pi^{ii}\pi^{jj} - \pi^{ij}\pi^{ji}\Big] = \frac{1}{2}(D-2)(D-3). \tag{61}$$

5 Conclusion

Detailed analysis of the massless fields of string theory was carried out in arbitrary dimensions of spacetime $D(>2)$. We have worked in the celebrated Coulomb gauge, ensuring positivity, and no constraints were imposed on the external sources thus generating the complete expressions for the corresponding propagators. Much emphasis was put on polarization aspects of the fields, and the number of independent degrees of freedom naturally followed. It is interesting to note that for $D = 4$, we recover our earlier expression of the graviton propagator derived in [4] by much more involved and a lengthy method. The importance of such propagators and their polarization aspects are expected to be useful in finding connections between string theory and field theory for computations and for the generation of non-trivial effective actions. These points will be taken up in subsequent reports, as well as the investigation of the massless fields in superstring theory.

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