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All the fundamental massless bosonic fields in superstring theory

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A systematic analysis of all the massless bosonic fields in superstring theory is carried out. Emphasis is put on the derivation of their propagators, their polarization aspects and the investigation of their underlying constraints as well as their number of degrees of freedom. The treatment is given in the presence of external sources, in the celebrated Coulomb gauge, ensuring the positivity of the formalism - a result which is also established in the process. The challenge here is the investigation involved in the self-dual fourth rank anti-symmetric tensor field. No constraints are imposed on the external sources so that their components may be varied independently, thus the complete expressions of the propagators may be obtained. As emphasized in our earlier work, the latter condition is an important one in dynamical theories with constraints giving rise to modifications as Faddeev-Popov factors. The analysis is carried out in 10 dimensions, not only because of the consistency requirement by the superstrings, but also in order to take into account of the self-duality character of the fourth rank anti-symmetric tensor field as spelled out in the paper.

1 Introduction

As we have pointed out in our earlier investigation [1], what is remarkable about string theory is that the fundamental fields that are required to describe the dynamics of elementary particles arise naturally in the mass spectra of oscillating springs and are not, a priori, assumed to exist or put in by hand in the underlying theories. String theory also generates some novel fields such as an anti-symmetric fourth rank massless tensor field as arising in the mass spectra of some superstring theories. Such anti-symmetric fields have been important in providing a hint of the existence of branes, as extended objects, with which such fields may interact. At present only the massless fields string modes are really physically relevant because of the enormous masses of the massive fields string-excitation modes. A massless vector field, for example, may be thought to acquire mass by some mechanism such as, for example, from open strings whose end points are attached to different branes and by acquiring any additional degree of freedom from the massless scalar field modes excitations. In this paper, we are interested in all the massless bosonic field excitations in *superstring* theory (c.f. [2–4]). These are: an anti-symmetric fourth rank self dual tensor field, anti-symmetric third and second rank tensor fields, a symmetric traceless second rank tensor field, a vector field, and finally, a scalar field. In the present work, as in the earlier one in [1], the scalar field presents no challenge and nothing new emerges about it here. We work in 10 dimensions, not only because this is the consistency condition emerging from superstring theory, but also there is a strict self-duality condition that is satisfied by the fourth rank anti-symmetric tensor field. This will be spelled out below.

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Our earlier analysis in [1] of the vector field, the symmetric traceless second rank tensor field, the second rank anti-symmetric tensor field were all investigated in arbitrary dimensions $D > 2$. Accordingly, there is no need to repeat their underlying analyses here. For details concerning these three massless fields, we refer the reader to [1].

Light-cone variables may be defined as follows: $x^{\pm} = (x^0 \pm x^9)/\sqrt{2},\ x^a, a = 1, \ldots, 8$. The coordinates $x^a, a = 1, \ldots, 8$ are sometimes referred to as transverse variables. The self-duality constraint of the fourth rank anti-symmetric tensor field $A^{\mu\nu\sigma\rho}$ may be then defined by

$$\frac{1}{4!}\,\varepsilon^{abcda'b'c'd'} A^{a'b'c'd'} = A^{abcd}, \tag{1}$$

giving restrictions between its various components. This constraint must be properly implemented in its underlying analysis. Here $\varepsilon^{abcda'b'c'd'}$ is totally anti-symmetric with $\varepsilon^{12345678} = +1$. We are interested in dynamical aspects of these fields. To this end, we are concerned about underlying constraints, the explicit expressions of their propagators, their polarization states and their inherit degrees of freedom. We work in the celebrated Coulomb gauge, which ensures the positivity of the formalism as will be established in each case. The fields are coupled to *non*-constrained external sources so that each of their components may be varied independently thus generating the complete expressions of the propagators. As emphasized in [5], the variations of each of the components of external sources is necessary in describing full field theory interactions, such as deriving Faddeev-Popov factors. A detailed demonstration of the importance of varying the components of the external sources independently with applications to gauge theories is given in Sect.3 of [5] for the convenience of the reader. The third rank anti-symmetric field is treated next in Sect.2, followed by the more difficult one of the fourth rank anti-symmetric tensor field in Sect.3. The Greek indices $\mu, \nu, \sigma, \ldots$ go over $0, 1, \ldots, 9$, while the Latin indices, in the middle of the alphabet, $i, j, k, \ldots$ go over $1, 2, \ldots, 9$, and finally Latin indices, in the beginning of the alphabet, $a, b, c, \ldots$ go over $1, 2, \ldots, 8$. Our metric is $[\eta^{\mu\nu}] = \mathrm{diag}[-1, 1, \ldots, 1]$. A summation over repeated indices is understood throughout.

2 Third rank anti-symmetric tensor field

The Lagrangian density of a third rank massless anti-symmetric tensor field $A^{\mu\nu\sigma}$, may be be defined by

$$\mathcal{L} = -\,\frac{1}{8}\,F^{\mu\nu\sigma\lambda} F_{\mu\nu\sigma\lambda}, \tag{2}$$

where $F^{\mu\nu\sigma\lambda}$ is totally anti-symmetric and is given by

$$F^{\mu\nu\sigma\lambda} = \partial^\mu A^{\nu\sigma\lambda} - \partial^\nu A^{\sigma\lambda\mu} + \partial^\sigma A^{\lambda\mu\nu} - \partial^\lambda A^{\mu\nu\sigma}, \tag{3}$$

defined as a cyclic permutation. The Lagrangian density is invariant under the gauge transformation

$$A^{\mu\nu\sigma} \;\to\; A^{\mu\nu\sigma} + \partial^\mu \varphi^{\nu\sigma} + \partial^\nu \varphi^{\sigma\mu} + \partial^\sigma \varphi^{\mu\nu}. \tag{4}$$

We work in a Coulomb-like gauge

$$\partial_i A^{ijk} = 0, \quad \partial_i A^{0ij} = 0, \tag{5}$$

and similarly defined with respect to the indices j, k.

We add a source contribution to (2), obtaining

$$\mathcal{L} = -\,\frac{1}{8}\,F^{\mu\nu\sigma\lambda} F_{\mu\nu\sigma\lambda} + A^{\mu\nu\sigma} J_{\mu\nu\sigma}, \tag{6}$$

where no constraints are imposed on the external source $J_{\mu\nu\sigma}$ so we may vary its components independently. Due to the constraints in (5), one cannot vary the components of A^{ijk}, as well as the components of A^{0jk}, independently. We may, however, introduce a field $\mathcal{A}^{ijk}$ that may be varied independently, and set

$$A^{ijk} = \pi^i_{[i'}\pi^j_{j'}\pi^k_{k']}\,\mathcal{A}^{i'j'k'}, \quad \pi^{ij} = \delta^{ij} - \frac{\partial^i\partial^j}{\boldsymbol{\partial}^2}, \tag{7}$$

and the square brackets in $\pi^i_{[i'}\pi^j_{j'}\pi^k_{k']}$, means an anti-symmetrization over the indices i', j', k'. Similarly, we may introduce a field $\mathcal{A}^{jk}$, and set

$$A^{0jk} = \pi^j_{[j'}\pi^k_{k']}\,\mathcal{A}^{j'k'}. \tag{8}$$

By varying the Lagrangian density in (6) with respect to $\mathcal{A}^{ijk}$, and $\mathcal{A}^{jk}$, we obtain

$$\partial^2 A^{ijk} = -\pi^i_{[i'}\pi^j_{j'}\pi^k_{k']}\,J^{i'j'k'}, \tag{9}$$

$$\boldsymbol{\partial}^2 A^{0jk} = -\pi^j_{[j'}\pi^k_{k']}\,J^{0j'k'}. \tag{10}$$

Clearly, only A^{ijk} may propagate.

We note from the right-hand side of (9), that if a constraint is imposed on the source J^{ijk}, the complete expression for the propagator does not follow. Upon equating the expectation value $\langle 0_+|A^{ijk}(x)|0_-\rangle$ and $(-\mathrm{i})\delta/\delta J^{ijk}(x)\langle 0_+|0_-\rangle$, and similarly carrying out for $\langle 0_+|A^{0jkl}(x)|0_-\rangle$, and integrating with respect to the source components, we obtain

$$\langle 0_+|0_-\rangle = \exp\left[+\frac{\mathrm{i}}{2}\int (dx)(dx')J_{\mu\nu\sigma}(x)\Delta_+^{\mu\nu\sigma,\mu'\nu'\sigma'}(x-x')J_{\mu'\nu'\sigma'}(x')\right], \tag{11}$$

and in the momentum description,

$$\Delta_+^{0jk,0j'k'}(p) = \frac{\pi^j_{[j'}\pi^k_{k']}}{\mathbf{p}^2}, \quad \pi^{ij} = \delta^{ij} - \frac{p^ip^j}{\mathbf{p}^2}, \tag{12}$$

$$\Delta_+^{ijk,i'j'k'}(p) = \frac{\pi^i_{[i'}\pi^j_{j'}\pi^k_{k']}}{p^2 - \mathrm{i}\epsilon}, \tag{13}$$

and $\Delta_+^{0ij,i'j'k'}(p) = 0$, $\Delta_+^{ijk,0j'k'}(p) = 0$. Clearly, $\Delta_+^{0jk,0j'k'}(p)$ contributes only a phase factor to $\langle 0_+|0_-\rangle$.

Upon introducing the completeness relation

$$\pi^{ij} = \sum_{a=1}^{8} e^i_a\,e^j_a, \tag{14}$$

in terms of polarization vectors e^i_a, where $p^ie^i_a = 0$, $e^i_{a_1}\,e^i_{a_2} = \delta_{a_1a_2}$, we may, after some grouping of the polarization vectors, re-write

$$\pi^i_{[i'}\pi^j_{j'}\pi^k_{k']} = e^i_{[a_1}e^j_{a_2}e^k_{a_3]}\,e^{i'}_{[a_1}e^{j'}_{a_2}e^{k'}_{a_3]}. \tag{15}$$

This suggests to introduce the polarizations

$$e^i_{[a_1}e^j_{a_2}e^k_{a_3]} \equiv e^{i\ j\ k}_{a_1a_2a_3}. \tag{16}$$

and define

$$J_{(a_1a_2a_3)} = J^{ijk}\, e^{i\ j\ k}_{a_1a_2a_3}. \tag{17}$$

The vacuum persistence probability then emerges as

$$|\langle 0_+|0_-\rangle|^2 = \exp\left[-\int \frac{d^9\mathbf{p}}{2|\mathbf{p}|(2\pi)^9}\Big(\sum_{a_1,a_2,a_3=1}^{8} |J_{(a_1a_2a_3)}(p)|^2\Big)\right] < 1, \quad p^0 = |\mathbf{p}|, \tag{18}$$

establishing the positivity of the formalism. The number of degrees of freedom is obtained as follows, giving

$$\begin{aligned}
&\sum_{a_1,a_2,a_3=1}^{8} e^{i\ j\ k}_{a_1a_2a_3}\, e^{i\ j\ k}_{a_1a_2a_3} \\
&= \frac{1}{3!}\Big(\pi^{ii}\pi^{jj}\pi^{kk} - \pi^{ij}\pi^{ij}\pi^{kk} - \pi^{ii}\pi^{jk}\pi^{kj} + \pi^{ij}\pi^{jk}\pi^{ki} - \pi^{ik}\pi^{jj}\pi^{ki} + \pi^{ik}\pi^{ji}\pi^{kj}\Big) \\
&= \frac{1}{6}(336) = 56
\end{aligned} \tag{19}$$

degrees of freedom.

3 Fourth rank anti-symmetric self-dual tensor field

The Lagrangian density of a fourth rank massless anti-symmetric tensor field $A^{\mu\nu\sigma\rho}$, may be defined by

$$\mathcal{L} = -\,\frac{1}{10}\, F^{\mu\nu\sigma\lambda\rho} F_{\mu\nu\sigma\lambda\rho}, \tag{20}$$

where $F^{\mu\nu\sigma\lambda\rho}$ is totally anti-symmetric and is given by

$$F^{\mu\nu\sigma\lambda\rho} = \partial^\mu A^{\nu\sigma\lambda\rho} + \partial^\nu A^{\sigma\lambda\rho\mu} + \partial^\sigma A^{\lambda\rho\mu\nu} + \partial^\lambda A^{\rho\mu\nu\sigma} + \partial^\rho A^{\mu\nu\sigma\lambda}, \tag{21}$$

defined as a cyclic permutation. The Lagrangian density is invariant under the gauge transformation

$$A^{\mu\nu\sigma\rho} \;\rightarrow\; A^{\mu\nu\sigma\rho} + \partial^\mu \varphi^{\nu\sigma\rho} - \partial^\nu \varphi^{\sigma\rho\mu} + \partial^\sigma \varphi^{\rho\mu\nu} - \partial^\rho \varphi^{\mu\nu\sigma}. \tag{22}$$

We work in a Coulomb-like gauge

$$\partial_i A^{ijk\ell} = 0, \quad \partial_i A^{0ijk} = 0, \tag{23}$$

and similarly defined with respect to the indices j, k, ℓ.

We add a source contribution to (20), obtaining

$$\mathcal{L} = -\,\frac{1}{10}\, F^{\mu\nu\sigma\lambda\rho} F_{\mu\nu\sigma\lambda\rho} + A^{\mu\nu\sigma\rho} J_{\mu\nu\sigma\rho}, \tag{24}$$

where no constraints are imposed on the external source $J_{\mu\nu\sigma\rho}$, so we may vary its components independently. Due to the constraints in (23), one cannot vary the components of $A^{ijk\ell}$, as well as the components of A^{0ijk}, independently. We may, however, introduce a field $\mathcal{A}^{ijk\ell}$, that may be varied independently, and set

$$A^{ijk\ell} = \Lambda^{i\,j\,k\,\ell}_{i'j'k'\ell'}\,\mathcal{A}^{i'j'k'\ell'}, \tag{25}$$

where $\Lambda^{i\,j\,k\,\ell}_{i'j'k'\ell'}$ is totally anti-symmetric in the indices $\{i,j,k,\ell\}$, as well as as in the indices $\{i',j',k',\ell'\}$, satisfying the following properties

$$\Lambda^{i\,j\,k\,\ell}_{i'j'k'\ell'} = \Lambda^{i'j'k'\ell'}_{i\,j\,k\,\ell}, \quad \Lambda^{i\,j\,k\,\ell}_{i'j'k'\ell'}\Lambda^{i'\,j'\,k'\,\ell'}_{i''j''k''\ell''} = \Lambda^{i\,\,j\,\,k\,\,\ell}_{i''j''k''\ell''}, \tag{26}$$

i.e., it is, in particular, a projection operator, and also satisfies orthogonality relations

$$\partial_i\Lambda^{i\,j\,k\,\ell}_{i'j'k'\ell'} = 0 = \Lambda^{i\,j\,k\,\ell}_{i'j'k'\ell'}\partial_i, \tag{27}$$

as well as in all of its indices j,k,ℓ,i',j',k',ℓ'. We will explicitly construct this operator below. We will see how the self-duality condition, spelled out below, may be also defined through this process. Similarly, we may set

$$A^{0ijk} = \Lambda^{i\,j\,k}_{i'j'k'}\,\mathcal{A}^{i'j'k'}, \tag{28}$$

where $\Lambda^{i\,j\,k}_{i'j'k'}$ is totally anti-symmetric in the indices $\{i,j,k\}$, as well as in the indices $\{i',j',k'\}$, and satisfies the properties

$$\Lambda^{i\,j\,k}_{i'j'k'} = \Lambda^{i'j'k'}_{i\,j\,k}, \quad \Lambda^{i\,j\,k}_{i'j'k'}\Lambda^{i'\,j'\,k'}_{i''j''k''} = \Lambda^{i\,\,j\,\,k}_{i''j''k''}, \tag{29}$$

i.e., it is, in particular, a projection operator, and also satisfies orthogonality relations

$$\partial_i\Lambda^{i\,j\,k}_{i'j'k'} = 0 = \Lambda^{i\,j\,k}_{i'j'k'}\partial_i, \tag{30}$$

as well as in all of its indices $j,k,\ i',j',k'$.

By using these properties, and varying the Lagrangian density in (24) with respect to the fields $\mathcal{A}^{ijk\ell}$, and $\mathcal{A}^{ijk}$, we obtain, after some labor, the equations

$$\partial^2 A^{ijk\ell} = -\Lambda^{i\,j\,k\,\ell}_{i'j'k'\ell'}\,J^{i'j'k'\ell'}, \tag{31}$$

$$\boldsymbol{\partial}^2 A^{0jk\ell} = -\Lambda^{j\,k\,\ell}_{j'k'\ell'}\,J^{0j'k'\ell'}. \tag{32}$$

Clearly, only $A^{ijk\ell}$ may propagate.

By taking the vacuum expectation value $\langle 0_+|\,.\,|0_-\rangle$ of (31), and carrying out a Fourier transform, gives

$$\langle 0_+|A^{ijkl}(x)|0_-\rangle = \int \frac{(dp)}{(2\pi)^{10}}\,\frac{1}{p^2-\mathrm{i}\epsilon}\Lambda^{i\,j\,k\,\ell}_{i'j'k'\ell'}\,J^{i'j'k'\ell'}(p)\,\langle 0_+|0_-\rangle, \tag{33}$$

and $\Lambda^{i\,j\,k\ell}_{i'j'k'\ell'}$, may be explicitly given in terms of mutually orthonormal polarization vectors e^i_a, $p^i e^i_a = 0$, $e^i_a e^i_{a'} = \delta_{aa'}$, $a,a' = 1,\ldots,8$, as follows

$$\Lambda^{i\,j\,k\,\ell}_{i'j'k'\ell'} = e^i_{[a}\,e^j_b\,e^k_c\,e^\ell_{d]}\,\Gamma^{a\,b\,c\,d}_{a'b'c'd'}\,e^{i'}_{[a'}\,e^{j'}_{b'}\,e^{k'}_{c'}\,e^{\ell'}_{d']}, \tag{34}$$

where $e^i_{[a}\, e^j_b\, e^k_c\, e^\ell_{d]}$ includes $4! = 24$ terms, with the square brackets in $[a\,b\,c\,d]$ defining an anti-symmetrization over the indices $a,\ b,\ c,\ d$. $\Gamma^{a\ b\ c\ d}_{a'b'c'd'}$ is explicitly given by

$$\Gamma^{a\ b\ c\ d}_{a'b'c'd'} = \frac{1}{2}\Big(\delta^a_{[a'}\,\delta^b_{b'}\,\delta^c_{c'}\,\delta^d_{d']} + \frac{1}{4!}\,\varepsilon^{abcda'b'c'd'}\Big), \tag{35}$$

where, we recall, $\varepsilon^{abcda'b'c'd'}$ is totally anti-symmetric with $\varepsilon^{12345678} = +1$. The following basic properties should be noted:

$$e^i_{[a}\, e^j_b\, e^k_c\, e^\ell_{d]}\ e^i_{[a'}\, e^j_{b'}\, e^k_{c'}\, e^\ell_{d']} = \delta^a_{[a'}\,\delta^b_{b'}\,\delta^c_{c'}\,\delta^d_{d']}, \tag{36}$$

$$p^i\,\Lambda^{i\ j\ k\ \ell}_{i'j'k'\ell'} = 0, \tag{37}$$

and similarly defined with respect to the indices $j, k, \ell, i', j', k', l'$,

$$\Gamma^{a\ b\ c\ d}_{a'b'c'd'}\,\Gamma^{a'\ b'\ c'\ d'}_{a''b''c''d''} = \Gamma^{a\ b\ c\ d}_{a''b''c''d''}, \tag{38}$$

and most importantly,

$$\frac{1}{4!}\,\varepsilon^{abcda'b'c'd'}\,\Gamma^{a'\ b'\ c'\ d'}_{a''b''c''d''} = \Gamma^{a\ \ b\ \ c\ \ d'}_{a''b''c''d''}. \tag{39}$$

All the properties in (26), (27) are now verified.

Now we are ready to discuss the self-duality condition. To this end, the Fourier transform in (33) reads

$$\langle 0_+|A^{ijkl}(p)|0_-\rangle = \frac{1}{p^2 - \mathrm{i}\epsilon}\Lambda^{i\ j\ k\ \ell}_{i'j'k'\ell'}\,J^{i'j'k'\ell'}(p)\,\langle 0_+|0_-\rangle. \tag{40}$$

Consider the momentum $\mathbf{p} = (0, 0, \ldots, 0, |\mathbf{p}|)$ with non-zero at the 9^{th} place. The polarization vectors may be written as $e^i_a = \delta^i_a$, and we simply obtain

$$\langle 0_+|A^{abcd}(p)|0_-\rangle = \frac{1}{p^2 - \mathrm{i}\epsilon}\Gamma^{a\ b\ c\ d}_{a'b'c'd'}\,J^{a'b'c'd'}(p)\,\langle 0_+|0_-\rangle. \tag{41}$$

The remarkable property in (39) gives rigorously,

$$\frac{1}{4!}\,\varepsilon^{abcda'b'c'd'}\,\langle 0_+|A^{a'b'c'd'}(p)|0_-\rangle = \langle 0_+|A^{abcd}(p)|0_-\rangle, \tag{42}$$

thus satisfying the self-duality restriction. For example, this gives $A^{1234} = +\,A^{5678}$, and for all disjoint sets $\{a, b, c, d\}$, $\{a', b', c', d'\}$, with unequal elements, $A^{abcd} = \pm\,A^{a'b'c'd'}$, where the signs are readily determined.

We note from the right-hand side of (31), that if a constraint is imposed on the source $J^{ijk\ell}$, the complete expression for the propagator does not follow. Upon equating the expectation value $\langle 0_+|A^{ijkl}(x)|0_-\rangle$ and $(-\mathrm{i})\,\delta/\delta J^{ijk\ell}(x)\langle 0_+|0_-\rangle$, and similarly carrying out this for $\langle 0_+|A^{0jk\ell}(x)|0_-\rangle$, and integrating with respect to the source components, we obtain

$$\langle 0_+|0_-\rangle = \exp\left[+\frac{\mathrm{i}}{2}\int (dx)(dx')J_{\mu\nu\sigma\rho}(x)\Delta_+^{\mu\nu\sigma\rho,\mu'\nu'\sigma'\rho'}(x-x')J_{\mu'\nu'\sigma'\rho'}(x')\right], \tag{43}$$

$$\Delta_+^{0ijk,0i'j'k'}(p) = \frac{\Lambda^{i\ j\ k}_{i'j'k'}}{\mathbf{p}^2}, \quad \Lambda^{i\ j\ k}_{i'j'k'} = e^i_{[a}\, e^j_b\, e^k_{c]}\ e^{i'}_{[a}\, e^{j'}_b\ e^{k'}_{c]}, \tag{44}$$

$$\Delta_+^{ijk\ell,i'j'k'\ell'}(p) = \frac{\Lambda^{i\ j\ k\ \ell}_{i'j'k'\ell'}}{p^2 - \mathrm{i}\epsilon}, \tag{45}$$

with all the other components equal to zero, and where $\Lambda^{i\ j\ k\ \ell}_{i'j'k'\ell'}$ is defined in (34), (35). Clearly, $\Delta_+^{0ijk,0i'j'k'}(p)$ gives rise to a phase factor to $\langle 0_+|0_-\rangle$.

The property in (26), together with the one in (38), suggest to introduce the polarizations

$$e^i_{[a'}\, e^j_{b'}\, e^k_{c'}\, e^\ell_{d']}\, \Gamma^{a'b'c'd'}_{a\ b\ c\ d} \ \equiv\ e^{ijk\ell}_{abcd}, \tag{46}$$

and set

$$e^{ijk\ell}_{abcd} J^{ijk\ell} \equiv J^{(abcd)}. \tag{47}$$

The vacuum persistence probability then emerges as

$$|\langle 0_+|0_-\rangle|^2 = \exp\left[-\int \frac{d^9\mathbf{p}}{2|\mathbf{p}|(2\pi)^9}\Big(\sum_{a,b,c,d=1}^{8} |J^{(abcd)}(p)|^2\Big)\right] < 1, \quad p^0 = |\mathbf{p}|, \tag{48}$$

establishing the positivity of the formalism.

The number of independent degrees of freedom may be obtained from the following, giving

$$\begin{aligned}
\sum_{a,b,c,d=1}^{8} e^{ijk\ell}_{abcd}\, e^{ijk\ell}_{abcd} &= e^i_{[a'}\, e^j_{b'}\, e^k_{c'}\, e^\ell_{d']}\, \Gamma^{a'b'c'd'}_{a\ b\ c\ d}\, e^i_{[a''}\, e^j_{b''}\, e^k_{c''}\, e^\ell_{d'']}\, \Gamma^{a''b''c''d''}_{a\ b\ c\ d} \\
&= \delta^{a'}_{[a''}\, \delta^{b'}_{b''}\, \delta^{c'}_{c''}\, \delta^{d'}_{d'']}\, \Gamma^{a''b''c''d''}_{a'\ b'\ c'\ d'} \\
&= \frac{1}{2} \sum_{a,b,c,d=1}^{8} \Big(\delta^a_{[a}\, \delta^b_b\, \delta^c_c\, \delta^d_{d]} + \frac{1}{4!}\, \delta^{a'}_{[a''}\, \delta^{b'}_{b''}\, \delta^{c'}_{c''}\, \delta^{d'}_{d'']}\, \varepsilon^{a'b'c'd'a''b''c''d''}\Big) \\
&= \frac{1}{2} \sum_{a,b,c,d=1}^{8} \Big(\delta^a_{[a}\, \delta^b_b\, \delta^c_c\, \delta^d_{d]} + 0\Big) = \frac{1}{2}\,(70) = 35
\end{aligned} \tag{49}$$

degrees of freedom, where we have used, in the process, the expression in (35), and inserted the summation signs in the last steps to emphasize that we are summing over repeated indices. Finally, in the last step, we have used the identity

$$\sum_{a,b,c,d=1}^{8} \delta^a_{[a}\, \delta^b_b\, \delta^c_c\, \delta^d_{d]} = 70. \tag{50}$$

4 Conclusion

A systematic analysis of all the massless bosonic fields in superstring theory have been carried out. We have worked out in the celebrated Coulomb gauge, ensuring positivity, paying special attention to constraints in the theory, while imposing no constraints on the external sources thus generating the complete expressions of the corresponding propagators. Much emphasis was put on polarization aspects of the fields, and the number of independent degrees of freedom naturally followed. It remains to investigate all the massless fermion fields in superstring theory. These require very special tools, beyond the scope of the present paper, and will be given in a separate report. The importance of the underlying propagators and their polarization aspects are expected to be useful in finding connections between string theory and field theory computations and for the generation of non-trivial effective actions. These points will be taken up in subsequent reports as well.

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References

[1] E. B. Manoukian, Fortschr. Phys. (2012) (in press).
[2] M. B. Green, J. H. Schwarz, and E. Witten, Superstring Theory, Vol. 1 and 2 (Cambridge University Press, Cambridge, 1998).
[3] K. Becker, M. Becker, and J. H. Schwarz, String Theory and M-Theory, (Cambridge University Press, Cambridge, 2007).
[4] J. Polchinski, String Theory, Vol. I and II (Cambridge University Press, Cambridge, 2005).
[5] E. B. Manoukian and S. Sukkhasena, Fortschr. Phys. **55**(11–12), 1281–1288 (2007).