

**Stability of the 3-form field during inflation**

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We consider the minimally coupled 3-form field, which has been considered as a candidate for realizing inflation. We have studied the conditions to avoid ghosts and Laplacian instabilities and found that some classes of potentials, e.g., the Mexican-hat, will in general be unstable. We then propose other classes of potentials which are instead free from any instability, drive a long enough slow-roll regime followed by an oscillatory epoch, and as a consequence, can provide successful inflation. Finally, we also provide stable potentials that lead to a small enough propagation speed for the scalar perturbations, making it possible for these models to produce non-Gaussianities.

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**I. INTRODUCTION**

The inflation paradigm was introduced in 1980 as a way to solve different issues, namely: the magnetic monopoles, the flatness, and the horizon problems [1]; however, it can also account for the observed temperature anisotropies in the cosmic microwave background [2] as well as the galaxy power spectrum [3]. In other words, a sufficiently long stage of accelerated expansion has been proposed as a way to solve all these problems at the same time. In order to explain this period of accelerated expansion, some new physics are introduced, and a scalar field [1,4] (or more than one [5,6]) is commonly used. However, the real mechanism for inflation is yet unknown, so it is interesting to explore different possibilities, which in general may lead to different predictions for several inflationary observables (i.e., spectral index, tensor-to-scalar ratio [7,8], non-Gaussianities parameter [9–12]).

Since fundamental scalar fields have not been discovered yet in nature, the idea of inflation might well be realized by other, higher-form fields. For example, vector inflation (or 1-form inflation) has been intensively investigated [13–16]. Unfortunately, most of the vector-field models encounter instabilities [17–20]. More generally, the N-form field inflation has been also investigated [21,22] and one of the results is that 1-form and 2-form fields are not stable, whereas the 3-form field can be stable [22]. Note also, that the 4-form field models correspond to the  $f(R)$  theories [22,23].

Recently, a form of inflation based on the evolution of a 3-form has been studied [24–27]. The origin of such a nonstandard form for the inflaton may come from a high-energy-scale theory, such as string theory. Indeed it is interesting to study such a model, as it may provide an alternative way to obtain inflation. Since the essence of the 3-form is by construction different from a single scalar

field, we expect this difference to play some role both at background and perturbation levels.

In fact, in this paper we will study the stability of a minimally coupled 3-form during inflation with a general expression for the potential. We will then find the conditions that avoid ghosts and Laplacian instabilities (i.e., we require a positive kinetic term and a non-negative speed of propagation for the independent linear perturbation modes). Once these conditions are obtained, we reconsider some models that have been recently introduced [26,28], and show that, if the potential is not carefully chosen, both ghosts and Laplacian instabilities will occur.

Hence, we provide some classes of potentials, which, by construction, are instead free from these instabilities, and, in this context, we study their background evolution, in order to confirm that a slow-roll period of inflation is then followed by a regime where the 3-form oscillates, ending inflation. We also investigate the possibility of having stable evolutions and, at the same time, a small enough speed of propagation for the scalar modes, making possible non-Gaussian signatures for these models. We will discuss the details of reheating, and the bounds on the inflationary parameters (spectral index, tensor-to-scalar ratio, and non-Gaussianities) in a future work.

The paper is organized as follows. In Sec. II, we introduce the Lagrangian of the model and write down the equations of motion. The linear perturbation theory for this model is studied in Sec. III, where we give the no-ghost conditions and the squared speed of propagation for the scalar, vector, and tensor modes. We present some classes of potentials that make the model free from ghosts and Laplacian instabilities in Sec. IV, where we also show that a slow-roll period of inflation is followed by an oscillatory regime that ends inflation. We present our conclusions in Sec. V.

## II. THE MODEL AND THE BACKGROUND EQUATIONS OF MOTION

Let us start with the following action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} R - \frac{1}{48} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta} - V(A_{\alpha\beta\gamma} A^{\alpha\beta\gamma}) \right], \quad (1)$$

where  $A_{\alpha\beta\gamma}$  is a 3-form, and  $\mathbf{F} = d\mathbf{A}$  is its Maxwell tensor [29], whose components can be written as

$$F_{\mu\nu\rho\sigma} = \nabla_\mu A_{\nu\rho\sigma} - \nabla_\sigma A_{\mu\nu\rho} + \nabla_\rho A_{\sigma\mu\nu} - \nabla_\nu A_{\rho\sigma\mu}. \quad (2)$$

### A. The background

In this subsection, we review the background equations for the 3-form field. All these equations and quantities were first derived in [22]. Let us now consider a flat Friedmann-Lemaître-Robertson-Walker (FLRW) manifold whose metric element can be written as

$$ds^2 = -dt^2 + a(t)^2 dx^2, \quad (3)$$

and, on this background, considering Eq. (A1) in the Appendix, the background 3-form  $A_{\alpha\beta\gamma}$  can be written as

$$A_{0ij} = 0, \quad A_{ijk} = a^3 \epsilon_{ijk} X, \quad (4)$$

where  $\epsilon_{ijk}$  is the three-dimensional Levi-Civita symbol (with  $\epsilon_{123} = 1$ ). Let us define the following quantities:

$$V = V(y), \quad V_{,y} \equiv \frac{dV(y)}{dy}, \\ V_{,yy} \equiv \frac{d^2V(y)}{dy^2}, \quad \text{where } y \equiv A_{\alpha\beta\gamma} A^{\alpha\beta\gamma}. \quad (5)$$

On a FLRW background, we have  $y = 6X^2$ . As a consequence of these definitions, on FLRW, we have

$$V(-X) = V(X), \quad (6)$$

$$V_{,y} = \frac{dX}{dy} V_{,X} = \frac{V_{,X}}{12X}, \quad (7)$$

$$\dot{V} = 12X\dot{X}V_{,y} \quad (8)$$

$$V_{,yy} = \frac{1}{12X} \frac{d}{dX} \left( \frac{V_{,X}}{12X} \right) = \frac{XV_{,XX} - V_{,X}}{144X^3}, \quad (9)$$

so that we will restrict the form of the potential  $V$  to even functions of  $X$ . In this case, the Friedmann equation can be written as

$$E_1 \equiv 3M_{\text{Pl}}^2 H^2 - \rho_X = 0, \quad (10)$$

where

$$\rho_X = \frac{1}{2}\dot{X}^2 + V + \frac{9}{2}H^2 X^2 + 3HX\dot{X} = \frac{1}{2}Y^2 + V \quad (11)$$

is the effective energy density of the 3-form, and we have defined  $Y \equiv \dot{X} + 3HX$ . The second Einstein equation reads as follows:

$$E_2 \equiv M_{\text{Pl}}^2 (2\dot{H} + 3H^2) + p_X = 0, \quad (12)$$

where  $p_X$  is the 3-form effective pressure defined as

$$p_X = -\left(\frac{1}{2}\dot{X}^2 + V + 3HX\dot{X} + \frac{9}{2}H^2 X^2 - 12V_{,y} X^2\right) \\ = 2V_{,y} - \rho_X. \quad (13)$$

The equation of motion for the field gives

$$E_X \equiv \ddot{X} + 3H\dot{X} + 3X\dot{H} + 12V_{,y} X = \dot{Y} + 12XV_{,y} = 0. \quad (14)$$

The equations of motion are not all independent, due to Bianchi identities: indeed, we have

$$\dot{E}_1 + 3H(E_1 - E_2) + YE_X = 0. \quad (15)$$

One consequence of the equations of motion is

$$M_{\text{Pl}}^2 \dot{H} = -V_{,y}, \quad (16)$$

so that the universe will be superaccelerating when  $V_{,y} < 0$ .

## III. LINEAR PERTURBATION THEORY

### A. Scalar modes

Let us consider now the metric for the scalar perturbations in the following form [30]:

$$ds^2 = -(1 + 2\alpha)dt^2 + 2\partial_i \psi dt dx^i + a^2(1 + 2\Phi)dx^2, \quad (17)$$

where we picked a spatial gauge so that the three-dimensional metric is diagonal. As for the 3-form, by using once more Eq. (A1) given in the Appendix<sup>1</sup>, we can use a time gauge to fix the scalar perturbations<sup>1</sup> as [26]

$$A_{0ij} = a\epsilon_{ijk}\partial_k\beta(t, \mathbf{x}), \quad A_{ijk} = a^3\epsilon_{ijk}X(t). \quad (18)$$

By expanding the action at second order in the fields we obtain

<sup>1</sup>Here the field  $\Phi$  in this gauge corresponds to the combination  $\Phi_{\text{GI}} = \Phi - H\alpha_0/Y - HX(\partial^2\tilde{\gamma})/Y$ , where, without fixing any gauge,  $\alpha_0$  is defined, following [26], as  $A_{ijk} = a^3\epsilon_{ijk}(X(t) + \alpha_0)$ , and  $\delta g_{ij} = a^2(2\Phi\delta_{ij} + 2\partial_i\partial_j\tilde{\gamma})$ . Then we can see that  $\Phi_{\text{GI}}$  is gauge invariant. In other words, we have completely fixed the gauge freedom by setting  $\alpha_0 = 0 = \tilde{\gamma}$ . This gauge-invariant field  $\Phi_{\text{GI}}$  is well defined as long as  $Y = \dot{X} + 3HX \neq 0$ . In particular, this gauge is well defined in  $X = 0 = y$ , as long as its speed does not vanish, that is,  $\dot{X} \neq 0$  at  $X = 0$ .

$$\begin{aligned}
S^{(2)} = & \int dt d^3x a^3 \left\{ \frac{6V_{,y}X^2}{a^2} (\partial\psi)^2 - 2M_{\text{Pl}}^2 (H\alpha - \dot{\Phi}) \frac{\partial^2\psi}{a^2} + \frac{1}{2} \frac{(\partial^2\beta)^2}{a^4} + 6V_{,y} \frac{(\partial\beta)^2}{a^2} + (Y\alpha + 12V_{,y}X\psi + 3Y\Phi) \frac{\partial^2\beta}{a^2} \right. \\
& - \frac{1}{2} (6M_{\text{Pl}}^2 H^2 - Y^2) \alpha^2 + \left[ 6M_{\text{Pl}}^2 H\dot{\Phi} - 2M_{\text{Pl}}^2 \frac{\partial^2\Phi}{a^2} + 3(Y^2 + 12V_{,y}X^2)\Phi \right] \alpha \\
& \left. - 3M_{\text{Pl}}^2 \dot{\Phi}^2 + M_{\text{Pl}}^2 \frac{(\partial\Phi)^2}{a^2} + \frac{9}{2} (Y^2 - 12V_{,y}X^2 - 144V_{,yy}X^4) \Phi^2 \right\}. \quad (19)
\end{aligned}$$

At a first look, this action has important differences with the general action (for the perturbations) of scalar-tensor theories [31]. First of all, the presence of the terms  $(\partial\psi)^2$  and  $\Phi^2$  which, for a second-order general scalar-tensor theory, vanish after using the equations of motion. Both these terms now vanish only when the 3-form is absent. Furthermore, the field  $\beta$  is not dynamical, and it can be integrated out in Fourier space (together with  $\alpha$  and  $\psi$ ).

In order to remove these auxiliary fields, it is convenient to work in Fourier space: in this case, we can integrate out the fields  $\alpha$ ,  $\psi$ , and  $\beta$ , by using their own equations of motion. In Fourier space, with  $\Phi(t, \mathbf{x}) = (2\pi)^{-3/2} \times \int d^3k \tilde{\Phi}_k e^{i\mathbf{k}\cdot\mathbf{x}}$ , with the reality condition  $\tilde{\Phi}_{-k} = \tilde{\Phi}_k^*$ , the equations of motion for the constraints give

$$12V_{,y}X^2\psi + 2M_{\text{Pl}}^2(H\alpha - \dot{\Phi}) - 12V_{,y}X\beta = 0, \quad (20)$$

$$\begin{aligned}
(Y^2 - 6M_{\text{Pl}}^2 H^2)\alpha + \frac{2M_{\text{Pl}}^2 H k^2 \psi}{a^2} + 6M_{\text{Pl}}^2 H\dot{\Phi} + 2M_{\text{Pl}}^2 \frac{k^2\Phi}{a^2} \\
+ 3(Y^2 + 12V_{,y}X^2)\Phi - \frac{Yk^2\beta}{a^2} = 0, \quad (21)
\end{aligned}$$

and

$$\frac{k^2}{a^2}\beta + 12V_{,y}\beta - Y\alpha - 12V_{,y}X\psi - 3Y\Phi = 0, \quad (22)$$

where we omitted the tilde of the Fourier modes for simplicity. This last equation can be solved for  $\beta$  as

$$\beta = \frac{a^2(Y\alpha + 12V_{,y}X\psi + 3Y\Phi)}{k^2 + 12V_{,y}a^2}, \quad (23)$$

so that we also have

$$\begin{aligned}
\psi = & \left( \frac{M_{\text{Pl}}^2}{6V_{,y}X^2} + \frac{2a^2M_{\text{Pl}}^2}{X^2k^2} \right) \dot{\Phi} + \frac{3a^2Y}{Xk^2} \Phi \\
& - \left( \frac{M_{\text{Pl}}^2H}{6V_{,y}X^2} + \frac{a^2(2M_{\text{Pl}}^2H - XY)}{k^2X^2} \right) \alpha, \quad (24)
\end{aligned}$$

and finally

$$\begin{aligned}
\alpha = & \frac{M_{\text{Pl}}^4 H k^2 + 6M_{\text{Pl}}^2 V_{,y} a^2 (3HX^2 + 2M_{\text{Pl}}^2 H - XY)}{M_{\text{Pl}}^2 H [M_{\text{Pl}}^2 k^2 H + 6V_{,y} a^2 (3HX^2 + 2M_{\text{Pl}}^2 H - 2XY)]} \dot{\Phi} \\
& + \frac{6V_{,y} M_{\text{Pl}}^2 k^2 X^2 + 18V_{,y} a^2 X (6X^3 V_{,y} + M_{\text{Pl}}^2 HY)}{M_{\text{Pl}}^2 H [M_{\text{Pl}}^2 k^2 H + 6V_{,y} a^2 (3HX^2 + 2M_{\text{Pl}}^2 H - 2XY)]} \Phi. \quad (25)
\end{aligned}$$

By substituting these expressions into the action written in Fourier space, we find

$$\tilde{S} = \int dt d^3k Q(t, k^2) \left[ \dot{\Phi}_k \dot{\Phi}_{-k} - c_X^2(t, k^2) \frac{k^2}{a^2} \Phi_k \Phi_{-k} \right]. \quad (26)$$

If  $Q > 0$ , we can define a canonical field  $\Psi$  as

$$\Phi = \sqrt{\frac{a}{2Q}} \Psi, \quad (27)$$

whose Lagrangian, in conformal time  $\eta$ , reads

$$\tilde{S} = \int d\eta d^3k \left[ \frac{1}{2} \Psi'_k \Psi'_{-k} - \frac{1}{2} c_X^2 k^2 \Psi_k \Psi_{-k} - \frac{1}{2} m_{\Psi}^2 \Psi_k \Psi_{-k} \right], \quad (28)$$

where  $m_{\Psi}^2$  is

$$m_{\Psi}^2 = \frac{1}{2} \left( \frac{a''}{a} - \frac{a'^2}{a^2} - \frac{Q''}{Q} + \frac{Q'^2}{Q^2} \right) - \frac{1}{4} \left( \frac{a'}{a} - \frac{Q'}{Q} \right)^2, \quad (29)$$

and a prime denotes differentiation with respect to conformal time.

### 1. No-ghost conditions

The no-ghost condition is then found to be

$$Q > 0, \quad (30)$$

where

$$Q = \frac{6a^5 M_{\text{Pl}}^2 V_{,y} Y^2}{M_{\text{Pl}}^2 k^2 H^2 + 6V_{,y} a^2 (3H^2 X^2 + 2M_{\text{Pl}}^2 H^2 - 2XYH)}. \quad (31)$$

It should be noted that condition (30) should hold at all times during inflation, whether or not the trajectory is in a slow-roll regime. Using the equations of motion (by replacing  $M_{\text{Pl}}^2 H^2$  with the Friedmann equation and then  $3HX = Y - \dot{X}$ ), we find that

$$3H^2 X^2 + 2M_{\text{Pl}}^2 H^2 - 2XYH = \frac{1}{3} \dot{X}^2 + \frac{2}{3} V, \quad (32)$$

so that

$$Q = \frac{6a^5 M_{\text{Pl}}^2 V_{,y} Y^2}{M_{\text{Pl}}^2 k^2 H^2 + 2V_{,y} a^2 (\dot{X}^2 + 2V)}. \quad (33)$$

This quantity must be positive for all  $k$ 's. For high  $k$ , we find the condition  $V_{,y} > 0$ . This condition must be satisfied

along the trajectory of motion. In some cases, it may be possible that for some (positive) values of  $y$ ,  $V_{,y}$  is negative, but such values of  $y$  are never reached: in this case, the model can still be viable. We note here that the condition  $V_{,y} > 0$ , when using Eq. (16), forbids the dynamics to be superaccelerating. It is worth noting that the condition  $V_{,y} > 0$  was also found in [26], by demanding the background condition  $p_X + \rho_X > 0$ . However, we argue that for general theories the positivity of the sum of the effective pressure and density does not necessarily imply the no-ghost condition (30) (see also [31]). In other words, condition (30) is a condition for the instability of the perturbation modes, and not a condition on the background dynamics.

For low  $k$ 's we find another requirement, that is,  $\dot{X}^2 + 2V > 0$ . Once more, this condition must be satisfied along the trajectory of motion. The bottom line is that the two conditions  $V \geq 0$ ,  $V_{,y} > 0$  are sufficient conditions for not having ghosts. If these conditions are not satisfied for all (positive)  $y$ 's, one should check that, at least for the values of  $y$  along the trajectory of motion for the model, the above-mentioned conditions still hold.

## 2. Speed of propagation

The speed of propagation is found as the large- $k$  limit of  $c_X^2(t, k^2)$  of Eq. (26). One can show that the speed of propagation, while using the background equations of motion, is given as

$$c_X^2 = \lim_{k \rightarrow \infty} c_X^2(t, k^2) = 1 + \frac{2V_{,yy}y}{V_{,y}} = \frac{XV_{,XX}}{V_{,X}}. \quad (34)$$

The speed of propagation found here corresponds to the one found by Koivisto and Nunes [26]. In general, only the simple quadratic potential  $V \propto y$ , implies a propagation with speed of light for all dynamics. Since  $y \geq 0$ , then a sufficient condition to also avoid Laplacian instabilities (besides the ghosts,  $V_{,y} > 0$ ) is  $V_{,yy} \geq 0$ .

## B. Vector modes

Let us define the metric perturbation for the vector modes as

$$\delta g_{0i} = aG_i, \quad \text{and} \quad \delta g_{ij} = a^2(C_{i,j} + C_{j,i}), \quad (35)$$

where  $G_{,i} = 0 = C_{,i}$ . We will also choose a gauge for which the 3-form has no vector perturbations (uniform field vector gauge). This choice completely fixes the gauge degrees of freedom. In this case, one can show that the action for the vector modes becomes

$$S = \int dt d^3x \left[ 6a^5 V_{,y} X^2 \dot{C}_i \dot{C}_i + 12a^4 V_{,y} X^2 \dot{C}_i Z_i + \frac{1}{4} M_{\text{Pl}}^2 a (\partial_j Z_i) (\partial_j Z_i) + 6a^3 V_{,y} X^2 Z_i Z_i \right], \quad (36)$$

where we introduced the field  $Z_i = G_i - a\dot{C}_i$ . By introducing Fourier modes, it is possible to integrate out the field  $Z_i$  as

$$\tilde{Z}_i(t, \mathbf{k}) = -\frac{24a^3 V_{,y} X^2 \dot{C}_i(t, \mathbf{k})}{M_{\text{Pl}}^2 k^2 + 24a^2 V_{,y} X^2}, \quad (37)$$

so that the action for the vector modes becomes

$$S = \int dt d^3k Q_V(t, k^2) [\dot{\tilde{C}}_i(t, \mathbf{k}) \dot{\tilde{C}}_i(t, -\mathbf{k})], \quad (38)$$

so that it is clear that the vector modes do not propagate.

### 1. No-ghost condition

The no-ghost condition for the vector modes corresponds to  $Q_V > 0$ , that is,

$$Q_V = \frac{6k^2 a^5 M_{\text{Pl}}^2 V_{,y} X^2}{M_{\text{Pl}}^2 k^2 + 24a^2 V_{,y} X^2} > 0, \quad (39)$$

implying

$$V_{,y} > 0, \quad (40)$$

which coincides to one of the conditions already found for the scalar modes.

### C. Tensor modes

The tensor modes are not affected by the presence of the 3-form, as this latter one is minimally coupled to gravity and it does not possess tensor degrees of freedom. To show this in greater detail, we choose the tensor perturbations as  $\delta g_{ij} = h_{ij}^T = h_+ e_{ij}^+ + h_\times e_{ij}^\times$ , where both the symmetric tensors  $e_{ij}$  are transverse and traceless. We also impose the normalization condition,  $e_{ij}(\mathbf{k}) e_{ij}(-\mathbf{k})^* = 1$ , for each polarization, whereas  $e_{ij}^+(\mathbf{k}) e_{ij}^\times(-\mathbf{k})^* = 0$ . Therefore, the second-order action can be written as

$$S_T = \sum_{\lambda=+, \times} \int dt d^3x a^3 \frac{M_{\text{Pl}}^2}{8} \left[ \dot{h}_\lambda^2 - \frac{1}{a^2} (\partial h_\lambda)^2 \right], \quad (41)$$

so that no stability condition comes from the tensor sector.

## IV. SUITABLE FORM OF POTENTIALS FOR 3-FORM INFLATION

According to the previous section, one of the no-ghost conditions can be written as  $V_{,y} > 0$ , where  $y = 6X^2 \geq 0$ . The existence of ghosts in the model depends on the shape of three form potential, but not on the sign of  $X$  (as  $y \propto X^2$ ). In fact, in order to search for the form of potentials, which makes the 3-form field ghost-free and without Laplacian instabilities ( $c_X^2 \geq 0$ ), we need to study more in detail the evolution of  $y$  (or, equivalently,  $X$ ). It is convenient for qualitative analysis to change variables to dimensionless variables and define some quantities. In the first part of this section we will define some quantities and use some

dimensionless variables as found in [26]. From the Friedmann equation, we have

$$\dot{H} = -\frac{1}{M_{\text{Pl}}^2} V_{,y} \dot{y} = -\frac{1}{2M_{\text{Pl}}^2} V_{,X} \dot{X}, \quad (42)$$

so that the 3-form field can play the role of a slow-rolling inflaton if  $V_{,X} X/M_{\text{Pl}}^2 \ll H^2$ . Substituting the above Eq. (42) into the evolution Eq. (14), we get

$$\ddot{X} + 3H\dot{X} + V_{\text{eff},X} = 0, \quad (43)$$

where

$$V_{\text{eff},X} = \frac{dV_{\text{eff}}}{dX} = V_{,X} \left(1 - \frac{3}{2} \frac{X^2}{M_{\text{Pl}}^2}\right), \quad (44)$$

so that the effective potential is given by

$$V_{\text{eff}}(X) = \int^X d\xi V_{,\xi} \left(1 - \frac{3}{2} \frac{\xi^2}{M_{\text{Pl}}^2}\right). \quad (45)$$

Using the dimensionless variables

$$x \equiv \frac{X}{M_{\text{Pl}}}, \quad \text{and} \quad w \equiv \frac{3x + x'}{\sqrt{6}}, \quad (46)$$

where the prime denotes a derivative with respect to  $N = \ln a$ , Eq. (43) can be written in the autonomous form as

$$x' = 3 \left[ \sqrt{\frac{2}{3}} w - x \right], \quad (47)$$

$$w' = \frac{3}{2} \lambda(x) (1 - w^2) \left( xw - \sqrt{\frac{2}{3}} \right), \quad (48)$$

where we have introduced the function

$$\lambda \equiv \frac{V_{,X}}{V}. \quad (49)$$

In these variables the slow-roll parameter can be written as

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2} \lambda (1 - w^2) x. \quad (50)$$

The accelerating expansion of the Universe is acquired by demanding  $\epsilon \ll 1$ . From this parameter, one can see that the kinetic term does not necessarily need to be small compared to the potential term, as for the standard picture of the inflaton scalar field. Conversely, it requires that  $w^2 \approx 1$  when  $x\lambda(x) \sim O(1)$ . In order to have inflation, one needs one more requirement to guarantee that the accelerating expansion is long enough. We introduce a parameter to characterize this behavior as

$$\eta \equiv \frac{\epsilon'}{\epsilon} - 2\epsilon = (1 + c_X^2) \frac{x'}{x}, \quad (51)$$

where the inflationary period requires that  $|\eta| \ll 1$ . Since  $c_X^2 > 0$ ,  $|\eta|$  will be small if  $x'/x$  is small, that is,  $x$  needs to be in a slow-roll regime. From Eq. (47), it implies that

$|\eta| \ll 1$  will be satisfied if  $x \approx \sqrt{2/3} w \approx \pm \sqrt{2/3}$ , where we have also imposed the first slow-roll condition  $|\epsilon| \ll 1$  when  $x\lambda(x) \sim O(1)$ .

We note that, as a consequence of the definition of  $w$ , we have  $Y/(M_{\text{Pl}} H) = \sqrt{6} w$ . Therefore, the Friedmann equation (10) implies

$$1 = \rho_X/(3M_{\text{Pl}}^2 H^2) = w^2 + V/(3M_{\text{Pl}}^2 H^2), \quad (52)$$

so that, if  $V \geq 0$ , then  $0 \leq w^2 \leq 1$ .

As we have already said, the field will slow roll when it reaches the points  $P \equiv (x, w) = (\pm\sqrt{2/3}, \pm 1)$  in phase space because these points are (de Sitter) fixed points (unless  $\lambda(x)$  is not finite at these points). There might be other fixed points,  $M$ , which correspond to the points where  $\lambda$  vanishes, that is,  $M \equiv (x, w) = (\bar{x}, \sqrt{3/2\bar{x}})$ , where  $\lambda(x = \bar{x}) = 0$ . It can be seen from Eq. (44) that the points  $P$  and  $M$  are the values of  $X$ , which correspond to the extrema of the effective potential.

### A. Stability of the fixed points

Let us start by studying the stability of the fixed point  $P$ . By choosing  $x = \pm\sqrt{2/3} + \delta x$ , and  $w = \pm 1 + \delta w$ , we can linearize the equations of motion with respect to the small quantities  $\delta x$  and  $\delta w$ , and we find

$$\delta x' = \sqrt{6} \delta w - 3 \delta x, \quad (53)$$

$$\delta w' = 0, \quad (54)$$

with solutions  $\delta w = b_1 = \text{constant}$ ,  $\delta x = \sqrt{2/3} b_1 + b_2 e^{-3N}$ , and  $b_{1,2}$  are (small) initial conditions. From the autonomous system in Eqs. (47) and (48), we find that the eigenvalues for this fixed point are  $(-3, 0)$ . The fact that one of the eigenvalues is zero, implies that, at linear order, we cannot deduce whether the fixed point is stable or not. In order to check the stability of this fixed point, one needs to study also the second-order solution. For the second-order perturbation, it is convenient to parametrize the perturbation variables in such that  $\delta x' = \delta w' = 0$ . This corresponds to choosing the perturbation variables along the eigenvector, which has zero eigenvalue. By using  $\delta x' = 0$ , one finds that  $\delta x = \sqrt{2/3} \delta w$ .

Therefore, by keeping the perturbations up to second order, we find

$$\delta w' = -2\sqrt{6} \lambda(\pm\sqrt{2/3}) \delta w^2, \quad (55)$$

which can be solved as

$$\delta w = \frac{\delta w_0}{1 + 2\sqrt{6} \lambda(\pm\sqrt{2/3}) \delta w_0 N}, \quad (56)$$

where  $\delta w_0 = \delta w(N=0)$ . To ensure the stability of the perturbation, one requires a condition

$$\lambda(\pm\sqrt{2/3}) \delta w_0 > 0. \quad (57)$$

Since we have  $-1 \leq w \leq 1$ ,  $\delta w_0$  must be negative at fixed point  $(+\sqrt{2/3}, +1)$  and  $\delta w_0$  must be positive at fixed point  $(-\sqrt{2/3}, -1)$ . Therefore, the condition above becomes

$$\lambda(+\sqrt{2/3}) = \frac{V_{,x}}{V} \Big|_{x=+\sqrt{2/3}} < 0, \quad (58)$$

$$\lambda(-\sqrt{2/3}) = \frac{V_{,x}}{V} \Big|_{x=-\sqrt{2/3}} > 0. \quad (59)$$

For a viable 3-form model, where  $xV_{,x} > 0$  and  $V > 0$ , these conditions show that, at second order, the fixed point is unstable. This second-order perturbation analysis is equivalent to the one in [26] and also agrees with the numerical calculation in [32]. We note that there is another method to finding the stability of the fixed point, which has zero eigenvalue as shown in [33].

The fact that this instability appears at second order means that the instability will in general evolve slowly. This instability will make inflation end eventually. Now we have one more condition for viable inflationary model from the 3-form field, which is that the point  $P$  must be unstable. This requirement is also compatible with the ghost-free condition. Furthermore, one can rule out some potential forms by using these condition. By considering various potentials that have been investigated in [26], one finds that the model with Mexican-hat potential,  $V = V_0(x^2 - c^2)$  is plagued by a ghost. In the case of  $c > \sqrt{2/3}$ , there is a ghost, and the point  $P$  is stable. For the case of  $c < \sqrt{2/3}$ , even though the point  $P$  is not stable, the field  $x$  will evolve to oscillate around  $x = c$  at the end of inflation and then a ghost eventually appears when  $x < c$ . We note that, for the shift-potential  $V = V_0(x^2 - c^2) + k$  where  $k$  is positive constant, a ghost will appear since the slope of the potential is the same. For the case of  $c < \sqrt{2/3}$ , the field  $x$  may not cross  $x = c$  if this point is a stable fixed point. However, the inflation will occur again since the field slowly moves to this fixed point.

To obtain the suitable potential form for inflation, there must contain the oscillating phase, which makes possible the reheating period. To avoid a ghost during the oscillating phase, the viable potential form must have only one minimum locating at  $x = 0$  which is not a stable fixed point. Therefore, we will find the property of this fixed point next.

As for the fixed point  $M = (x, w) = (\bar{x}, \sqrt{3/2}\bar{x})$ , where  $\lambda(\bar{x}) = 0$ , by choosing  $x = \bar{x} + \delta x$ , and  $w = \sqrt{3/2}\bar{x} + \delta w$ , we find the linearized equations

$$\delta x' = \sqrt{6}\delta w - 3\delta x, \quad (60)$$

$$\delta w' = -\frac{\sqrt{6}}{8}(2 - 3\bar{x}^2)^2\bar{\Gamma}\delta x, \quad (61)$$

where

$$\bar{\Gamma} = \frac{V_{,xx}}{V} - \left(\frac{V_{,x}}{V}\right)^2 \Big|_{x=\bar{x}} = \frac{V_{,xx}}{V} \Big|_{x=\bar{x}}. \quad (62)$$

The solution leads to

$$\delta x = d_1 e^{-N(3+\gamma)/2} + d_2 e^{-N(3-\gamma)/2}, \quad (63)$$

where

$$\gamma = \sqrt{9 - 3\bar{\Gamma}(3\bar{x}^2 - 2)^2}. \quad (64)$$

An instability will appear if  $\gamma > 3$ , or  $\bar{\Gamma} < 0$ . For the fixed point which  $\bar{x} = 0$ , one found that  $\bar{\Gamma} > 0$  for the positive even potential. Thus, this fixed point is always stable. One of the ways to obtain the oscillating phase is that  $\lambda(x = 0)$  must be not finite. Therefore, one requires more condition for the potential form that  $V$  must vanish at  $x = 0$ ,  $V(x = 0) = 0$ . This requirement will rule out the potential forms that have been investigated in [26] such that  $V = V_0 e^{\alpha x^2}$ ,  $V = V_0(x^2 + \alpha)$  and  $V = V_0(x^4 + \alpha)$ , where  $\alpha$  is a positive constant. Now we can summarize that the viable potential forms that have been investigated in [26] are only  $V = V_0 x^2$  and  $V = V_0 x^4$ .

Generally, a power-law potential of the form  $V \propto y^p = x^{2p}$  will be a suitable potential form for the inflationary model from the 3-form. However, when choosing a power-law potential of the form  $V \propto y^p$ , we immediately notice that  $Q \propto y^{p-1}$ , which in general vanishes (for  $p > 1$ ) or diverges (for  $p < 1$ ) as  $y \rightarrow 0$ , unless  $p = 1$ . Since we will focus on the values  $p \geq 1$ , most of the potential will allow the field to cross this value ( $y = 0$ ), so that  $Q$  will vanish in the origin. This property represents a problem, in general, as this means that, at that point, the second-order Lagrangian vanishes (as  $c_X^2$  remains finite for  $V \propto y^p$ ), and the theory becomes strongly coupled, i.e., higher order corrections become dominant. In fact, the metric curvature perturbation, in order for perturbation theory to make sense, needs to be smaller than unity for all dynamics. Therefore, in the limit that  $Q \rightarrow 0$ , the whole action, if  $c_X^2$  remains finite, will tend to vanish. It is worth recalling that at the point  $V_{,y} = 0$  where  $Q$  vanishes, the chosen gauge is in general well defined, unless at that point we have that  $Y = \dot{X} + 3HX = 0$ , which for  $X = 0$ , it implies  $\dot{X} = 0$ . But  $X = 0 = \dot{X}$  is not a point that is reached by the dynamics in a finite interval of time, in general.

In order to avoid this possible strong-coupling issue, we propose the following generalized power-law potential:<sup>2</sup>

$$V(x^2) = V_0[(x^2)^p + bx^2], \quad (65)$$

where  $p$  is a constant which can be, as for now, positive or negative, whereas  $b > 0$ . Note that for the potential form,  $V = V_0 x^2$ , it has been investigated in detail in [28]. We can

<sup>2</sup>In general, we can choose a larger class of potentials given as  $V(y) = V_0(cy + \sum_i c_i y^{p_i})$ , where  $c > 0$ ,  $c_i \geq 0$ , and  $p_i \geq 1$ .

also modify the Gaussian potential in order to satisfy the conditions as

$$V(x^2) = V_0[e^{\nu x^2} - 1], \quad (66)$$

where  $\nu$  is a positive constant parameter. We will investigate the properties of these potential forms in the next subsection.

### B. Power-law potential

We now investigate cosmological behavior for the potential

$$V(x^2) = V_0[(x^2)^p + bx^2], \quad (67)$$

where  $p$  is a constant which can be, as for now, positive or negative, whereas  $b > 0$ . For this form of the potential, we have that the no-ghost condition

$$\frac{V_{,x}}{x} = 2V_0[p(x^2)^{p-1} + b] > 0 \quad (68)$$

is always positive for  $p \geq 0$ , and also finite for  $p \geq 1$ . For this reason, from now on, we will only consider the case of  $p \geq 1$ . On the other hand, since

$$c_X^2 = \frac{(2p-1)p(x^2)^{p-1} + b}{p(x^2)^{p-1} + b}, \quad (69)$$

$c_X^2$  will be always positive and finite for  $p \geq 1$ . The bottom line is that for  $p \geq 1$  the model is free from instabilities for any real value of  $x$ , that is, for any dynamics. It should be noted that for the value  $p = 1$ , the potential reduces to a quadratic power-law potential. On the de Sitter fixed point  $P$ , we have

$$c_X^2(x = \pm\sqrt{2/3}) = \frac{(2p-1)p(2/3)^{p-1} + b}{p(2/3)^{p-1} + b} \geq 1, \quad (70)$$

and the inequality holds for  $p \geq 1$ , and  $b > 0$ .

Because of Eq. (67), it can be shown that

$$V_{\text{eff},x} = 2V_0(1 - \frac{3}{2}x^2)[px^{2p-1} + bx], \quad (71)$$

and

$$V_{\text{eff}} = V_0 \left[ \frac{x^{2p}}{2(p+1)} [2 + p(2 - 3x^2)] + bx^2 \left( 1 - \frac{3}{4}x^2 \right) \right], \quad (72)$$

so that the extremum points of this potential occur at  $x = \pm\sqrt{2/3}$ , and  $x = 0$ . We also notice that

$$\lambda(x)_x = \frac{2[p(x^2)^{p-1} + b]}{[(x^2)^{p-1} + b]}, \quad (73)$$

which, for  $p \geq 1$ , is positive and finite for all  $x$ . Furthermore,  $\lim_{x \rightarrow 0} \lambda x = 2$ .

For illustration, we consider here the simplest case of  $p = 1$  whereas, in the next section, we will describe in

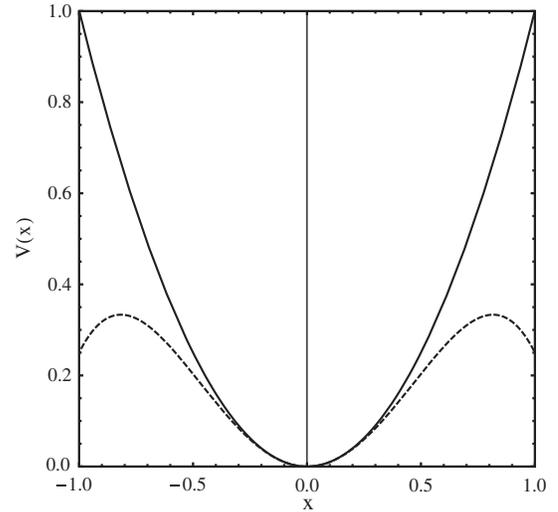


FIG. 1. The potential  $V(x) \propto x^2$  is shown here. In the figure, the bare potential and the effective potential are represented by a solid and dashed line, respectively.

greater detail the case of  $p = 2$  (quartic potential). The potential and effective potential for the  $p = 2$  case are plotted in Fig. 1. The field that starts at  $|x| > \sqrt{2/3}$  with  $|w| \sim 1$  is able to drive long enough inflation. However, this time, the field rolls down the potential from point A or B and then oscillates around the minimum of the potential without ghosts ( $V_{,y} = V_0(1+b)/6 > 0$ ) or Laplacian instabilities ( $c_X^2 = 1$ ). We also use a direct numerical integration to confirm both the slow-roll and the oscillatory regimes as shown in Fig. 2. From the evolution of  $\epsilon$  in the right panel, inflation ends at  $N \sim 72$  corresponding to  $\epsilon \sim 1$ .

### C. Quartic potential

We study here a particular case of the power-law potential introduced in the previous section, namely

$$V = V_0(x^4 + bx^2), \quad (74)$$

and we plot it (together with its effective potential) in Fig. 3.

In this case, we also have

$$\frac{3M_{\text{pl}}^2 H^2}{V_0} = \frac{x^4 + bx^2}{1 - w^2}, \quad (75)$$

so that, by introducing a dimensionless cosmic time  $t$ , we can write

$$\frac{dx}{dt} = \frac{M_{\text{pl}} H}{\sqrt{V_0/3}} \frac{dx}{dN} = \sqrt{\frac{x^4 + bx^2}{1 - w^2}} \frac{dx}{dN}. \quad (76)$$

In Fig. 4, we show the evolution for both  $x$  and  $w$ . The field slow rolls until an oscillatory regime starts, making inflation end.

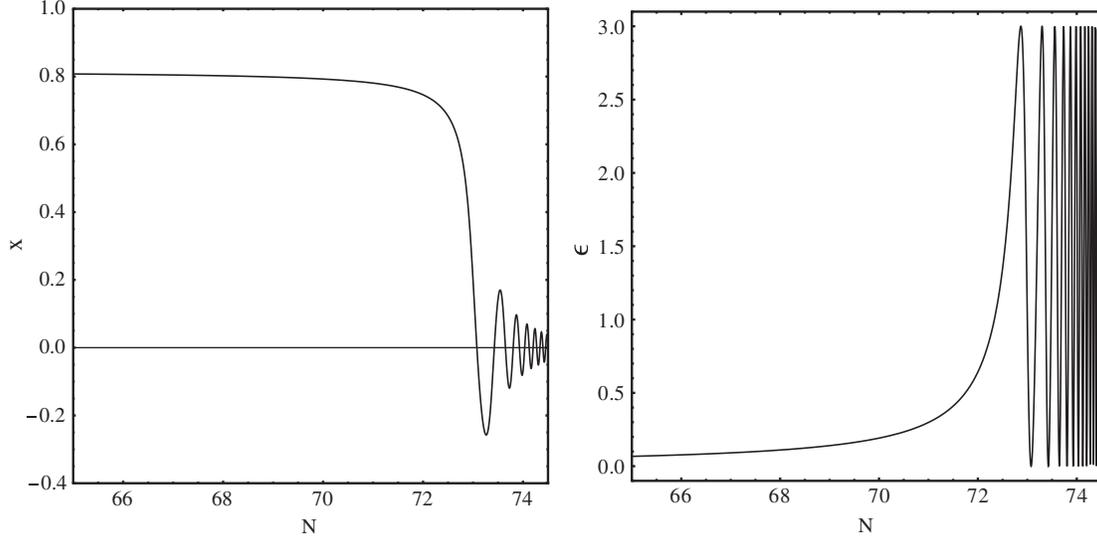


FIG. 2. The evolution of  $x$  and  $\epsilon$  for the potential  $V = V_0(1 + b)x^2$  and  $p = 1$ . We chose initial conditions  $w(0) = 0.99$  and  $x(0) = 3\sqrt{2/3}w(0)$ . When the field  $x$  starts to oscillate around the minimum, the parameter  $\epsilon$  will start to oscillate around  $\epsilon = 3/2$ . Thus the inflation will end when  $\epsilon \sim 1$  corresponding to  $N \sim 72$ .

During the slow-roll regime, the propagation speed takes the value  $c_X^2 \approx \frac{3(4+b)}{4+3b}$ , whereas, as the solution starts oscillating,  $c_X^2 \rightarrow 1$ . This behavior is confirmed in Fig. 5.

Finally, we show in Fig. 6 that after inflation ends, there is an oscillatory regime that mimics a dust dominated universe, as we have  $H^2 \propto a^{-3} \propto e^{-3N}$ . This behavior is similar to the standard single-field inflationary models, and this is not surprising, because as  $x \rightarrow 0$ , we find  $V_{\text{eff}} \approx bx^2$ ; so that the equation of motion for the field, Eq. (14), reduces to

$$\ddot{x} + 3H\dot{x} \approx -bx, \quad \text{for } |x| \ll 1, \quad (77)$$

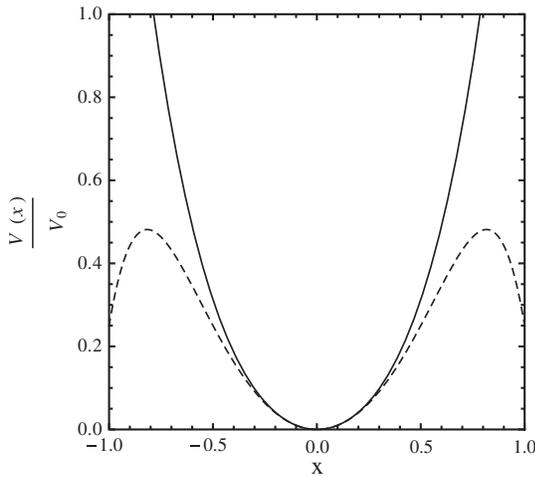


FIG. 3. The potential  $V = V_0(x^4 + bx^2)$ , with  $b = 1$  (the continuous and black curve), together with the effective potential  $V_{\text{eff}}$  (represented by the dashed curve).

which exactly matches the equation of motion for standard inflation in the presence of a quadratic inflaton potential. In other words, the dynamics of the 3-form, for  $x \rightarrow 0$ , tends to be more and more identical to the dynamics of a single scalar field oscillating around the minimum of a quadratic potential.

After inflation ends, during the oscillatory regime, in Fig. 4, we see that  $x \rightarrow 0$ , whereas  $w$  oscillates between  $-1$  and  $1$ . Furthermore, we also find that  $dx/dt \rightarrow 0$  together with  $x$ , whereas  $dw/dt$  keeps oscillating, remaining finite, as shown in Fig. 7.

#### D. Gaussian potential

We now consider the exponential potential

$$V = V_0(e^{\nu y/6} - 1) = V_0(e^{\nu x^2} - 1), \quad (78)$$

where  $\nu$  is a constant parameter that can be positive or negative. For this form of potential, we have

$$\frac{V_{,x}}{x} = 2\nu V_0 e^{\nu x^2}, \quad (79)$$

so that the ghost will not exist if  $\nu$  is positive. The speed of propagation in this case is given by

$$c_X^2 = 1 + 2\nu x^2. \quad (80)$$

This implies that if the ghost does not exist,  $c_X^2$  is always positive. Substituting Eq. (78) into Eq. (44), one gets

$$V_{\text{eff},x} = 2\nu x V_0 e^{\nu x^2} (1 - \frac{3}{2}x^2), \quad (81)$$

or

$$V_{\text{eff}} = -\frac{V_0}{2\nu} \{3 + 2\nu + e^{\nu x^2} [(3x^2 - 2)\nu - 3]\}, \quad (82)$$

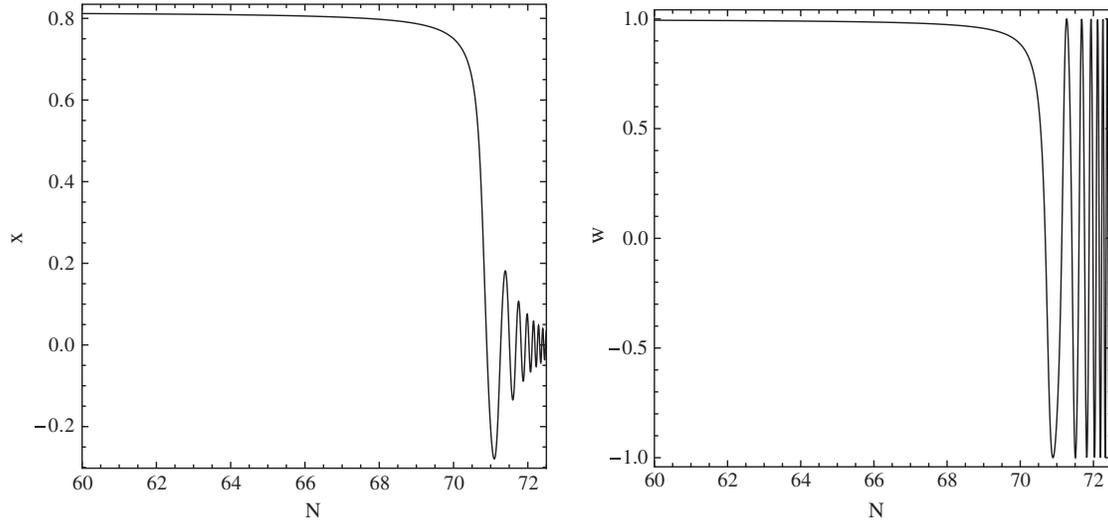


FIG. 4. The evolution of  $x$  and  $w$  for the potential  $V = V_0(x^4 + bx^2)$ , and  $b = 1$ . We chose initial conditions  $w(0) \approx 0.9991$ , and  $x(0) \approx 0.8158$ .

and we plot it, together with the bare potential, in Fig. 8. We also notice that as  $x \rightarrow 0$ , then  $V_{\text{eff}} \approx V_0 \nu x^2$ , so that we expect an oscillatory regime to take place, ending inflation.

It is easy to see that the effective potential has the extremum at  $x = \pm\sqrt{2/3}$  and  $x = 0$ . Similar to the analysis for the previous potentials, the field can drive inflation when we initially put it in the region satisfying the condition  $\epsilon \ll 1$ , e.g.,  $|x| \gtrsim \sqrt{2/3}$  and  $|w| \sim 1$ . The condition  $|\eta| \ll 1$  will be satisfied when the field is frozen nearly  $x = \pm\sqrt{2/3}$ . Since  $x = \pm\sqrt{2/3}$  are not stable fixed points, the field can continuously evolve through  $x = \pm\sqrt{2/3}$  and then oscillates about  $x = 0$  eventually. This behavior is also shown by using numerical integration methods as seen in Fig. 9. Because of this behavior, the speed of propaga-

tion will be approximately equal to  $c_X^2 \approx 1 + 4\nu/3$  in the slow-roll regime, whereas  $c_X^2 \rightarrow 1$ , as  $x \rightarrow 0$ . In Fig. 10, we also show the behavior of the Hubble parameter during the oscillatory regime, confirming that a matter-dominated era takes place during this epoch.

Finally, we show the trajectory of  $dx/dt$  and  $x$ , together with  $dw/dt$  and  $w$  in Fig. 11.

### E. General form of potential

From the investigation of the previous subsections, one can see that the viable 3-form models can be characterized

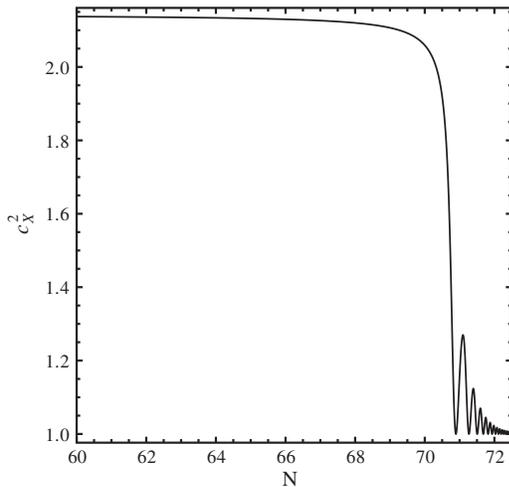


FIG. 5. The evolution of  $c_X^2$  for the potential  $V = V_0(x^4 + bx^2)$ , and  $b = 1$ . The model does not possess any instability.

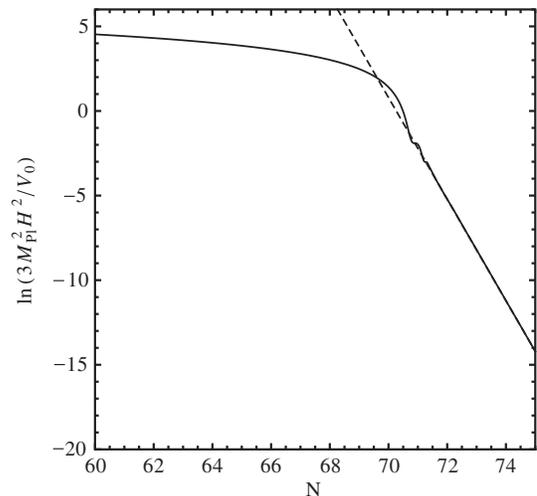


FIG. 6. The evolution of  $3M_{\text{pl}}^2 H^2 / V_0$  for the potential  $V = V_0(x^4 + bx^2)$ , and  $b = 1$  (continuous black curve). This figure shows that after inflation ends (around  $N \approx 70$ ), the universe enters a matter-dominated epoch, as the curve approaches a dashed line, which represents the line  $\ln(3M_{\text{pl}}^2 H^2 / V_0) = -3N + \text{constant}$ . This means that after inflation  $H^2 \propto e^{-3N} \propto a^{-3}$ .

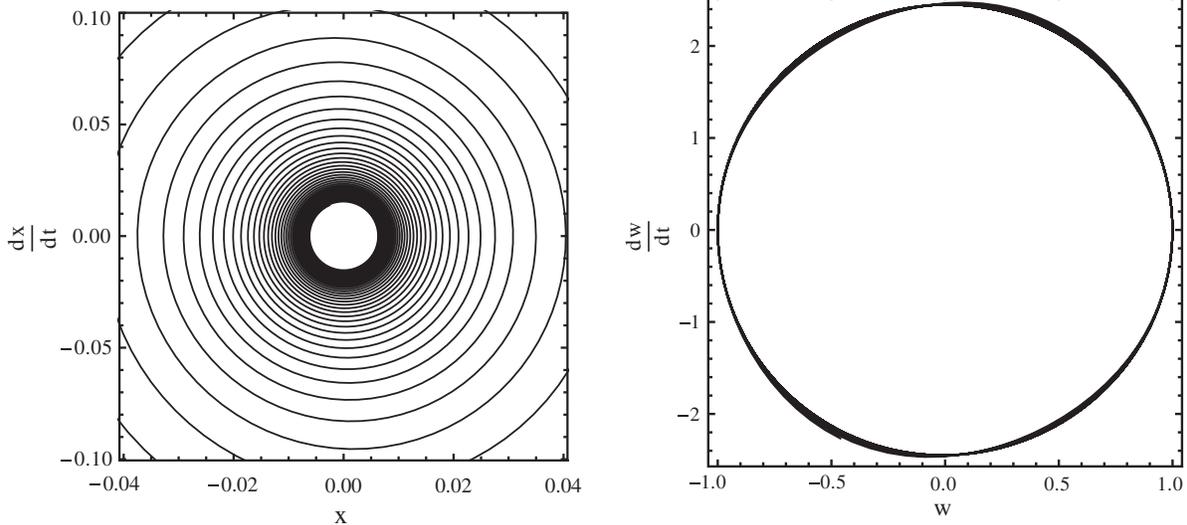


FIG. 7. The evolution of  $dx/dt$  as a function of  $x$ , and  $dw/dt$  as a function of  $w$  for the potential  $V = V_0(x^4 + bx^2)$ , and  $b = 1$  during the oscillatory regime. We chose initial conditions so that at  $t = 1$ , the values of  $x$  and  $w$  correspond, respectively, to  $x(N = 72) \approx 0.0644$ , and  $w(N = 72) \approx -0.46$  of Fig. 4. We stop the integration at  $t = 100$ . This figure shows that  $x$  spiralizes, whereas  $w$  continues oscillating during the matter-dominated regime.

by the shape of their potential. The study of power-law potentials suggests that the viable potential form which is free from ghosts and Laplacian instability should have the local minimum at  $x = 0$ . This is because when  $V_{,x}$  changes sign around the minimum point (as  $V$  is, by construction, an even function of  $x$ ,  $V(x) = V(-x)$ ),  $x$  also changes sign such that  $V_{,x}/x = 12V_{,y}$  is always positive (where  $y = 6X^2$ , and  $x = X/M_{\text{Pl}}$ ). In this situation,  $c_X^2 > 0$  around  $x = 0$  because  $V_{,xx} > 0$ . The speed of propagation is still positive as long as  $x$  remains significantly far from the nearest local maximum (if it exists) of the potential along the trajectory of motion. Hence, if the bare potential has no

local maxima between  $x = \pm x_s$ , where  $x_s$  is the initial value of  $x$ , the field can evolve between  $x = \pm x_s$  without giving rise to ghosts or Laplacian instabilities. In order to avoid the stable fixed point at  $x = 0$ , providing the oscillation phase at this point, our investigation also suggests that the value of the potential should be zero,  $V(x = 0) = 0$ .

We have introduced a class of potentials that are always free of instabilities by construction. However, this is not the only possibility. In fact, there might be regions of the potentials that can lead to instabilities; nonetheless, those same regions are never reached by the dynamics. This fact, can in principle, enlarge the possible inflationary scenarios for these models, especially when we look for particular predictions on some inflationary observables.

In other words, one can search for potentials that may give rise to some interesting behavior of inflaton. For example, some models of inflation can provide the possibility of generating non-Gaussianities in the cosmic microwave background data. The non-Gaussianities can be characterized by a parameter  $f_{\text{NL}}$  which, at least for scalar-tensor theories, can lead to observable signatures, whenever the speed of propagation for the field  $c_X^2$  is positive but less than unity. In most of the single-field models studied so far, the smaller  $c_X^2$ , the larger  $f_{\text{NL}}$  [31].

Although a more detailed study is necessary to determine  $f_{\text{NL}}$  for 3-forms, it is interesting to see whether stable and ghost-free 3-forms can lead to a small speed of propagation  $c_X^2$ . For the potential we have investigated so far, such as  $V = V_0(x^4 + bx^2)$ , if we allow the negative sign of the first term, the non-Gaussianities may be generated. Let us consider, as an example, the potential  $V = V_0 \tanh(\nu x^2)$  where  $\nu$  is a positive constant parameter.

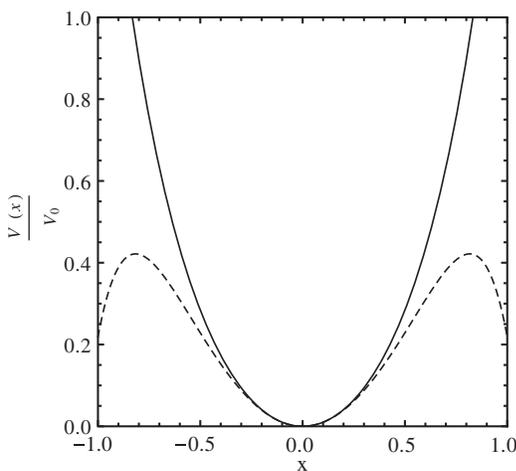


FIG. 8. The potential  $V(x) = V_0(e^{\nu x^2} - 1)$  is represented by a solid line (for  $\nu = 1$ ), whereas the effective potential is represented by a dashed line.

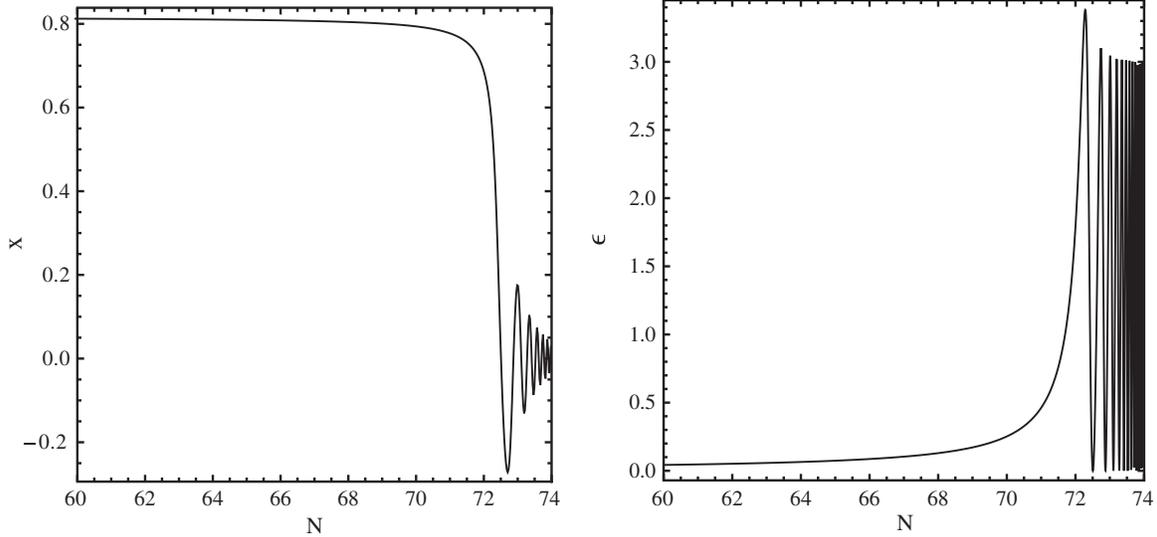


FIG. 9. The evolution for  $x$  (left panel) and  $\epsilon$  (right panel) for the potential  $V_0(e^{\nu x^2} - 1)$ , and  $\nu = 1$ . We chose the initial conditions  $w(0) \approx 0.999$ , and  $x(0) \approx 0.815$ .

For the potential  $V = V_0 \tanh(\nu x^2)$ , the propagation speed takes the form

$$c_X^2 = 1 - 4\nu \tanh(\nu x^2)x^2. \quad (83)$$

There is a region for the parameters in which the model is not viable (as  $c_X^2 < 0$ ). The condition for excluding this region depends on the values of  $X$  and  $\nu$ . If we demand the model to be viable in the region inside  $X < M_{\text{Pl}}$ , one can set  $\nu = 0.52$ . Then we obtain  $c_X^2(x \approx \sqrt{2/3}) \approx 0.534$ . Smaller values for  $c_X^2$  will be obtained by restricting the viable region narrower, nearly the fixed point  $x = \sqrt{2/3}$ . For example, we obtain  $c_X^2 \sim 0.076$  during inflation when

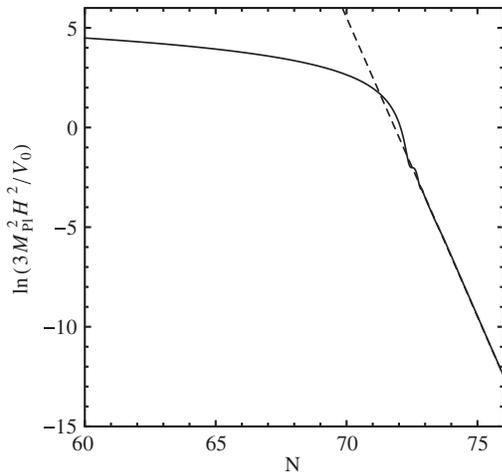


FIG. 10. The evolution of  $3M_{\text{Pl}}^2 H^2 / V_0$  for the potential  $V = V_0(e^{\nu x^2} - 1)$ , and  $\nu = 1$  (continuous black curve). This figure shows that after inflation ends (around  $N \approx 72$ ), the universe enters a matter-dominated epoch, as the curve approaches a dashed line, which represents the line  $\ln(3M_{\text{Pl}}^2 H^2 / V_0) = -3N + \text{constant}$ . This means that after inflation  $H^2 \propto e^{-3N} \propto a^{-3}$ .

we set  $\nu = 3/4$  as shown in Fig. 12. Therefore, the speed of propagation,  $c_X^2$ , can be small (but positive) during inflation, however, finally,  $c_X^2 \sim 1$  during the oscillating phase, as expected.

There are other possible potential forms which can give the speed of propagation less than one such as  $V = V_0(x^2 - bx^4 + \epsilon x^6)$  with small  $\epsilon$ . However, the results are not significantly different from the form we have investigated here. We note that the suitable form of the potential that provides small enough  $c_X^2$  satisfies the condition  $yV_{,yy}/V_{,y} \sim \text{constant}$  during  $\sqrt{2/3} < x < 1$ .

## V. GENERAL CONSIDERATIONS AND CONCLUSIONS

We have proposed a class of potentials that are free of instabilities, can drive inflation, and provide a final stage of a matter-dominated-like oscillatory epoch, during which reheating can occur. In order to avoid a ghost and instabilities, these potentials should have a local minimum at  $x = 0$  and have no local maximum along the trajectory of motion. The 3-form field  $x$  can oscillate around this minimum if the potential vanishes at  $x = 0$ , i.e., the fixed point  $M = (x, w) = (0, 0)$  is unstable. A simple example for such a potential is

$$V = V_0(bx^2 + (x^2)^p), \quad \text{with } b > 0, p \geq 1. \quad (84)$$

We have introduced this form for the potential because, for simple power-law monomials, i.e.,  $V \propto y^p$ , with  $p > 1$ , the second-order action for the perturbations given in Eq. (26) will vanish at  $y = 0$  since  $Q \propto V_{,y} = 0$ . This corresponds, in general, to a strong-coupling limit for the theory. One can avoid this situation by modifying the power-law potential as in Eq. (84). There is no fixed point at  $x = 0$  for this form of the potential. Therefore, the field can oscillate

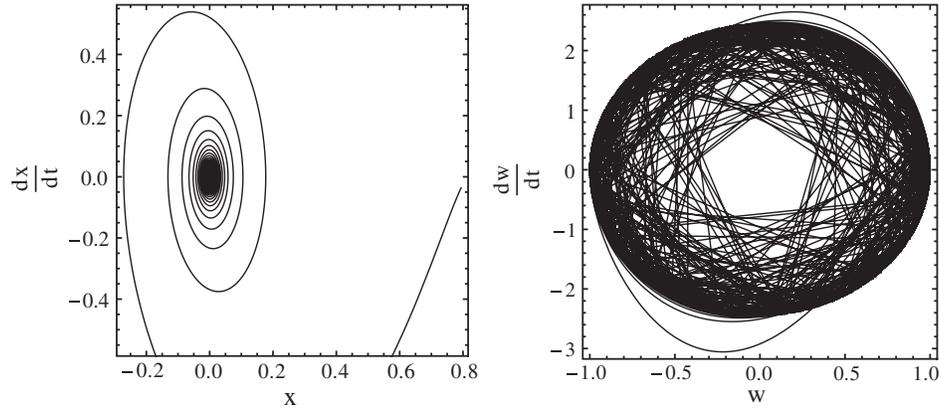


FIG. 11. Phase space plot for the variables  $dx/dt$  and  $x$  (left panel), and for  $dw/dt$  and  $w$  (right panel), during the oscillatory epoch.

around  $x = 0$  to provide the mechanism to end the inflation without reaching  $Q = 0$  at  $y = 0$ . More in detail, according to the previous section, the points in region  $|x| > \sqrt{2/3}$  (unstable slow-roll fixed point of the dynamical equations of motion) will be forced to move to the region  $|x| < \sqrt{2/3}$ , and the inflationary period will be long enough if the field  $x$  starts at  $|x| = x_s > \sqrt{2/3}$  with  $|w| \sim 1$ , where  $w \propto x' + 3x$ . The bottom line is that, in general, the 3-form field can drive long enough inflation without the ghosts or instabilities if its potential has local minimum at  $x = 0$  and has no local maximum between  $x = \pm x_s$ .

We also give another working example, the Gaussian potential, here defined as

$$V = V_0(e^{\nu x^2} - 1), \quad (85)$$

which has similar properties to the power-law case discussed above. In fact, a long enough slow-roll regime is followed by an oscillatory epoch where inflation ends.

Even if avoiding ghosts ( $Q > 0$ ) and Laplacian instabilities ( $c_X^2 < 0$ ) are necessary conditions to be satisfied, they are not, however, sufficient, in general, to have a successful period of inflation. In other words, it is not assured that inflation ends for other classes of potentials which are, on the other end, free from instabilities.

If the potential  $V(y)$  is such that for  $y \geq 0$ , it satisfies the conditions  $V \geq 0$ ,  $V_{,y} > 0$ , and  $V_{,yy} \geq 0$ , then no instabilities arise, as already said. However, if we also impose that as  $y \rightarrow 0$ , we have  $V(y) \simeq cy$ , where  $c$  is a positive constant, then for  $x \approx 0$  (and this point can be reached),  $V_{\text{eff}} \propto x^2$ , so that in general, an oscillatory epoch can take place, ending inflation.

In the last subsection in Sec. IV, a possibility to find non-Gaussianities from the 3-form model of inflation is investigated. Our results show that some potential forms can provide a small enough speed of propagation for the scalar modes,  $c_X^2$ . However, in order to achieve small values for  $c_X^2$  and to keep at the same time a stable evolution, we had

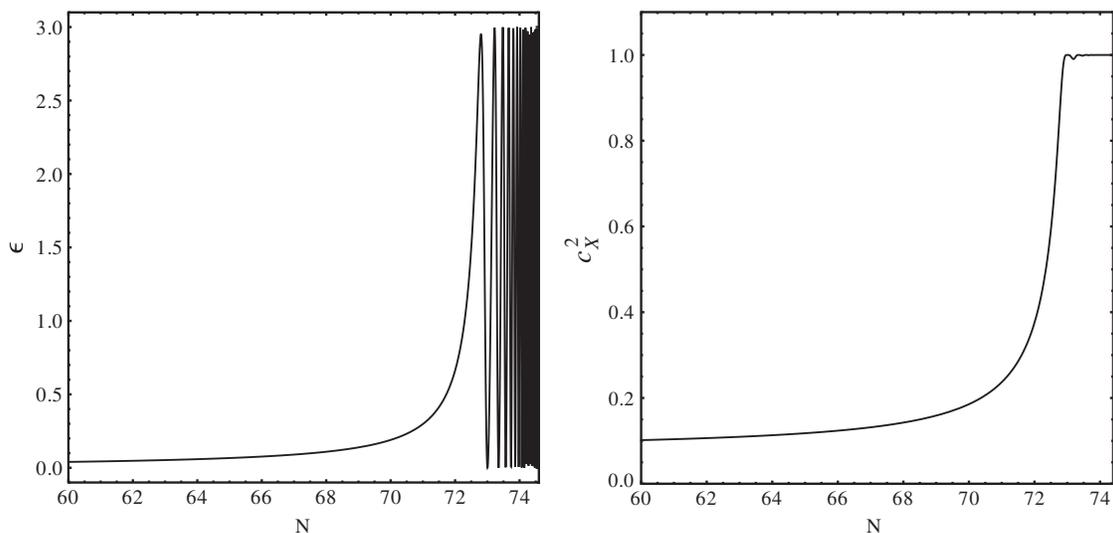


FIG. 12. In the left panel, this plot shows the evolution of the slow-roll parameter,  $\epsilon$ , for potential  $V = V_0 \tanh(\nu x^2)$  with  $\nu = 3/4$ . In the right panel, this plot shows the evolution of speed of propagation,  $c_X^2$ , for the potential  $V = V_0 \tanh(\nu x^2)$  with  $\nu = 3/4$ .

to restrict the allowed interval for the field dynamics such that  $X \lesssim \sqrt{2/3}M_{\text{Pl}}$ .

We have investigated the stability of the perturbations for a minimally coupled 3-form, whose action has a standard kinetic term and a generic potential function. We have found the conditions for which the inflationary dynamics can be stable, and gave some classes of potentials that can provide enough inflation without generating ghosts or Laplacian instabilities. We will leave the question of constraining the parameter space for these potentials by using the bounds on the spectral index and tensor-to-scalar ratio to a future research project.

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### APPENDIX: THE DUAL THEORY

It is possible to define the 1-form or vector dual to the 3-form as

$$A_{\alpha\beta\gamma} = E_{\alpha\beta\gamma\delta} B^\delta = \sqrt{-g} \epsilon_{\alpha\beta\gamma\delta} B^\delta, \quad (\text{A1})$$

where  $E_{\alpha\beta\gamma\delta}$  is the Levi-Civita antisymmetric tensor on curved backgrounds, which on Minkowski reduces to  $\epsilon_{\alpha\beta\gamma\delta}$  (with  $\epsilon_{0123} = 1 = -\epsilon^{0123}$ ). Then we also have  $E^{\alpha\beta\gamma\delta} = \epsilon^{\alpha\beta\gamma\delta} / \sqrt{-g}$ . It is easy to show that  $\nabla_\mu E_{\alpha\beta\gamma\delta} = 0$ .

In the following we will make use of the following relations:  $\epsilon^{\alpha\beta\gamma\delta} \epsilon_{\mu\beta\gamma\delta} = -6\delta_\mu^\alpha$ , and  $\epsilon^{\alpha\beta\gamma\delta} \epsilon_{\mu\nu\gamma\delta} = -2(\delta_\mu^\alpha \delta_\nu^\beta - \delta_\nu^\alpha \delta_\mu^\beta)$ . Therefore, we obtain

$$A_{\alpha\beta\gamma} A^{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma\delta} \epsilon^{\alpha\beta\gamma\mu} B_\mu B^\delta = -6B_\mu B^\mu. \quad (\text{A2})$$

We also have

$$\begin{aligned} & -\frac{1}{48} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta} \\ &= -\frac{1}{2} F_{0123} F^{0123} \\ &= -\frac{1}{2} (\epsilon_{1230} \nabla_0 B^0 - \epsilon_{0123} \nabla_3 B^3 + \epsilon_{3012} \nabla_2 B^2 - \epsilon_{2301} \nabla_1 B^1) \\ &= \times (\epsilon^{1230} \nabla^0 B_0 - \epsilon^{0123} \nabla^1 B_1 + \epsilon^{3012} \nabla^2 B_2 - \epsilon^{2301} \nabla^1 B_1) \\ &= \frac{1}{2} (\nabla^\mu B_\mu)^2, \end{aligned} \quad (\text{A3})$$

so that the action is equivalent to the following one:

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} R + \frac{1}{2} (\nabla^\mu B_\mu)^2 - V(B_\mu^2) \right], \quad (\text{A4})$$

which shows that the 3-form action is classically equivalent to a particular class of vector-tensor theories. Relation (A1) can be inverted to give

$$B^\mu = \frac{1}{3!} \frac{1}{\sqrt{-g}} \epsilon^{\mu\alpha\beta\gamma} A_{\alpha\beta\gamma}; \quad (\text{A5})$$

therefore, once the tensor  $A$  is known we can uniquely find  $B$ . At the level of the perturbations we find

$$\delta B^\mu = \frac{1}{3!} \frac{\epsilon^{\mu\alpha\beta\gamma}}{\sqrt{-g}} \left[ \frac{A_{\alpha\beta\gamma}}{2} g_{\rho\sigma} \delta g^{\rho\sigma} + \delta A_{\alpha\beta\gamma} \right], \quad (\text{A6})$$

which is valid on any background.

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