Power-Law Cosmology: Tachyonic Potential

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Abstract

Dark energy arising from the unstable D-brane system, the tachyon field, is considered in this work as an acceleration source of the power-law cosmology model, $a \propto t^{\alpha}$. We investigate the universe with tachyonic scalar field and dust dominate at late time. We use data derived from WMAP7 data and WMAP7+BAO+H_0 combined dataset which give α = 0.99±0.02 (WMAP7+BAO+H_0) and 0.99±0.04 (WMAP7) to construct the tachyon potential and scalar field solution.

Keywords: tachyon, power-law, cosmology

Introduction

Recently, it is know that the universe is expanding with an acceleration rate [1,2] which could be an effect of unknown form of energy dubbed dark energy. Many ideas [3,4,5] proposed as models of dark energy. Tachyonic scalar field is one of them. The tachyonic field, motivated from unstable D-brane system, can renders a tachyonic fluid with negative pressure [6,7] which is one of wanted aspects for dark energy candidates. Tachyonic field when evolving under potentials [8,9,10,11] has smooth equation of state parameter, w_ϕ, running between -1 and 0 [12].

Power-law cosmology where the scale factor is a function of the cosmic time as $a \propto t^{\alpha}$ where $0 \le \alpha \le \infty$ can describe the acceleration of the universe as $\alpha > 1$ and is able to remove the horizon and flatness problems as in the simplest inflationary model [13]. Furthermore, it also resolves the age problem of the CDM model [14].

In this work, we study FLRW power-law cosmology with barotropic (dust) fluid and tachyonic field components evolving under potential. The situation considered is at the late time sufficiently long after matter and radiation equality. We find tachyonic field solution and construct the potential $V(\phi)$. In the construction, we use derived WMAP7 and combined WMAP7+BAO+H_0 datasets to find those parameters.

Tachyonic power-law

Let us considering the universe filled with two ingredients: tachyon scalar field, ϕ, and barotropic perfect fluid. The energy density ρ_ϕ and the pressure p_ϕ of the tachyon in the flat FLRW universe are defined as

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \qquad (1)$$

$$p_\phi = -V(\phi)\sqrt{1-\dot{\phi}^2}. \qquad (2)$$

Dynamics of the barotropic fluid is described by the fluid equation,

$$\dot{\rho}_{\rm m} = -3H\rho_{\rm m}, \qquad (3)$$

since $w_{\rm m}$ is constant and

$$\rho_{\rm m} = \frac{D}{a^n}, \qquad (4)$$

where a constant $n = 3(1 + w_{\rm m})$ and $D \ge 0$ is a proportional constant. Equation of motion of the tachyon field describing energy conservation as the universe expands is

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V_{,\phi}}{V} = 0, \qquad (5)$$

where $V_{,\phi}$ is a derivative of the potential $V(\phi)$ with respect to field ϕ, dV/dϕ. The Friedmann equation of the system is

$$H^2 = \frac{8\pi G}{3}\rho_{\rm tot} - \frac{kc^2}{a^2}, \qquad (6)$$

where $\rho_{\rm tot}$ is the total energy density. It can be shown directly that

$$V(\phi) = \frac{3}{8\pi G}\left(H^2 + \frac{kc^2}{a^2} - \frac{D}{a^3}\right)\sqrt{\frac{3H^2 + 2\dot{H} + \frac{kc^2}{a^2}}{3H^2 - 8\pi G\frac{D}{a^3} + 3\frac{kc^2}{a^2}}} \qquad (7)$$

and the kinetic term is

$$\dot{\phi}^2 = -\frac{2\dot{H} + 8\pi G\frac{D}{a^3} - 2\frac{kc^2}{a^2}}{3H^2 - 8\pi G\frac{D}{a^3} + 3\frac{kc^2}{a^2}}. \qquad (8)$$

For dust $n = 3$ ($w_{\rm m} = 0$). In the power-law cosmology, the scaling factor is defined as

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\alpha}. \qquad (9)$$

Then Hubble parameter and its time derivative are

$H(t)=\dot{a}/a=\alpha/t$ and $\dot{H}=-\alpha/t^2$ respectively. By considering flat FLRW universe ($k=0$), we are set the scale factor at the present time a_0 to unity. Substituting H and $\dot{H}$ into equations (7) and (8), we obtain the potential and kinetic term

$$V(t)=\left(\frac{3\alpha^2}{8\pi Gt^2}-D\frac{t_0^{3\alpha}}{t^{3\alpha}}\right)\sqrt{\frac{3\alpha^2-2\alpha}{3\alpha^2-8\pi GDt_0^{3\alpha}t^{2-3\alpha}}}, \quad (10)$$

$$\dot{\phi}^2=\frac{2\alpha-8\pi GDt_0^{3\alpha}t^{2-3\alpha}}{3\alpha^2-8\pi GDt_0^{3\alpha}t^{2-3\alpha}}, \quad (11)$$

with solution to be integrated,

$$\phi(t)=\pm\int\sqrt{\frac{2\alpha-8\pi GDt_0^{3\alpha}t^{2-3\alpha}}{3\alpha^2-8\pi GDt_0^{3\alpha}t^{2-3\alpha}}}dt. \quad (12)$$

We will use only positive value.

Cosmological Parameters and Results

In power-law cosmology, α relates to the Hubble parameter H_0 as $\alpha=H_0t_0$. The other quantities can be derived from the cosmological parameters at present i.e. by setting $a_0=1$, the constant D in equation (4) is $D=\rho_{m,0}$. The values of those derived parameters from the WMAP7 [15] and WMAP7+BAO+H_0 [16] dataset are presented in Table 1 [17].

Table 1: Derived parameters for tachyon-field power-law cosmology.

parameter	WMAP7+BAO +H_0	WMAP7
t_0 (sec)	$(4.34\pm0.03)\times10^{17}$	$(4.35\pm0.04)\times10^{17}$
H_0 (km/s/Mpc)	70.4 ± 1.4	70.3 ± 2.5
α	0.99 ± 0.02	0.99 ± 0.04
$\rho_{m,0}$ (kg/m^3)	$(2.53\pm0.37)\times10^{-27}$	$(2.52\pm0.31)\times10^{-27}$

In SI units by using the parameters in Table 1, the potential and the kinetic terms are significantly the same for both datasets (combined WMAP7+BAO +H_0 and WMAP7),

$$V(t)=\left(1.75\times10^{9}-6.12\times10^{25}t^{-0.97}\right)t^{-2}\sqrt{\frac{0.96}{2.94-1.03\times10^{17}t^{-0.97}}} \quad (13)$$

$$\dot{\phi}^2(t)=\frac{1.98-1.03\times10^{17}t^{-0.97}}{2.94-1.03\times10^{17}t^{-0.97}}, \quad (14)$$

which gives the tachyon field for both data from combined WMAP7+BAO+H_0 and WMAP7. From equation (12), one can approximate $D\approx0$ at late time but still keeping D-term in the potential. We obtain

$$\phi(t)\approx\int_t^{t_0}\sqrt{\frac{2}{3\alpha}}dt=\phi_0-\sqrt{\frac{2}{3\alpha}}t. \quad (15)$$

In above equation, we define $\phi_0\equiv\sqrt{2/3\alpha}\,t_0$. Hence

$$t\approx\sqrt{\frac{3\alpha}{2}}(\phi_0-\phi). \quad (16)$$

Substituting back to equation (10), we obtain tachyon potential as a function of ϕ as

$$V(\phi)\approx\left[\frac{\alpha}{4\pi G(\phi_0-\phi)^2}-\frac{D\phi_0^{3\alpha}}{(\phi_0-\phi)^{3\alpha}}\right]\times\sqrt{\frac{3\alpha^2-2\alpha}{3\alpha^2-12\pi GD\alpha\phi_0^{3\alpha}(\phi_0-\phi)^{2-3\alpha}}} \quad (17)$$

This potential can be plotted with two datasets

$$V(\phi)\approx0.98\left[\frac{1.18\times10^{9}}{(3.56\times10^{17}-\phi)^2}-\frac{3.40\times10^{25}}{(3.56\times10^{17}-\phi)^{2.97}}\right]\times\sqrt{\frac{1}{2.94-8.46\times10^{16}(3.56\times10^{17}-\phi)^{-0.97}}} \quad (18)$$

$$\phi_0\approx3.56\times10^{17}, \quad (19)$$

for WMAP7+BAO+H_0 and

$$V(\phi)\approx0.98\left[\frac{1.18\times10^{9}}{(3.57\times10^{17}-\phi)^2}-\frac{3.41\times10^{25}}{(3.57\times10^{17}-\phi)^{2.97}}\right]\times\sqrt{\frac{1}{2.94-8.49\times10^{16}(3.57\times10^{17}-\phi)^{-0.97}}} \quad (20)$$

$$\phi_0\approx3.57\times10^{17}, \quad (21)$$

for WMAP7. These potential are plotted in Fig. (1)

Conclusions

In this paper, we study a power-law cosmology in universe filled with tachyon scalar field and dust components at late time. Considering the power-law cosmology with tachyon scalar field evolving under potential, we derive the potential as a function of time and as a function of field, ϕ. When tachyonic field dominant, $V(\phi)\propto\phi^{-2}$ and the condition $\alpha>2/3$.

Approximating $D\approx0$ at late time in the tachyonic field integral, we obtain tachyonic potential as a function of the field. The potential obtain is useful for further investigation for interacting case of dark matter and dark energy.

Acknowledgements

In this work, R. R. is granted by the Royal Golden Jubilee Ph.D. Program. B. G. is sponsored by the National Research Council of Thailand, Basic

Research Council of the Thailand, Basic Research Grant of the Thailand Research Fund and ThEP.

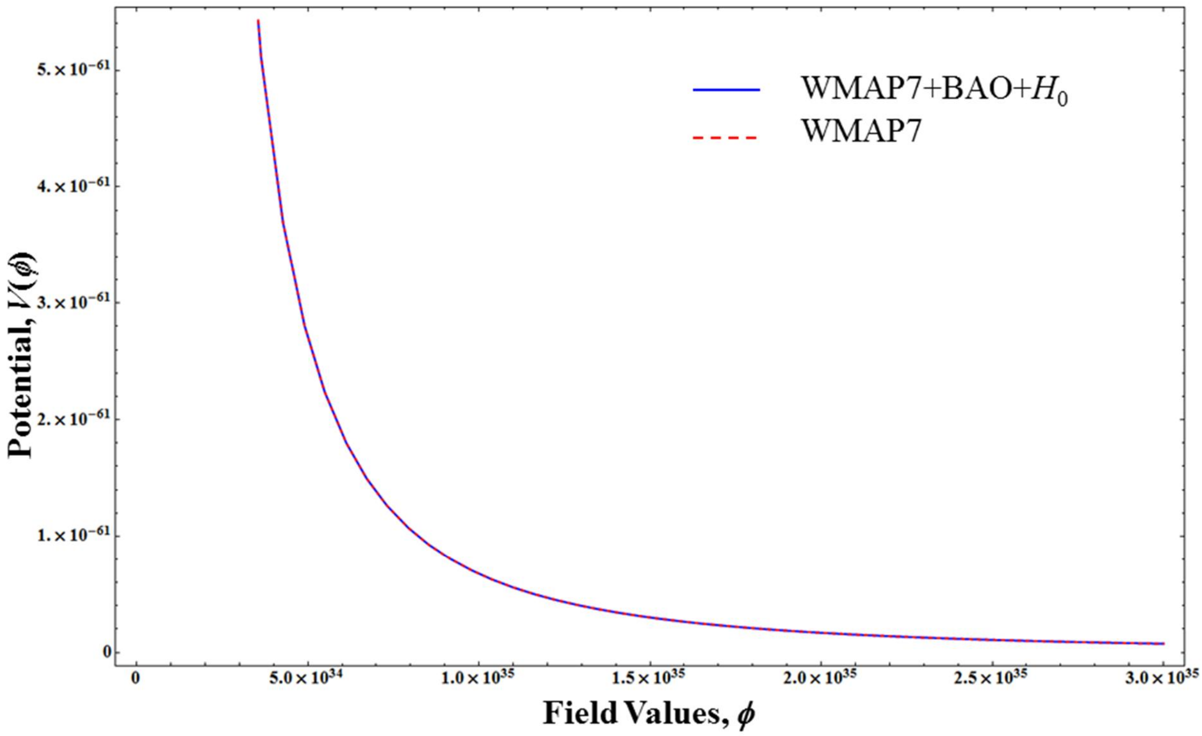

FIG. (1). Tachyon potential plotted as a function of field ϕ with two datasets: WMAP7+BAO+H_0 (blue solid line) and WMAP7 (red dash line).

References

1. A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland, C. J. Hogan, S. Jha, R. P. Kirshner, B. Leibundgut, M. M. Phillips, D. Reiss, B. P. Schmidt, R. A. Schommer, R. C. Smith, J. Spyromilio, C. Stubbs, N. B. Suntzeff and J. Tonry, "Observational evidence from supernovae for an accelerating universe and a cosmological constant", The Astronomical Journal **116** (1998) 1009.
2. S. J. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, D. E. Groom, I. M. Hook, A. G. Kim, M. Y. Kim, J. C. Lee, N. J. Nunes, R. Pain, C. R. Pennypacker, R. Quimby, C. Lidman, R.S. Ellis, M.Irwin, R. G. McMahon, P. Ruiz-Lapuente, N. Walton, B. Schaefer, B. J. Boyle, A. V. Filippenko, T. Matheson, A. S. Fruchter, N. Panagia, H. J. M. Newberg, W. J. Couch, and The Supernova Cosmology Project, "Measurements of Omega and Lambda from 42 high redshift supernovae", The Astronomical Journal **517** (1999) 565.
3. T. Padmanabhan, "Dark energy: The cosmological challenge of the millennium", Current Science **88** (2005) 1057.
4. E. J. Copeland, M. Sami and S. Tsujikawa, "Dynamics of dark energy", International Journal of Modern Physics D **15** (2006) 1753.
5. T. Padmanabhan, "Dark Energy: Mystery of the Millennium", AIP Conference Proceedings **861** (2006) 179.
6. A. Sen, "Rolling Tachyon", Journal of High Energy Physics **04** (2002) 048.
7. A. Sen, "Tachyon Matter", Journal of High Energy Physics **07** (2002) 065.
8. T. Padmanabhan, "Accelerated expansion of the universe driven by tachyonic matter", Physical Review D **66** (2002) 021301(R).
9. J. S. Bagla, H. K. Jassal and T. Padmanabhan, "Cosmology with tachyon field as dark energy", Physical Review D **67** (2003) 063504.
10. Z. K. Guo and Y. Z. Zhang, "Cosmological scaling solutions of multiple tachyon fields with inverse square potentials", Journal of Cosmology and Astroparticle Physics **0408** (2004) 010.
11. E. J. Copeland, M. R. Garousi, M. Sami and S. Tsujikawa, "What is needed of a tachyon if it is

to be the dark energy?", Physical Review D **71** (2005) 043003.

12. G. W. Gibbons, "Cosmological evolution of the rolling tachyon", Physics Letters B **537** (2002) 1.
13. F. Lucchin and S. Matarrese, "Power-law inflation", Physical Review D **32** (1985) 1316.
14. E. W. Kolb, "A coasting cosmology", The Astrophysical Journal **344** (1989) 543.
15. D. Larson, J. Dunkley, G. hinshaw, E. Komatsu, M. R. Nolta, C. L. Bennett, B. Gold, M. Halpern, R. S. Hill, N. Jarosik, A. Kogut, M. Limon, S. S. Meyer, N. Odegard, L. Page, K. M. Smith, D. N. Spergel, G. S. Tucker, J. L. Weiland, E. Wollack and E. L. Wright, "Seven-Year *Wilkinson Microwave Anisotropy Probe* (*WMAP*) Observations: Power Spectra and *WMAP*-Derived Parameters", The Astrophysical Journal Supplement Series **192** (2011) 16.
16. E. Komatsu, K. M. Smith, J. Dunkley, C. L. Bennett, B. Gold, G. Hinshaw, N. Jarosik, D. Larson, M. R. Nolta, L. Page, D. N. Spergel, M. Halpern, R. S. Hill, A. Kogut, M. Limon, S. S. Meyer, N. Odegard, G. S. Tucker, J. L. Weiland, E. Wollack and E. L. Wright, "Seven-Year *Wilkinson Microwave Anisotropy Probe* (*WMAP*) Observations: Cosmological Interpretation", The Astrophysical Journal Supplement Series **192** (2011) 18.
17. B. Gumjudpai and C. Kaeonikhom, "Observational constraints on scalar-field power-law cosmology", arXiv:1201.3499 [astro-ph].