Physics Letters B 695 (2011) 45–54

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Observational constraints on phantom power-law cosmology

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ARTICLE INFO

Article history:
Received 21 September 2010
Received in revised form 30 October 2010
Accepted 18 November 2010
Available online 24 November 2010
Editor: S. Hannestad

Keywords:
Phantom power-law cosmology
Dark energy
WMAP7 dataset

ABSTRACT

We investigate phantom cosmology in which the scale factor is a power law, and we use cosmological observations from Cosmic Microwave Background (CMB), Baryon Acoustic Oscillations (BAO) and observational Hubble data, in order to impose complete constraints on the model parameters. We find that the power-law exponent is $\beta \approx -6.51^{+0.24}_{-0.25}$, while the Big Rip is realized at $t_s \approx 104.5^{+1.9}_{-2.0}$ Gyr, in 1σ confidence level. Providing late-time asymptotic expressions, we find that the dark-energy equation-of-state parameter at the Big Rip remains finite and equal to $w_{DE} \approx -1.153$, with the dark-energy density and pressure diverging. Finally, we reconstruct the phantom potential.

1. Introduction

Recent cosmological observations obtained by SNIa [1], WMAP [2], SDSS [3] and X-ray [4] indicate that the observable universe experiences an accelerated expansion. Although the simplest way to explain this behavior is the consideration of a cosmological constant [5], the known fine-tuning problem [6] led to the dark energy paradigm. The dynamical nature of dark energy, at least in an effective level, can originate from a variable cosmological "constant" [7], or from various fields, such is a canonical scalar field (quintessence) [8], a phantom field, that is a scalar field with a negative sign of the kinetic term [9,10], or the combination of quintessence and phantom in a unified model named quintom [11]. Finally, an interesting attempt to probe the nature of dark energy according to some basic quantum gravitational principles is the holographic dark energy paradigm [12] (although the recent developments in Horava gravity could offer a dark energy candidate with perhaps better quantum gravitational foundations [13]).

The advantage of phantom cosmology, either in its simple or in its quintom extension, is that it can describe the phantom state of the universe, that is when the dark energy equation-of-state parameter lies below the phantom divide -1, as it might be the case according to observations [1]. Additionally, a usual consequence of phantom cosmology in its basic form, is the future Big Rip [14] or similar singularities [15], and thus one needs additional non-conventional mechanism if he desires to avoid such a possibility [16].

On the other hand power-law cosmology, where the scale factor is a power of the cosmological time, proves to be a very good phenomenological description of the universe evolution, since according to the value of the exponent it can describe the radiation epoch, the dark matter epoch, and the accelerating, dark energy epoch [17–19]. Additionally, it was found to be consistent with nucleosynthesis [20,21], with the age of high-redshift objects such as globular clusters [21,22], with the SNIa data [23,24], and with X-ray gas mass fraction measurements of galaxy clusters [25,26]. Furthermore, in the context of the power-law model, one can describe the gravitational lensing statistics [27], the angular size-redshift data of compact radio sources [28], and the SNIa magnitude-redshift relation [23,27].

In this work we desire to impose observational constraints on phantom power-law cosmology, that is on the scenario of a phantom scalar field along with the matter fluid in which the scale factor is a power law. In particular, we use cosmological observations from

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0370-2693/$ – see front matter
doi:10.1016/j.physletb.2010.11.047

Cosmic Microwave Background (CMB), Baryon Acoustic Oscillations (BAO) and observational Hubble data (H_0), in order to impose complete constraints on the model parameters, focusing on the power-law exponent and on the Big Rip time.

This Letter is organized as follows. In Section 2 we construct the scenario of phantom power-law cosmology. In Section 3 we use observational data in order to impose constraints on the model parameters, and in Section 4 we discuss the physical implications of the obtained results. Finally, Section 5 is devoted to the conclusions.

2. Phantom cosmology with power-law expansion

In this section we present phantom cosmology under power-law expansion. Throughout the work we consider the homogeneous and isotropic Friedmann–Robertson–Walker (FRW) background geometry with metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\, d\Omega_2^2\right], \tag{1}$$

where t is the cosmic time, r is the spatial radius coordinate, Ω_2 is the 2-dimensional unit sphere volume, and k characterizes the curvature of 3-dimensional space of which $k = -1, 0, 1$ corresponds to open, flat and closed universe respectively. Finally, as usual, $a(t)$ is the scale factor.

The action of a universe constituted of a phantom field ϕ, minimally coupled to gravity, reads [10]:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi) + L_{\mathrm{m}}\right], \tag{2}$$

where $V(\phi)$ is the phantom field potential, R the Ricci scalar and G the gravitational constant. The term L_{m} accounts for the total (dark plus baryonic) matter content of the universe, which is assumed to be a barotropic fluid with energy density ρ_m and pressure p_m, and equation-of-state parameter $w_m = p_m/\rho_m$. Finally, since we focus on small redshifts the radiation sector is neglected, although it could be straightforwardly included.

The Friedmann equations, in units where the speed of light is 1, write:

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_\phi) - \frac{k}{a^2}, \tag{3}$$

$$\dot{H} = -4\pi G(\rho_m + p_m + \rho_\phi + p_\phi) + \frac{k}{a^2}, \tag{4}$$

where a dot denotes the derivative with respect to t and $H \equiv \dot{a}/a$ is the Hubble parameter. In these expressions, ρ_ϕ and p_ϕ are respectively the energy density and pressure of the phantom field, which are given by:

$$\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{5}$$

$$p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{6}$$

The evolution equation of the phantom field, describing its energy conservation as the universe expands, is

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0, \tag{7}$$

or written equivalently in field terms:

$$\ddot{\phi} + 3H\dot{\phi} - \frac{dV}{d\phi} = 0. \tag{8}$$

Note that as we mentioned in the Introduction, in phantom cosmology the dark energy sector is attributed to the phantom field, that is $\rho_{DE} \equiv \rho_\phi$ and $p_{DE} \equiv p_\phi$, and thus its equation-of-state parameter is given by

$$w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} = \frac{p_\phi}{\rho_\phi}. \tag{9}$$

Finally, the equations close by considering the evolution of the matter density:

$$\dot{\rho}_m + 3H(1 + w_m)\rho_m = 0, \tag{10}$$

with straightforward solution

$$\rho_m = \frac{\rho_{m0}}{a^n}, \tag{11}$$

where $n \equiv 3(1 + w_m)$ and $\rho_{m0} \geqslant 0$ is the value at present time t_0.

Lastly, note that we can extract two helpful relations, namely from (3) we obtain

$$\rho_\phi = \frac{3}{8\pi G}\left(H^2 - \frac{8\pi G}{3}\rho_m + \frac{k}{a^2}\right), \tag{12}$$

while from (4), (5) we acquire

$$\dot{\phi}^2 = \frac{1}{4\pi G}\left(\dot{H} - \frac{k}{a^2}\right) + \rho_m \frac{n}{3}. \tag{13}$$

Let us now incorporate the power-law behavior of the scale factor. In the case of quintessence cosmology, the power-law ansatz takes the usual form

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\beta}, \tag{14}$$

with a_0 the value of the scale factor at present time t_0. However, in the case of phantom scenario, the power-law ansatz must be slightly modified, in order to acquire self-consistency. In particular, one rescales time as $t \to t_s - t$, with t_s a sufficiently positive reference time, and thus the scale factor becomes [17,29]:

$$a(t) = a_0\left(\frac{t_s - t}{t_s - t_0}\right)^{\beta}, \tag{15}$$

while the Hubble parameter and its time-derivative read:

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} = -\frac{\beta}{t_s - t}, \tag{16}$$

$$\dot{H} = -\frac{\beta}{(t_s - t)^2}. \tag{17}$$

Therefore, for $\beta < 0$ we have an accelerating ($\ddot{a}(t) > 0$) and expanding ($\dot{a}(t) > 0$) universe, which possesses additionally a positive $\dot{H}(t)$ that is it exhibits super-acceleration [30]. That is, in phantom power-law cosmology, expansion is always accompanied by acceleration. Furthermore, with $\beta < 0$, at $t = t_s$ the scale factor and the Hubble parameter diverge, that is the universe results to a Big Rip. These behaviors are common in phantom cosmology [10,31] and their realization is a self-consistency test of our work. On the other hand, note that the quintessence-ansatz (14) cannot lead to acceleration or to Big Rip and this was the reason for the introduction of the phantom power-law ansatz (15) in [17,29].

Having introduced the power-law ansatz that is suitable for phantom cosmology, we can easily extract the time-dependence of the various quantities, which re-expressed as functions of the redshift can be confronted by the observational data. In particular, substituting (11), (12), (13) in (5) we obtain

$$V(\phi) = \frac{3}{8\pi G}\left(H^2 + \frac{\dot{H}}{3} + \frac{2k}{3a^2}\right) + \left(\frac{n-6}{6}\right)\frac{\rho_{m0}}{a^n}. \tag{18}$$

In the following we consider as usual the matter (dark plus baryonic) component to be dust, that is $w_m \approx 0$ or equivalently $n = 3$. Thus, using the ansatz (15), and restoring the SI units using also $M_{\mathrm{P}}^2 = \hbar c/8\pi G$, we obtain

$$V(t) = \frac{M_{\mathrm{P}}^2 c}{\hbar}\left[\frac{3\beta^2 - \beta}{(t_s - t)^2} + \frac{2kc^2(t_s - t_0)^{2\beta}}{a_0^2(t_s - t)^{2\beta}}\right] - \frac{\rho_{m0}c^2}{2}\frac{(t_s - t_0)^{3\beta}}{a_0^3(t_s - t)^{3\beta}}. \tag{19}$$

Additionally, solving Eq. (13) for the phantom field and inserting the power-law scale factor, gives

$$\phi(t) = \int \sqrt{\frac{2M_{\mathrm{P}}^2 c}{\hbar}\left[-\frac{\beta}{(t_s - t)^2} - \frac{kc^2(t_s - t_0)^{2\beta}}{a_0^2(t_s - t)^{2\beta}}\right] + \frac{\rho_{m0}c^2(t_s - t_0)^{3\beta}}{a_0^3(t_s - t)^{3\beta}}}\, dt. \tag{20}$$

Finally, the time-dependence of the phantom energy density and pressure can be extracted from (5) and (6), using (19) and (20), namely:

$$\rho_\phi = \frac{M_{\mathrm{P}}^2 c}{\hbar}\left[\frac{3\beta^2}{(t_s - t)^2} + \frac{3kc^2(t_s - t_0)^{2\beta}}{a_0^2(t_s - t)^{2\beta}}\right] - \frac{\rho_{m0}c^2(t_s - t_0)^{3\beta}}{a_0^3(t_s - t)^{3\beta}}, \tag{21}$$

$$p_\phi = -\frac{M_{\mathrm{P}}^2 c}{\hbar}\left[\frac{3\beta^2 - 3\beta}{(t_s - t)^2}\right] - \frac{\rho_{m0}c^2(t_s - t_0)^{3\beta}}{2a_0^3(t_s - t)^{3\beta}}, \tag{22}$$

and thus we can straightforwardly extract the time evolution of the dark energy equation-of-state parameter through (9) as $w_{DE}(t) = p_\phi(t)/\rho_\phi(t)$. Note that at $t \to t_s$, apart from the scale factor, ρ_ϕ and p_ϕ diverge too, however w_{DE} remains finite. This is exactly the Big Rip behavior according to the classification of singularities of [15].

All the aforementioned time-dependencies can be expressed in terms of the redshift z. In particular, since $1 + z = a_0/a$, in phantom power-law cosmology we have

$$t = t_s - (t_s - t_0)(1 + z)^{-1/\beta}. \tag{23}$$

Therefore, using this relation we can extract the z-dependence of all the relevant quantities of the scenario at hand, which can then straightforwardly be confronted by the data.

3. Observational constraints

In the previous section we presented the cosmological scenario in which the dark energy sector is attributed to a phantom scalar field, and where the scale factor is a power law of the cosmic time. Thus, in the present section we can proceed to confrontation with observations. In particular, we use Cosmic Microwave Background (CMB), Baryon Acoustic Oscillations (BAO) and Observational Hubble Data (H_0), in order to impose constraints on the model parameters, and especially to the power-law exponent β and to the Big Rip time t_s. Finally, we first obtain our results using only the CMB-WMAP7 data [32], and then we perform a combined fit using additionally the BAO [33] and H_0 ones [34].

We mention that in the present work we prefer not to use SNIa data as in the combined WMAP5 + BAO + SNIa dataset [35]. This is because the combined WMAP5 dataset uses SNIa data from [36,37] which do not include systematic error, and the cosmological parameters derived from the combined WMAP5 dataset also differ from those derived from other compilations of SNIa data [38]. Inclusion of the SNIa systematic error which is comparable to the its statistical error can significantly alter the value of the equation of state [39]. Furthermore, recent analysis shows that the value of the equation-of-state parameter derived from two different light-curve fitters could be different from the one derived from two different datasets. This could make it difficult to identify if w_{DE} is phantom, since its obtained values from the two fitters are different [40]. A very recent critics on SNIa data analysis has been presented in [41]. Definitely the incorporation of SNIa data in constraining phantom cosmology is a subject that deserves further investigation.

Similarly to the non-phantom case [42], the exponent β can be straightforwardly expressed as

$$\beta = -H_0(t_s - t_0), \tag{24}$$

where, as usual, we use the subscript 0 to denote the value of a quantity at present, and we moreover set a_0 to 1. Furthermore, we introduce the usual density parameter $\Omega_m \equiv 8\pi G\rho_m/(3H^2)$, and we split Ω_m in its baryonic and cold dark matter part, Ω_b and Ω_{CDM} respectively ($\Omega_m = \Omega_b + \Omega_{\mathrm{CDM}}$). Lastly, it proves convenient to introduce the critical density $\rho_c = 3H^2/8\pi G$, and thus we can use the relation $\rho_{m0} = \Omega_{m0}\rho_{c0}$.

In a general, non-flat geometry the Big Rip time t_s cannot be calculated, bringing a large uncertainty to the observational fitting. However, one could estimate it, performing some very plausible assumptions [9]. In particular, assuming a flat geometry, which is a very good approximation [39], and assuming that at late times the phantom dark energy will dominate the universe, which is always the case in phantom models, t_s can be expressed as [9]

$$t_s \simeq t_0 + \frac{2}{3}|1 + w_{DE}|^{-1} H_0^{-1}(1 - \Omega_{m0})^{-1/2}. \tag{25}$$

Here we have to mention that there is one last assumption in extracting this relation, namely that at late times the dark energy equation-of-state parameter w_{DE} approaches a constant value. Fortunately, this is always the case in flat power-law phantom cosmology examined in this work, as can be seen from (21), (22) for $k = 0$, recalling also that β is always negative in an expanding universe. In this case, at late times we indeed have:

$$w_{DE} \simeq -1 + \frac{1}{\beta}, \tag{26}$$

which lies always below the phantom divide as expected.[1] In addition, one can straightway extract $[H(t)^2]$ through (16) as

$$\big[H(t)^2\big] = H_0^2\left(\frac{t_s - t_0}{t_s - t}\right)^2. \tag{28}$$

Finally, as we have mentioned, the time-functions can be expressed as redshift-functions using (23).

Having all the required information, we proceed to the data fitting. For the case of the WMAP7 data alone we use the maximum likelihood parameter values for H_0, t_0, $\Omega_{\mathrm{CDM}0}$ and Ω_{b0} [39], focusing on the flat geometry. Additionally, we perform a combined observational fitting, using WMAP7 data, along with Baryon Acoustic Oscillations (BAO) in the distribution of galaxies, and Observational Hubble Data (H_0). The details and the techniques of the construction are presented in Appendix A.

4. Results and discussions

In the previous section we presented the method that allows for the confrontation of power-law phantom cosmology with the data. In the present section we perform such an observational fitting, presenting our results, and discussing their physical implications.

First of all, in Table 1, we show for completeness the maximum likelihood values for the present time t_0, the present Hubble parameter H_0, the present baryon density parameter Ω_{b0} and the present cold dark matter density parameter $\Omega_{\mathrm{CDM}0}$, that was used in our fitting [39], in WMAP7 as well as in the combined fitting. In the same table we also provide the 1σ bounds of every parameter. In Table 2 we present the maximum likelihood values and the 1σ bounds for the derived parameters, namely the power-law exponent β, the present matter energy density value ρ_{m0}, the present critical energy density value ρ_{c0} and the Big Rip time t_s. As we observe, β is negative, as expected in consistent phantom cosmology. We mention here that the phantom power-law ansatz (15) is technically different from the

[1] Note that if instead of w_{DE} we consider the effective w_{eff}, that is including the weighted contribution of matter, then we have

$$w_{\mathrm{eff}} \to -1 + \frac{2}{3\beta} \tag{27}$$

at $t \to t_s$ [43], for any curvature value.

Table 1
Observational maximum likelihood values in 1σ confidence level for the present time t_0, the present Hubble parameter H_0, the present baryon density parameter Ω_{b0} and the present cold dark matter density parameter Ω_{CDM0}, for WMAP7 as well as for the combined fitting WMAP7 + BAO + H_0. The values are taken from [39].

Parameter	WMAP7 + BAO + H_0	WMAP7
t_0	13.78 ± 0.11 Gyr [$(4.33 \pm 0.04) \times 10^{17}$ sec]	13.71 ± 0.13 Gyr [$(4.32 \pm 0.04) \times 10^{17}$ sec]
H_0	$70.2^{+1.3}_{-1.4}$ km/s/Mpc	71.4 ± 2.5 km/s/Mpc
Ω_{b0}	0.0455 ± 0.0016	0.0445 ± 0.0028
Ω_{CDM0}	0.227 ± 0.014	0.217 ± 0.026

Table 2
Derived maximum likelihood values in 1σ confidence level for the power-law exponent β, the present matter energy density value ρ_{m0}, the present critical energy density value ρ_{c0} and the Big Rip time t_s, for WMAP7 as well as for the combined fitting WMAP7 + BAO + H_0.

Parameter	WMAP7 + BAO + H_0	WMAP7
β	$-6.51^{+0.24}_{-0.25}$	-6.5 ± 0.4
ρ_{m0}	$(2.52 \pm 0.26) \times 10^{-27}$ kg/m^3	$(2.50 \pm 0.30) \times 10^{-27}$ kg/m^3
ρ_{c0}	$(9.3^{+0.3}_{-0.4}) \times 10^{-27}$ kg/m^3	$(9.57 \pm 0.67) \times 10^{-27}$ kg/m^3
t_s	$104.5^{+1.9}_{-2.0}$ Gyr [$(3.30 \pm 0.06) \times 10^{18}$ sec]	102.3 ± 3.5 Gyr [$(3.23 \pm 0.11) \times 10^{18}$ sec]

quintessence one (14), and thus one cannot straightforwardly compare the exponent values of the two cases (for example a similar w_{DE} is produced by significantly different exponents in the two scenarios [23]). Now, note that the Big Rip time is one order of magnitude larger than the present age of the universe, which shows that such an outcome is unavoidable in phantom cosmology, unless one include additional mechanisms for the exit from phantom phase [16], an approach that was not taken into account in this work.

Let us discuss in more detail the values and the evolution of some quantities of interest. For the combined data WMAP7 + BAO + H_0, the potential (19) is fitted as

$$V(t) \approx \frac{6.47 \times 10^{27}}{(3.30 \times 10^{18} - t)^2} - 2.51 \times 10^{-371}\big(3.30 \times 10^{18} - t\big)^{19.54}, \tag{29}$$

while WMAP7 data alone give

$$V(t) \approx \frac{6.37 \times 10^{27}}{(3.23 \times 10^{18} - t)^2} - 1.99 \times 10^{-368}\big(3.23 \times 10^{18} - t\big)^{19.39}. \tag{30}$$

Note that the second terms in these expressions, although very small at early times, they become significant at late times, that is close to the Big Rip. In particular, the inflection happens at $22.4^{+1.9}_{-2.0}$ Gyr (WMAP7 + BAO + H_0) and $22.0^{+3.5}_{-3.5}$ Gyr (WMAP7), after which we obtain a rapid increase.

Now, concerning the scalar field evolution $\phi(t)$, at late times ($t \to t_s$) the ρ_{m0}-term in (20) can be neglected. Thus, (20) reduces to

$$\phi(t) \approx \int \sqrt{-\frac{2M_{\mathrm{P}}^2 c}{\hbar}\frac{\beta}{(t_s - t)^2}}\, dt, \tag{31}$$

which can be fitted using combined WMAP7 + BAO + H_0 giving

$$\phi(t) \approx -2.64 \times 10^{13} \ln\big(3.30 \times 10^{18} - t\big), \tag{32}$$

while for WMAP7 dataset alone we obtain

$$\phi(t) \approx -2.63 \times 10^{13} \ln\big(3.23 \times 10^{18} - t\big). \tag{33}$$

As expected, both the phantom field and its kinetic energy ($-\dot{\phi}^2/2$) diverge at the Big Rip.

Having fitted the phantom potential $V(t)$ and the phantom field itself $\phi(t)$, it is now straightforward to obtain the potential as a function of the phantom field, namely $V(\phi)$. In particular, (32) and (33) can be easily inverted, giving $t(\phi)$, and thus substitution to (29) and (30) respectively provides $V(\phi)$. Doing so, for the combined data WMAP7 + BAO + H_0 the potential is fitted as

$$V(\phi) \approx 6.47 \times 10^{27} e^{0.75 \times 10^{-13}\phi} - 2.51 \times 10^{-371} e^{-7.4 \times 10^{-13}\phi}, \tag{34}$$

while for WMAP7 dataset alone we obtain

$$V(\phi) \approx 6.37 \times 10^{27} e^{0.76 \times 10^{-13}\phi} - 1.99 \times 10^{-368} e^{-7.4 \times 10^{-13}\phi}. \tag{35}$$

In order to provide a more transparent picture, in Fig. 1 we present the corresponding plot for $V(\phi)$, for both the WMAP7 + BAO + H_0 as well as the WMAP7 case.

Let us now consider the equation-of-state parameter for the phantom field, that is for the dark energy sector. As we mentioned in the end of Section 2, it is given by $w_{DE}(t) = p_\phi(t)/\rho_\phi(t)$, with $p_\phi(t)$ and $\rho_\phi(t)$ given by relations (22) and (21) respectively. Finally, one can extract the redshift dependence using (23). One can therefore use WMAP7 and WMAP7 + BAO + H_0 observational data in order to fit the evolution of $w_{DE}(z)$ at late times, that is for $t \to t_s$, or equivalently for $z \to -1$. For the WMAP7 + BAO + H_0 combined dataset we find

$$w_{DE}(z) \approx \frac{1}{2} - \frac{6.068}{3.670 - (1+z)^{3.307}}, \tag{36}$$

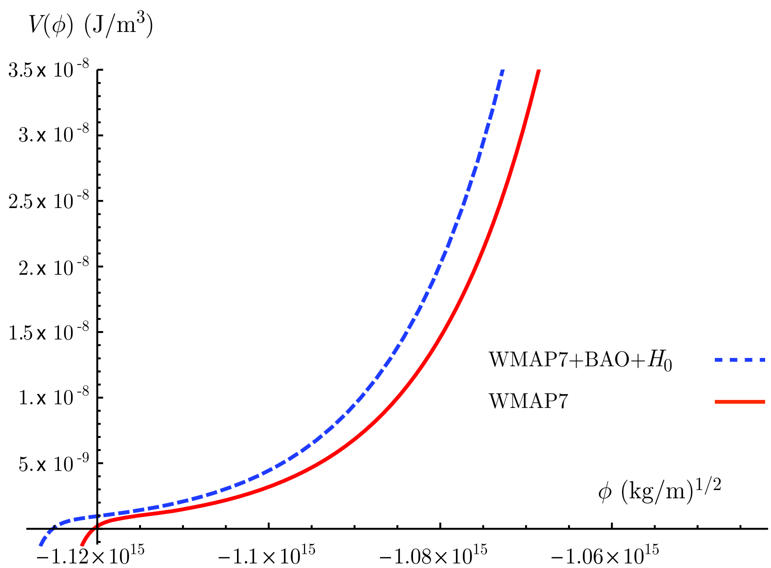

Fig. 1. The phantom potential obtained from observational data fitting of WMAP7 and WMAP7 + BAO + H_0.

while for the WMAP7 dataset alone we have

$$w_{DE}(z) \approx \frac{1}{2} - \frac{6.328}{3.824 - (1+z)^{3.309}}. \tag{37}$$

As we observe, at $t \to t_s$, w_{DE} becomes -1.153 for the combined dataset and -1.155 for the WMAP7 dataset alone. However, as we have already discussed in the end of Section 2, at $t \to t_s$ despite the finiteness of w_{DE}, the phantom dark energy density and pressure become infinite. These behaviors are the definition of a Big Rip [15], and this acts as a self-consistency test of our model.

5. Conclusions

In this work we investigated phantom cosmology in which the scale factor is a power law. After constructing the scenario, we used observational data in order to impose constraints on the model parameters, focusing on the power-law exponent β and on the Big Rip time t_s.

Using the WMAP7 dataset alone, we found that the power-law exponent is $\beta \approx -6.5 \pm 0.4$ while the Big Rip is realized at $t_s \approx 102.3 \pm 3.5$ Gyr, in 1σ confidence level. Additionally, the dark-energy equation-of-state parameter w_{DE} lies always below the phantom divide as expected, and at the Big Rip it remains finite and equal to -1.155. However, both the phantom dark-energy density and pressure diverge at the Big Rip.

Using WMAP7 + BAO + H_0 combined observational data we found that $\beta \approx -6.51^{+0.24}_{-0.25}$, while $t_s \approx 104.5^{+1.9}_{-2.0}$ Gyr, in 1σ confidence level. Moreover, w_{DE} at the Big Rip becomes -1.153. Finally, in order to present a more transparent picture, we provided the reconstructed phantom potential.

In summary, we observe that phantom power-law cosmology can be compatible with observations, exhibiting additionally the usual phantom features, such is the future Big Rip singularity. However, it exhibits also the known disadvantage that the dark-energy equation-of-state parameter lies always below the phantom divide, by construction. In order to acquire a more realistic picture, describing also the phantom divide crossing, as it might be the case according to observations, one should proceed to the investigation of quintom power-law cosmology, considering apart from the phantom a canonical scalar field, too. Such a project is left for future investigation.

Finally, let us make a comment on the nature of the investigated scenarios. Although the classical behavior of phantom fields has a very rich phenomenology and can be compatible with observations, as it is known the discussion about the construction of quantum field theory of phantoms is still open in the literature. For instance in [44] the authors reveal the causality and stability problems and the possible spontaneous breakdown of the vacuum into phantoms and conventional particles in four dimensions. However, on the other hand, there have also been serious attempts in overcoming these difficulties and construct a phantom theory consistent with the basic requirements of quantum field theory [45], with the phantom fields arising as an effective description. The present analysis is just a first approach on phantom power-law cosmology. Definitely, the subject of quantization of such scenarios is open and needs further investigation.

Acknowledgements

We thank Kiattisak Thepsuriya and the referee for useful discussions and comments. C.K. is supported by a research studentship funded by Thailand Toray Science Foundation (TTSF) and the Thailand Center of Excellence in Physics (ThEP). B.G. is sponsored by the Thailand Research Fund's Basic Research Grant (TRF Advanced Research Scholar), TTSF and ThEP.

Appendix A. Observational data and constraints

In this appendix we briefly review the main sources of observational constraints used in this work, namely WMAP7 Cosmic Microwave Background (CMB), Baryon Acoustic Oscillations (BAO), and Observational Hubble Data (H_0). In our calculations we take the total likelihood $L \propto e^{-\chi^2/2}$ to be the product of the separate likelihoods of BAO, CMB and H_0. Thus, the total χ^2 is

$$\chi^2(p_s) = \chi^2_{\mathrm{CMB}} + \chi^2_{\mathrm{BAO}} + \chi^2_{H_0}. \tag{A.1}$$

A.1. CMB constraints

We use the CMB data to impose constraints on the parameter space, following the recipe described in [35]. The "CMB shift parameters" [46] are defined as:

$$R \equiv \sqrt{\Omega_{m0}} H_0 r(z_*), \quad l_a \equiv \pi r(z_*)/r_s(z_*). \tag{A.2}$$

R can be physically interpreted as a scaled distance to recombination, and l_a can be interpreted as the angular scale of the sound horizon at recombination. $r(z)$ is the comoving distance to redshift z defined as

$$r(z) \equiv \int_0^z \frac{1}{H(z)}\, dz, \tag{A.3}$$

while $r_s(z_*)$ is the comoving sound horizon at decoupling (redshift z_*), given by

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{1}{H(z)\sqrt{3(1+R_b/(1+z))}}\, dz. \tag{A.4}$$

The quantity R_b is the ratio of the energy density of photons to baryons, and its value can be calculated as $R_b = 31500\Omega_{b0}h^2(T_{\mathrm{CMB}}/2.7\ \mathrm{K})^{-4}$ (Ω_{b0} being the present day density parameter for baryons) using $T_{\mathrm{CMB}} = 2.725$ [35]. The redshift at decoupling $z_*(\Omega_{b0}, \Omega_{m0}, h)$ can be calculated from the following fitting formula [47]:

$$z_* = 1048\left[1 + 0.00124\left(\Omega_{b0}h^2\right)^{-0.738}\right]\left[1 + g_1\left(\Omega_{m0}h^2\right)^{g_2}\right], \tag{A.5}$$

with g_1 and g_2 given by:

$$g_1 = \frac{0.0783(\Omega_{b0}h^2)^{-0.238}}{1 + 39.5(\Omega_{b0}h^2)^{0.763}},$$
$$g_2 = \frac{0.560}{1 + 21.1(\Omega_{b0}h^2)^{1.81}}.$$

Finally, the χ^2 contribution of the CMB reads

$$\chi^2_{\mathrm{CMB}} = \mathbf{V}^{\mathbf{T}}_{\mathrm{CMB}} \mathbf{C}_{\mathrm{inv}} \mathbf{V}_{\mathrm{CMB}}. \tag{A.6}$$

Here $\mathbf{V}_{\mathrm{CMB}} \equiv \mathbf{P} - \mathbf{P}_{\mathrm{data}}$, where $\mathbf{P}$ is the vector (l_a, R, z_*) and the vector $\mathbf{P}_{\mathrm{data}}$ is formed from the WMAP 5-year maximum likelihood values of these quantities [35]. The inverse covariance matrix $\mathbf{C}_{\mathrm{inv}}$ is also provided in [35].

A.2. Baryon acoustic oscillations constraints

In this case the measured quantity is the ratio $d_z = r_s(z_d)/D_V(z)$, where $D_V(z)$ is the so-called "volume distance", defined in terms of the angular diameter distance $D_A \equiv r(z)/(1+z)$ as

$$D_v(z) \equiv \left[\frac{(1+z)^2 D_A^2(z) z}{H(z)}\right]^{1/3}, \tag{A.7}$$

and z_d is the redshift of the baryon drag epoch, which can be calculated from the fitting formula [48]:

$$z_d = \frac{1291(\Omega_{m0}h^2)^{0.251}}{1 + (\Omega_{M0}h^2)^{0.828}}\left[1 + b_1\left(\Omega_{b0}h^2\right)^{b_2}\right], \tag{A.8}$$

where b_1 and b_2 are given by

$$b_1 = 0.313\left(\Omega_{m0}h^2\right)^{-0.419}\left[1 + 0.607\left(\Omega_{m0}h^2\right)^{0.674}\right],$$
$$b_2 = 0.238\left(\Omega_{m0}h^2\right)^{0.223}.$$

We use the two measurements of d_z at redshifts $z = 0.2$ and $z = 0.35$ [33]. We calculate the χ^2 contribution of the BAO measurements as:

$$\chi^2_{\mathrm{BAO}} = \mathbf{V}^{\mathbf{T}}_{\mathrm{BAO}} \mathbf{C}_{\mathrm{inv}} \mathbf{V}_{\mathrm{BAO}}. \tag{A.9}$$

Here the vector $\mathbf{V}_{\mathrm{BAO}} \equiv \mathbf{P} - \mathbf{P}_{\mathrm{data}}$, with $\mathbf{P} \equiv (d_{0.2}, d_{0.35})$, and $\mathbf{P}_{\mathrm{data}} \equiv (0.1905, 0.1097)$, the two measured BAO data points [33]. The inverse covariance matrix is provided in [33].

Table 3
The observational $H(z)$ data [34].

z	0	0.1	0.17	0.27	0.4	0.48	0.88	0.9	1.30	1.43	1.53	1.75
$H(z)$ (km s^{-1} Mpc^{-1})	74.2	69	83	77	95	97	90	117	168	177	140	202
1σ uncertainty	±3.6	±12	±8	±14	±17	±60	±40	±23	±17	±18	±14	±40

A.3. Observational Hubble data constraints

The observational Hubble data are based on differential ages of the galaxies [49]. In [50], Jimenez et al. obtained an independent estimate for the Hubble parameter using the method developed in [49], and used it to constrain the equation of state of dark energy. The Hubble parameter, depending on the differential ages as a function of the redshift z, can be written as

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}. \tag{A.10}$$

Therefore, once dz/dt is known, $H(z)$ is directly obtained [51]. By using the differential ages of passively-evolving galaxies from the Gemini Deep Deep Survey (GDDS) [52] and archival data [53], Simon et al. obtained $H(z)$ in the range of $0 \lesssim z \lesssim 1.8$ [51]. We use the twelve observational Hubble data from [34] listed in Table 3.

The best-fit values of the model parameters from observational Hubble data [51] are determined by minimizing

$$\chi^2_{H_0}(p_s) = \sum_{i=1}^{12} \frac{[H_{th}(p_s; z_i) - H_{\text{obs}}(z_i)]^2}{\sigma^2(z_i)}, \tag{A.11}$$

where p_s denotes the parameters contained in the model, H_{th} is the predicted value for the Hubble parameter, H_{obs} is the observed value, $\sigma(z_i)$ is the standard deviation measurement uncertainty, and the summation runs over the 12 observational Hubble data points at redshifts z_i.

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