Late-time Equation of State of Phantom Power-law Cosmology

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Abstract

Phantom scalar field model of dark energy is a model for describing the accelerating universe today. We are interested in a model whose power-law exponent of scale factor is negative. We use observation datasets from Cosmic Microwave Background (CMB), Baryon Acoustic Oscillations (BAO) and observational Hubble Space-Telescope data (HST, H_0), in order to impose complete constraints on the model parameters. We find that the dark-energy equation-of-state parameter at the Big Rip remains finite and equal to $w_{\rm DE} \approx -1.153$ for combined dataset and $w_{\rm DE} \approx -1.155$ for the WMAP7 dataset alone.

Keywords: Phantom Power-law Cosmology, Scalar Field Dark Energy

Introduction

Cosmological observations today indicate that the observable universe is accelerately expanding. Although simplest way to explain this behavior is to consider cosmological constant [1] but the cosmological constant suffers from the fine-tuning problem [2]. Hence dark energy should be dynamical. This could be originated in form of variable cosmological "constant" [3], or in form of scalar fields, such as canonical scalar field (quintessence) [4] and a phantom field, which is a canonical scalar field with negative kinetic term [5, 6].

In this work we find late-time equation-of-state parameter of phantom power-law cosmology and show that it is finite at the Big Rip. Cosmological observations are taken from Cosmic Microwave Background (CMB), Baryon Acoustic Oscillations (BAO) and observational Hubble Space Telescope data (HST, H_0), in order to impose complete constraint on the model parameters and to determine value of equation-of-state parameter of the phantom dark energy at late time.

Phantom Dark Energy with Power-Law Expansion

In this work, we consider homogeneous and isotropic Friedmann–Robertson–Walker (FRW) universe with signature [+ – – –]. The action of FRW universe constituted of a phantom field ϕ reads [6]

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left[\frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi) + \mathcal{L}_{\rm m}\right], \quad (1)$$

where $V(\phi)$, R, G are the phantom field potential, Ricci scalar and gravitational constant respectively. $\mathcal{L}_{\rm m}$ is the total matter content in the universe (including cold dark matter–CDM). The matter content is assumed to be perfect fluid with energy density $\rho_{\rm m}$, pressure $p_{\rm m}$, and constant equation-of-state parameter $w_{\rm m} = p_{\rm m}/\rho_{\rm m}$.

Cosmological equations, in units with unity speed of light, are taken in the form

$$H^2 = \frac{8\pi G}{3}\left(\rho_{\rm m} + \rho_\phi\right) - \frac{k}{a^2}, \text{ and} \quad (2)$$

$$\dot{H} = -4\pi G(\rho_{\rm m} + p_{\rm m} + \rho_\phi + p_\phi) + \frac{k}{a^2}, \quad (3)$$

where a dot denotes derivative with respect to cosmic time t, a is scale factor, k is the curvature of the universe with $k = -1, 0, +1$ corresponding to open, flat and closed universe respectively. $H \equiv \dot{a}/a$ is the Hubble parameter. Note that subscript "ϕ" is for phantom scalar field.

The evolution equation describing the conservation of matter density is taken as

$$\dot{\rho}_{\rm m} + 3H\left(1 + w_{\rm m}\right)\rho_{\rm m} = 0, \quad (4)$$

with $\rho_{\rm m} = \rho_{\rm m0}/a^n$, where $n \equiv 3(1 + w_{\rm m})$ and $\rho_{\rm m0} \geq 0$ is the value at present time t_0. Varying the action Eq. (1) with respect to the metric gives phantom field energy density and pressure:

$$\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi), \; p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (5)$$

The equation-of-state parameter of phantom field dark energy is given by $w_{\rm DE} \equiv p_{\rm DE}/\rho_{\rm DE} = p_\phi/\rho_\phi$, and thus its conservation equation of the phantom field is taken in the form:

$$\dot{\rho}_{\rm DE} + 3H\left(1 + w_{\rm DE}(t)\right)\rho_{\rm DE} = 0. \quad (6)$$

For phantom power-law cosmology, the scale factor must be modified to slightly difference from the case of quintessence. The scale factor which power-law time exponent is [7, 8]

$$a(t) = \left(\frac{t_{\rm s} - t}{t_{\rm s} - t_0}\right)^{\beta} \quad (7)$$

where $t_{\rm s}$ is a largely positive reference time. Thus the Hubble parameter and its time-derivative read:

$$H = -\frac{\beta}{t_{\rm s} - t}, \; \dot{H} = -\frac{\beta}{(t_{\rm s} - t)^2}. \quad (8)$$

In particular, by using the Eq. (2), (3), (5), and the integrals of Eq. (4), we obtain the phantom potential [9]:

$$V(\phi) = \frac{3}{8\pi G}\left(H^2 + \frac{\dot{H}}{3} + \frac{2k}{3a^2}\right) + \left(\frac{n-6}{6}\right)\frac{\rho_{\rm m0}}{a^n}. \quad (9)$$

The equation-of-state parameter of matter can be considered as dust, i.e. pressureless fluid ($w_{\rm m} = 0$ or equivalently $n = 3$). Using the phantom potential (9), restoring SI units and using $M_{\rm P}^2 = \hbar c / 8\pi G$, we obtain [3]

$$V(t) = \frac{M_{\rm P}^2 c}{\hbar}\left[\frac{3\beta^2 - \beta}{(t_{\rm s} - t)^2} + \frac{2kc^2 (t_{\rm s} - t_0)^{2\beta}}{a_0^2 (t_{\rm s} - t)^{2\beta}}\right] - \frac{\rho_{m0} c^2}{2}\frac{(t_{\rm s} - t_0)^{3\beta}}{(t_{\rm s} - t)^{3\beta}}. \quad (10)$$

Finally, the phantom energy density and pressure can be found from Eq. (5), by using Eq. (3), Eq. (8) and Eq. (10), namely:

$$\rho_{\phi} = \frac{M_{\rm P}^2}{\hbar}\left[\frac{3\beta^2}{(t_{\rm s} - t)^2} + \frac{3kc^2 (t_{\rm s} - t_0)^{2\beta}}{a_0^2 (t_{\rm s} - t)^{2\beta}}\right] - \frac{\rho_{m0} c^2 (t_{\rm s} - t_0)^{3\beta}}{a_0^3 (t_{\rm s} - t)^{3\beta}}, \quad (11)$$

$$p_{\phi} = -\frac{3M_{\rm P}^2 c}{\hbar}\left[\frac{\beta^2 - \beta}{(t_{\rm s} - t)^2}\right] - \frac{\rho_{\rm m0} c^2 (t_{\rm s} - t_0)^{3\beta}}{2a_0^2 (t_{\rm s} - t)^{3\beta}}. \quad (12)$$

Therefore we can determine equation-of-state parameter of dark energy via $w_{\rm DE} = p_{\phi}/\rho_{\phi}$.

Observational Constraints

In this section we use observational data from CMB, BAO and H_0, in order to impose constraints on the model parameters. We use the CMB-WMAP7 dataset [10], and [11] a combined dataset of WMAP7 with the BAO and H_0 [12]. Considering Friedmann equation (2) in form of density parameter $\Omega \equiv \rho/\rho_{\rm c} = 8\pi G\rho/3H^2$ assuming flat universe. The Friedmann equation hence reads

$$H^2 = H_0^2\left[\Omega_{\rm m0} a^{-3} + \Omega_{\rm DE0} a^{-3(1+w_{\rm DE})}\right], \quad (13)$$

where a subscript "0" denotes the value at present time t_0.The geometry of the universe measured today is close flat [13], thus we can approximate that $\Omega_{\rm DE0} \approx 1 - \Omega_{\rm m0}$. Observation today [13] tells us that the universe is already dark-energy-dominated (as seen in Table 1), thus the matter term in Eq. (13) is negligible. The Eq. (13) becomes

$$\int_{a_0}^{\infty} a^{3(1+w_{\rm DE})/2-1} {\rm d}a = \int_{t_0}^{t_{\rm s}} H_0 \left(1 - \Omega_{\rm m}\right)^{1/2} {\rm d}t, \quad (14)$$

where $t_{\rm s}$ is the Big Rip time. Phantom energy ($w_{\rm DE} <$ –1) will dominates the universe at the late time. Thus $t_{\rm s}$ can be calculated as [5]

$$t_{\rm s} \cong t_0 + \frac{2}{3}\left|1 + w_{\rm DE0}\right|^{-1} H_0^{-1}\left(1 - \Omega_{\rm m0}\right)^{-1/2}. \quad (15)$$

Note that at the Big Rip time $t_{\rm s}$, the scale factor blows up. One can find $t_{\rm s}$ using the present observable value of $w_{\rm DE0}$.

Results and Discussion

In Table 1, being self-contained, the present time maximum-likelihood values of cosmological parameters are presented together. The cosmological parameters are derived using [9, 13] WMAP7 dataset and the combined dataset, providing the 1σ bounds of every parameter. As β is negative, the phantom power-law cosmology (7) is technically different from the quintessence.

Table 1: Observational maximum likelihood values in 1σ confidence level. The values are taken from [9, 13]

Parameter	**WMAP7+BAO+H_0**	**WMAP**
t_0	13.78 ± 0.11 Gyr	13.71 ± 0.13 Gyr
H_0	$70.2^{+1.3}_{-1.4}$ km/s/Mpc	71.4 ± 2.5 km/s/Mpc
Ω_{b0}	0.0455 ± 0.0016	0.0445 ± 0.0028
Ω_{CDM0}	0.227 ± 0.014	0.217 ± 0.026
β	$-6.51^{+0.24}_{-0.25}$	-6.5 ± 0.4
ρ_{m0}	$(2.52 \pm 0.26) \times 10^{-27}$ kg/m^3	$(2.50 \pm 0.30) \times 10^{-27}$ kg/m^3
ρ_{c0}	$9.3^{+0.3}_{-0.4} \times 10^{-27}$ kg/m^3	$(9.57 \pm 0.67) \times 10^{-27}$ kg/m^3
t_s	$104.5^{+1.9}_{-2.0}$ Gyr	102.3 ± 3.5 Gyr

Let us discuss in more detail the values and the evolution of some quantities of interest. For the combined data WMAP7+BAO+H_0, the potential (10) is fitted as [9]

$$V(t) = \frac{6.47\times10^{27}}{\left(3.30\times10^{18}-t\right)^2} - 2.51\times10^{-371}\left(3.30\times10^{18}-t\right)^{19.54}, \quad (16)$$

while WMAP7 data alone gives

$$V(t) = \frac{6.37\times10^{27}}{\left(3.23\times10^{18}-t\right)^2} - 1.99\times10^{-368}\left(3.23\times10^{18}-t\right)^{19.39}. \quad (17)$$

In particular, the inflection happens at $22.4^{+1.9}_{-2.0}$ Gyr (WMAP7+BAO+ H_0) and 22.0 ± 3.5 Gyr (WMAP7) [9], after these time scales the potential rapidly increases.

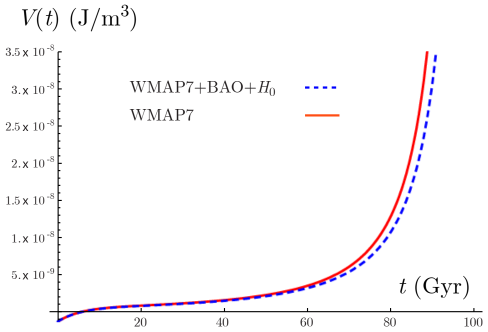

Figure 1. The phantom scalar field potential obtained from observational datasets of WMAP7 and WMAP7 +BAO+H_0.

Let us now consider the equation-of-state parameter of the phantom dark energy. It can be constructed by using the relations (11) and (12) via $w_{DE}(t) = p_{DE}(t) / \rho_{DE}(t)$. Using the relation of redshift z and the function of scale factor therefore,

$$1+z = \left(\frac{t_s - t_0}{t_s - t}\right)^{\beta}. \quad (18)$$

One can write the equation-of-state parameter as a function of redshift as

$$w_{DE} = \frac{-\frac{M_P^2}{\hbar}\left[\frac{3\beta^2-3\beta}{(t_s-t_0)^2}\right](1+z)^{2/\beta} - \frac{\rho_{m0}c^2}{2a_0^3}(1+z)^3}{\frac{M_P^2}{\hbar}\left[\frac{3\beta^2(1+z)^{2/\beta}}{(t_s-t_0)^2} + \frac{3kc^2}{a_0^2}(1+z)^2\right] - \frac{\rho_{m0}c^2}{a_0^3}(1+z)^3}. \quad (19)$$

Using WMAP7+BAO+H_0 observational data in order to fit the evolution of $w_{DE}(z)$ at late time, $t \to t_s$, or equivalently for $z \to -1$ for flat universe, we find

$$w_{DE}(z) \approx \frac{1}{2} - \frac{6.068}{3.676-(1+z)^{3.307}} \quad (20)$$

while for the WMAP7 dataset alone,

$$w_{DE}(z) \approx \frac{1}{2} - \frac{6.328}{3.824-(1+z)^{3.309}}. \quad (21)$$

As seen in the plot (Fig. 2), at $t \to t_s$, $w_{DE} \to -1.153$ for the combined dataset and -1.155 for the WMAP7 dataset alone. These behaviors are of the Big Rip [8].

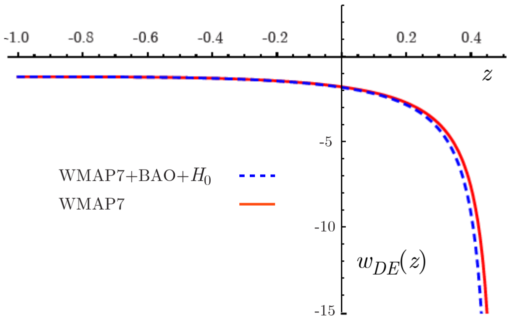

Figure 2. The equation-of-state parameters of phantom dark energy obtained from observational dataset of WMAP7 and WMAP7 +BAO + H_0. For the late time (z → –1) they have finite value.

Conclusions

We use observational data in order to impose constraints on the model parameters. For WMAP7+ BAO+H_0 combined observational dataset, t_s is $104.5^{+1.9}_{-2.0}$ Gyr, and t_s is 102.3 ± 3.5 Gyr for WMAP7 alone [3]. We plot the phantom scalar field potential as function of time which is not shown in [9]. Moreover, we use both datasets to plot $w_{DE}(z)$ in Fig. 2. This is to illustrate the temporal (i.e. redshift) evolution of equation of state parameter. We found that at the Big-Rip time, w_{DE} approaches −1.153 with WMAP7+ BAO+H_0 combined dataset and −1.155 with WMAP7 dataset.

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References

1. V. Sahni, A. Starobinsky, Int. J. Mod. Phys. D **9** (2000) 373;
   P.J. Peebles, B. Ratra, Rev. Mod. Phys. **75** (2003) 559.
2. P. J. Steinhardt, Critical Problems in Physics, Princeton University Press, 1997.
3. J. Sola, H. Stefancic, Phys. Lett. B **624** (2005) 147;
   I. L. Shapiro, J. Sola, Phys. Lett. B **682** (2009) 105.
4. B. Ratra, P.J.E. Peebles, Phys. Rev. D **37** (1988) 3406;
   C. Wetterich, Nucl. Phys. B **302** (1988) 668;
   A. R. Liddle, R. J. Scherrer, Phys. Rev. D **59** (1999) 023509;
   I. Zlatev, L. M. Wang, P.J. Steinhardt, Phys. Rev. Lett. **82** (1999) 896;
   Z. K. Guo, N. Ohta, Y.Z. Zhang, Mod. Phys. Lett. A **22** (2007) 883;
   S. Dutta, E. N. Saridakis, R. J. Scherrer, Phys. Rev. D **79** (2009) 103005;
   E. N. Saridakis, S. V. Sushkov, Phys. Rev. D **81** (2010) 083510.
5. R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003).
6. R. R. Caldwell, Phys. Lett. B **545** (2002) 23;
   S. Nojiri, S.D. Odintsov, Phys. Lett. B **562** (2003) 147;
   P. Singh, M. Sami, N. Dadhich, Phys. Rev. D **68** (2003) 023522;
   J. M. Cline, S. Jeon, G.D. Moore, Phys. Rev. D **70** (2004) 043543;
   V. K. Onemli, R. P. Woodard, Phys. Rev. D **70** (2004) 107301;
   W. Hu, Phys. Rev. D **71** (2005) 047301;
   M. R. Setare, E.N. Saridakis, JCAP **0903** (2009) 002;
   E. N. Saridakis, Nucl. Phys. B **819** (2009) 116;
   S. Dutta, R.J. Scherrer, Phys. Lett. B **676** (2009) 12.
7. S. Nojiri, S.D. Odintsov, Gen. Rel. Grav. **38** (2006) 1285;
   I.P. Neupane, H. Trowland, arXiv:0902.1532 [gr-qc];
   I.P. Neupane, C. Scherer, JCAP **0805** (2008) 009.
8. S. Nojiri, S.D. Odintsov, M. Sasaki, Phys. Rev. D **71** (2005) 123509.
9. C. Kaeonikhom, B. Gumjudpai and Emmanuel N. Saridakis, Phys. Lett. B **695**, 45 (2011).
10. D. Larson, et al., arXiv:1001.4635 [astro-ph.CO].
11. W. J. Percival, et al., Mon. Not. Roy. Astron. Soc. **401** (2010) 2148.
12. A. G. Riess, et al., Astrophys. J. **699** (2009) 539;
    D. Stern, et al., JCAP **1002** (2010) 008.
13. E. Komatsu, et al., arXiv:1001.4538 [astro-ph.CO].
14. S. Nojiri, S. D. Odintsov, S. Tsujikawa, Phys. Rev. D **71** (2005) 063004.