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Possible existence of viable models of bi-gravity with detectable graviton oscillations by gravitational wave detectors

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We discuss graviton oscillations based on the ghost-free bi-gravity theory. We point out that this theory possesses a natural cosmological background solution that is very close to the case of general relativity. Furthermore, the interesting parameter range of the graviton mass, which can be explored by observations of gravitational waves, is not at all excluded by the constraint from solar system tests. Therefore, a graviton oscillation with a possible inverse chirp signal would be an interesting scientific target for KAGRA, Advanced LIGO, Advanced Virgo, and GEO.

Subject Index E02, E03

1. Introduction

A great deal of work has been done on the detection possibility of the modified propagation of gravitational waves due to the finite graviton mass [1–4]. However, adding mass to the graviton was thought to be theoretically problematic due to the so-called Boulware–Deser (BD) ghost [5].

Recently, Hassan and Rosen proposed the first example of ghost-free bi-gravity models [6], based on the fully nonlinear massive gravity theory in which the Boulware–Deser ghost is removed by construction [7–9]. We consider two metrics expressed by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu.$$

We introduce a ghost-free action $S = \int d^4x \mathcal{L}$ with

$$\mathcal{L} = \sqrt{-g}\left[M_{\rm G}^2\left(\frac{R}{2} - m^2 \sum_{n=0}^{4} c_n V_n(Y^\mu_\nu),\right) + L_{\rm m}\right] + \frac{\kappa M_{\rm G}^2}{2}\sqrt{-\tilde{g}}\tilde{R},$$

where $M_{\rm G}^2 = 1/(8\pi G_N)$; G_N is the gravitational constant; $Y^\mu_\nu = \sqrt{g^{\mu\alpha}\tilde{g}_{\alpha\nu}}$; R and $\tilde{R}$ are the Ricci scalars with respect to $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, respectively; g and $\tilde{g}$ are the determinants of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, respectively; κ is a constant that expresses the ratio between the two gravitational constants for $\tilde{g}_{\mu\nu}$ and $g_{\mu\nu}$; c_n ($n = 0, \ldots, 4$) are dimensionless constants; and $L_{\rm m}$ is the Lagrangian of the matter that interacts only with $g_{\mu\nu}$. By expressing the trace of Y^n as $[Y^n] = {\rm tr}(Y^n) = Y^{\alpha_0}_{\alpha_1} Y^{\alpha_1}_{\alpha_2} \cdots Y^{\alpha_{n-1}}_{\alpha_0}$, we can write V_n as

$$V_0 = 1, \quad V_1 = [Y], \quad V_2 = [Y]^2 - [Y^2],$$
$$V_3 = [Y]^3 - 3[Y][Y^2] + 2[Y^3],$$
$$V_4 = [Y]^4 - 6[Y]^2[Y^2] + 8[Y][Y^3] + 3[Y^2]^2 - 6[Y^4].$$

The variation of the action with respect to $g^{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ yields the field equations as follows:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + B_{\mu\nu} = M_{\rm G}^{-2} T_{\mu\nu},$$
$$\kappa \left[\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R} \right] + \tilde{B}_{\mu\nu} = 0,$$

where $T_{\mu\nu}$ is the energy momentum tensor of ordinary matter, whereas $B_{\mu\nu}$ and $\tilde{B}_{\mu\nu}$ come from the variations of the mass term. $B_{\mu\nu}$ and $\tilde{B}_{\mu\nu}$, as well as $T_{\mu\nu}$, satisfy conservation laws, which are explicitly given by

$$\nabla_\mu B^\mu_\nu = 0, \quad \nabla_\mu T^\mu_\nu = 0, \quad \tilde{\nabla}_\mu \tilde{B}^\mu_\nu = 0, \tag{1}$$

where ∇ and $\tilde{\nabla}$ are the covariant derivative operators with respect to $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, respectively.

2. The cosmological background

The background cosmology of this theory has been widely studied in Refs. [10–12], but here our focus is on a particularly healthy branch. We assume that the two metrics can be written as

$$ds^2 = a^2(-dt^2 + d\boldsymbol{x}^2), \quad d\tilde{s}^2 = \tilde{a}^2(-\tilde{c}^2 dt^2 + d\boldsymbol{x}^2),$$

where a, $\tilde{a}$, and $\tilde{c}$ are functions of the time coordinate t. The Friedmann equation for the physical metric reads

$$3H^2 = \frac{\rho_{\rm m} + \rho_V}{M_{\rm G}^2}, \tag{2}$$

where we have introduced the Hubble parameter $H \equiv \dot{a}/a^2$, the matter energy density $\rho_{\rm m}$ including the dark energy, and the energy density due to the mass term

$$\rho_V(\xi) \equiv M_{\rm G}^2 m^2 (c_0 + 3\xi c_1 + 6\xi^2 c_2 + 6\xi^3 c_3)\,,$$

with $\xi \equiv \tilde{a}/a$. The Friedmann equation for the hidden metric reads as follows:

$$\frac{3}{\tilde{c}^2 a^2} \left(\frac{\dot{\tilde{a}}}{\tilde{a}} \right)^2 = \frac{m^2}{\kappa} \left(\frac{c_1}{\xi} + 6c_2 + 18\xi c_3 + 24\xi^2 c_4 \right). \tag{3}$$

Writing down the first equation in Eq. (1), we have

$$3\Gamma(\xi)[\tilde{c} a H - (\dot{\tilde{a}}/\tilde{a})] = 0,$$

where $\Gamma(\xi) \equiv c_1\xi + 4c_2\xi^2 + 6c_3\xi^3$. This equation can be solved by imposing $\Gamma(\xi) = 0$ or $\tilde{c}aH - (\dot{\tilde{a}}/\tilde{a}) = 0$, which implies the existence of two branches. In the following we will discuss the *physical branch*, defined by the latter condition, since the other branch is pathological. (The degrees of freedom of the theory reduce [12]. This will lead to a similar phenomenology to that observed in the original ghost-free single-metric massive gravity, which is characterized by the presence of a scalar non-perturbative ghost [13].) Combining this condition with Eqs. (2) and (3), we obtain an algebraic equation for ξ:

$$\frac{\rho_{\rm m}}{M_{\rm G}^2 m^2} = \left[\frac{c_1}{\kappa\xi} + \left(\frac{6c_2}{\kappa} - c_0\right) + \left(\frac{18c_3}{\kappa} - 3c_1\right)\xi + \left(\frac{24c_4}{\kappa} - 6c_2\right)\xi^2 - 6c_3\xi^3\right]. \tag{4}$$

If $m^2 \gg \rho_{\rm m}/M_{\rm G}^2$, the r.h.s. of Eq. (4) should be very small. Denoting a value of ξ at which the right-hand side vanishes by ξ_c, we focus on a cosmological background solution for which ξ asymptotes to ξ_c for $\rho_{\rm m} \to 0$. As we can absorb the constant part of $\rho_V(\xi)$ into the cosmological constant in $\rho_{\rm m}$, we also assume that $\rho_V(\xi_c) = 0$.

For this type of solution, we can expand ξ around ξ_c at low energies. Keeping only the linear order in $\xi - \xi_c$, Eq. (4) becomes

$$\frac{\xi - \xi_c}{\xi_c} \approx -\frac{\rho_{\rm m}}{3m^2 M_{\rm G}^2 \Gamma_c}\frac{\kappa\xi_c^2}{1+\kappa\xi_c^2},$$

where $\Gamma_c \equiv \Gamma(\xi_c)$. Substituting this relation into Eq. (2), we recover the usual Friedmann equation as

$$3H^2 \approx \tilde{M}_{\rm G}^{-2}\rho_{\rm m}, \tag{5}$$

with the effective gravitational constant given by

$$\tilde{M}_{\rm G}^2 \equiv M_{\rm G}^2(1+\kappa\xi_c^2).$$

This result has been found after linearizing the dynamical equations. However, it can be proven that the same result still holds at order $(\xi - \xi_c)^2$. In particular, we notice here that the effective cosmological Newton constant is time-independent[1].

On using the definition of ξ, the relation $\tilde{c}aH = \dot{\tilde{a}}/\tilde{a}$ implies $\dot{\xi} = (\tilde{c} - 1)aH\xi$. Substituting the differentiation of Eq. (4) into this relation, we obtain

$$\tilde{c} \approx 1 + \frac{\kappa\xi_c^2(\rho_{\rm m} + P_{\rm m})}{\Gamma_c m^2 \tilde{M}_{\rm G}^2}$$

at low energies, where $P_{\rm m}$ is the matter pressure density. The above relation implies that the light cone of the hidden metric automatically gets closer to the physical one as the matter energy density is diluted.

3. Propagation of gravitational waves

We now discuss the propagation of gravitational waves. We introduce tensor-type perturbations as $g_{ij} = a^2(h_+\varepsilon^+_{ij} + h_\times\varepsilon^\times_{ij})$ and $\tilde{g}_{ij} = \tilde{a}^2(\tilde{h}_+\varepsilon^+_{ij} + \tilde{h}_\times\varepsilon^\times_{ij})$, with $\mathrm{tr}(\varepsilon^+\varepsilon^+) = 1 = \mathrm{tr}(\varepsilon^\times\varepsilon^\times)$ and $\mathrm{tr}(\varepsilon^+\varepsilon^\times) = 0$. The gravitational waves propagate at the speed of light for the physical sector, and at the speed $\tilde{c} \approx 1 + O(H^2/m^2)$ for the hidden sector. However, the physical and hidden gravitons, because of the coupling through the mass term, will oscillate from one to the other. Keeping only the

[1] In fact, Eq. (4) reduces to

$$\frac{\rho_{\rm m}}{M_{\rm G}^2 m^2} \approx -\frac{3(\xi - \xi_c)\Gamma_c(\xi_c^2\kappa + 1)}{\xi_c^2\kappa} - \frac{3}{2}\frac{(\xi - \xi_c)^2 C\Gamma_c(\xi_c^2\kappa + 1)}{\xi_c^2\kappa},$$

where C is defined later on in Eq. (4) and is supposed to be large. Therefore, we expect a background similar to general relativity to hold up to $\rho_{\rm m}/M_{\rm G}^2 \lesssim m^2$, when $C\Gamma_c \sim 1$.

leading effect of the deviation of $\tilde{c}$ from unity, and neglecting the cosmic expansion effects, we write the propagation equations as [12]

$$\ddot{h} - \triangle h + m^2\Gamma_c(h - \tilde{h}) = 0, \tag{6}$$

$$\ddot{\tilde{h}} - \tilde{c}^2\triangle\tilde{h} + \frac{m^2\Gamma_c}{\kappa\xi_c^2}(\tilde{h} - h) = 0, \tag{7}$$

where we have omitted the $+/\times$ index. For this set of equations, we write down the dispersion relation, assuming $\tilde{c} - 1 \ll 1$, but the magnitude of

$$x \equiv \frac{2(2\pi f)^2(\tilde{c} - 1)}{\mu^2},$$

is moderate, where we have defined

$$\mu^2 \equiv \lambda_\mu^{-2} = \frac{(1 + \kappa\xi_c^2)\Gamma_c m^2}{\kappa\xi_c^2}.$$

Then, for a given gravitational wave frequency f, two eigen wave numbers are given by

$$k_{1,2}^2 = (2\pi f)^2 - \frac{\mu^2}{2}\left(1 + x \mp \sqrt{1 + 2x\frac{1 - \kappa\xi_c^2}{1 + \kappa\xi_c^2} + x^2}\right),$$

and the corresponding eigenfunctions h_1 and h_2 are related to h and $\tilde{h}$ as

$$h_1 = \cos\theta_g h + \sin\theta_g\sqrt{\kappa}\xi_c\tilde{h},$$
$$h_2 = -\sin\theta_g h + \cos\theta_g\sqrt{\kappa}\xi_c\tilde{h},$$

with the mixing angle

$$\theta_g = \frac{1}{2}\cot^{-1}\left(\frac{1 + \kappa\xi_c^2}{2\sqrt{\kappa}\xi_c}x + \frac{1 - \kappa\xi_c^2}{2\sqrt{\kappa}\xi_c}\right).$$

We find that μ is the graviton mass of the second mode in the Minkowski limit ($x \to 0$).

When we consider the propagation over a distance D, the phase shifts, due to the modified dispersion relation for their respective modes, are given by

$$\delta\Phi_{1,2} = -\frac{\mu D\sqrt{\tilde{c} - 1}}{2\sqrt{2x}}\left(1 + x \mp \sqrt{1 + x^2 + 2x\frac{1 - \kappa\xi^2}{1 + \kappa\xi^2}}\right).$$

Notice that this factor is symmetric under the replacement $x \to 1/x$. In the limit $x \to 0$, the first mode becomes massless. Although this mode also has a non-trivial dispersion relation, its magnitude of modification tends to be suppressed. The factor $\mu D\sqrt{\tilde{c} - 1} = \sqrt{3(1 + \kappa\xi_c^2)\Omega_0}HD$ becomes $O(1)$ only after propagating over a cosmological distance unless $\kappa\xi_c^2$ is extremely large, where Ω_0 is the energy fraction of the dust matter at the present epoch. On the other hand, the remaining factor takes the maximum value $2^{-1/2} - (2 + 2\kappa\xi^2)^{-1/2}$ at $x = 1$, which is also at most $O(1)$. In contrast to the first mode, the phase shift of the second mode can be significantly large when x is small or large. Here we plot $\delta\Phi_{1,2}$ in Fig. 1 for $\kappa\xi_c^2 = 0.2$, 1, and 100.

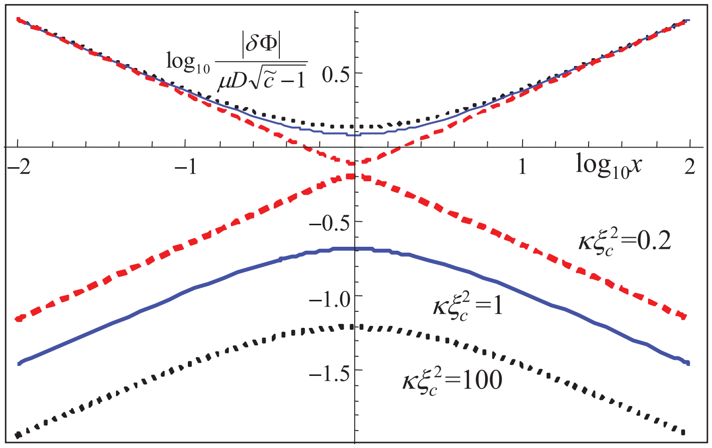

Fig. 1. $|\delta\Phi_{1,2}|$ as a function of x for $\kappa\xi_c^2 = 0.2$ (dotted, black), 1 (blue), and 100 (dashed, red). Thick and thin curves represent $|\delta\Phi_1|$ and $|\delta\Phi_2|$, respectively.

4. Gravitational potential around a star in the Minkowski limit

In the above, we find that, unless $\kappa\xi_c^2$ is extremely large, a relatively small value of λ_μ together with the excitation of the second mode is required for an observable magnitude of the phase shifts due to the non-trivial dispersion relation. Here we show that, in the present bi-gravity models, even with such a small value of λ_μ, we can easily avoid the solar system constraint from the precision measurement of gravity. In the low-energy limit, it would be natural to assume the hierarchy $k^2 \gg \mu^2 \gg H^2$. Since the limit $H \to 0$ is smooth, the H-dependent terms in the action appear as a positive power in H. Since such terms will not give any dominant contribution under the assumption of the above hierarchy, we set $H = 0$ from the beginning here.

Let us now consider static spherical symmetric perturbations for both metrics induced by the non-relativistic matter energy density $\rho_{\rm m}$, which is coupled only to the physical metric. We can write the respective perturbed metrics as

$$ds^2 = -e^{u-v}dt^2 + e^{u+v}(dr^2 + r^2 d\Omega^2),$$
$$d\tilde{s}^2 = -\xi_c^2 e^{\tilde{u}-\tilde{v}}dt^2 + \xi_c^2 e^{\tilde{u}+\tilde{v}}(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2),$$

without loss of generality. Here $\tilde{r}$ is related to r by $\tilde{r} = e^{\mathcal{R}}(r)r$, and $\mathcal{R}(r)$ is another perturbation variable. We have adopted the parametrization such that u vanishes in the case of general relativity. Now we write down the equations of motion and eliminate the variables on the hidden metric side, $\tilde{u}$, $\tilde{v}$, and $\mathcal{R}$. However, doing this is not so straightforward. In order to simplify the manipulation, we truncate the perturbation equations at second order and also neglect higher-order terms in μ appropriately.

When we compute the terms second order in perturbation, we notice that there are terms enhanced by the factor $1/\mu^2$. If we scrutinize these terms, some of them contain the factor

$$C \equiv \left.\frac{d(\log\Gamma)}{d\log\xi}\right|_{\xi=\xi_c}.$$

Our assumption here is that the energy scale of the bi-gravity theory itself is relatively high but the graviton mass μ is suppressed by a certain mechanism. Under this assumption, we pick up only the terms enhanced by the factor C/μ^2 from the second-order terms in the equations of motion. Then, after a little calculation, we obtain

$$(\triangle - \mu^2)u - \frac{3[(\triangle u)^2 - (\partial_i\partial_j u)^2]}{8}\frac{\bar{C}}{\mu^2} = \frac{\kappa\xi_c^2\rho_{\rm m}}{3\tilde{M}_G^2}, \quad (8)$$

$$\triangle v + 3\triangle u - [(\triangle u)^2 - (\partial_i\partial_j u)^2]\frac{\bar{C}}{\mu^2} = -\frac{\rho_{\rm m}}{\tilde{M}_G^2}, \quad (9)$$

where ∂_i is the differentiation with respect to the coordinates $r(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ and $\triangle \equiv \partial_i\partial^i$, the standard 3D Laplacian operator, and we have defined $\bar{C} \equiv C(1+\kappa\xi_c^2)/(\kappa\xi_c^2)$. At this level, the expressions are recast into a form that does not assume spherical symmetry, where there is no ambiguity.

Although we have truncated the equations at second order for simplicity, the higher-order terms will not be suppressed once the second-order terms become important. Nevertheless, such higher-order terms will not change the following discussion as to the order of magnitude estimate on the correction to Newton's law.

First we focus on Eq. (8). Notice that non-vanishing u is the origin of the van Dam–Veltman–Zakharov (vDVZ) discontinuity [14,15]. This equation tells us that the Vainshtein radius [16], within which the second term dominates the first term on the left-hand side in Eq. (8), is given by

$$r_V = O((Cr_g\lambda_\mu^2)^{1/3}),$$

where r_g is the gravitational radius of the star. From the above estimate, we find that the Vainshtein radius can be made arbitrarily large even with a large graviton mass, if C is sufficiently large. Thus, the solar system can be easily contained within the Vainshtein radius, where the second- or even higher-order terms on the left-hand side of Eq. (8) dominate. Then, we have $u \le O\left([\kappa\xi_c^2/(1+\kappa\xi_c^2)]\sqrt{r_g r/C\lambda_\mu^2}\right)$. Even if we require u to be smaller than 10^{-9} in the solar system $r \approx 10^{13}$ cm, λ_μ^2 can be left arbitrarily small depending on the value of C [17].

Once u is suppressed on the scale of the solar system, Eq. (9) tells us that the equation for v does not deviate greatly from the one in the Newtonian case: $\Delta v = -\tilde{M}_G^{-2}\rho_{\rm m}$, and the gravitational constant is not different from the cosmological one. The fact that, inside the Vainshtein radius, the effective Newton's constant exactly matches the time-independent cosmological value indicates that the theory is not affected by time-dependent bounds on $G_{\rm eff}$; see, e.g., Refs. [18,19]. In Eq. (9) the terms second order in u are eliminated with the aid of Eq. (8) to make the effective gravitational source for v manifest. Solving the equation, we find that the correction to v is at most $O(u)$. Notice that the mass term for v in Eq. (9) is absent. Therefore, v does not suffer from the Yukawa-type correction.

Hence, one can conclude that the correction to the Newtonian potential is at most $O\left(\sqrt{\kappa\xi_c^2 r_g r/C\lambda_\mu^2}\right)$. Namely, we can avoid the constraint from the test in the solar system, keeping the graviton mass sufficiently large. In the above we assumed that C is large. On the other hand, in constructing the cosmological background, we have used a linear approximation for the deviation from the conformal equivalence between the two metrics, i.e. $\tilde{c} - 1 \ll 1$. If we further expand the background equations in terms of $\tilde{c} - 1$, we find terms enhanced by the factor C at second order. However, as long as $C\lambda_\mu^2 < H^{-2}$ is satisfied, we can verify that the formulæ for the background metric remain approximately valid. In the early universe, where H is larger, the nonlinear terms necessarily become important. However, the terms second order in $\xi - \xi_c$ do not alter the effective Newton constant for the homogeneous background cosmology.

The equations for $\tilde{u}$ and $\tilde{v}$ can be obtained similarly as

$$\tilde{u} = -\frac{u}{\kappa\xi_c^2}, \quad \tilde{v} = v + \frac{3(1+\kappa\xi_c^2)}{\kappa\xi_c^2}u.$$

Once u is suppressed, i.e. if the Vainshtein mechanism is at work, we find $\tilde{v} \approx v$, which implies that metric perturbations on both sides are equally excited inside the Vainshtein radius.

5. Graviton oscillations and inverse chirp signal

Here we begin with discussing the generation of gravitational waves. We have found that the metric excitations are almost conformal within the Vainshtein radius of a star. If we consider the junction between the near-zone metric perturbation with the far-zone metric described as gravitational waves, both h and $\tilde{h}$ are excited exactly as in the case of general relativity. This implies that both eigenmodes h_1 and h_2 are excited unless $x = 0$. (Recall that $h_2 \propto h - \tilde{h}$ when $x = 0$.)

One may suspect that the linear approximation to the gravitational wave perturbation equations (7) is not valid within the Vainshtein radius. However, the effective energy momentum tensor coming from the variation of the mass term, which gives corrections to the case of general relativity, is greatly enhanced only for the terms purely composed of u (or equivalently $\tilde{u}$), which behave as clouds around localized matter sources. Namely, it just contributes as the source of gravitational waves but does not change the wave propagation. The other corrections are suppressed as long as the amplitude of the deviation of the metric from the case of general relativity remains small.

Next, we analyze the gravitational waveform from inspirals of NS–NS binaries at a distance. For the current bi-gravity model, our detector signal becomes a linear combination of two components, whose relative amplitudes are determined by the mixing angle θ_g. For simplicity, we here neglect the time dependence of θ_g as well as all the cosmological effects. Using the stationary phase approximation and flux conservation, the observed signal is given in Fourier space as

$$h(f) = A(f)e^{i\Phi(f)}\left[B_1 e^{i\delta\Phi_1(f)} + B_2 e^{i\delta\Phi_2(f)}\right], \tag{10}$$

where the amplitude $A(f)$ (after angular average), $B_{1,2}$, and the phase function $\Phi(f, g)$ (truncated at 1.5PN order) are given by

$$\begin{aligned}
A(f) &= \sqrt{\frac{5\pi}{24}}\frac{\mathcal{M}^2}{(8\pi M_{\rm G}^2)^2 D}\, y^{-7/6},\\
B_1 &= \cos\theta_g(\cos\theta_g + \sqrt{\kappa}\xi_c \sin\theta_g),\\
B_2 &= \sin\theta_g(\sin\theta_g - \sqrt{\kappa}\xi_c \cos\theta_g),\\
\Phi(f) &\equiv 2\pi f t_c - \Phi_c - \pi/4 + \frac{3}{128}y^{-5/3}\\
&\quad + \frac{5}{96}\left(\frac{743}{336} + \frac{11}{4}\eta\right)\eta^{-2/5}y^{-1} - \frac{3\pi}{8}\eta^{-3/5}y^{-2/3},
\end{aligned}$$

with $y \equiv \mathcal{M}f/(8\tilde{M}_{\rm G}^2)$, the chirp mass $\mathcal{M} \equiv (m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$, and the reduced mass ratio $\eta = m_1 m_2/(m_1+m_2)^2$. The first and second terms in Eq. (10) show the contributions of h_1 and h_2, respectively. Here we plot $B_{1,2}$ in Fig. 2 for $\kappa\xi_c^2 = 0.2$, 1, and 100.

For $x \ll 1$, the excitation of the second mode h_2 is suppressed. Furthermore, $\delta\Phi_1$ is suppressed in this regime. Therefore, the propagation of gravitational waves is similar to the case of general relativity. For $x \gg 1$, both h_1 and h_2 are equally excited. However, since the gravitational wave

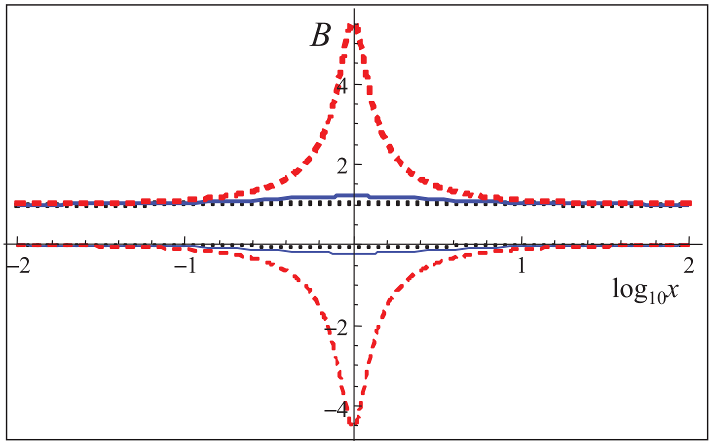

Fig. 2. $B_{1,2}$ as a function of x for $\kappa\xi_c^2 = 0.2$ (dotted, black), 1 (blue), and 100 (dashed, red). Thick and thin curves represent B_1 and B_2, respectively.

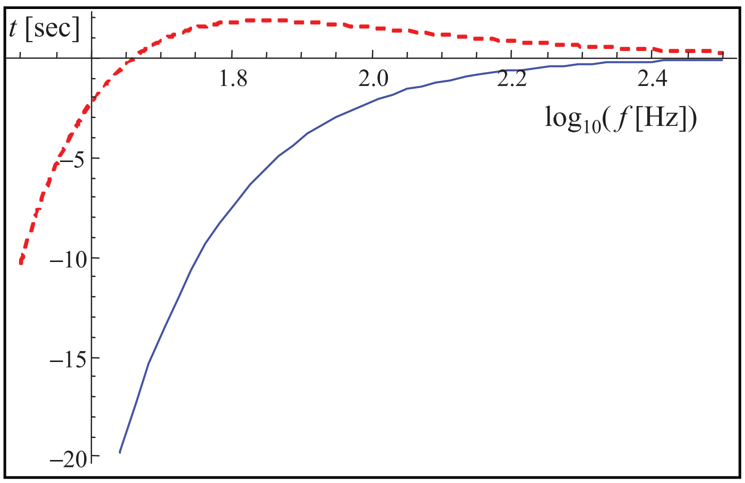

Fig. 3. The arrival time as a function of the frequency f for respective modes of a $1.4\,M_\odot + 1.4\,M_\odot$ binary inspiral with $\kappa\xi_c^2 = 100$, $D = 300$ Mpc, $H = 67.3$ km s^{-1} Mpc^{-1}, $\Omega_0 = 0.315$, and $\lambda_\mu = 0.001$ pc. The blue solid curve is for the first mode, while the dashed red one is for the second mode.

detector can detect the perturbation of the physical metric only, we can observe only h_1. Therefore, the frequencies at which both modes can be observed are limited to $x \approx 1$. This is the meaning of Fig. 2.

When both modes are observable, graviton oscillations due to the interference between the two modes can be detected beyond the distance scale where $\delta\Phi_2 - \delta\Phi_1$ becomes $O(1)$. The difference of the phases $\delta\phi_1 - \delta\phi_2$ is minimum at $x = 1$, as shown in Fig. 1, which is evaluated as $\delta\phi_1 - \delta\phi_2|_{x=1} = \sqrt{6\Omega_0}HD$. Therefore, one may think that the effect is really small as long as $D \lesssim H^{-1}$. However, the average density of the universe is much lower than the average density in galaxies, where binaries are embedded. The adiabatic change of the background density does not change the amplitude of each propagating mode. Therefore, gravitational waves experience a much lower value of x, typically $x \approx 10^{-8}$, during the propagation. Roughly speaking, $\delta\Phi_2 - \delta\Phi_1 \approx \sqrt{3(1+\kappa\xi_c^2)\Omega_0/2x}\,HD$ for $x \ll 1$. Hence, the effect can be greatly enhanced.

Once $d(\delta\Phi_2 - \delta\Phi_1)/df$ becomes sufficiently large, the arrival times of the two modes are different. Then, we may observe two chirp signals. Using the stationary phase approximation, the relation between the arrival time of the wave and the frequency is determined by $t = d(\Phi + \delta\Phi_i)/d(2\pi f)$. For illustrative purposes, in Fig. 3 we show the shifts of the arrival time compared with the case of general relativity for $\kappa\xi_c^2 = 100$, $D = 300\,\mathrm{Mpc}$, $H = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_0 = 0.315$, and $\mu = (0.001\,\mathrm{pc})^{-1}$, for which $x \approx 4 \times 10^{-8}$ at 100 Hz. One can see that the relation between the arrival time and the frequency is reversed for the second mode, i.e. an inverse chirp signal may occur.

Since a large graviton mass can be consistent with the solar system test in our current model, there is a possibility that we may detect graviton oscillations or even an inverse chirp signal with next-generation gravitational wave detectors. In the present model, measurable effects are expected only when $\kappa\xi_c^2$ is large. This requirement may cause some conflict with observations but at first analysis there seems to be no severe constraint. We think that this model gives the first proof of existence of models in which a measurable deviation from general relativity in the gravitational wave propagation can be expected.

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