# Inflationary model in minimally modified gravity theories

Jakkrit Sangtawee[*]
*The Institute for Fundamental Study "The Tah Poe Academia Institute," Naresuan University, Phitsanulok 65000, Thailand*

Khamphee Karwan[†]
*The Institute for Fundamental Study "The Tah Poe Academia Institute," Naresuan University, Phitsanulok 65000, Thailand and Thailand Center of Excellence in Physics, Ministry of Higher Education, Science, Research and Innovation, Bangkok 10400, Thailand*

(Received 25 March 2021; accepted 13 June 2021; published 8 July 2021)

We investigate an inflationary model constructed from minimally modified gravity (MMG) theories. We study MMG theory in the form of $f(\mathbf{H}) \propto \mathbf{H}^{1+p}$ gravity, where $\mathbf{H}$ is the Hamiltonian constraint in Einstein gravity and $p$ is a constant. Inflation is difficult to achieve in this theory of gravity unless an additional scalar field playing the role of the inflaton is introduced in the model. We find that the inflaton with an exponential potential can drive inflation with a graceful exit, different from the case of Einstein gravity. The slow-roll parameter for both the exponential and power-law potentials is inversely proportional to number of $e$-foldings, similar to the case of Einstein gravity. We also find for the scalar perturbation that the curvature perturbation on super-Hubble-radius scales grows rapidly during inflation unless $p \sim 0$. For the tensor modes, the amplitude of the perturbations is constant on large scales up to the lowest order in the slow-roll parameter, and the sound speed of the perturbations can deviate from unity and vary with time depending on the form of $f(\mathbf{H})$.

DOI: 10.1103/PhysRevD.104.023511

## I. INTRODUCTION

Cosmic inflation [1–3] is a standard framework that addresses issues in the hot big bang model and provides a mechanism for the generation of primordial density perturbations. In the standard scenario, inflation can be achieved by introducing extra degrees of freedom in the Universe. In the case of Einstein gravity, the extra degrees of freedom may be in the form of fields minimally coupled to gravity, called the inflaton. Alternatively, the extra degrees of freedom can be parts of degrees of freedom of the gravitational interaction. The extra degrees of freedom of gravity can be obtained by assuming a nonminimal coupling between the extra field and curvature terms in the action. This class of theories is called scalar-tensor theories of gravity [4]. Moreover, the extra degrees of freedom of the gravitational interaction can also be obtained due to nonlinear curvature terms in the action. The simplest example of this class of gravity is $f(R)$ gravity [5].

However, in the cuscuton models [6–8] it has been shown that the acceleration of the Universe can be achieved even though the minimally coupled extra degree of freedom is a nondynamical field. This implies that theories that have two dynamical degrees of freedom can also drive the acceleration of the Universe. Alternative theories to Einstein's theory of gravity, which have two degrees of freedom as the Einstein's theory, have been studied in various contexts [9–17]. Such theories can be constructed by supposing that the temporal diffeomorphism is broken, while the spatial diffeomorphism is still invariant. In general, if the diffeomorphism invariant is broken in this way, the theories can have an extra degree of freedom, similar to scalar-tensor theories of gravity [18]. However, if the Lagrangians of theories are linear functions of the lapse function, the theories can have two tensorial degrees of freedom for gravity under suitable conditions. This class of theories is called minimally modified gravity (MMG) theories [10,13]. Nevertheless, these conditions cannot be satisfied if matter appears in the action. To ensure that this class of theories still has two tensorial degrees of freedom for gravity when matter appears in the theories, we have to impose the gauge-fixing condition [11,19,20]. Cosmology with this class of theories has been investigated in Refs. [8,20]. In Ref. [20], it has been shown that a late-time Universe with this class of gravity theories is more preferred by observational data than the ΛCDM model. Matter coupling in this class of theories has been discussed in Refs. [20–22].

[*]jakkrits60@nu.ac.th
[†]khampheek@nu.ac.th

 © 2021 American Physical Society

Here we investigate inflation due to this class of gravity theories. This work is organized as follows. First, we review MMG theories in the next section. We investigate background inflation in Sec. III. We study cosmological perturbation in Sec. IV, and we conclude in the last section.

## II. MINIMALLY MODIFIED GRAVITY THEORIES

Minimally modified gravity theories are the modified theories propagating two tensorial degrees of freedom, like Einstein's theory of gravity. Generally, most popular modified theories of gravity always generate extra degrees of freedom in the theories. The extra degrees of freedom can be related to the broken diffeomorphism invariant in the construction of the theories. However, we can construct theories that have two tensorial degrees of freedom even if the full diffeomorphism invariant is broken. We can construct MMG theories by supposing that the Hamiltonian of the theories is linear in the lapse function and imposing a suitable constraint. Square root gravity and exponential gravity are the MMG theories that we obtain using this method [10]. However, there is an interesting class of MMG theories, $f(\mathbf{H})$ theory, in which the Lagrangian of the theory is a function of the Hamiltonian constraint $\mathbf{H}$ in Einstein gravity This class of MMG theories can be constructed in another way using the Hamiltonian construction [13].

In order to construct the MMG theories we break the temporal diffeomorphism invariant, which is conveniently represented by the Arnowitt-Deser-Misner (ADM) decomposition. In the ADM formalism, one can write the line element in the form

$$\begin{aligned} ds^2 = &(-\mathcal{N}^2 + \mathcal{N}_i\mathcal{N}^i)dt^2 \\ &+ h_{ij}(\mathcal{N}^i dt + \mathcal{N} dx^i)(\mathcal{N}^j dt + \mathcal{N} dx^j), \end{aligned} \tag{1}$$

where $h_{ij}$, $\mathcal{N}$, and $\mathcal{N}^i$ are the three-dimensional induced metric, the lapse function, and the shift vector, respectively. We are interested in MMG theories in the form of $f(\mathbf{H})$ theory, the action of which can be written in the form

$$\begin{aligned} &S[h_{ij}, \mathcal{N}, \mathcal{N}^i] \\ &= \frac{m_p^2}{2}\int d^4x\mathcal{N}\sqrt{h}\mathcal{L}_G \\ &= \frac{m_p^2}{2}\int d^4x\mathcal{N}\sqrt{h}\left[\frac{2}{f_{,c}(C)}(K_{ij}K^{ij} - K^2) - f(C)\right], \end{aligned} \tag{2}$$

where $m_p = 1/\sqrt{8\pi G}$ is the reduced Planck mass, $h$ is the determinant of the metric $h_{ij}$, and

$$K_{ij} = \frac{1}{2\mathcal{N}}(\dot{h}_{ij} - D_j\mathcal{N}_i - D_i\mathcal{N}_j). \tag{3}$$

Here, $D_i$ is the covariant derivative compatible with the metric $h_{ij}$ and a dot denotes a derivative with respect to time $t$. The variable $C$ can be computed from

$$C = \frac{K_{ij}K^{ij} - K^2}{[f_{,c}(C)]^2} - R. \tag{4}$$

From the above expressions, $f(C)$ is an arbitrary function of $C$, $f_{,c}$ denotes the derivative of $f(C)$ with respect to $C$, and we see that $C$ has the same dimension as the three-dimensional Ricci scalar $R$, i.e., its dimension is mass$^2$. Moreover, the action reduces to the action for Einstein gravity if $f_{,c} = 1$.

To study possible models of inflation from this theory of gravity, we add an extra scalar field to the above action as

$$S[h_{ij}, \mathcal{N}, \mathcal{N}^i, \phi] = \int d^4x\mathcal{N}\sqrt{h}\left[\frac{m_p^2}{2}\mathcal{L}_G + X - V(\phi)\right]. \tag{5}$$

Here we suppose that the field has a standard kinetic term, where $X = -\partial_\mu\phi\partial^\mu\phi/2$ is the kinetic term of the scalar field and $V$ is the potential term. However, the degrees of freedom in the theory increase when the scalar field is simply added to the action. To ensure that the theory still has two tensorial degrees of freedom for gravity, we have to fix the gauge degree of freedom in the theory. Using the choice of gauge presented in Ref. [20], the Hamiltonian of the gauge-fixing term is written in the form

$$H_{gf} = \int d^3x\sqrt{h}\tilde{\lambda}^i\partial_i\left(\frac{\pi}{\sqrt{h}}\right), \tag{6}$$

where $\tilde{\lambda}^i$ is a Lagrange multiplier and $\pi$ is the trace of the momentum conjugate to the induced metric. Imposing this gauge fixing, the action for $f(\mathbf{H})$ becomes

$$\begin{aligned} S = &\frac{1}{2}\int d^4x\mathcal{N}\sqrt{h}\Big\{m_p^2\Big[(C+R)[2-\lambda_0 f_{,c}(C)]f_{,c}(C) - f(C) \\ &+\lambda_0\Big[K^{ij}K_{ij} - K^2 - \frac{2K}{\mathcal{N}}D_k\tilde{\lambda}^k - \frac{3}{2\mathcal{N}^2}(D_k\tilde{\lambda}^k)^2\Big]\Big] \\ &+2X - 2V(\phi)\Big\}, \end{aligned} \tag{7}$$

where $\lambda_0$ is another Lagrange multiplier, and in this case $C$ becomes

$$C = \frac{1}{[f_{,c}(C)]^2}\left[K^{ij}K_{ij} - K^2 - \frac{2K}{\mathcal{N}}D_k\tilde{\lambda}^k - \frac{3}{2\mathcal{N}^2}(D_k\tilde{\lambda}^k)^2\right] - R. \tag{8}$$

The above expression for $C$ can be obtained by varying the action (7) with respect to $\lambda_0$. Varying the action with respect to $C$, $\mathcal{N}$, and $\mathcal{N}^k$ yields, respectively,

$$\lambda_0 = \frac{1}{f_{,c}}, \tag{9}$$

$$0 = f(C) - \frac{2}{m_p^2}\left[X - V - \frac{1}{\mathcal{N}^2}(\dot{\phi} - \mathcal{N}^i \partial_i \phi)^2\right], \tag{10}$$

$$0 = D_i K^{ik} - h^{ik} D_i K - h^{ik} D_i D_m \lambda_0^m - \frac{1}{m_p^2 \mathcal{N}}(\dot{\phi} - \mathcal{N}^i \partial_i \phi)\partial_k \phi. \tag{11}$$

Variation with respect to the scalar field gives us the evolution equation for the scalar field as

$$\frac{\partial}{\partial t}\left[\frac{\sqrt{h}}{\mathcal{N}}(\dot{\phi} - \mathcal{N}^j \partial_j \phi)\right] - \partial_i\left[\frac{\sqrt{h}}{\mathcal{N}}\mathcal{N}^i \dot{\phi} + \mathcal{N}\sqrt{h}\left(h^{ij} - \frac{\mathcal{N}^i \mathcal{N}^j}{\mathcal{N}^2}\right)\partial_j \phi\right] + \mathcal{N}\sqrt{h} V_\phi = 0, \tag{12}$$

where the subscript $_\phi$ denotes a derivative with respect to the scalar field $\phi$.

## III. BACKGROUND EVOLUTION

We now consider the evolution of the spatially flat Friedmann universe for the theory described in the previous section. Due to the homogeneity and isotropy of the Friedmann universe, $\mathcal{N} = 1$, $\mathcal{N}^i = 0$, and therefore

$$ds^2 = -dt^2 + a^2(t)\delta_{ij} dx^i dx^j, \tag{13}$$

where $a(t)$ is a cosmic scale factor. For the Friedmann universe, the constraint in Eq. (10) and the expression for $C$ in Eq. (8) are given by

$$f = -\frac{2}{m_p^2}(X + V) = -\frac{1}{m_p^2}(\dot{\phi}^2 + 2V), \tag{14}$$

$$C f_{,c}^2 = -6H^2, \tag{15}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter. The evolution equation for the scalar field inflaton in the Friedmann universe is

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0. \tag{16}$$

The slow-roll parameter $\epsilon \equiv -\dot{H}/H^2$ can be computed by differentiating Eq. (15) with respect to time to obtain $\dot{C}$, and substituting the resulting $\dot{C}$ into the time derivative of Eq. (14). The result is

$$\epsilon = \frac{\eta f_{,c}}{2}\left(1 + 2\frac{C f_{,cc}}{f_{,c}}\right), \tag{17}$$

where $\eta \equiv \dot{\phi}^2/(H^2 m_p^2)$. The above relation reduces to the usual relation for $\epsilon$ for Einstein gravity when $f_{,c} = 1$. It follows from Eq. (17) that $\epsilon \ll 1$, which is required during inflation, when $\eta \ll 1$ or $|f_{,c} + 2C f_{,cc}| \ll 1$. However, the latter condition is difficult to achieve, so slow-roll inflaton is needed for inflation in this theory. The case $\eta \ll 1$ corresponds to the slow-roll evolution of the inflaton field $\phi$. Under the slow-roll approximation, $|\ddot{\phi}| \ll |H\dot{\phi}|$, Eq. (16) becomes

$$\frac{d\phi}{dN} = -\frac{V_\phi}{3H^2}, \tag{18}$$

where $N \equiv \ln a$ is the number of $e$-foldings.

In order to study the evolution of the background universe, we have to specify the form of $f(C)$. Here, we suppose

$$f(C) = -\Lambda\left(-\frac{C}{\Lambda}\right)^{1+p}, \tag{19}$$

where $\Lambda$ is a constant with dimension of mass$^2$ and $p$ is a constant parameter. We then obtain from Eq. (15) that

$$f = -\Lambda\left[\frac{6H^2}{\Lambda(1+p)^2}\right]^{\frac{1+p}{2p+1}}, \tag{20}$$

$$f_{,c} = (1+p)\left[\frac{6H^2}{\Lambda(1+p)^2}\right]^{\frac{p}{2p+1}}. \tag{21}$$

Hence, we obtain the modified Friedmann equation by substituting the above expression into Eq. (14) as

$$\left[\frac{6H^2}{\Lambda(1+p)^2}\right]^{\frac{1+p}{2p+1}} = \frac{1}{m_p^2 \Lambda}(\dot{\phi}^2 + 2V(\phi)). \tag{22}$$

Using the slow-roll condition, $V \gg \dot{\phi}$, we can write Eq. (22) as

$$H^2 = \frac{2^{\frac{2p+1}{1+p}}(1+p)^2 \Lambda}{6}\left(\frac{V}{m_p^2 \Lambda}\right)^{\frac{2p+1}{1+p}}. \tag{23}$$

Substituting Eq. (23) into Eq. (18), we get

$$\frac{d\phi}{dN} = -\frac{2^{-p/(1+p)}}{\Lambda(1+p)^2}\frac{V_\phi}{\tilde{V}^{\frac{2p+1}{1+p}}}, \tag{24}$$

where $\tilde{V} \equiv V/(m_p^2 \Lambda)$. The above equation can be written in integral form as

$$\int_0^{N_N} dN = 2^{p/(1+p)}\Lambda(1+p)^2 \int_{\phi_e}^{\phi_N} d\phi \frac{\tilde{V}^{\frac{2p+1}{1+p}}}{V_\phi}, \tag{25}$$

where the subscript $_e$ denotes evaluation at the end of inflation, while the subscript $_N$ denotes evaluation at the moment when particular modes of cosmological perturbations generated during inflation cross the horizon. For the form of $f$ given by Eq. (19), the slow-roll parameter $\epsilon$ in the slow-roll approximation is

$$\epsilon = \frac{2^{-\frac{2p+1}{1+p}}(2p+1)}{m_p^2\Lambda^2(1+p)^3}\frac{V_\phi^2}{\tilde{V}^{\frac{3p+2}{1+p}}}. \tag{26}$$

In the slow-roll approximation, we can write $f_{,c}$ in terms of the potential as

$$f_{,c} = (1+p)2^{\frac{p}{1+p}}\tilde{V}^{\frac{p}{1+p}}, \tag{27}$$

$$C = -\Lambda 2^{\frac{1}{1+p}}\tilde{V}^{\frac{1}{1+p}}. \tag{28}$$

To integrate Eq. (25) and compute $\epsilon$ in terms of the number of $e$-foldings, we have to specify the potential $V$ of the scalar field. As illustrative examples, we consider two cases where $V$ takes either an exponential or power-law form.

### A. Exponential potential

We first consider the potential in the form

$$V(\phi) = V_0\Lambda m_p^2 e^{\lambda\tilde{\phi}}, \tag{29}$$

where $\tilde{\phi} \equiv \phi/m_p$, while $V_0$ and $\lambda$ are dimensionless constants. Substituting the above potential into Eq. (25) and performing an integration, we get

$$N_N = \frac{2^{p/(1+p)}(1+p)^3}{\lambda^2 p V_0^{-p/(1+p)}}\left[e^{\lambda\tilde{\phi}_N p/(1+p)} - e^{\lambda\tilde{\phi}_e p/(1+p)}\right]. \tag{30}$$

We can calculate $\phi_e$ by using the slow-roll parameter. Since $\epsilon = 1$ at the end of inflation, we get from Eq. (26) that

$$e^{\lambda\tilde{\phi}_e p/(1+p)} = \frac{\lambda^2(2p+1)}{2^{\frac{2p+1}{p+1}}(1+p)^3 V_0^{p/(1+p)}}. \tag{31}$$

Substituting the above equation into Eq. (30), we get

$$N_N + N_* = \frac{2^{p/(1+p)}(1+p)^3}{\lambda^2 p V_0^{-p/(1+p)}} e^{\lambda\tilde{\phi}_N p/(1+p)}, \tag{32}$$

where

$$N_* \equiv \frac{2p+1}{2p}. \tag{33}$$

Inserting Eq. (32) into Eqs. (26) and (18), we can write $\epsilon$ and $\eta$ in terms of the number of $e$-foldings as

$$\epsilon_N = \frac{N_*}{N_N + N_*}, \qquad \eta_N = \frac{(p+1)^2}{\lambda^2 p^2 (N_N + N_*)^2}. \tag{34}$$

Using Eqs. (27) and (32), we have

$$f_{,c}(N) = f_{,c*}(N_N + N_*) = \frac{\lambda^2(2p+1)}{2(1+p)^2}\frac{1}{\epsilon}, \tag{35}$$

where $f_{,c*}$ is defined as

$$f_{,c*} \equiv \frac{\lambda^2 p}{(1+p)^2}. \tag{36}$$

It follows from the above calculations that the inflaton with an exponential potential has a graceful exit in this theory of gravity. This result is different from that in Einstein's theory of gravity. The moment of the graceful exit is described by Eq. (31).

### B. Power-law potential

In this section, we apply a potential of the form

$$V(\phi) = V_0 m_p^2 \Lambda \tilde{\phi}^q \tag{37}$$

to Eq. (25). After integrating, we obtain

$$N_N = \frac{2^{p/(1+p)}(1+p)^3 V_0^{p/(1+p)}}{q(pq+2p+2)}\left[\tilde{\phi}_N^{\frac{pq+2p+2}{1+p}} - \tilde{\phi}_e^{\frac{pq+2p+2}{1+p}}\right]. \tag{38}$$

Using the condition $\epsilon = 1$ at the end of inflation, we can calculate $\tilde{\phi}_e$ as

$$\tilde{\phi}_e^{\frac{pq+2p+2}{1+p}} = \frac{2^{-\frac{2p+1}{1+p}}q^2(2p+1)}{(1+p)^3 V_0^{p/(1+p)}}. \tag{39}$$

Substituting the above expression into Eq. (38), we get

$$N_N + N_* = \frac{2^{p/(1+p)}(1+p)^3 V_0^{p/(1+p)}}{q(pq+2p+2)}\tilde{\phi}_N^{\frac{pq+2p+2}{1+p}}, \tag{40}$$

where

$$N_* \equiv \frac{q(2p+1)}{2(pq+2p+2)}. \tag{41}$$

Then, we can calculate

$$\epsilon_N = \frac{N_*}{N_N + N_*}, \quad \text{and}$$
$$\eta_N = \left[\frac{q^{2p+2}(1+p)^{2(pq-p-1)}}{2^{2p}V_0^{2p}(pq+2p+2)^{2(pq+p+1)}} \times (N_N + N_*)^{-2(pq+1+p)}\right]^{1/(pq+2p+2)}. \quad (42)$$

Using Eqs. (27) and (40), we have

$$f_{,c}(N) = f_{,c*}(N+N_*)^{\frac{pq}{pq+2p+2}} = f_{,c*}\left(\frac{q(2p+1)}{2(pq+2p+2)\epsilon}\right)^{\frac{pq}{pq+2p+2}}, \quad (43)$$

where

$$f_{,c*} \equiv \left(\frac{4^p V_0^{2p} q^{qp}(pq+2p+2)^{qp}}{(1+p)^{2qp-2p-2}}\right)^{1/(pq+2p+2)}. \quad (44)$$

### C. Numerical results

In this subsection we solve the evolution equations for the background universe numerically and plot the results in Figs. 1–3. The models in our plots are shown in Table I. In Fig. 1, we plot the evolution of $\epsilon$ for both the exponential and power-law potential cases. From this figure, we see that for both forms of potential, the inflationary epoch can take place such that the slow-roll parameter $\epsilon$ increases from a small value during the early stage towards one at the end of inflation. The main different feature of different models comes from the different evolution of $f_{,c}$. As will be seen in the next section, $f_{,c}$ controls the evolution of the curvature perturbation during inflation. The evolutions of $f_{,c}$ are plotted in Figs. 2 and 3. According to Eq. (35), $f_{,c}$ is proportional to $1/\epsilon$ for the exponential potential, so for this form of potential $f_{,c}$ can increase several orders of magnitude throughout the inflationary epoch. This conclusion agrees with the plot in Fig. 2. However, for the power-law potential, Eq. (43) shows that the rate of change of $f_{,c}$ decreases when $q$ and $p$ decrease. When $p \to 0$, the model for the power-law case reduces to Einstein gravity such that $f_{,c} = 1$. Nevertheless, it follows from Eq. (34) that there is no Einstein limit for the case of the exponential potential. The dependence of $f_{,c}$ on the parameters $p$ and $q$ for the case of the power-law potential is shown in Figs. 2

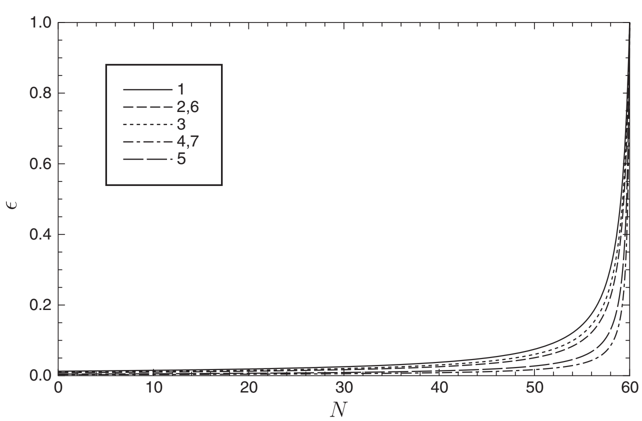


FIG. 1. Plots of the slow-roll parameter $\epsilon$ as a function of the number of $e$-foldings for the models 1–7. In the plots, models 1–7 correspond to lines 1–7, respectively.

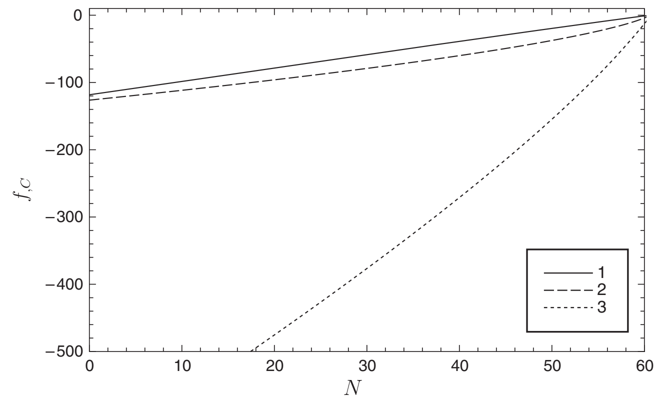


FIG. 2. Plots of $f_{,c}$ as a function of the number of $e$-foldings. In the plots, lines 1–3 represent models 1–3, respectively.

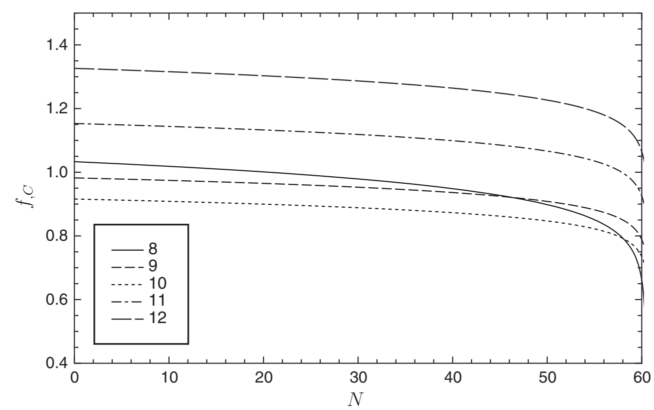


FIG. 3. Plots of $f_{,c}$ as a function of the number of $e$-foldings. In the plots, lines 8–12 represent models 8–12, respectively.

TABLE I. Models used in the numerical calculation.

| No. Model | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Potential | $e^{\phi}$ | $\phi^2$ | $\phi^4$ | $0.7\phi^{1/2}$ | $0.085\phi^{1/2}$ | $0.002\phi^2$ |
| $p$ | 1 | 1 | 1 | 1 | 1/5 | 1/21 |

| No. Model | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|
| Potential | $0.05\phi^1$ | $0.02\phi^1$ | $0.0005\phi^2$ | $0.0001\phi^2$ | $0.02\phi^2$ | $0.5\phi^2$ |
| $p$ | 1/10 | 1/5 | 1/21 | 1/21 | 1/21 | 1/21 |

and 3. From the figures, we see that the variation of $f_{,c}$ decreases when $p$ and $q$ decrease.

## IV. EVOLUTION OF PRIMORDIAL DENSITY PERTURBATIONS

In this section we consider the evolution of primordial perturbations generated in the inflationary model introduced in Sec. II. In the following consideration, we concentrate on scalar and tensor perturbations which provide the model predictions, and we set $m_p = 1$.

### A. Scalar perturbations

To study linear perturbations in this theory, we parametrize perturbations in the lapse function and shift vector as

$$\mathcal{N} = a(\tau)(1+\alpha), \qquad \mathcal{N}_i = a(\tau)\partial_i\chi, \tag{45}$$

where $\alpha$ and $\chi$ are scalar perturbations, $\tau = \int dt/a$ is the conformal time, and the background of the lapse function is the scale factor $a(\tau)$. The induced metric can be decomposed in terms of scalar perturbations as

$$h_{ij} = a(\tau)^2((1+2\xi)\delta_{ij} + 2\partial_i\partial_j E), \tag{46}$$

where $\xi$ and $E$ are the other scalar-perturbation variables. Due to the spatial diffeomorphism invariant, we can set spatial gauge degrees of freedom such that $E = 0$. Here, we use the perturbation variables introduced in Ref. [20]:

$$\frac{\delta\rho_\phi}{\rho_\phi} \equiv \delta_\phi + 3\frac{a'}{a^2}(1+w_\phi)\chi, \qquad \alpha \equiv \Psi - \frac{\chi'}{a},$$
$$\xi \equiv -\Phi - \frac{a'}{a^2}\chi, \qquad v_\phi \equiv -\frac{a}{k^2}\theta_\phi + \chi, \tag{47}$$

where $\Psi$, $\Phi$, $\delta_\phi$, and $\theta_\phi$ describe the two metric perturbations, the density contrast in the inflaton field, and the velocity perturbation in the inflaton field, respectively. In the above and subsequent expressions, the wave number of the perturbation modes in Fourier space is denoted by $k$, and a prime denotes a derivative with respect to conformal time. These perturbation variables can reduce to gauge-invariant combinations in Newtonian gauge in the context of Einstein gravity. Usually, perturbations during inflation are described by the curvature perturbation in the comoving gauge, because the curvature perturbation in this case is constant on large scales when entropy perturbations and isotropic perturbations disappear [23]. For this reason, we study the scalar perturbations using the following perturbation variables:

$$\tilde{\delta}_\phi \equiv \delta_\phi + \frac{3}{k^2}\frac{a'}{a}(1+w_\phi)\theta_\phi, \qquad \zeta \equiv \Phi + \frac{a'}{a}\frac{\theta_\phi}{k^2},$$
$$\bar{\alpha} \equiv \Psi - \frac{a'}{a}\frac{\theta_\phi}{k^2} - \frac{\theta'_\phi}{k^2}. \tag{48}$$

The above perturbation variables can become gauge-invariant combinations in the comoving gauge for the case of Einstein gravity.

In principle, to investigate the primordial density perturbations generated during inflation, we should construct the action for second-order perturbations in which the primordial perturbations are described by canonical variables. However, the action for perturbations for this theory is rather complicated due to the scale dependence of the gauge-fixing term in the action. Thus, instead of constructing this action, we concentrate on the evolution of the curvature perturbation on large scales, whose evolution equation can be obtained from the evolution equations for perturbations presented in Ref. [20]. The necessary equations are

$$0 = \Phi' + \mathcal{H}\Psi + \frac{4k^2\mathcal{H}f_{,CC}}{a^2 f_{,C}}\Phi + \frac{3}{\Gamma_1 k^2}\left[6\Gamma_2\mathcal{H}^2\frac{f_{,CC}}{a^2 f_{,C}} - \frac{1}{2}\Gamma a^2 f_{,C}^2 - \frac{k^2\Gamma_1}{9\Gamma_2}\right]\eta\mathcal{H}^2\theta_\phi, \tag{49}$$

$$0 = -\frac{2}{3}f_{,C}\frac{k^2}{a\mathcal{H}}\Phi - \frac{a}{3\mathcal{H}}\rho_\phi\delta_\phi - \frac{3a\Gamma_2\Gamma}{k^2\Gamma_1}\theta_\phi, \tag{50}$$

$$0 = \Psi + \frac{(4f_{,CC}k^2 - a^2 f_{,C})\Gamma_1}{a^2\Gamma_2}\Phi - \frac{a^2(f_{,C}^2-1)}{\Gamma_2}c_e^2\delta_\phi$$
$$+ \frac{1}{k^2\Gamma_1\Gamma_2 a^2}\left[a^4 f_{,C}(f_{,C}^2-1)(\Gamma' + 2\mathcal{H}\Gamma) + 2\mathcal{H}\frac{f_{,CC}}{f_{,C}}(2f_{,C}^2 k^2\Gamma_1 - 9a^2\Gamma\Gamma_2)\right]\eta\mathcal{H}^2\theta_\phi, \tag{51}$$

where $\mathcal{H} \equiv a'/a$ and

$$\Gamma \equiv \frac{1}{3}(\rho_\phi + p_\phi), \tag{52}$$

$$\Gamma_1 \equiv \Gamma a^2 f_{,C} + \frac{2}{9}k^2, \tag{53}$$

$$\Gamma_2 \equiv \Gamma a^2 + \frac{2}{9}k^2 f_{,C}. \tag{54}$$

Equations (49)–(51) are completed by the conservation equations for the perturbations,

$$0 = \delta'_\phi + (1+w_\phi)\theta_\phi - 3\mathcal{H}(w_\phi - c_e^2)\delta_\phi - 3(1+w_\phi)\Phi', \tag{55}$$

$$0 = \theta'_\phi - k^2\Psi - \frac{c_e^2}{1+w_\phi}k^2\delta_\phi + \mathcal{H}(1-3c_e^2)\theta_\phi. \tag{56}$$

After straightforward calculation, we obtain the evolution equation for $\zeta$ as

$$\zeta'' + K_1\zeta' + K_2\zeta = 0, \tag{57}$$

where the coefficients $K_1$ and $K_2$ are functions of the number of $e$-foldings, wave number $k$, and $\mathcal{H}$. The explicit expressions for these coefficients are presented in the Appendix. The curvature perturbation $\zeta$ is related to the perturbation in the scalar field as

$$\zeta = -\frac{a^2}{2f_{,c}k^2}\rho_\phi\delta_\phi + \left(\mathcal{H} - \frac{3a^2\mathcal{H}\Gamma\Gamma_2}{2k^2 f_{,c}\Gamma_1}\right)\frac{\theta_\phi}{k^2}. \tag{58}$$

In the region where $k^2/\mathcal{H}^2 > \mathcal{O}(\epsilon)$, Eq. (57) can be written up to the lowest order in slow-roll parameters as

$$v'' - \frac{z''}{z}v + c_s^2 k^2 v = 0, \tag{59}$$

where $v = z\zeta$, and in this case

$$\frac{z''}{z} = \frac{1}{4}(8 + 18f_{,c} - 9f_{,c}^2 - 18f_{,c}^3 + 9f_{,c}^4)\mathcal{H}^2,$$
$$\text{and} \quad c_s^2 = 1 + \mathcal{O}(\epsilon). \tag{60}$$

The expression for $z$ is computed from

$$z = a\exp\left\{\int d\tau\left(\frac{3}{2}f_{,c}\mathcal{H}(1-f_{,c})\right)\right\}, \tag{61}$$

where $z$ reduces to $z = a$ in the Einstein limit. For the subhorizon modes, $k \gg \mathcal{H}$, Eq. (59) is satisfied by the solution [24]

$$v = \frac{e^{-ikc_s\tau}}{\sqrt{2c_s k}}. \tag{62}$$

For the superhorizon modes, where $k \ll \mathcal{H}$ but $k^2/\mathcal{H}^2$ is still larger than $\mathcal{O}(\epsilon)$, Eq. (59) is solved by the solution $v \propto z$, where the proportional constant can be computed by matching the solution for the subhorizon limit with that for the superhorizon limit. However, we are not interested in such a calculation here because the condition $k/\mathcal{H} > \mathcal{O}(\epsilon)$ is violated just a few $e$-foldings after the horizon crossing. When this condition is violated, the evolution of $\zeta$ is time dependent, as we will see below. For the case where $k^2/\mathcal{H}^2 < \mathcal{O}(\epsilon)$, the evolution equation for the curvature perturbation up to the dominant contribution from $k/\mathcal{H}$ can be written in the form

$$\frac{d^2\zeta_k}{dN^2} + (3+A)\frac{d\zeta_k}{dN} + (\Xi + B)\zeta_k = 0. \tag{63}$$

Here,

$$\begin{aligned} A \equiv \frac{1}{18\eta(f_{,c}-1)^2}[&\eta^2 f_{,c}^2\{9 + f_{,c}(\Xi - 9)\} + 4\{\epsilon^2(4f_{,c} - 3) - \eta_1(3f_{,c}^5 - 6f_{,c}^4 + 6f_{,c}^2 + \epsilon f_{,c} \\ &- 6f_{,c} - \epsilon + 3) - 6(3f_{,c}^5 - 6f_{,c}^4 + 6f_{,c}^2 - 4f_{,c} + 1) + \epsilon(6f_{,c}^5 - 12f_{,c}^4 + 12f_{,c}^2 - 17f_{,c} \\ &+ \epsilon_1(f_{,c} - 1) + 8)\}\Xi + 2\eta\{5f_{,c}^2\Xi - 2f_{,c}\Xi - \epsilon(18f_{,c}^4 - 36f_{,c}^3 + f_{,c}^2(5\Xi - 27) \\ &- 3f_{,c}(\Xi - 27) - 36) + 27f_{,c}^4 - 27f_{,c}^3 + 9\eta_1(f_{,c} - 1)^3(f_{,c} + 1) - 81f_{,c}^2 + 135f_{,c} - 54\}], \end{aligned} \tag{64}$$

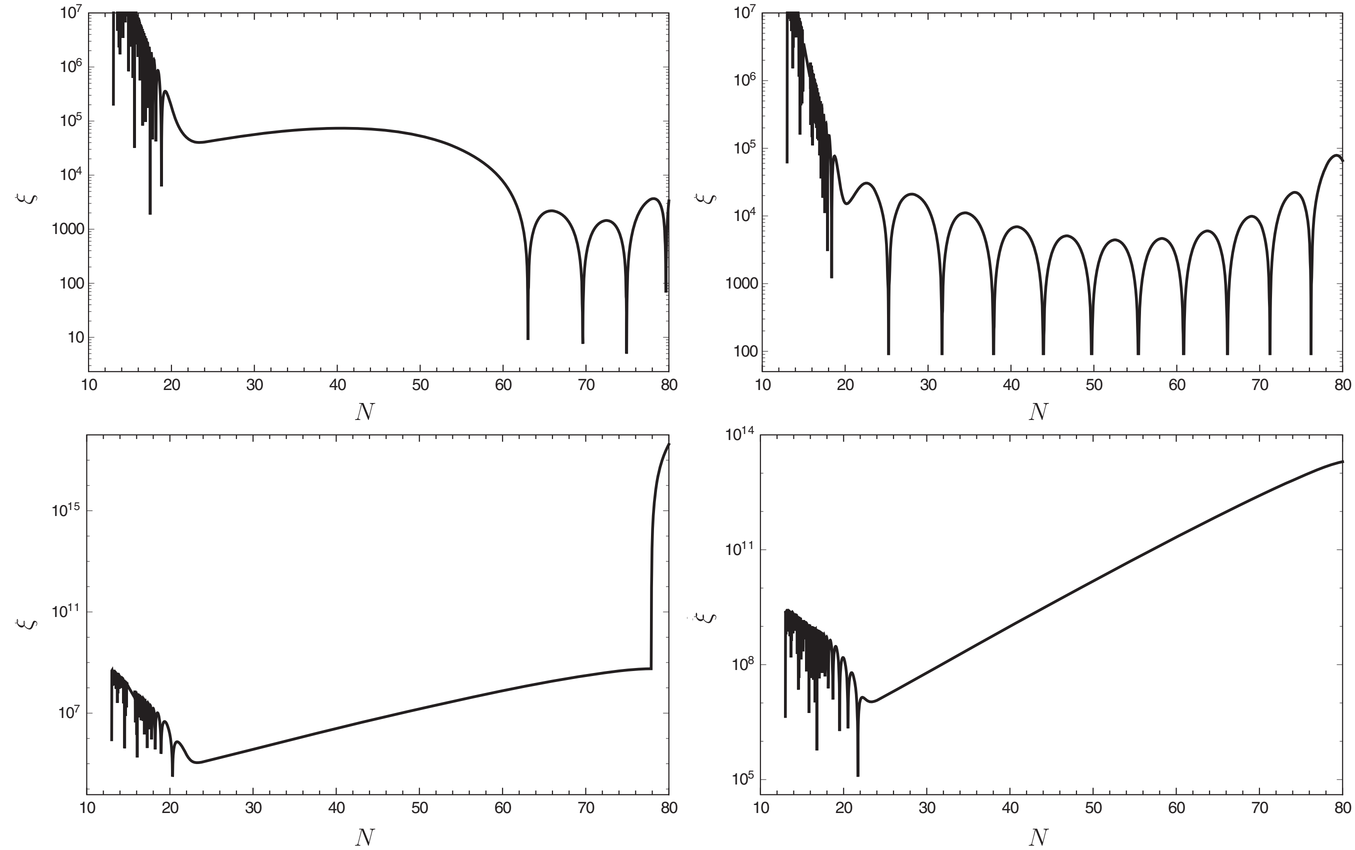


FIG. 4. Plots of $\zeta$ as a function of the number of $e$-foldings. The top left, top right, bottom left, and bottom right panels represent the evolution of $\zeta$ for models 8, 10, 11, and 12, respectively. In all plots, the perturbation crosses the Hubble radius at $N = 20$.

$$\begin{aligned} B \equiv {} & -\frac{1}{54\eta^2(f_{,c}-1)^2} f_{,c}[4\eta\Xi\{\epsilon(-\epsilon_1(f_{,c}-1)(f_{,c}(\Xi-9)+9)-6f_{,c}^6\Xi+12f_{,c}^5\Xi+108f_{,c}^4 \\ & -12f_{,c}^3(\Xi+18)+f_{,c}^2(37\Xi-99)+f_{,c}(387-19\Xi)-180)+\eta_1(f_{,c}-1)(\epsilon(9f_{,c}^3-9f_{,c}^2 \\ & +f_{,c}(\Xi-18)+18)+3(f_{,c}^5\Xi-f_{,c}^4\Xi-f_{,c}^3(\Xi+9)+f_{,c}^2(\Xi+9)-f_{,c}(\Xi-18)-18)) \\ & +\epsilon^2(-18f_{,c}^4+36f_{,c}^3-9f_{,c}^2(\Xi-4)+f_{,c}(6\Xi-99)+45)+3(6f_{,c}^6\Xi-12f_{,c}^5\Xi \\ & -f_{,c}^4(\Xi+54)+2f_{,c}^3(7\Xi+54)+f_{,c}^2(18-13\Xi)+2f_{,c}(\Xi-72)+\Xi+72)\} \\ & +8(\epsilon-3)\Xi^2\{\epsilon(\epsilon_1(f_{,c}-1)+6f_{,c}^5-12f_{,c}^4+12f_{,c}^2-17f_{,c}+8)+\eta_1(-\epsilon f_{,c}+\epsilon-3f_{,c}^5 \\ & +6f_{,c}^4-6f_{,c}^2+6f_{,c}-3)+\epsilon^2(4f_{,c}-3)-6(3f_{,c}^5-6f_{,c}^4+6f_{,c}^2-4f_{,c}+1)\} \\ & -2\eta^2\{9f_{,c}^5\Xi(\eta_1-2\epsilon+6)-18f_{,c}^4\Xi(\eta_1-2\epsilon+6)+f_{,c}^2(9\Xi(2\eta_1-10\epsilon+13) \\ & +(3\epsilon-2)\Xi^2+243)-9f_{,c}(\Xi(\eta_1-4\epsilon+5)+27)+f_{,c}^3((8-6\epsilon)\Xi^2+18(2\epsilon-1)\Xi-81) \\ & +81\}+\eta^3(-f_{,c}^2)\Xi(f_{,c}^2\Xi-9f_{,c}+9)]-\Xi, \end{aligned} \tag{65}$$

where $\Xi \equiv k^2/\mathcal{H}^2 < \mathcal{O}(\epsilon) \ll 1$, $\epsilon_1 \equiv \dot{\epsilon}/(H\epsilon)$, and $\eta_1 \equiv \dot{\eta}/(H\eta)$. Since it is difficult to compute the analytic solution for the above equation due to the time dependence of $f_{,c}$, which is not necessarily slowly varying with time, we will study the important features of the solution for this equation numerically in the next section. However, from the structure of this equation, we expect that the dominant solution for Eq. (63) should be time dependent unless

$f_{,c} = 1$. One can check that for $f_{,c} = 1$, the coefficients $A$ and $B$ vanish, which corresponds to Einstein gravity. This also indicates that $\zeta$ is nearly constant on large scales when $f_{,c}$ is sufficiently close to unity.

### B. Numerical result

To confirm the rough analytic estimation in the previous section, we solve the evolution equation for the curvature perturbation numerically. We start the numerical integration at the time when the physical wavelength of perturbations is well inside the Hubble radius. The initial conditions are chosen according to Eq. (62) by splitting $\zeta$ into real and imaginary parts. We integrate Eq. (57) for the real ($\zeta_{\rm real}$) and imaginary ($\zeta_{\rm imaginary}$) parts of $\zeta$ separately, and plot the absolute value $\zeta = \sqrt{\zeta^2_{\rm real} + \zeta^2_{\rm imaginary}}$ in the following figures. According to the discussion in the previous section, the main features of the $\zeta$ evolution depend on $f_{,c}$. Hence, we consider the evolution of $\zeta$ for models 8, 10, 11, and 12, in which $f_{,c}$ varies by a few percent, and $f_{,c}$ is nearly constant with $f_{,c} \lesssim 1$, $f_{,c} \sim 1$, and $f_{,c} \gtrsim 1$. From the plots in Fig. 4, we see that $\zeta$ can rapidly grow on super-Hubble-radius scales, although $f_{,c}$ changes by only a few percent around one throughout inflation. These results could be the consequence of unknown sources of entropy and anisotropic perturbations. On the other hand, the growth of perturbations on large scales may arise due to the possibility that $\zeta$ is not equivalent to the curvature perturbation in the comoving gauge in the standard cosmological perturbation theory.

### C. Tensor perturbations

To study the tensor modes of perturbations, we write the metric tensor in the form of the background metric and tensor perturbations as

$$h_{ij} = a^2(\delta_{ij} + \gamma_{ij}), \qquad h^{ij} = a^{-2}(\delta^{ij} - \gamma^{ij}), \tag{66}$$

where $\gamma^i_i = 0$ and $\partial_i \gamma^{ij} = 0$. Since the gauge-fixing term does not depend on the tensor perturbation, the tensor perturbation computed from Eqs. (7) and (5) are equivalent. Hence, for convenience we insert the metric from Eq. (66) into Eq. (5) and expand the action up to second order in perturbations. We obtain the second-order action for the tensor perturbation as

$$S^{(2)}_T = \int dt dx^3 a^3 \left( \frac{1}{8f'} \dot{\gamma}_{ij} \dot{\gamma}^{ij} - \frac{f'}{8} \partial_i \gamma^{kl} \partial^i \gamma_{kl} \right), \tag{67}$$

where the divergent term is omitted. The tensor perturbation $\gamma_{ij}$ can be expanded in terms of the polarization tensors as

$$\gamma_{ij} = \int \frac{d^3k}{(2\pi)^3} \sum_{s=\pm} \epsilon^s_{ij}(k) \gamma^s_k(\tau) e^{i\vec{k}\cdot\vec{x}}, \tag{68}$$

where $\epsilon_{ii} = k^i \epsilon_{ij} = 0$ and $\epsilon^s_{ij}(k) \epsilon^{s'}_{ij}(k) = 2\delta_{ss'}$. According to the action (67), each of the mode functions $\gamma^s_k(\tau)$ obeys

$$\gamma^{s\prime\prime}_k + \frac{(a^2/f_{,c})'}{a^2/f_{,c}} \gamma^{s\prime}_k + k^2 c^2_T \gamma^s_k = 0, \tag{69}$$

where $c^2_T = f^2_{,c}$ is the sound speed squared of the tensor perturbations. As in the usual calculation, we define [24]

$$v^s_T \equiv z_T \gamma^s_k, \quad \text{where } z^2_T \equiv \frac{a^2}{4f_{,c}}, \tag{70}$$

so that Eq. (69) becomes

$$v^{s\prime\prime}_T + k^2 c^2_T v^s_T - \frac{z''_T}{z_T} v^s_T = 0. \tag{71}$$

Applying the standard calculation, we have

$$|v^s_T|^2_c = \left. \frac{1}{2c_T k} \right|_c, \tag{72}$$

which implies that the amplitude of tensor perturbations is constant on large scales up to the lowest order in the slow-roll parameter, and we can compute the power spectrum for the tensor perturbations as

$$P^T_k \equiv \frac{k^3}{2\pi^2} (|\gamma^+_k|^2_c + |\gamma^-_k|^2_c) = \frac{2}{\pi^2} \frac{H^2}{f^2_{,c}}, \tag{73}$$

where the tensor perturbations cross the sound horizon at $aH = c_T k$. The spectral index for the tensor perturbations can be computed as

$$n_T \equiv \frac{d \ln P^T_k}{d \ln k} = -2\epsilon - 2\frac{\dot{f}_{,c}}{H f_{,c}}, \tag{74}$$

where $c^2_T \equiv f^2_{,c}$. Using $\dot{c}_s/(H c_s) \simeq \dot{f}_{,c}/(H f_{,c})$ and Eqs. (22), (30), and (38), the tensor spectral index for the exponential and power-law potentials is given by

$$n_T = -\frac{2}{(2p+1)} \frac{N_*}{N_N + N_*}, \tag{75}$$

where $N_*$ for the exponential potential is given by Eq. (33) and $N_*$ for the power-law potential is given by Eq. (41).

## V. CONCLUSIONS

We have studied models of inflation in the MMG theory. We have concentrated on $f(\mathbf{H})$ gravity in the form

$f(C) = -\Lambda(-C/\Lambda)^{1+p}$, where $p$ and $\Lambda$ are constant, while $C$ is given by Eq. (8). It can be checked that this theory reduces to Einstein gravity when $p = 0$. It is difficult for this theory to drive inflation without introducing an inflaton field. We have examined inflationary models in which the potential of the inflaton takes the exponential and power-law forms. We have found that the slow-roll parameter $\epsilon$ is inversely proportional to the number of $e$-foldings, similar to the case of Einstein gravity. The expression for $\epsilon$ in the case of the power-law potential takes the same form as in Einstein gravity when $p = 0$. Nevertheless, there is no Einstein limit for the case of the exponential potential. According to the evolution equation for the perturbations, it can be seen that the evolution equation of perturbations depends on $f_{,c}$. For the case of the exponential potential, $f_{,c}$ is inversely proportional to $\epsilon$, so that $f_{,c}$ can vary by a few orders of magnitude throughout inflation. However, for the case of the power-law potential, $f_{,c}$ becomes nearly constant when $p$ is close to zero. From the numerical integration, we have found that the curvature perturbation on large scales can grow extremely large if $f_{,c}$ can significantly vary with time. The curvature perturbation becomes constant on large scales when $f_{,c} \sim 1$. It could be possible that the curvature perturbation on large scales is not conserved in this model because the curvature perturbation used here is not equivalent to the curvature perturbation in the comoving gauge in standard cosmological perturbation theory. On the other hand, the nonconservation of the curvature perturbation on large scales could be the consequence of entropy and anisotropic perturbations. In general, it is difficult to define a curvature perturbation that is conserved on large scales similar to the curvature perturbation in the comoving gauge in the Einstein theory, because it is not clear whether the entropy and anisotropic perturbations disappear in this $f(\mathbf{H})$ theory. These questions are left for future investigation. For the tensor perturbation, the sound speed of the tensor mode can significantly deviate from unity and vary with time if $p \neq 0$.

## ACKNOWLEDGMENTS

J. S. was supported by Development and Promotion of Science and Technology Talents Project (DPST) scholarship from The Institute for the Promotion of Teaching Science and Technology (IPST) for his MSc study.

## APPENDIX: EXPRESSIONS FOR THE COEFFICIENTS $\alpha$ AND $\beta$

In this Appendix, we present the explicit forms of the coefficients $\alpha$ and $\beta$ in Eq. (57). First, we decompose them as

$$K_1 = \frac{n_1}{d_1}, \qquad K_2 = \frac{n_2}{d_2}, \tag{A1}$$

where the expressions for $n_1$, $n_2$, $d_1$, and $d_2$ are given by

$$\begin{aligned}
n_1 &= a_1 + a_2 k_H^2 + a_3 k_H^4 + a_4 k_H^6 + a_5 k_H^8 + a_6 k_H^8, \\
d_1 &= 4(3\eta + 2f_{,c}k_H^2)(-8(-3+\epsilon)f_{,c}^2 k_H^6 + 9\eta^3(-9 + f_{,c}(9 + k_H^2)) \\
&\quad + 4\eta f_{,c}k_H^4(18 - 6\epsilon - 9f_{,c} + f_{,c}^2(9 + k_H^2)) + 6\eta^2 k_H^2(9 - 3\epsilon - 18f_{,c} + k_H^2 + f_{,c}^2(18 + k_H^2))), \\
n_2 &= b_1 + b_2 k_H^2 + b_3 k_H^4 + b_4 k_H^6 + b_5 k_H^8 + b_6 k_H^{10} + b_7 k_H^{12}, \\
d_2 &= 6(3\eta + 2f_{,c}k_H^2)^2(-8(-3+\epsilon)f_{,c}^2 k_H^6 + 9\eta^3(-9 + f_{,c}(9 + k_H^2)) \\
&\quad + 4\eta f_{,c}k_H^4(18 - 6\epsilon - 9f_{,c} + f_{,c}^2(9 + k_H^2)) + 6\eta^2 k_H^2(9 - 3\epsilon - 18f_{,c} + k_H^2 + f_{,c}^2(18 + k_H^2))).
\end{aligned}$$

Here, $a_i$ are

$$\begin{aligned}
a_1 &= -96f_{,c}^3(-2 - 3f_{,c} + 3f_{,c}^2)\mathcal{H}, \\
a_2 &= -432\eta f_{,c}^3(-3 + 2f_{,c} - 2f_{,c}^2 + f_{,c}^3)\mathcal{H} + 16f_{,c}^3(6\eta_1 + (2\epsilon - \eta f_{,c})(-2 - 3f_{,c} + 3f_{,c}^2))\mathcal{H}, \\
a_3 &= 16\epsilon f_{,c}^3(2\epsilon_1 - 2\eta_1 + 2\epsilon - \eta f_{,c})\mathcal{H} - 648\eta^2 f_{,c}(2 - 5f_{,c} + 4f_{,c}^2 - 4f_{,c}^3 + f_{,c}^4)\mathcal{H} \\
&\quad + 72\eta f_{,c}^2(2\eta_1(2 + f_{,c}^2) + 2\epsilon(2 - 5f_{,c} + 2f_{,c}^2) - \eta(-1 + f_{,c} - 2f_{,c}^2 + f_{,c}^3))\mathcal{H}, \\
a_4 &= 324\eta^3(-4 + 15f_{,c} - 18f_{,c}^2 + 6f_{,c}^3 + 3f_{,c}^4)\mathcal{H} + 24\eta f_{,c}(f_{,c}(6\epsilon_1\epsilon + \eta^2 - \epsilon\eta f_{,c}(3 + 2f_{,c}^2) \\
&\quad + \epsilon^2(2 + 4f_{,c}^2)) + \eta_1(-2\epsilon f_{,c}(2 + f_{,c}^2) + \eta(-1 + f_{,c}^4)))\mathcal{H} - 36\eta^2(\eta(2 - 3f_{,c} + f_{,c}^2 - 6f_{,c}^3 + 9f_{,c}^4) \\
&\quad - 6f_{,c}(\epsilon(6 - 11f_{,c} + 6f_{,c}^2 + 2f_{,c}^3 - 2f_{,c}^4) + \eta_1(1 + f_{,c} + f_{,c}^2 - f_{,c}^3 + f_{,c}^4)))\mathcal{H},
\end{aligned}$$

$$
\begin{aligned}
a_5 &= 36\eta^2 f_{,c}(6\epsilon_1\epsilon - 2\epsilon^2 - 3\epsilon\eta f_{,c} + 8\epsilon^2 f_{,c}^2 + \eta^2 f_{,c}^2 - 2\epsilon\eta f_{,c}^3 + \eta_1(\eta f_{,c}(-1+f_{,c}^2) \\
&\quad - 2\epsilon(1+2f_{,c}^2)))\mathcal{H} - 54\eta^3(\epsilon(-20+66f_{,c} - 60f_{,c}^2 - 24f_{,c}^3 + 24f_{,c}^4) + f_{,c}(\eta(4-3f_{,c}+6f_{,c}^2) \\
&\quad - 6\eta_1(2 - f_{,c} - 2f_{,c}^2 + 2f_{,c}^3)))\mathcal{H}, \\
a_6 &= 54\eta^3(2\epsilon_1\epsilon + \eta_1\eta f_{,c}(-1+f_{,c}^2) + \epsilon^2(-2+4f_{,c}^2) + \epsilon f_{,c}(\eta - 2\eta_1 f_{,c} - 2\eta f_{,c}^2))\mathcal{H},
\end{aligned} \tag{A2}
$$

and $b_i$ are 9

$$
\begin{aligned}
b_1 &= 576 f_{,c}^4\mathcal{H}^2 + 5184 f_{,c}^4(-2+f_{,c}+f_{,c}^2)\mathcal{H}^2, \\
b_2 &= 192 f_{,c}^4(-2\epsilon + \eta f_{,c})\mathcal{H}^2 + 2592\eta f_{,c}^3(-18+18f_{,c} - f_{,c}^2 - 4f_{,c}^3 + 5f_{,c}^4)\mathcal{H}^2 \\
&\quad + 864 f_{,c}^3(\eta(1 - f_{,c} + f_{,c}^3 + 2f_{,c}^4) - 2f_{,c}(\eta_1 - \eta_1 f_{,c} + \epsilon(-6+3f_{,c}+2f_{,c}^2)))\mathcal{H}^2, \\
b_3 &= 16 f_{,c}^4(-2\epsilon+\eta f_{,c})^2\mathcal{H}^2 + 7776\eta^2 f_{,c}^2(-9+18f_{,c} - 11f_{,c}^2 - 5f_{,c}^3 + 7f_{,c}^4)\mathcal{H}^2 \\
&\quad + 144 f_{,c}^2(\eta^2(1+4f_{,c}^2 - f_{,c}^3 + 4f_{,c}^4 + f_{,c}^6) - 4\epsilon f_{,c}^2(\epsilon_1(-1+f_{,c})f_{,c} + \epsilon(6-3f_{,c}-2f_{,c}^2) \\
&\quad + \eta_1(-2+f_{,c}+f_{,c}^2)) + 2\eta f_{,c}(\epsilon(-3+f_{,c} - 3f_{,c}^3 - 3f_{,c}^4) + 2\eta_1(-1+f_{,c}^4)))\mathcal{H}^2 \\
&\quad + 432\eta f_{,c}^2(\eta(-5-4f_{,c} - 3f_{,c}^2 + 10f_{,c}^3 + 8f_{,c}^4 - 6f_{,c}^5 + 6f_{,c}^6) + 6f_{,c}(2\epsilon(9-8f_{,c}+f_{,c}^2 \\
&\quad + 2f_{,c}^3 - 2f_{,c}^4) + \eta_1(-3+5f_{,c} - 2f_{,c}^2 - f_{,c}^3 + f_{,c}^4)))\mathcal{H}^2, \\
b_4 &= 34992\eta^3(-1+f_{,c})^2 f_{,c}(-1+3f_{,c}+2f_{,c}^2)\mathcal{H}^2 + 648\eta^2 f_{,c}(\eta(-5+f_{,c} - 14f_{,c}^2 + 17f_{,c}^3 - 5f_{,c}^4 \\
&\quad - 18f_{,c}^5 + 18f_{,c}^6) + 18f_{,c}(\eta_1(-1+f_{,c})^2(-1+f_{,c}+f_{,c}^2) + 2\epsilon(3-5f_{,c}+3f_{,c}^2+2f_{,c}^3-2f_{,c}^4)))\mathcal{H}^2 \\
&\quad + 72\eta f_{,c}(\eta^2(-1+12f_{,c}^2 - 6f_{,c}^3 + 31f_{,c}^4 - 6f_{,c}^5 + 6f_{,c}^6) + 2\eta f_{,c}(2\epsilon(1+3f_{,c}+2f_{,c}^2 - 18f_{,c}^3 + 3f_{,c}^5 \\
&\quad - 3f_{,c}^6) + 3\eta_1(-3-2f_{,c}^2 + f_{,c}^3 + 4f_{,c}^4 - f_{,c}^5 + f_{,c}^6)) - 12\epsilon f_{,c}^2(\eta_1(-6+5f_{,c}+f_{,c}^2 - f_{,c}^3 + f_{,c}^4) \\
&\quad + 2(2\epsilon_1(-1+f_{,c})f_{,c} + \epsilon(9-7f_{,c}+f_{,c}^3 - f_{,c}^4))))\mathcal{H}^2 - 24f_{,c}^2(16\epsilon^3 f_{,c}^2(-1+f_{,c}^2) \\
&\quad - \eta^2 f_{,c}(1+f_{,c}^2)(\eta + 2\eta f_{,c}^2 + 2\eta_1 f_{,c}(-1+f_{,c}^2)) - 4\epsilon^2 f_{,c}(2\eta_1 f_{,c}(-1+f_{,c}^2) + \eta(1+f_{,c}^2+4f_{,c}^4)) \\
&\quad + 2\epsilon\eta(\eta(1+f_{,c}^2)^2(1+2f_{,c}^2) + 2f_{,c}(\epsilon_1 - \epsilon_1 f_{,c}^2 + 2\eta_1(-1+f_{,c}^4))))\mathcal{H}^2, \\
b_5 &= 8748\eta^4 f_{,c}(9-13f_{,c}+2f_{,c}^2+2f_{,c}^3)\mathcal{H}^2 + 972\eta^3 f_{,c}(6\eta_1(-1+7f_{,c} - 6f_{,c}^2 - 3f_{,c}^3 + 3f_{,c}^4) \\
&\quad - 12\epsilon(-3+12f_{,c} - 11f_{,c}^2 - 6f_{,c}^3 + 6f_{,c}^4) + \eta(2-11f_{,c}+16f_{,c}^2 - 16f_{,c}^3 - 18f_{,c}^4 + 18f_{,c}^5))\mathcal{H}^2 \\
&\quad + 36\eta f_{,c}^2(-4\epsilon^2\eta + 48\epsilon^3 f_{,c} - 8\epsilon\eta^2 f_{,c} + 4\epsilon^2\eta f_{,c}^2 + 5\eta^3 f_{,c}^2 - 48\epsilon^3 f_{,c}^3 - 24\epsilon\eta^2 f_{,c}^3 + 40\epsilon^2\eta f_{,c}^4 \\
&\quad + 5\eta^3 f_{,c}^4 - 8\epsilon\eta^2 f_{,c}^5 + 8\epsilon_1\epsilon\eta(-1+f_{,c}^2) + 2\eta_1(-1+f_{,c}^2)(12\epsilon^2 f_{,c} + \eta^2 f_{,c}(1+2f_{,c}^2) \\
&\quad - 2\epsilon\eta(3+5f_{,c}^2)))\mathcal{H}^2 + 108\eta^2 f_{,c}(72\epsilon^2 f_{,c}(-3+4f_{,c} - 2f_{,c}^2 - f_{,c}^3 + f_{,c}^4) + \eta^2 f_{,c}(-2-3f_{,c} \\
&\quad + 23f_{,c}^2 - 15f_{,c}^3 + 9f_{,c}^4) + 6\eta_1(-1+f_{,c})(-6\epsilon f_{,c}(2-f_{,c}+f_{,c}^3) + \eta(1+f_{,c}+5f_{,c}^2+2f_{,c}^3+3f_{,c}^5)) \\
&\quad + \epsilon(-72\epsilon_1(-1+f_{,c})f_{,c}^2 + \eta(10+6f_{,c}+44f_{,c}^2 - 114f_{,c}^3 + 66f_{,c}^4 + 36f_{,c}^5 - 36f_{,c}^6)))\mathcal{H}^2, \\
b_6 &= 4374\eta^4 f_{,c}(2\eta_1(2-2f_{,c} - f_{,c}^2 + f_{,c}^3) + \eta f_{,c}(2-3f_{,c} - 2f_{,c}^2 + 2f_{,c}^3) - 2\epsilon(7-8f_{,c} - 4f_{,c}^2 + 4f_{,c}^3))\mathcal{H}^2 \\
&\quad + 54\eta^2 f_{,c}(-4\epsilon^2\eta + 48\epsilon^3 f_{,c} - 12\epsilon^2\eta f_{,c}^2 + \eta^3 f_{,c}^2 - 48\epsilon^3 f_{,c}^3 - 16\epsilon\eta^2 f_{,c}^3 + 40\epsilon^2\eta f_{,c}^4 + 5\eta^3 f_{,c}^4 - 8\epsilon\eta^2 f_{,c}^5 \\
&\quad + 4\epsilon_1\epsilon\eta(-1+f_{,c}^2) + 4\eta_1(-1+f_{,c}^2)(6\epsilon^2 f_{,c} + \eta^2 f_{,c}^3 - \epsilon(\eta + 5\eta f_{,c}^2)))\mathcal{H}^2 - 972\eta^3 f_{,c}(-\eta^2(-2+f_{,c})f_{,c}^3 \\
&\quad - 4\epsilon^2(-3+9f_{,c} - 8f_{,c}^2 - 3f_{,c}^3 + 3f_{,c}^4) + \eta_1(-1+f_{,c})(\eta f_{,c}(-2+f_{,c} - 3f_{,c}^3) + 2\epsilon(2-5f_{,c}+3f_{,c}^3)) \\
&\quad + 2\epsilon f_{,c}(4\epsilon_1(-1+f_{,c}) + \eta(-3+7f_{,c} - 7f_{,c}^2 - 3f_{,c}^3 + 3f_{,c}^4)))\mathcal{H}^2, \\
b_7 &= 81\eta^3 f_{,c}(-2\epsilon+\eta f_{,c})^2(\eta f_{,c} + 2\eta_1(-1+f_{,c}^2) - 4\epsilon(-1+f_{,c}^2))\mathcal{H}^2 - 729\eta^4 f_{,c}(-20\epsilon^2 + 4\epsilon_1\epsilon(-1+f_{,c}) \\
&\quad + 24\epsilon^2 f_{,c} + 8\epsilon\eta f_{,c} + 8\epsilon^2 f_{,c}^2 - 12\epsilon\eta f_{,c}^2 + \eta^2 f_{,c}^2 - 8\epsilon^2 f_{,c}^3 - 4\epsilon\eta f_{,c}^3 + 4\epsilon\eta f_{,c}^4 + 2\eta_1(-1+f_{,c})(\eta f_{,c} - \eta f_{,c}^3 \\
&\quad + 2\epsilon(-2+f_{,c}^2)))\mathcal{H}^2.
\end{aligned}
$$

In the above expressions, $k_H = k/\mathcal{H}$, and we use Eqs. (14), (15), and (17) to write $f_{,cc}$ as

$$f_{,cc} = -\frac{1}{12X} f_{,c}^2 \left( \frac{\epsilon - f_{,c}\eta}{2} \right). \tag{A3}$$


[1] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[2] A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[3] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[4] Y. Fujii and K.-i Maeda, *The Scalar-Tensor Theory of Gravitation* (Cambridge University Press, Cambridge, England, 2007).
[5] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Phys. Rep. **513**, 1 (2012).
[6] N. Afshordi, D. J. H. Chung, M. Doran, and G. Geshnizjani, Phys. Rev. D **75**, 123509 (2007).
[7] A. Iyonaga, K. Takahashi, and T. Kobayashi, J. Cosmol. Astropart. Phys. 12 (2018) 002.
[8] A. Iyonaga, K. Takahashi, and T. Kobayashi, J. Cosmol. Astropart. Phys. 07 (2020) 004.
[9] J. Khoury, G. E. J. Miller, and A. J. Tolley, Phys. Rev. D **85**, 084002 (2012).
[10] C. Lin and S. Mukohyama, J. Cosmol. Astropart. Phys. 10 (2017) 033.
[11] K. Aoki, C. Lin, and S. Mukohyama, Phys. Rev. D **98**, 044022 (2018).
[12] K. Aoki, A. De Felice, C. Lin, S. Mukohyama, and M. Oliosi, J. Cosmol. Astropart. Phys. 01 (2019) 017.
[13] S. Mukohyama and K. Noui, J. Cosmol. Astropart. Phys. 07 (2019) 049.
[14] A. De Felice, A. Doll, and S. Mukohyama, J. Cosmol. Astropart. Phys. 09 (2020) 034.
[15] X. Gao and Z.-B. Yao, Phys. Rev. D **101**, 064018 (2020).
[16] Z.-B. Yao, M. Oliosi, X. Gao, and S. Mukohyama, Phys. Rev. D **103**, 024032 (2021).
[17] Y.-M. Hu and X. Gao, Spatially covariant gravity with two degrees of freedom: Perturbative analysis, arXiv:2104.07615v1.
[18] X. Gao, Phys. Rev. D **90**, 081501 (2014).
[19] R. Carballo-Rubio, F. Di Filippo, and S. Liberati, J. Cosmol. Astropart. Phys. 06 (2018) 026.
[20] K. Aoki, A. De Felice, S. Mukohyama, K. Noui, M. Oliosi, and M. C. Pookkillath, Eur. Phys. J. C **80**, 708 (2020).
[21] C. Lin, J. Cosmol. Astropart. Phys. 05 (2019) 037.
[22] C. Lin and Z. Lalak, arXiv:1911.12026.
[23] D. Wands, K. A. Malik, D. H. Lyth, and A. R. Liddle, Phys. Rev. D **62**, 043527 (2000).
[24] J. Garriga and V. F. Mukhanov, Phys. Lett. B **458**, 219 (1999).