Journal of Physics: Conference Series

PAPER • OPEN ACCESS

The greybody factor for the monopole and odd-parity modes of the Proca field in the Schwarzschild black hole spacetimes

To cite this article: Supanat Bunjusuwan and Chun-Hung Chen 2023 *J. Phys.: Conf. Ser.* **2653** 012022

View the article online for updates and enhancements.

You may also like

- Evaporation of large black holes in AdS: greybody factor and decay rate
Jorge V. Rocha
- Black hole evaporation in de Sitter space
Ruth Gregory, Ian G Moss, Naritaka Oshita et al.
- A calibrated dual-wavelength infrared thermometry approach with non-greybody compensation for machining temperature measurements
A Hijazi, S Sachidanandan, R Singh et al.

This content was downloaded from IP address 110.170.245.55 on 29/02/2024 at 07:00

The greybody factor for the monopole and odd-parity modes of the Proca field in the Schwarzschild black hole spacetimes

Supanat Bunjusuwan* **and Chun-Hung Chen**

The Institute for Fundamental Study, Naresuan University, Phitsanulok 65000, Thailand

E-mail: Supanatb64@nu.ac.th*

Abstract. The greybody factor is one of the famous results of the black hole perturbation theory, which describes the transmission probability of a particle radiated by a black hole into spatial infinity. In this work, we separated the angular parts of the equations of motion and derived the radial equations for the Proca field in the Schwarzschild black hole spacetime. The radial equations for the monopole and odd-parity modes are fully decoupled in the Schrödinger-like form. We study the greybody factor by determining the rigorous bound.

1. Introduction

The greybody factor is the transmission probability of a particle emitting from the black hole to spatial infinity, the behavior of the particle can be understood as Hawking radiation. The Hawking radiation involves considering the quantum effects of particle creation and annihilation near the event horizon of a black hole, which was first performed by Stephen Hawking in 1975 [1]. Since we cannot predict which spin of particles would be the candidates for Hawking radiation, the greybody factor of various spin particles in various black hole spacetimes has become an important research topic. Therefore in the current proceeding, we are interested in the greybody factor for the Proce field (massive vector field) in the Schwarzschild black hole spacetime.

This work is organized as follows. In Section 2, we present our analysis for the Proca field in the spherically symmetric black hole spacetimes and introduce the rigorous bound method [2] for studying the greybody factor. We show our results and conclusion in Section 3.

2. Analysis and method

2.1. Equations of motion for Proca field in Spherically symmetric black hole spacetime

Alexandru Proca modified the electromagnetic Lagrangian density by adding the "mass" term as

$$\mathcal{L}_{Proca} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{8\pi}A_\mu A^\mu, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the anti-symmetric field strength tensor, A_μ is the vector potential and m is mass term of a spin-1 particle. The equation of motion is given by

$$\partial_\mu F^{\mu\nu} = m^2 A^\nu. \tag{2}$$

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

We next consider the equations of motion in the spherically symmetric black hole spacetimes by replacing the partial derivatives with the covariant derivatives. The line element is given by

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{3}$$

where $f(r)$ is a function of r which depends on the black hole spacetimes, then the equations of motion for Proca field can be written as

$$\nabla_\mu \nabla^\mu A^\nu - \nabla_\mu \nabla^\nu A^\mu - m^2 A^\nu = 0. \tag{4}$$

To further analyze equation (4), we take the Lorenz condition $\nabla_\mu A^\mu = 0$. The second term of the left-hand side can be simplified as

$$\nabla_\mu \nabla^\nu A^\mu \;=\; g^{\nu\rho} R^\sigma{}_{\mu\sigma\rho} A^\mu = R_\mu{}^\nu A^\mu, \tag{5}$$

which related to the Ricci tensor. Then the equations of motion can be written as

$$\nabla_\mu \nabla^\mu A^\nu - m^2 A^\nu - R_\mu{}^\nu A^\mu = 0. \tag{6}$$

It is worth noting that the Lorenz condition comes naturally for the Proca field in both flat and curved spacetimes, one may check this condition by taking the contraction derivative on both sides of equation (2). For the flat spacetime case, the anti-symmetry of $F^{\mu\nu}$ and the commutative relation of partial derivatives will lead to $\partial_\mu A^\mu = 0$. For the curved spacetime cases, we have $\nabla_\nu \nabla_\mu F^{\mu\nu} \sim R_{\mu\nu} F^{\mu\nu} = 0$ since $R_{\mu\nu} = R_{(\mu\nu)}$ and $F^{\mu\nu} = F^{[\mu\nu]}$, as such the Lorenz condition must hold.

2.2. Separation variables and radial equations

To obtain the radial equations, we separate the angular parts of equation (6) by introducing the ansatz of the vector potential as in [3]

$$A_\mu(t,r,\theta,\phi) = \frac{1}{r}\sum_{i=1}^{4}\sum_{lm} c_i u^{lm}_{(i)}(t,r) Z^{(i)lm}_\mu(\theta,\phi), \tag{7}$$

where $c_1 = c_2 = 1$, $c_3 = c_4 = [l(l+1)]^{-1/2}$ and $Z^{(i)lm}_\mu(\theta,\phi)$ is the basis of four vector spherical harmonics. This expression represents the vector potential that can be decomposed into a series of angular basis functions, the coefficients c_i determining the amplitude of each mode, and the functions $u^{lm}_{(i)}(t,r)$ and $Z^{(i)lm}_\mu(\theta,\phi)$ representing time-radial and angular dependencies, respectively. We further defined the angular dependence as follows

$$Z^{(1)lm}_\mu \;=\; [1,0,0,0]Y^{lm}(\theta,\phi), \tag{8}$$

$$Z^{(2)lm}_\mu \;=\; [0,f^{-1},0,0]Y^{lm}(\theta,\phi), \tag{9}$$

$$Z^{(3)lm}_\mu \;=\; \frac{r}{\sqrt{l(l+1)}}[0,0,\partial_\theta,\partial_\phi]Y^{lm}(\theta,\phi), \tag{10}$$

$$Z^{(4)lm}_\mu \;=\; \frac{r}{\sqrt{l(l+1)}}[0,0,\frac{1}{\sin\theta}\partial_\phi,-\sin\theta\partial_\theta]Y^{lm}(\theta,\phi). \tag{11}$$

Substituting the vector potential from equations (7)-(11) to equation (6). The radial equation for general static spherically symmetric black holes can be written as

$$\hat{\mathcal{D}}^2 u_1 + f'(\dot{u}_2 - \partial_{r_*} u_1) = 0, \tag{12}$$

$$\hat{\mathcal{D}}^2 u_2 + f'\left(\dot{u}_1 - \partial_{r_*} u_2\right) - \frac{2f^2}{r^2}\left(u_2 - u_3\right) = 0, \tag{13}$$

$$\hat{\mathcal{D}}^2 u_3 + \frac{2f}{r^2} l(l+1) u_2 = 0, \tag{14}$$

$$\hat{\mathcal{D}}^2 u_4 = 0, \tag{15}$$

where $\hat{\mathcal{D}}^2 = -\partial_t^2 + \partial_{r_*}^2 - f\left[l(l+1)/r^2 + m^2\right]$, $\dot{u_i} = \partial_t u_i$, $f' = \partial_r f$ and r_* is the tortoise coordinate defined as $dr_* = f^{-1}dr$. The first three radial equations (12)-(14) are coupled, which describe the even-parity sector. Equation (15) takes the same form as the Regge-Wheeler equation, which is completely decoupled from the first three equations and describes the odd-parity sector. One may notice that the Ricci scalar in equation (6) is eliminated in equations (12)-(15), which is always true for both Ricci-flat and non-Ricci-flat cases. As a remark that for deriving equations (14) and (15), it is more convenient to take the linear combination of equation (6) as $(\partial_\theta + cot\theta) \times (\nu = \theta) + \partial_\phi\,(\nu = \phi) = 0$ and $(2\cos\theta + \sin\theta\partial_\theta) \times (\nu = \phi) - \sin^{-1}\theta\partial_\phi\,(\nu = \theta) = 0$. With further consideration of the Lorenz condition, we obtain one more coupled equation of u_1, u_2, and u_3,

$$-\dot{u_1} + \partial_{r_*}u_2 + \frac{f}{r}(u_2 - u_3) = 0, \tag{16}$$

which allow us to simplify equation.(13) as

$$\hat{\mathcal{D}}u_2 + \frac{f}{r}(f' - \frac{2f}{r})(u_2 - u_3) = 0. \tag{17}$$

However, equations (14) and (17) are still coupled equations. We may pay the price of increasing the order of partial differential equations to decouple the pair of equations as mentioned in [4]. A complete discussion of the even-parity modes beyond the scope of the current proceeding, which we left to the future direction. Therefore in the later sections, we mainly focus on two special cases. Firstly for the odd-parity modes, we took $u_4(t,r) = u_{odd}(r)e^{-i\omega t}$ in the equation (15), where ω is the energy parameter for the massive spin-1 particles, and the radial equation may be obtained as

$$\left[\partial_{r_*}^2 + \omega^2 - f(r)\left(\frac{l(l+1)}{r^2} + m^2\right)\right]u_{odd}(r) = 0. \tag{18}$$

Secondary for the monopole modes we took $u_3(t,r) = u_4(t,r) = 0$, $l = 0$ and $u_2(t,r) = u_m(r)e^{-i\omega t}$, equation (17) can be simplified as a decoupled equation as [5]

$$\left[\partial_{r_*}^2 + \omega^2 - f(r)\left(\frac{2\,(r-3)}{r^3} + m^2\right)\right]u_m(r) = 0. \tag{19}$$

Note that for the third term of the left-hand side of equations (18) and (19) were understood as the effective potentials for each case, which we express them with V_{eff} in the later discussions.

2.3. The rigorous bounds on the greybody factors

We studied the greybody factor by the analysis of rigorous bound [2,6–10]. The general form of the rigorous bound on the greybody factor is given by

$$T \geq \mathrm{sech}^2 \int_{-\infty}^{\infty} \frac{\sqrt{[\partial_{r_*}h(r_*)]^2 + [\omega^2 - V_{eff}(r_*) - h^2(r_*)]^2}}{2h(r_*)}dr_*, \tag{20}$$

where $h(r_*)$ is a positive function that must satisfy the condition $h(-\infty) = h(+\infty) = \omega$. Two special cases that yield beneficial practical results are $h(r_*) = \omega$ and $h(r_*) = \sqrt{\omega^2 - V_{eff}}$ [2]. However, in this work, the first choice of $h = \omega$ may not be sufficient for evaluating the bounds because of the non-vanishing of the effective potential in the asymptotic infinity, $V_{eff}(\infty) \sim m^2$, then $T \to 0$. The second choice of $h(r_*) = \sqrt{\omega^2 - V_{eff}}$ presented a sufficient bound in the region of $\omega^2 \geq V_{eff}|_{peak}$, but lost the information of $V_{eff}|_{peak} > \omega^2 > m^2$. Therefore, we select $h(r_*) = \sqrt{\omega^2 - f(r)m^2}$ in this work and the efficient area shall be $\omega^2 > m^2$.

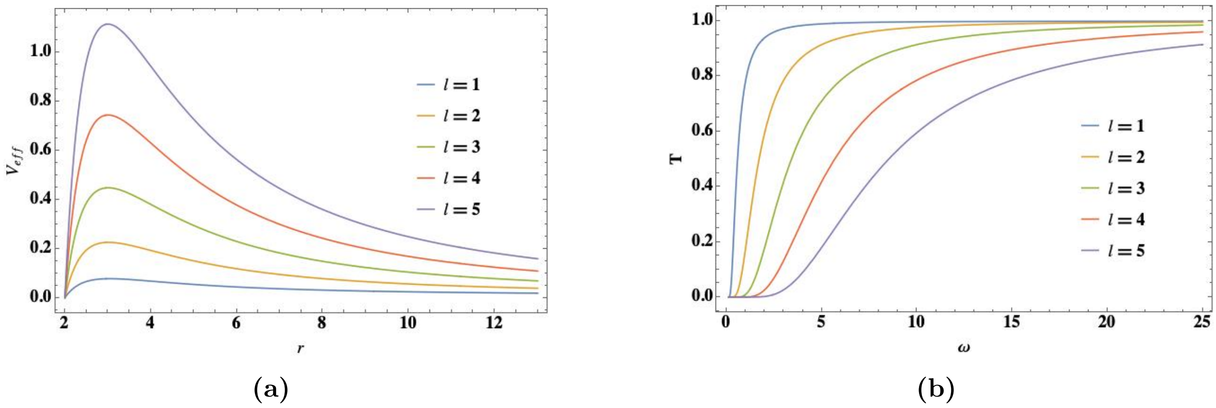


Figure 1. (a). The effective potential for the odd-parity modes with $m = 0.1$ and variation l. (b). The rigorous bound of the greybody factor for the odd-parity modes with $m = 0.1$ and variation l.

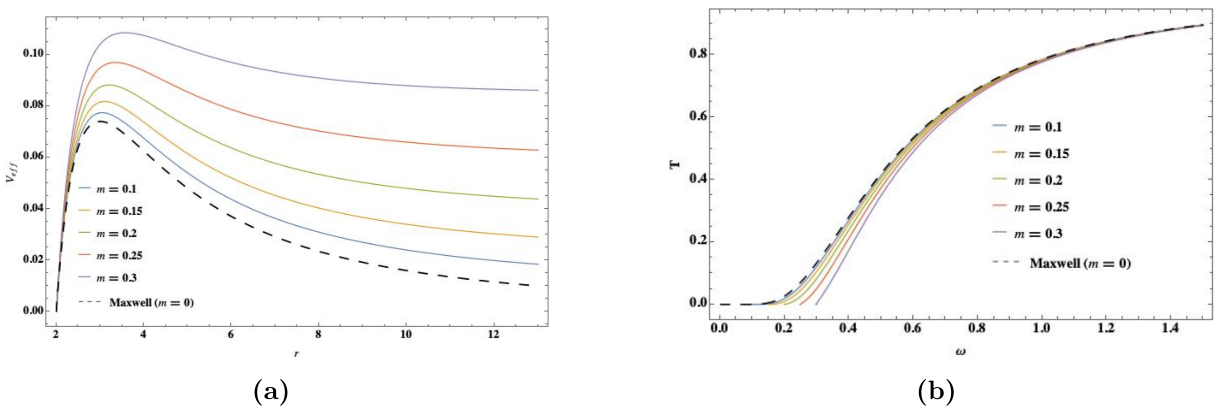


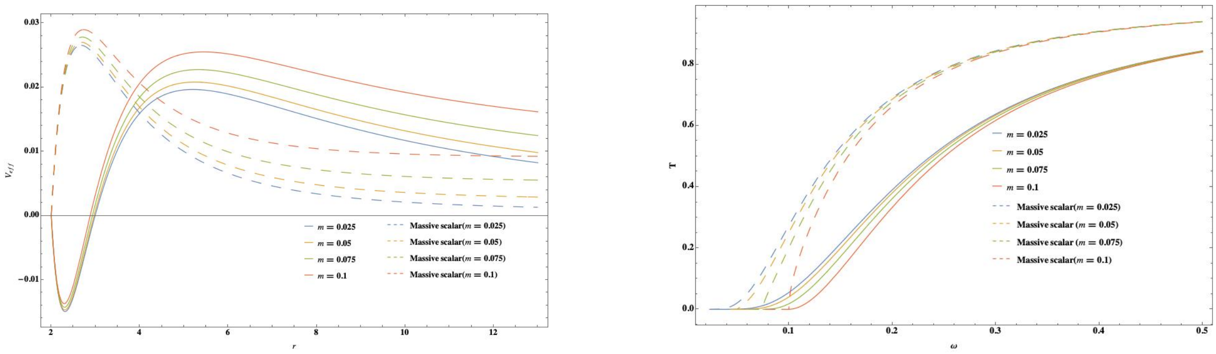
Figure 2. (a). The effective potential for the odd-parity modes with $l = 1$ and variation m. (b). The rigorous bound of the greybody factor for the odd-parity modes with $l = 1$ and variation m.

3. Results and conclusion

3.1. Results

For presenting the corresponding results in the Schwarzschild background, we took $f(r) = 1-\frac{2M}{r}$ and $M = 1$ for the following calculations. The effective potential and the corresponding greybody factor for the odd-parity modes may be obtained by taking equation (18) in equation (20), the results are presented in figure 1 and figure 2.

In figure 1a, we fixed the mass parameter $m = 0.1$ of the Proca field and took the angular parameter from $l = 1$ to $l = 5$, the maximum value of the effective potentials increase when l increase. The corresponding greybody factors shift from lower energy to higher energy as presented in figure 1b. The physical meaning for this shifting indicates that, with the maximum value of the effective potential increase, a particle with a given energy includes less transmission probability. In figure 2a, we fixed the angular parameter $l = 1$, the effective potential increases when the mass parameter increases. Note that the effective potential for the Maxwell field is presented on the same figure with a black dashed line. The effective potential for the Maxwell field shall be the massless limit for the Proca effective potential, and the exact formula was given in [2]. The corresponding greybody factors shift from lower energy to higher energy as presented



(a) **(b)**

Figure 3. (a). The effective potential for the monopole modes with the variation of m. (b). The rigorous bound of the greybody factor for the monopole modes with the variation of m.

in figure 2b. For the monopole modes, the results may be obtained by taking equation (19) in equation (20). The effective potential with the variation of m and the associated greybody factors are presented in figure 3a and figure 3b. The comparisons with the massive scalar field are presented with dashed lines as well, where the effective potentials were given in [10]. The spectrum of effective potentials included many differences, however, one may observe that for the greybody factor, with the Proca mass getting smaller, the spectrum approach to the massive scalar cases, which is consistent with the discussion in [3].

3.2. Conclusions
We may conclude our results as follows. In our current studies, the greybody factors are influenced by the maximum value of the effective potentials within a specific given field. When the peak of the effective potential is smaller, a particle with fixed energy includes a higher transmission probability, which means the greybody factor increases.

In summary, we derived four radial equations for the Proca field in the spherically symmetric black hole spacetimes. The first three equations were coupled, which described the even-parity sector, and the monopole mode shall be a special case simplified from them. The last equation was fully decoupled from others and described the odd-parity sector. We obtained the rigorous bound of the greybody factor for the odd-parity and monopole modes, and the general studies for the even-parity modes shall keep as the future direction.

Acknowledgments
This work was supported by the Naresuan University research grant (R2566C048).

References
[1] Hawking S W 1975 *Commun. Math. Phys.* **43** 199–220
[2] Boonserm P and Visser M 2008 *Phys. Rev.* D **78** 101502 (*Preprint* 0806.2209)
[3] Rosa J G and Dolan S R 2012 *Phys. Rev.* D **85** 044043 (*Preprint* 1110.4494)
[4] Fernandes T V, Hilditch D, Lemos J P S and Cardoso V 2022 *Phys. Rev.* D **105** 044017 (*Preprint* 2112.03282)
[5] Konoplya R A 2006 *Phys. Rev.* D **73** 024009 (*Preprint* gr-qc/0509026)
[6] Visser M 1999 *Phys. Rev.* A **59** 427–38 (*Preprint* quant-ph/9901030)
[7] Ngampitipan T and Boonserm P 2013 *J. Phys.: Conf. Ser.* **435** 012027 (*Preprint* 1301.7527)
[8] Boonserm P, Chen C H, Ngampitipan T and Wongjun P 2021 *Phys. Rev.* D **104** 084054 (*Preprint* 2105.07589)
[9] Boonserm P, Ngampitipan T and Wongjun P 2018 *Eur. Phys. J.* C **78** 492 (*Preprint* 1705.03278)
[10] Boonserm P, Phalungsongsathit S, Sansuk K and Wongjun P 2023 *Eur. Phys. J.* C **83** 657 (*Preprint* 2305.00459)