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Towards consistent extension of quasidilaton massive gravity

Antonio De Felice [a,b,*], Shinji Mukohyama [c]

[a] ThEP's CRL, NEP, The Institute for Fundamental Study, Naresuan University, Phitsanulok 65000, Thailand
[b] Thailand Center of Excellence in Physics, Ministry of Education, Bangkok 10400, Thailand
[c] Kavli Institute for the Physics and Mathematics of the Universe (WPI), Todai Institutes for Advanced Study, University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8583, Japan

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ABSTRACT

We present the first example of a unitary theory of Lorentz-invariant massive gravity, with all degrees of freedom propagating on a strictly homogeneous and isotropic, self-accelerating de Sitter background. The theory is a simple extension of the quasidilaton theory, respecting the symmetry of the original theory but allowing for a new type of coupling between the massive graviton and the quasidilaton scalar.

Introduction

Since the pioneering work of Fierz and Pauli in 1939 [1], it has been a long-standing question in theoretical physics whether a graviton can have a non-vanishing mass. Recently a fully nonlinear theory of massive gravity was found by de Rham, Gabadadze and Tolley (dRGT) [2,3] and has provided a positive answer to this fundamental question.

The study of massive gravity is motivated not only by the above mentioned theoretical question but also by the observed acceleration of cosmic expansion, one of the greatest mysteries in modern cosmology. There is a possibility that a finite graviton mass might be the source of accelerated expansion of the universe. In this respect, it is important to establish a theoretically consistent and observationally viable cosmological scenario in massive gravity. However, it was recently shown that all homogeneous and isotropic cosmological solutions in the dRGT theory are unstable [4].

This no-go result suggests two possible directions: (i) to break either homogeneity [5] or isotropy [6,7] of the cosmological background, or (ii) to extend the theory [8,9] (see also [10] for a non-self-accelerating bi-gravity extension). The purpose of the present Letter is to explore the second possibility and to establish a stable self-accelerating homogeneous and isotropic cosmological solution. The hope is that this theory will provide a theoretically acceptable setup to start studying the phenomenology of this theory and its potential imprints in the experimental data.

The model

The *quasidilaton*, denoted hereafter as σ, is an additional scalar field in the context of an extended dRGT massive gravity [8], introduced to realize a new global symmetry

$$\sigma \to \sigma + \sigma_0, \qquad \phi^a \to e^{-\sigma_0/M_{\rm Pl}}\phi^a, \tag{1}$$

where ϕ^a $(a = 0, \ldots, 3)$ are four scalar fields called *Stückelberg fields* and σ_0 is an arbitrary constant. The theory also enjoys the Poincare symmetry in the space of Stückelberg fields

$$\phi^a \to \phi^a + c^a, \qquad \phi^a \to \Lambda^a_b \phi^b, \tag{2}$$

so that ϕ^a enter the action only through the so-called Minkowski *fiducial metric* defined as

$$f_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b. \tag{3}$$

We extend the quasidilaton theory by adding a new type of coupling between the massive graviton and the quasidilaton. This is achieved by replacing $f_{\mu\nu}$ in the action of the original theory with

$$\tilde{f}_{\mu\nu} \equiv f_{\mu\nu} - \frac{\alpha_\sigma}{M_{\rm Pl}^2 m_g^2} e^{-2\sigma/M_{\rm Pl}}\partial_\mu\sigma\partial_\nu\sigma, \tag{4}$$

where α_σ is a new coupling constant[1] and m_g is the graviton mass introduced in (10) below. Note that the factor $e^{-2\sigma/M_{\rm Pl}}$ in the second term was introduced so that $f_{\mu\nu}$ and $\tilde{f}_{\mu\nu}$ share the same

[1] We expect $\alpha_\sigma = O(1)$. In other words, the (technically natural) suppression scale of the new term is $\Lambda_2 \sim (M_{\rm Pl} m_g)^{1/2}$ and thus is higher than $\Lambda_3 \sim (M_{\rm Pl} m_g^2)^{1/3}$. The reason for this is because the original quasidilaton (i.e. the theory with $\alpha_\sigma = 0$) in the Λ_3 decoupling limit enjoys an enhanced Galileon symmetry [8].

* Corresponding author.

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scaling property under (1):

$$f_{\mu\nu} \to e^{-2\sigma_0/M_{\rm Pl}} f_{\mu\nu}, \qquad \tilde{f}_{\mu\nu} \to e^{-2\sigma_0/M_{\rm Pl}} \tilde{f}_{\mu\nu}. \tag{5}$$

Having defined $\tilde{f}_{\mu\nu}$ in this way, a building block for the action of extended quasidilaton massive gravity is constructed as

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - e^{\sigma/M_{\rm Pl}} \left(\sqrt{g^{-1}\tilde{f}}\right)^\mu{}_\nu, \tag{6}$$

where g^{-1} represents the inverse $g^{\mu\nu}$ of the physical metric $g_{\mu\nu}$. It is easy to see from (5) that the tensor $\mathcal{K}^\mu{}_\nu$ is invariant under (1). We then build the following terms, which provide a mass to the graviton.

$$\mathcal{L}_2 \equiv \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right), \tag{7}$$

$$\mathcal{L}_3 \equiv \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right), \tag{8}$$

$$\mathcal{L}_4 \equiv \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right), \tag{9}$$

where square brackets denote a trace.

Note that the dependence of the extended fiducial metric (4) on the time-derivative of the quasidilaton alters the Hamiltonian structure of the system, and one might worry about possible reappearance of the Boulware–Deser (BD) ghost [11]. Since the type of theory considered in the present Letter does not fall into a class of models that was claimed to be free from the BD ghost [12], it remains an open question whether the BD ghost is absent at fully nonlinear level. (Note that a failure to prove the absence of the BD ghost does not necessarily imply the presence of it.) In the following, instead of providing a fully nonlinear study, we shall satisfy ourselves by explicitly showing the absence of the BD ghost at the level of linear perturbations around a self-accelerating solution.

After introducing a canonical kinetic term for the quasidilaton field σ, we are ready to write down the full Lagrangian as

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R - 2\Lambda - \frac{\omega}{M_{\rm Pl}^2}\partial_\mu\sigma\partial^\mu\sigma + 2m_g^2(\mathcal{L}_2 + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4)\right]. \tag{10}$$

This action can be further extended, e.g. by introducing shift-symmetric covariant Galileon-type kinetic terms for the quasidilaton field, or/and by introducing other massive gravity Lagrangians with different values of α_σ, α_3, and α_4. One can also add an extra term $\xi\sqrt{-\tilde{f}}e^{4\sigma/M_{\rm Pl}}$ invariant under (1). In the present Letter, however, we shall focus our attention to the simplest extension provided by (10).

In the limit $\alpha_\sigma \to 0$, the action (10) reduces to the one in the original theory of quasidilaton, but it was shown in [13,14] that the original theory suffers from ghost instability in the scalar sector. In the following we shall show that the inclusion of the α_σ term can render the extended quasidilaton theory stable.

The background

Let us consider here a flat Friedmann–Lemaître–Robertson–Walker (FLRW) ansatz for the theory defined in Eq. (10), that is

$$ds^2 = -N(t)^2\,dt^2 + a(t)^2\delta_{ij}\,dx^i\,dx^j, \tag{11}$$

$$\phi^0 = \phi^0(t), \qquad \phi^i = x^i, \qquad \sigma = \bar{\sigma}(t). \tag{12}$$

The extended fiducial metric (4) then reduces to

$$\tilde{f}_{00} = -n(t)^2, \qquad \tilde{f}_{ij} = \delta_{ij}, \tag{13}$$

where

$$n(t)^2 \equiv \left(\dot{\phi}^0\right)^2 + \frac{\alpha_\sigma}{M_{\rm Pl}^2 m_g^2} e^{-2\bar{\sigma}/M_{\rm Pl}}\dot{\bar{\sigma}}^2. \tag{14}$$

We introduce the following quantities characterizing the background solution.

$$H \equiv \frac{\dot{a}}{Na}, \qquad X \equiv \frac{e^{\bar{\sigma}/M_{\rm Pl}}}{a}, \qquad r \equiv \frac{n}{N}a. \tag{15}$$

We consider here a to be a dimensionless quantity, so as n, N, X, ω, r and α_σ. Also $[\phi^a] = M^{-1}$, $[H] = M$, and $[\sigma] = M$. As we shall see below, the three independent equations of motion for the background allow for an attractor solution on which H, X, and r are constants.

Varying the action w.r.t. $\phi^0(t)$ and then setting $n(t) = 1$ leads to

$$\partial_t\left[a^4 X(1-X)J\dot{\phi}^0/n\right] = 0, \tag{16}$$

where

$$J \equiv 3 + 3(1-X)\alpha_3 + (1-X)^2\alpha_4. \tag{17}$$

This implies that $X(1-X)J\dot{\phi}^0/n \propto 1/a^4 \to 0$ as the universe expands (i.e. $a\to\infty$). We thus have three cases: $X = 0$, $X = 1$ and $J = 0$. We would not consider the case with $X = 0$ since it would lead to a strong coupling [8]. The case with $X = 1$ is not interesting since it does not lead to a self-accelerating solution but corresponds to a solution driven by the bare cosmological constant Λ. Therefore, in this Letter we shall consider the case with

$$J = 0. \tag{18}$$

This, together with

$$r = 1 + \frac{\omega H^2}{m_g^2 X^2[\alpha_3(X-1) - 2]}, \tag{19}$$

$$\left(3 - \frac{\omega}{2}\right)H^2 = \Lambda + \Lambda_X, \tag{20}$$

leads to a self-accelerating solution. The de Sitter solution we have just found is supposed to describe the final state of the dynamics of our universe, when all matter fields can be safely neglected. Here,

$$\Lambda_X \equiv m_g^2(X-1)\left[6 - 3X + (X-4)(X-1)\alpha_3 + (X-1)^2\alpha_4\right]. \tag{21}$$

Eq. (20), together with the requirement that $\partial(H^2)/\partial\Lambda > 0$, or, in other words, the positivity of the effective Newton's constant for the background evolution, implies that

$$\omega < 6. \tag{22}$$

This study shows that it is possible, in general, for this theory to possess self accelerating solutions with effective cosmological constant given by Λ_X. It should be noticed that the new component in the extended fiducial metric (4), i.e. the term proportional to α_σ, does not enter in the background dynamics. However, the parameter α_σ, as we will see later on, will play a crucial role in order to stabilize the propagation of the perturbation fields.

Scalar perturbations

We have shown the existence of a self-accelerating solution for this extended quasidilaton theory. In the following analysis of perturbations, we choose the unitary gauge: we set the Stückelberg fields to their background values. This choice completely fixes the gauge freedom.

As for the scalar sector we introduce the metric in the form

$$\delta g_{00} = -2N^2\Phi, \qquad \delta g_{0i} = Na\partial_i B, \tag{23}$$

$$\delta g_{ij} = a^2\left[2\delta_{ij}\Psi + \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\partial_l\partial^l\right)E\right], \tag{24}$$

whereas the quasidilaton field is perturbed as

$$\sigma = \bar{\sigma} + M_{\rm Pl}\delta\sigma. \tag{25}$$

After decomposing each perturbation variable into Fourier modes, and expanding the action up to second order, we find that B and Φ do not have kinetic terms as expected. We can thus integrate them out. Furthermore, on introducing the field redefinition

$$\delta\sigma = \Psi + \bar{\delta\sigma}, \tag{26}$$

we notice that Ψ also becomes an auxiliary field. This feature is due to the specific structure of the graviton mass term in the action, which has been constructed in such a way that the Boulware–Deser ghost [11] is removed. After integrating out the field Ψ as well, the theory admits only two propagating scalar modes.

No-ghost condition

We then obtain the kinetic matrix K_{IJ} $(I, J = 1, 2)$ in the total Lagrangian $\mathcal{L} \ni K_{11}|\dot{\bar{\delta\sigma}}|^2 + K_{22}|\dot{E}|^2 + K_{12}(\dot{\bar{\delta\sigma}}^\dagger\dot{E} + \text{h.c.})$. In order to avoid a ghost degree of freedom, we demand that $\det K_{IJ} > 0$ and $K_{22} > 0$. By requiring these two inequalities for all momenta, and noting the condition (22) from the background evolution, we obtain the following conditions

$$0 < \omega < 6, \qquad X^2 < \frac{\alpha_\sigma H^2}{m_g^2} < r^2X^2. \tag{27}$$

It should be pointed out that the latter condition implies that $r > 1$ (note that r is positive by definition) and that $\alpha_\sigma H^2/m_g^2 > 0$. In particular, if $\alpha_\sigma = 0$ then there is always a ghost in the scalar sector [13,14]. In this sense the α_σ term introduced in the present Letter plays a key role to establish the stability of the quasidilaton theory. We also notice that $(\dot{\phi}^0/n)^2 = 1 - \alpha_\sigma H^2/(m_g^2r^2X^2)$ and that the last inequality in (27) is equivalent to $(\dot{\phi}^0/n)^2 > 0$. We have thus shown the existence of a parameter regime in which the scalar sector is free from ghost.

Speed of propagation

In order to find the speed of propagation for the scalar modes, we find it convenient to diagonalize the kinetic matrix by defining the fields $q_{1,2}$ as

$$\bar{\delta s} \equiv kq_1, \qquad E \equiv \frac{q_2}{k^2} - \frac{K_{12}}{K_{22}}kq_1, \tag{28}$$

where k is the size of the comoving momentum. The k-dependence in this field redefinition has been introduced so that, for the new kinetic matrix, the diagonal elements tend to finite (and non-zero) values for large k.

The new kinetic matrix $\mathcal{T}_{IJ}$ is diagonal as

$$\mathcal{L} \ni \mathcal{T}_{11}(t,k)|\dot{q}_1|^2 + \mathcal{T}_{22}(t,k)|\dot{q}_2|^2, \tag{29}$$

where

$$\mathcal{T}_{11} = (\det K_{IJ})k^2/K_{22}, \qquad \mathcal{T}_{22} = K_{22}/k^4, \tag{30}$$

and, when the no-ghost conditions (27) hold we find

$$\mathcal{T}_{11} > 0, \quad \text{and} \quad \mathcal{T}_{22} > 0. \tag{31}$$

When $k/a \gg H$ and $k/a \gg m_g$, we can safely ignore time-dependence of each coefficient in the equations of motion. At the leading order in large k expansion we thus obtain the following structure of the equations of motion.

$$\mathcal{T}_{11}\ddot{q}_1 + k\mathcal{B}\dot{q}_2 \simeq 0, \tag{32}$$

$$\mathcal{T}_{22}\ddot{q}_2 - k\mathcal{B}\dot{q}_1 + k^2\mathcal{C}q_2 \simeq 0, \tag{33}$$

where $\mathcal{T}_{11}$, $\mathcal{T}_{22}$, and other coefficients $\mathcal{B}$ and $\mathcal{C}$ are k-independent. All other terms in the equations of motion are suppressed by inverse powers of $k/(aH)$ or $k/(am_g)$. Then one can read off the speed of propagation as

$$c_s^2 = \frac{\mathcal{B}^2 + \mathcal{C}\mathcal{T}_{11}}{\mathcal{T}_{11}\mathcal{T}_{22}}\frac{a^2}{N^2} = 1 \tag{34}$$

for one mode and $c_s^2 = 0$ for the other mode. Thus, scalar modes with $k/a \gg \max(H, m_g)$ do not develop gradient instabilities. For a self-accelerating solution ($\Lambda = 0$ and thus $H \sim m_g$) this means that there is no gradient instability parametrically faster than the cosmological timescale. Therefore the study of the Laplace instabilities does not add any new constraint to the model.

Vector perturbations

The vector modes in the theory consist of the vector modes of the metric tensor, that is

$$\delta g_{0i} = aNB_i^T, \qquad \delta g_{ij} = \frac{a^2}{2}\left(\partial_i E_j^T + \partial_j E_i^T\right), \tag{35}$$

where $\partial^i B_i^T = \partial^i E_i^T = 0$. As we have seen, the new α_σ term does not affect the background evolution. It does not affect the vector modes either and the results should agree with the case with $\alpha_\sigma = 0$ already studied in [13,14]. In fact we find that the field B_i can be integrated out and the reduced Lagrangian becomes

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{16}a^3N\left[\frac{\mathcal{T}_V}{N^2}\left|\dot{E}_i^T\right|^2 - k^2M_{\rm GW}^2\left|E_i^T\right|^2\right], \tag{36}$$

where

$$\mathcal{T}_V \equiv \frac{2k^2\omega H^2a^2}{k^2(r^2-1) + 2\omega H^2a^2}, \tag{37}$$

$$M_{\rm GW}^2 \equiv \frac{(r-1)X^3m_g^2}{X-1} + \frac{\omega H^2(rX + r - 2)}{(X-1)(r-1)}. \tag{38}$$

The speed of propagation for large k reduces to $c_V^2 = (M_{\rm GW}^2/H^2)\cdot(r^2-1)/(2\omega)$. Thus the stability for vector modes is ensured if $\mathcal{T}_V > 0$ and $c_V^2 > 0$. These conditions, together with the no-ghost conditions for the scalar modes (27), impose

$$M_{\rm GW}^2 > 0. \tag{39}$$

This condition does not depend on α_σ, however it constrains the other parameters in the theory.

Tensor perturbations

As for the tensor modes, defined in the metric tensor as

$$\delta g_{ij} = a^2 h_{ij}^{TT}, \tag{40}$$

with $\delta^{ij} h_{ij}^{TT} = 0$, and $\partial^j h_{ij}^{TT} = 0$, we also find the same results as in [13,14]. Namely, their Lagrangian reduces to

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{8} a^3 N \left[\frac{|\dot{h}_{ij}^{TT}|^2}{N^2} - \left(\frac{k^2}{a^2} + M_{\rm GW}^2 \right) |h_{ij}^{TT}|^2 \right]. \tag{41}$$

This sector is well behaved and the graviton acquires a mass $M_{\rm GW}^2$, as expected.

Self-acceleration

For a self-accelerating background without a bare cosmological constant (i.e. setting $\Lambda = 0$), all stability conditions are satisfied if

$$\begin{aligned} &[0 < X < 1 \text{ and } 1 < r \leqslant \bar{r} \text{ and } 0 < \omega < 6] \\ \text{or} \quad &[0 < X < 1 \text{ and } r > \bar{r} \text{ and } 0 < \omega < \bar{\omega}] \\ \text{or} \quad &[X > 1 \text{ and } \bar{\omega} < \omega < 6], \end{aligned} \tag{42}$$

where $\bar{r} \equiv \frac{2+X}{1+2X}$ and $\bar{\omega} = \frac{6(r-1)^2 X^3}{[r^2+2r-1]X^3 - 6rX^2 + 6X + 2r - 4}$, provided that α_σ is chosen to satisfy the second of (27).

Summary

We have presented the first example of a unitary theory of Lorentz-invariant massive gravity, with all degrees of freedom propagating on a self-accelerating de Sitter background. The theory is a simple extension of the quasidilaton theory, respecting the symmetry of the original theory but allowing for a new type of coupling between the massive graviton and the quasidilaton scalar. We have found that: (i) there exist non-trivial flat FLRW solutions; (ii) a self-accelerating de Sitter universe is realized as an attractor of the system; and (iii) for a range of parameters all degrees of freedom on the attractor have healthy kinetic terms and there is no gradient instability parametrically faster than the cosmological time scale.

In [13,14] it was shown that the self-accelerating solution in the original quasidilaton theory, even including some additional interactions such as Galileon terms and Goldstone-type terms, always suffers from ghost instability. Our finding in the present Letter, i.e. the stability of the self-accelerating solution in the extended theory, can be considered as an important step towards a consistent theory of quasidilaton massive gravity. While it was argued in [14] that properties of perturbations in quasidilaton theories are generically UV sensitive, the existence of a stable extended theory is quite encouraging, and at the very least provides an existence proof of a unitary theory with a self-accelerating cosmological background. The setup in the present Letter also provides a framework in which cosmological and phenomenological implications of massive gravity can be tested.

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