IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Shapes of primordial non-Gaussianities in the Horndeski's most general scalar-tensor theories

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

JCAP03(2013)030

(http://iopscience.iop.org/1475-7516/2013/03/030)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 202.28.21.92
The article was downloaded on 05/04/2013 at 04:59

Please note that terms and conditions apply.

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

Shapes of primordial non-Gaussianities in the Horndeski's most general scalar-tensor theories

Antonio De Felice[a,b] **and Shinji Tsujikawa**[c]

[a]ThEP's CRL, NEP, The Institute for Fundamental Study, Naresuan University, Phitsanulok 65000, Thailand
[b]Thailand Center of Excellence in Physics, Ministry of Education, Bangkok 10400, Thailand
[c]Department of Physics, Faculty of Science, Tokyo University of Science, 1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan

E-mail: antoniod@nu.ac.th, shinji@rs.kagu.tus.ac.jp

Received January 24, 2013
Accepted February 27, 2013
Published March 22, 2013

Abstract. In the Horndeski's most general scalar-tensor theories, we derive the three-point correlation function of scalar non-Gaussianities generated during single-field inflation in the presence of slow-variation corrections to the leading-order term. Unlike previous works, the resulting bispectrum is valid for any shape of non-Gaussianities. In the squeezed limit, for example, this gives rise to the same consistency relation as that derived by Maldacena in standard single-field slow-roll inflation. We estimate the shape close to the squeezed one at which the effect of the term inversely proportional to the scalar propagation speed squared begins to contribute to the bispectrum. We also show that the leading-order bispectrum can be expressed by the linear combination of two convenient bases whose shapes are highly correlated with equilateral and orthogonal types respectively. We present concrete models in which the orthogonal and enfolded shapes can dominate over the equilateral one.

Keywords: inflation, non-gaussianity, modified gravity, cosmological perturbation theory

JCAP03(2013)030

© 2013 IOP Publishing Ltd and Sissa Medialab srl
doi:10.1088/1475-7516/2013/03/030

Contents

1 Introduction 1

2 Equations of motion for the background and linear perturbations 3

3 Three-point correlation functions in the presence of correction terms 6

4 Local, equilateral, and enfolded non-Gaussianities 14
- 4.1 Local non-Gaussianities 14
- 4.2 Equilateral non-Gaussianities 15
- 4.3 Enfolded non-Gaussianities 16

5 Shapes of non-Gaussianities 17

6 Shapes of non-Gaussianities in concrete models 21
- 6.1 Power-law k-inflation 21
- 6.2 k-inflation with the term $G_3(X)$ 22
- 6.3 k-inflation with the term $G_4(X)$ 23
- 6.4 k-inflation with the term $G_5(X)$ 24

7 Conclusions 25

A Corrections to the bispectrum 26

B Expressions for $\mathcal{U}$ and $\mathcal{V}$ 29

1 Introduction

The potential presence of primordial non-Gaussianities in the CMB temperature anisotropies can be a powerful probe for the physics in the early Universe — especially for inflation [1]. The inflationary paradigm generally predicts nearly scale-invariant density perturbations [2] with a suppressed tensor-to-scalar ratio, whose prediction is consistent with the CMB power spectrum measured by COBE [3] and WMAP [4, 5]. The detection of scalar non-Gaussianities not only breaks the degeneracy among many inflationary models, but it also offers the possibility to discriminate between the inflationary paradigm and other alternative scenarios (such as curvaton [6]) [7]–[34].

There are several different shapes of non-Gaussianities depending on the wave numbers $\boldsymbol{k}_1$, $\boldsymbol{k}_2$, and $\boldsymbol{k}_3$ satisfying the condition $\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 = 0$ [35]–[45]. The simplest one is the so-called local shape, which has a peak in the squeezed limit (i.e., the limit where the modulus of the momenta approaches $k_3 \to 0$ and $k_1 \simeq k_2$). The second shape corresponds to the equilateral configuration with a peak at $k_1 = k_2 = k_3$. A factorizable shape whose scalar product with the equilateral template vanishes is called the orthogonal one. There is another shape dubbed the enfolded one, which is a linear combination of the equilateral and orthogonal templates.

From the bispectrum $\mathcal{A}_\mathcal{R}$ of the three-point correlation function of curvature perturbations $\mathcal{R}$, the non-linear parameter characterizing the strength of non-Gaussianities is defined

JCAP03(2013)030

by $f_{\rm NL} = (10/3)\,\mathcal{A}_{\mathcal{R}}/\sum_{i=1}^{3} k_i^3$. For purely adiabatic Gaussian perturbations we have that $f_{\rm NL} = 0$, but the presence of non-Gaussian perturbations leads to the deviation from $f_{\rm NL} = 0$. The WMAP 9 year data provide the following bounds on the non-linear parameters of local, equilateral, and orthogonal non-Gaussianities, respectively [46]:

$$f_{\rm NL}^{\rm local} = 37.2 \pm 19.9 \quad (68\,\%\,{\rm CL})\,, \qquad f_{\rm NL}^{\rm local} = 37 \pm 40 \quad (95\,\%\,{\rm CL})\,, \tag{1.1}$$

$$f_{\rm NL}^{\rm equil} = 51 \pm 136 \quad (68\,\%\,{\rm CL})\,, \qquad f_{\rm NL}^{\rm equil} = 51 \pm 272 \quad (95\,\%\,{\rm CL})\,, \tag{1.2}$$

$$f_{\rm NL}^{\rm ortho} = -245 \pm 100 \quad (68\,\%\,{\rm CL})\,, \qquad f_{\rm NL}^{\rm ortho} = -245 \pm 200 \quad (95\,\%\,{\rm CL})\,. \tag{1.3}$$

Since the non-linear parameter of the enfolded shape is given by $f_{\rm NL}^{\rm enfold} = (f_{\rm NL}^{\rm equil} - f_{\rm NL}^{\rm ortho})/2$ [44, 45], we obtain the following bounds from eqs. (1.2) and (1.3):

$$f_{\rm NL}^{\rm enfold} = 148 \pm 118 \quad (68\,\%\,{\rm CL})\,, \qquad f_{\rm NL}^{\rm enfold} = 148 \pm 236 \quad (95\,\%\,{\rm CL})\,. \tag{1.4}$$

For the local, orthogonal, and enfolded shapes the model with purely Gaussian perturbations ($f_{\rm NL} = 0$) is outside the 68 % observational contour, but, apart from the orthogonal case, it is still consistent with the WMAP constraints at 95 % CL.

In standard single-field inflation based on a canonical scalar field, Maldacena [12] showed that the non-linear parameter in the squeezed limit is given by $f_{\rm NL}^{\rm local} = (5/12)(1-n_{\mathcal{R}})$, where $n_{\mathcal{R}}$ is the scalar spectral index. Creminelli and Zaldarriaga [47] pointed out that the same non-Gaussianity consistency relation holds for any single-field model under the condition that only one mode of curvature perturbations survives after the Hubble radius crossing while the other one decays[1] (see refs. [50–53] for related works).

In the context of single-field k-inflation [54], the bispectrum of curvature perturbations was first derived by Seery and Lidsey in 2005 [18]. Since the scalar propagation speed squared c_s^2 can be much smaller than 1 [55], it is possible to realize the large equilateral non-linear parameter $|f_{\rm NL}^{\rm equil}| \sim 1/c_s^2 \gg 1$. If we naively take the squeezed limit for the leading-order bispectrum derived in refs. [18, 56], the term proportional to $1/c_s^2$ does not disappear. This comes from the fact that the slow-variation corrections to the bispectrum need to be taken into account to estimate the local-type non-Gaussianity correctly. In fact, Chen et al. [56] showed that the Maldacena's consistency relation is recovered in the squeezed limit by carefully computing all the possible slow-variation corrections to the leading-order bispectrum. Thus the slow-variation single-field k-inflation models with $c_s^2 \ll 1$ lead to small local non-Gaussianities, even though the equilateral non-linear parameter can be large.

In the Horndeski's most general scalar-tensor theories with second-order equations of motion [57–60], the leading-order three-point correlation function of curvature perturbations was derived on the quasi de Sitter background [61, 62] (see refs. [63–65] for the scalar non-Gaussianities in related Galileon models and refs. [66] for the bispectrum of tensor perturbations in the Horndeski's theories). Although the result is valid for the estimation of the equilateral non-linear parameter, the bispectrum is not general enough to be used for any shape of non-Gaussianities. In this paper we take into account all the possible slow-variation corrections to the leading-order bispectrum in the Horndeski's theories.[2] Not only we repro-

[1] If the decaying mode is non-negligible relative to the growing mode, the Maldacena's consistency relation can be violated [48, 49].

[2] In the effective field theory of inflation (which allows the equations of motion higher than second order), a similar approach was taken by Cheung et al. [50] to show that the Maldacena's consistency relation holds in the squeezed limit. While the authors in this paper mainly focused on the local shape, we derive the full bispectrum in the Horndeski's theory which can be used for any shape of non-Gaussianities.

JCAP03(2013)030

duce the Maldacena's consistency relation in the squeezed limit, but we identify the shape close to the squeezed one at which the term $1/c_s^2$ begins to contribute to the bispectrum.

Given our general expression of the bispectrum in the most general single-field scalar-tensor theories, we can evaluate the non-linear parameters of several different shapes to confront each inflationary model with observations. In particular the result $|f_{\rm NL}^{\rm local}| \ll 1$ is robust for any slow-variation single-field model, so the detection of non-Gaussianities in the squeezed limit will allow us to falsify the slow-variation single-field scenario. Note that in realistic observations the shape is not completely squeezed, in which case the bispectrum can be affected by the appearance of the term $1/c_s^2$ mentioned above. Our results are useful to distinguish such difference accurately.

If $c_s^2 \ll 1$, then the non-linear parameters $|f_{\rm NL}^{\rm equil}|$, $|f_{\rm NL}^{\rm ortho}|$, and $|f_{\rm NL}^{\rm enfold}|$ can be much larger than the order of 1. Which shape dominates over the other ones depends on the models of inflation. In ref. [62] it was shown that the correlation between the equilateral template and the shapes arising from the Horndeski's theories is quite high, but linear combinations of equilateral operators can give rise to a significantly different shape for a wide range of coefficients [44, 45]. In this regard we anticipate that there may be some models in which the shape orthogonal to the equilateral template provides an important contribution to the bispectrum.

In this paper we show that the leading-order three-point correlation function in the Horndeski's theories can be expressed by a linear combination of two bases whose shapes are highly correlated with equilateral and orthogonal shapes respectively. This decomposition is useful because the contributions from the equilateral and orthogonal shapes can be easily estimated for concrete models of inflation. We show that in k-inflation with the covariant Galileon terms there are cases in which the correlations with the orthogonal and enfolded templates are larger than that with the equilateral one. Thus the shapes of non-Gaussianities allow us to discriminate such models from observations.

This paper is organized as follows. In section 2 we review the background and linear perturbation equations in the Horndeski's theories. In section 3 we derive the three-point correlation function of curvature perturbations in the presence of slow-variation corrections to the leading-order bispectrum. In section 4 the non-linear parameter $f_{\rm NL}$ is evaluated in the squeezed, equilateral, and enfolded limits, respectively. In section 5 we express the leading-order bispectrum in terms of equilateral and orthogonal bases. In section 6 we show concrete models of inflation in which the orthogonal and enfolded shapes can dominate over the equilateral one. Section 7 is devoted to conclusions. In appendix we show the details of the slow-variation corrections to the bispectrum.

2 Equations of motion for the background and linear perturbations

The action corresponding to the most general scalar-tensor theories is given by [57, 58]

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + P(\phi,X) - G_3(\phi,X)\,\Box\phi + \mathcal{L}_4 + \mathcal{L}_5\right], \tag{2.1}$$

where g is the determinant of the metric $g_{\mu\nu}$, $M_{\rm pl}$ is the reduced Planck mass, R is a Ricci scalar, and

$$\mathcal{L}_4 = G_4(\phi,X)\,R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right], \tag{2.2}$$

$$\mathcal{L}_5 = G_5(\phi,X)\,G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)]\,. \tag{2.3}$$

JCAP03(2013)030

Here P and G_i's ($i = 3, 4, 5$) are functions in terms of ϕ and $X = -\partial^\mu\phi\partial_\mu\phi/2$ with the partial derivatives $G_{i,X} \equiv \partial G_i/\partial X$, and $G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu}R/2$ is the Einstein tensor ($R_{\mu\nu}$ is the Ricci tensor).

We consider the following ADM metric [67] with scalar metric perturbations α, ψ, and $\mathcal{R}$ about the flat Friedmann-Lemaître-Robertson-Walker (FLRW) background

$$ds^2 = -[(1+\alpha)^2 - a(t)^{-2}\, e^{-2\mathcal{R}}\, (\partial\psi)^2]\, dt^2 + 2\partial_i\psi\, dt\, dx^i + a(t)^2 e^{2\mathcal{R}} d\boldsymbol{x}^2\,, \tag{2.4}$$

where $a(t)$ is the scale factor with cosmic time t. We choose the uniform field gauge $\delta\phi = 0$, which fixes the time-component of a gauge-transformation vector ξ^μ. The spatial part of ξ^μ is fixed by gauging away a perturbation E that appears as a form $E_{,ij}$ in the metric (2.4).

The background equations of motion are given by

$$\begin{aligned}
&3M_{\rm pl}^2H^2F + P + 6HG_{4,\phi}\dot\phi + \left(G_{3,\phi} - 12\,H^2G_{4,X} + 9\,H^2G_{5,\phi} - P_{,X}\right)\dot\phi^2 \qquad (2.5)\\
&+\left(6\,G_{4,\phi X} - 3\,G_{3,X} - 5\,G_{5,X}H^2\right)H\dot\phi^3 + 3\,(G_{5,\phi X} - 2\,G_{4,XX})\,H^2\dot\phi^4 - H^3G_{5,XX}\dot\phi^5 = 0\,,\\
&(1 - 4\delta_{G4X} - 2\delta_{G5X} + 2\delta_{G5\phi})\epsilon =\\
&\quad \delta_{PX} + 3\delta_{G3X} - 2\delta_{G3\phi} + 6\,\delta_{G4X} - \delta_{G4\phi} - 6\,\delta_{G5\phi} + 3\,\delta_{G5X} + 12\,\delta_{G4XX} + 2\,\delta_{G5XX}\\
&\quad -10\,\delta_{G4\phi X} + 2\,\delta_{G4\phi\phi} - 8\,\delta_{G5\phi X} + 2\,\delta_{G5\phi\phi} - \delta_\phi(\delta_{G3X} + 4\,\delta_{G4X} - \delta_{G4\phi}\\
&\quad +8\,\delta_{G4XX} + 3\,\delta_{G5X} - 4\,\delta_{G5\phi} + 2\,\delta_{G5XX} - 2\delta_{G4\phi X} - 4\,\delta_{G5\phi X})\,, \qquad (2.6)
\end{aligned}$$

where $H = \dot a/a$ is the Hubble parameter (a dot represents a derivative with respect to t), $F = 1 + 2G_4/M_{\rm pl}^2$, and

$$\begin{aligned}
&\epsilon = -\frac{\dot H}{H^2}\,,\ \delta_\phi = \frac{\ddot\phi}{H\dot\phi}\,,\ \delta_{PX} = \frac{P_{,X}X}{M_{\rm pl}^2H^2F}\,,\ \delta_{G3X} = \frac{G_{3,X}\dot\phi X}{M_{\rm pl}^2HF}\,,\ \delta_{G3\phi} = \frac{G_{3,\phi}X}{M_{\rm pl}^2H^2F}\,,\ \delta_{G4X} = \frac{G_{4,X}X}{M_{\rm pl}^2F}\,,\\
&\delta_{G4\phi} = \frac{G_{4,\phi}\dot\phi}{M_{\rm pl}^2HF}\,,\ \delta_{G4\phi X} = \frac{G_{4,\phi X}\dot\phi X}{M_{\rm pl}^2HF}\,,\ \delta_{G4\phi\phi} = \frac{G_{4,\phi\phi}X}{M_{\rm pl}^2H^2F}\,,\ \delta_{G4XX} = \frac{G_{4,XX}X^2}{M_{\rm pl}^2F}\,,\ \delta_{G5\phi} = \frac{G_{5,\phi}X}{M_{\rm pl}^2F}\,,\\
&\delta_{G5X} = \frac{G_{5,X}H\dot\phi X}{M_{\rm pl}^2F}\,,\ \delta_{G5XX} = \frac{G_{5,XX}H\dot\phi X^2}{M_{\rm pl}^2F}\,,\ \delta_{G5\phi X} = \frac{G_{5,\phi X}X^2}{M_{\rm pl}^2F}\,,\ \delta_{G5\phi\phi} = \frac{G_{5,\phi\phi}\dot\phi X}{M_{\rm pl}^2HF}\,, \qquad (2.7)
\end{aligned}$$

whose magnitudes are much smaller than 1 during inflation. The terms $\delta_{G4\phi X}$, $\delta_{G4\phi\phi}$, $\delta_{G5\phi X}$, $\delta G_{5\phi\phi}$ as well as $\delta_{G3\phi X} = G_{3,\phi X}X^2/(M_{\rm pl}^2H^2F)$ and $\delta_{G3\phi\phi} = G_{3,\phi\phi}\dot\phi X/(M_{\rm pl}^2H^3F)$ are second-order of ϵ. From eq. (2.6) it follows that

$$\epsilon = \delta_{PX} + 3\delta_{G3X} - 2\delta_{G3\phi} + 6\,\delta_{G4X} - \delta_{G4\phi} - 6\,\delta_{G5\phi} + 3\,\delta_{G5X} + 12\,\delta_{G4XX} + 2\,\delta_{G5XX} + \mathcal{O}(\epsilon^2)\,. \tag{2.8}$$

For the quantity $\delta_F = \dot F/(HF)$ we have

$$\delta_F = 2\delta_{G4\phi} + \mathcal{O}(\epsilon^2)\,. \tag{2.9}$$

Using the relations between $\mathcal{R}$, ψ, and α that follows from Hamiltonian and momentum constraints, the second-order action for perturbations reduces to [60–62, 68]

$$\mathcal{S}_2 = \int dtd^3x\, a^3Q\left[\dot{\mathcal{R}}^2 - \frac{c_s^2}{a^2}\,(\partial\mathcal{R})^2\right]\,, \tag{2.10}$$

JCAP03(2013)030

where

$$Q = \frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2}\,, \tag{2.11}$$

$$c_s^2 = \frac{3(2w_1^2w_2H - w_2^2w_4 + 4w_1\dot{w}_1w_2 - 2w_1^2\dot{w}_2)}{w_1(4w_1w_3 + 9w_2^2)}\,, \tag{2.12}$$

and

$$w_1 = M_{\rm pl}^2F - 4XG_{4,X} - 2HX\dot{\phi}G_{5,X} + 2XG_{5,\phi}\,, \tag{2.13}$$

$$\begin{aligned} w_2 = {} & 2M_{\rm pl}^2HF - 2X\dot{\phi}G_{3,X} - 16H(XG_{4,X} + X^2G_{4,XX}) + 2\dot{\phi}(G_{4,\phi} + 2XG_{4,\phi X}) \\ & - 2H^2\dot{\phi}(5XG_{5,X} + 2X^2G_{5,XX}) + 4HX(3G_{5,\phi} + 2XG_{5,\phi X})\,, \end{aligned} \tag{2.14}$$

$$\begin{aligned} w_3 = {} & \\ & -9M_{\rm pl}^2H^2F + 3(XP_{,X} + 2X^2P_{,XX}) + 18H\dot{\phi}(2XG_{3,X} + X^2G_{3,XX}) - 6X(G_{3,\phi} + XG_{3,\phi X}) \\ & + 18H^2(7XG_{4,X} + 16X^2G_{4,XX} + 4X^3G_{4,XXX}) - 18H\dot{\phi}(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2G_{4,\phi XX}) \\ & + 6H^3\dot{\phi}(15XG_{5,X} + 13X^2G_{5,XX} + 2X^3G_{,5XXX}) - 18H^2X(6G_{5,\phi} + 9XG_{5,\phi X} + 2X^2G_{5,\phi XX}), \end{aligned} \tag{2.15}$$

$$w_4 = M_{\rm pl}^2F - 2XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}\,. \tag{2.16}$$

For later convenience we introduce the following parameter

$$\epsilon_s \equiv \frac{Qc_s^2}{M_{\rm pl}^2F} \simeq \epsilon + \delta_{G3X} + \delta_{G4\phi} + 8\delta_{G4XX} + \delta_{G5X} + 2\delta_{G5XX} + \mathcal{O}(\epsilon^2)\,. \tag{2.17}$$

At linear level the curvature perturbation obeys the equation of motion

$$\left.\frac{\delta\mathcal{L}_2}{\delta\mathcal{R}}\right|_1 \equiv -2\left[\frac{d}{dt}(a^3Q\dot{\mathcal{R}}) - aQc_s^2\partial^2\mathcal{R}\right] = 0\,. \tag{2.18}$$

We decompose $\mathcal{R}$ into the Fourier components, as

$$\mathcal{R}(\tau,\boldsymbol{x}) = \frac{1}{(2\pi)^3}\int d^3k\,\mathcal{R}(\tau,\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,, \qquad \mathcal{R}(\tau,\boldsymbol{k}) = u(\tau,\boldsymbol{k})a(\boldsymbol{k}) + u^*(\tau,-\boldsymbol{k})a^\dagger(-\boldsymbol{k})\,, \tag{2.19}$$

where $\tau = \int a^{-1}\,dt$, $\boldsymbol{k}$ is the comoving wave number, $a(\boldsymbol{k})$ and $a^\dagger(\boldsymbol{k})$ are the annihilation and creation operators, respectively, satisfying the commutation relations $\left[a(\boldsymbol{k}_1), a^\dagger(\boldsymbol{k}_2)\right] = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1 - \boldsymbol{k}_2)$ and $[a(\boldsymbol{k}_1), a(\boldsymbol{k}_2)] = \left[a^\dagger(\boldsymbol{k}_1), a^\dagger(\boldsymbol{k}_2)\right] = 0$.

Introducing a rescaled field $v = zu$ with $z = a\sqrt{2Q}$, it follows that

$$v'' + \left(c_s^2k^2 - \frac{z''}{z}\right)v = 0\,, \tag{2.20}$$

where a prime represents a derivative with respect to τ. Under the slow-variation approximation the term z''/z can be expressed as

$$\frac{z''}{z} = 2(aH)^2\left(1 - \frac{1}{2}\epsilon + \frac{3}{4}\eta_{sF} - \frac{3}{2}s\right) + \mathcal{O}(\epsilon^2)\,, \tag{2.21}$$

where

$$\eta_{sF} \equiv \frac{(\epsilon_sF)^\cdot}{H(\epsilon_sF)} = \eta_s + \delta_F\,, \qquad \eta_s \equiv \frac{\dot{\epsilon}_s}{H\epsilon_s}\,, \qquad s \equiv \frac{\dot{c}_s}{Hc_s}\,. \tag{2.22}$$

JCAP03(2013)030

Taking the dominant contribution in eq. (2.21) and using the approximate relation $a \simeq -1/(H\tau)$, we have $z''/z \simeq 2/\tau^2$. The solution to eq. (2.20), which recovers the Bunch-Davies vacuum state ($v = e^{-ic_sk\tau}/\sqrt{2c_sk}$) in the asymptotic past ($k\tau \to -\infty$), is given by

$$u(\tau, k) = \frac{i\,H\,e^{-ic_sk\tau}}{2(c_sk)^{3/2}\sqrt{Q}}\,(1 + ic_sk\tau)\,. \tag{2.23}$$

The slow-variation terms in eq. (2.21) provide the corrections to the mode function (2.23). Later we shall discuss the effect of such corrections on the primordial non-Gaussianities.

The power spectrum $\mathcal{P}_\mathcal{R}(k_1)$ of curvature perturbations, some time after the Hubble radius crossing, is defined by $\langle 0|\mathcal{R}(0,\boldsymbol{k}_1)\mathcal{R}(0,\boldsymbol{k}_2)|0\rangle = (2\pi^2/k_1^3)\mathcal{P}_\mathcal{R}(k_1)\,(2\pi)^3\delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2)$. From eq. (2.23) it follows that

$$\mathcal{P}_\mathcal{R} = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon_s F c_s}\,, \tag{2.24}$$

which should be evaluated at $c_sk = aH$. The spectral index $n_\mathcal{R}$ is given by

$$n_\mathcal{R} - 1 \equiv \left.\frac{d\ln\mathcal{P}_\mathcal{R}}{d\ln k}\right|_{c_sk=aH} = -2\epsilon - \eta_{sF} - s\,. \tag{2.25}$$

The corrections to the solution (2.23) only give rise to the $\mathcal{O}(\epsilon^2)$ terms in eq. (2.25).

Similarly the power spectrum $\mathcal{P}_h$ and the spectral index n_t of gravitational waves are given, respectively, by [60, 62]

$$\mathcal{P}_h = \frac{H^2}{2\pi^2 Q_t c_t^3} \simeq \frac{2H^2}{\pi^2 M_{\rm pl}^2 F}\,, \qquad n_t = \left.\frac{d\ln\mathcal{P}_h}{d\ln k}\right|_{c_tk=aH} = -2\epsilon - \delta_F\,, \tag{2.26}$$

where $Q_t = w_1/4 = (M_{\rm pl}^2F/4)(1 - 4\delta_{G4X} - 2\delta_{G5X} + 2\delta_{G5\phi})$ and $c_t^2 = w_4/w_1 \simeq 1 + 4\delta_{G4X} + 2\delta_{G5X} - 4\delta_{G5\phi}$. When both $\mathcal{P}_\mathcal{R}$ and $\mathcal{P}_h$ remain constant, the tensor-to-scalar ratio can be evaluated as

$$r = \frac{\mathcal{P}_h}{\mathcal{P}_\mathcal{R}} \simeq 16 c_s\epsilon_s\,. \tag{2.27}$$

3 Three-point correlation functions in the presence of correction terms

In the Horndeski's theories the third-order action of perturbations was derived in ref. [61, 62]. Here we do not repeat the details, but we summarize the main results. Under the approximation that all of the slow-variation terms in eq. (2.7) are much smaller than 1, the third-order action reads

$$\begin{aligned}\mathcal{S}_3 = \int dt\,d^3x \Big\{ & a^3\mathcal{C}_1 M_{\rm pl}^2 \mathcal{R}\dot{\mathcal{R}}^2 + a\,\mathcal{C}_2 M_{\rm pl}^2 \mathcal{R}(\partial\mathcal{R})^2 + a^3\mathcal{C}_3 M_{\rm pl}\dot{\mathcal{R}}^3 + a^3\mathcal{C}_4\dot{\mathcal{R}}(\partial_i\mathcal{R})(\partial_i\mathcal{X}) \\ & + a^3(\mathcal{C}_5/M_{\rm pl}^2)\partial^2\mathcal{R}(\partial\mathcal{X})^2 + a\mathcal{C}_6\dot{\mathcal{R}}^2\partial^2\mathcal{R} + \mathcal{C}_7\left[\partial^2\mathcal{R}(\partial\mathcal{R})^2 - \mathcal{R}\partial_i\partial_j(\partial_i\mathcal{R})(\partial_j\mathcal{R})\right]/a \\ & + a(\mathcal{C}_8/M_{\rm pl})\left[\partial^2\mathcal{R}\partial_i\mathcal{R}\partial_i\mathcal{X} - \mathcal{R}\partial_i\partial_j(\partial_i\mathcal{R})(\partial_j\mathcal{X})\right] + \mathcal{F}_1\left.\frac{\delta\mathcal{L}_2}{\delta\mathcal{R}}\right|_1 \Big\}\,,\end{aligned} \tag{3.1}$$

JCAP03(2013)030

where $\partial^2\mathcal{X} = Q\dot{\mathcal{R}}$. The dimensionless coefficients $\mathcal{C}_i$ $(i = 1, \cdots, 8)$ and the coefficient $\mathcal{F}_1$ are [62]

$$\mathcal{C}_1 = -\frac{3F\epsilon_s}{c_s^2}\left(\frac{1}{c_s^2}-1\right) + \frac{F\epsilon_s}{c_s^4}(\epsilon_s - \eta_s - 4\delta_{G3X} - 12\delta_{G4X} \\ - 32\delta_{G4XX} + 12\delta_{G5\phi} - 10\delta_{G5X} - 8\delta_{G5XX}) + \mathcal{O}(\epsilon^3), \tag{3.2}$$

$$\mathcal{C}_2 = F\epsilon_s\left(\frac{1}{c_s^2}-1\right) + \frac{F\epsilon_s}{c_s^2}(\epsilon_s + \eta_s - 2s + 4\delta_{G4X} + 2\delta_{G5X} - 4\delta_{G5\phi}) + \mathcal{O}(\epsilon^3), \tag{3.3}$$

$$\mathcal{C}_3 = \frac{F\epsilon_s}{c_s^2}\frac{M_{\rm pl}}{H}\left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right) + \frac{F\epsilon_s}{c_s^2}\frac{M_{\rm pl}}{H}\left\{\frac{1}{c_s^2}(\delta_{G3X} + 4\delta_{G4X} + 3\delta_{G5X} - \delta_{G4\phi} - 4\delta_{G5\phi} \right. \\ + 8\delta_{G4XX} + 2\delta_{G5XX}) - (3+2\lambda_{3X})\,\delta_{G3X} - 8\,(5+2\lambda_{4X})\,\delta_{G4XX} - 4(4+\lambda_{5X})\delta_{G5XX} \\ \left. + \delta_{G4\phi} + 8\delta_{G5\phi} - 8\delta_{G4X} - 9\delta_{G5X} - 6\frac{c_s^2}{\epsilon_s}\left[(1+\lambda_{3X})\delta_{G3X}^2 + \xi(\delta^2)\right]\right\} + \mathcal{O}(\epsilon^3), \tag{3.4}$$

$$\mathcal{C}_4 = -\frac{2\epsilon_s}{c_s^2} + \mathcal{O}(\epsilon^2)\,, \tag{3.5}$$

$$\mathcal{C}_5 = \frac{1}{4F}(\epsilon_s - 4\delta_{G3X} - 8\delta_{G4XX} + 8\delta_{G5X} + 4\delta_{G5XX}) + \mathcal{O}(\epsilon^2)\,, \tag{3.6}$$

$$\mathcal{C}_6 = 2F\left(\frac{M_{\rm pl}}{H}\right)^2[(1+\lambda_{3X})\delta_{G3X} + 4(3+2\lambda_{4X})\delta_{G4XX} + \delta_{G5X} + (5+2\lambda_{5X})\delta_{G5XX}] + \mathcal{O}(\epsilon^2), \tag{3.7}$$

$$\mathcal{C}_7 = -\frac{2}{3}F\left(\frac{M_{\rm pl}}{H}\right)^2(\delta_{G3X} + 6\delta_{G4XX} + \delta_{G5X} + \delta_{G5XX}) + \mathcal{O}(\epsilon^2), \tag{3.8}$$

$$\mathcal{C}_8 = 2\frac{M_{\rm pl}}{H}(\delta_{G3X} + 4\delta_{G4XX}) + \mathcal{O}(\epsilon^2)\,, \tag{3.9}$$

$$\mathcal{F}_1 = -\frac{L_1\mu_1 + 6X\dot{\phi}G_{5,X}}{6w_1^2}\{(\partial_k\mathcal{R})(\partial_k\mathcal{X}) - \partial^{-2}\partial_i\partial_j[(\partial_i\mathcal{R})(\partial_j\mathcal{X})]\} - \frac{L_1}{c_s^2}\mathcal{R}\dot{\mathcal{R}} \\ + \frac{L_1(L_1\mu_1 + 12X\dot{\phi}G_{5,X})}{12w_1a^2}\{(\partial\mathcal{R})^2 - \partial^{-2}\partial_i\partial_j[(\partial_i\mathcal{R})(\partial_j\mathcal{R})]\}\,, \tag{3.10}$$

where $\lambda_{3X} = XG_{3,XX}/G_{3,X}$, $\lambda_{4X} = XG_{4,XXX}/G_{4,XX}$, $\lambda_{5X} = XG_{5,XXX}/G_{5,XX}$, $\mu_1 = 3M_{\rm pl}^2F - 24(XG_{4,X} + X^2G_{4,XX}) - 6H\dot{\phi}(5XG_{5,X} + 2X^2G_{5,XX}) + 6X(3G_{5,\phi} + 2XG_{5,\phi X})$, $L_1 = 2w_1/w_2$, and

$$\Sigma = \frac{w_1(4w_1w_3 + 9w_2^2)}{12M_{\rm pl}^4}\,, \tag{3.11}$$

$$\lambda = \frac{F^2}{3}[3X^2P_{,XX} + 2X^3P_{,XXX} + 3H\dot{\phi}(XG_{3,X} + 5X^2G_{3,XX} + 2X^3G_{3,XXX}) \\ - 2(2X^2G_{3,\phi X} + X^3G_{3,\phi XX}) + 6H^2(9X^2G_{4,XX} + 16X^3G_{4,XXX} + 4X^4G_{4,XXXX}) \\ - 3H\dot{\phi}(3XG_{4\phi,X} + 12X^2G_{4,\phi XX} + 4X^3G_{4,\phi XXX}) \\ + H^3\dot{\phi}(3XG_{5,X} + 27X^2G_{5,XX} + 24X^3G_{5,XXX} + 4X^4G_{5,XXXX}) \\ - 6H^2(6X^2G_{5,\phi X} + 9X^3G_{5,\phi XX} + 2X^4G_{5,\phi XXX})]\,. \tag{3.12}$$

JCAP03(2013)030

The explicit form of the second-order term $\xi(\delta^2)$ in eq. (3.4) is given in appendix of ref. [62]. The coefficient $\mathcal{F}_1$ involves the terms with the spatial and time derivatives of $\mathcal{R}$ and $\mathcal{X}$. These provide the corrections to the three-point correlation function higher than first order in slow-variation parameters.[3] Since we are interested in the bispectrum up to first order, we neglect the contribution of the term $\mathcal{F}_1(\delta\mathcal{L}_2/\delta\mathcal{R})|_1$ in the following discussion. We also evaluated other boundary terms and found that they only lead to the contribution higher than the order ϵ.

The vacuum expectation value of $\mathcal{R}$ for the three-point operator in the asymptotic future ($\tau \to 0$) is

$$\langle \mathcal{R}(\boldsymbol{k}_1)\mathcal{R}(\boldsymbol{k}_2)\mathcal{R}(\boldsymbol{k}_3)\rangle = -i\int_{-\infty}^{0} d\tau\, a\, \langle 0|\left[\mathcal{R}(0,\boldsymbol{k}_1)\mathcal{R}(0,\boldsymbol{k}_2)\mathcal{R}(0,\boldsymbol{k}_3), \mathcal{H}_{\rm int}(\tau)\right]|0\rangle\,. \tag{3.13}$$

The interacting Hamiltonian $\mathcal{H}_{\rm int}$ is related to the third-order Lagrangian $\mathcal{L}_3$ as $\mathcal{H}_{\rm int} = -\mathcal{L}_3$, where $\mathcal{S}_3 = \int dt\, \mathcal{L}_3$. We write the three-point correlation function in the form

$$\langle \mathcal{R}(\boldsymbol{k}_1)\mathcal{R}(\boldsymbol{k}_2)\mathcal{R}(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)(\mathcal{P}_{\mathcal{R}})^2\mathcal{F}_{\mathcal{R}}(k_1,k_2,k_3)\,, \tag{3.14}$$

where

$$\mathcal{F}_{\mathcal{R}}(k_1,k_2,k_3) = \frac{(2\pi)^4}{\prod_{i=1}^3 k_i^3}\mathcal{A}_{\mathcal{R}}(k_1,k_2,k_3)\,. \tag{3.15}$$

If we use the leading-order solution (2.23) for the mode function and neglect the variation of the terms $\mathcal{C}_i$'s for the integration of eq. (3.13) with the approximation $a \simeq -1/(H\tau)$, the resulting bispectrum is [61, 62]

$$\begin{aligned}\mathcal{A}_{\mathcal{R}} \supset\ & \frac{c_s^2}{4\epsilon_s F}\mathcal{C}_1 S_1 + \frac{1}{4\epsilon_s F}\mathcal{C}_2 S_2 + \frac{3c_s^2}{2\epsilon_s F}\frac{H}{M_{\rm pl}}\mathcal{C}_3 S_3 + \frac{1}{8}\mathcal{C}_4 S_4 + \frac{\epsilon_s F}{4c_s^2}\mathcal{C}_5 S_5 + \frac{3}{\epsilon_s F}\left(\frac{H}{M_{\rm pl}}\right)^2\mathcal{C}_6 S_6 \\ & + \frac{1}{2\epsilon_s F c_s^2}\left(\frac{H}{M_{\rm pl}}\right)^2\mathcal{C}_7 S_7 + \frac{1}{8c_s^2}\frac{H}{M_{\rm pl}}\mathcal{C}_8 S_8\,,\end{aligned} \tag{3.16}$$

where

$$\begin{aligned} S_1 &= \frac{2}{K}\sum_{i>j}k_i^2k_j^2 - \frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3, \qquad S_2 = \frac{1}{2}\sum_i k_i^3 + \frac{2}{K}\sum_{i>j}k_i^2k_j^2 - \frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3, \\ S_3 &= \frac{(k_1k_2k_3)^2}{K^3}, \qquad S_4 = \sum_i k_i^3 - \frac{1}{2}\sum_{i\neq j}k_ik_j^2 - \frac{2}{K^2}\sum_{i\neq j}k_i^2k_j^3\,, \\ S_5 &= \frac{1}{K^2}\left[\sum_i k_i^5 + \frac{1}{2}\sum_{i\neq j}k_ik_j^4 - \frac{3}{2}\sum_{i\neq j}k_i^2k_j^3 - k_1k_2k_3\sum_{i>j}k_ik_j\right], \\ S_6 &= S_3, \qquad S_7 = \frac{1}{K}\left(1+\frac{1}{K^2}\sum_{i>j}k_ik_j + \frac{3k_1k_2k_3}{K^3}\right)\left[\frac{3}{4}\sum_i k_i^4 - \frac{3}{2}\sum_{i>j}k_i^2k_j^2\right], \\ S_8 &= \frac{1}{K^2}\left[\frac{3}{2}k_1k_2k_3\sum_i k_i^2 - \frac{5}{2}k_1k_2k_3K^2 - 6\sum_{i\neq j}k_i^2k_j^3 - \sum_i k_i^5 + \frac{7}{2}K\sum_i k_i^4\right], \end{aligned} \tag{3.17}$$

[3] Note that in ref. [12] the term $\mathcal{R}^2$ is present in the expression of $\mathcal{F}_1$, which gives rise to the first-order contribution η_s. We absorb this term to other coefficients, so that the field definition in ref. [12] is unnecessary.

JCAP03(2013)030

and $K = k_1 + k_2 + k_3$. The five shape functions S_i's ($i = 1, \cdots, 5$) are present in the context of k-inflation [18, 56]. In the Horndeski's theories the additional functions S_7 and S_8 appear, but they can be expressed by using other shape functions as [69]

$$S_7 = -\frac{3}{2}(3S_1 - S_2) + 18S_3\,, \qquad S_8 = 3S_1 - S_2 + 3S_4\,. \tag{3.18}$$

Since the three functions S_6, S_7, and S_8 vanish in the limit $k_3 \to 0$, the last three terms in eq. (3.16) do not contribute to the local non-Gaussianities.

The bispectrum $\mathcal{A}_\mathcal{R}$ coming from the contributions of $\mathcal{C}_1$, $\mathcal{C}_2$, $\mathcal{C}_3$, $\mathcal{C}_6$, $\mathcal{C}_7$ are 0-th order of ϵ, while the bispectrum from $\mathcal{C}_4$ and $\mathcal{C}_8$ are first order. Since the term $\mathcal{C}_5$ leads to the bispectrum at the order of ϵ^2, we can neglect its contribution. In the case where the leading-order terms of $\mathcal{A}_\mathcal{R}$ vanish (which occurs for local non-Gaussianities), we need to take into account next-order corrections to the bispectrum coming from the integrals that involve the terms $\mathcal{C}_1$, $\mathcal{C}_2$, $\mathcal{C}_3$, $\mathcal{C}_6$, $\mathcal{C}_7$ in eq. (3.1). Using the linear equation of motion $\delta\mathcal{L}_2/\delta\mathcal{R}|_1 = 0$, the $\mathcal{C}_6$, $\mathcal{C}_7$, and $\mathcal{C}_8$ dependent terms can be absorbed into the first five terms in eq. (3.1) [69]. Then the third-order action (3.1) reads

$$\begin{aligned}\mathcal{S}_3 = \int dt\, d^3x \Big\{ & a^3\tilde{\mathcal{C}}_1 M_{\rm pl}^2 \mathcal{R}\dot{\mathcal{R}}^2 + a\,\tilde{\mathcal{C}}_2 M_{\rm pl}^2 \mathcal{R}(\partial\mathcal{R})^2 + a^3\tilde{\mathcal{C}}_3 M_{\rm pl}\dot{\mathcal{R}}^3 \\ & + a^3\tilde{\mathcal{C}}_4\dot{\mathcal{R}}(\partial_i\mathcal{R})(\partial_i\mathcal{X}) + a^3(\tilde{\mathcal{C}}_5/M_{\rm pl}^2)\partial^2\mathcal{R}(\partial\mathcal{X})^2 \Big\}\,.\end{aligned} \tag{3.19}$$

The coefficients $\tilde{\mathcal{C}}_i$ ($i = 1, \cdots, 5$), which give rise to the corrections up to the order of ϵ in $\mathcal{A}_\mathcal{R}$, are [69]

$$\tilde{\mathcal{C}}_1 = \mathcal{C}_1 - \frac{3H^2}{2c_s^4 M_{\rm pl}^2}(6 + 2\epsilon + 7\eta_{sF} - 5\eta_7)\mathcal{C}_7 + \frac{3H\epsilon_s F}{2c_s^4 M_{\rm pl}}\mathcal{C}_8\,, \tag{3.20}$$

$$\tilde{\mathcal{C}}_2 = \mathcal{C}_2 + \frac{3H^2}{2c_s^2 M_{\rm pl}^2}(2 - 2\epsilon + \eta_{sF} + \eta_7 - 4s)\mathcal{C}_7 - \frac{H\epsilon_s F}{2c_s^2 M_{\rm pl}}\mathcal{C}_8\,, \tag{3.21}$$

$$\tilde{\mathcal{C}}_3 = \mathcal{C}_3 + \frac{H}{3c_s^2 M_{\rm pl}}(6 + 3\eta_{sF} - 4s - \eta_6)\mathcal{C}_6 + \frac{H}{c_s^4 M_{\rm pl}}(6 + 3\eta_{sF} - s - 2\eta_7)\mathcal{C}_7\,, \tag{3.22}$$

$$\tilde{\mathcal{C}}_4 = \mathcal{C}_4 + \frac{3H}{c_s^2 M_{\rm pl}}\mathcal{C}_8\,, \tag{3.23}$$

$$\tilde{\mathcal{C}}_5 = \mathcal{C}_5\,, \tag{3.24}$$

where $\eta_i \equiv \dot{\mathcal{C}}_i/(H\mathcal{C}_i)$, with $i = 6, 7, 8$. The three-point correlation function analogous to (3.16) is given by

$$\mathcal{A}_\mathcal{R} \supset \frac{c_s^2}{4\epsilon_s F}\tilde{\mathcal{C}}_1 S_1 + \frac{1}{4\epsilon_s F}\tilde{\mathcal{C}}_2 S_2 + \frac{3c_s^2}{2\epsilon_s F}\frac{H}{M_{\rm pl}}\tilde{\mathcal{C}}_3 S_3 + \frac{1}{8}\tilde{\mathcal{C}}_4 S_4\,, \tag{3.25}$$

where we dropped the $\tilde{\mathcal{C}}_5$-dependent term. The difference between eqs. (3.16) and (3.25) is that the bispectrum (3.25) includes the corrections coming from the time-variations of $\mathcal{C}_6$ and $\mathcal{C}_7$. However eqs. (3.16) and (3.25) are equivalent at leading order. The terms $\tilde{\mathcal{C}}_i$

JCAP03(2013)030

$(i = 1, \cdots, 4)$ can be expressed as

$$\tilde{\mathcal{C}}_1 = \tilde{\mathcal{C}}_1^{\rm lead} + \frac{F}{c_s^4}\left[\epsilon_s \delta\mathcal{C}_1 + (2\epsilon + 7\eta_{sF} - 5\eta_7)\delta\mathcal{C}_7 + 3\epsilon_s\delta\mathcal{C}_8\right] , \tag{3.26}$$

$$\tilde{\mathcal{C}}_2 = \tilde{\mathcal{C}}_2^{\rm lead} + \frac{F}{c_s^2}\left[\epsilon_s \delta\mathcal{C}_2 + (2\epsilon - \eta_{sF} - \eta_7 + 4s)\delta\mathcal{C}_7 - \epsilon_s\delta\mathcal{C}_8\right] , \tag{3.27}$$

$$\tilde{\mathcal{C}}_3 = \tilde{\mathcal{C}}_3^{\rm lead} + \frac{FM_{\rm pl}}{c_s^2 H}\left[\epsilon_s \delta\mathcal{C}_3 + \frac{2}{3}\left(3\eta_{sF} - 4s - \eta_6\right)\delta\mathcal{C}_6 - \frac{2}{3c_s^2}(3\eta_{sF} - s - 2\eta_7)\delta\mathcal{C}_7\right] , \tag{3.28}$$

$$\tilde{\mathcal{C}}_4 = -\frac{2\epsilon_s}{c_s^2} + \frac{6}{c_s^2}\delta\mathcal{C}_8 , \tag{3.29}$$

where the leading-order terms are

$$\tilde{\mathcal{C}}_1^{\rm lead} = -\frac{3F}{c_s^2}\left(\frac{1}{c_s^2} - 1\right)\epsilon_s + \frac{6F}{c_s^4}\delta\mathcal{C}_7 , \tag{3.30}$$

$$\tilde{\mathcal{C}}_2^{\rm lead} = F\left(\frac{1}{c_s^2} - 1\right)\epsilon_s - \frac{2F}{c_s^2}\delta\mathcal{C}_7 = -\frac{c_s^2}{3}\tilde{\mathcal{C}}_1^{\rm lead} , \tag{3.31}$$

$$\tilde{\mathcal{C}}_3^{\rm lead} = \frac{FM_{\rm pl}}{c_s^2 H}\left[\left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right)\epsilon_s + 4\delta\mathcal{C}_6 - \frac{4}{c_s^2}\delta\mathcal{C}_7\right] , \tag{3.32}$$

and $\delta\mathcal{C}_i$'s are the first-order slow-variation terms given by

$$\delta\mathcal{C}_1 = \epsilon_s - \eta_s - 4\delta_{G3X} - 12\delta_{G4X} - 32\delta_{G4XX} + 12\delta_{G5\phi} - 10\delta_{G5X} - 8\delta_{G5XX} , \tag{3.33}$$

$$\delta\mathcal{C}_2 = \epsilon_s + \eta_s - 2s + 4\delta_{G4X} + 2\delta_{G5X} - 4\delta_{G5\phi} , \tag{3.34}$$

$$\begin{aligned}\delta\mathcal{C}_3 = {} & \frac{1}{c_s^2}(\delta_{G3X} + 4\delta_{G4X} + 3\delta_{G5X} - \delta_{G4\phi} - 4\delta_{G5\phi} + 8\delta_{G4XX} + 2\delta_{G5XX}) \\ & - (3 + 2\lambda_{3X})\,\delta_{G3X} - 8\,(5 + 2\lambda_{4X})\,\delta_{G4XX} - 4(4 + \lambda_{5X})\delta_{G5XX} \\ & + \delta_{G4\phi} + 8\delta_{G5\phi} - 8\delta_{G4X} - 9\delta_{G5X} - 6\frac{c_s^2}{\epsilon_s}\left[(1 + \lambda_{3X})\delta_{G3X}^2 + \xi(\delta^2)\right] ,\end{aligned} \tag{3.35}$$

$$\delta\mathcal{C}_6 = (1 + \lambda_{3X})\delta_{G3X} + 4(3 + 2\lambda_{4X})\delta_{G4XX} + \delta_{G5X} + (5 + 2\lambda_{5X})\delta_{G5XX} , \tag{3.36}$$

$$\delta\mathcal{C}_7 = \delta_{G3X} + 6\delta_{G4XX} + \delta_{G5X} + \delta_{G5XX} , \tag{3.37}$$

$$\delta\mathcal{C}_8 = \delta_{G3X} + 4\delta_{G4XX} . \tag{3.38}$$

In order to derive the full expression of $\mathcal{A}_\mathcal{R}$ to the order of ϵ, we need to compute the corrections to the first three integrations in eq. (3.19). As studied in ref. [56] in the context of k-inflation, there are several corrections to the bispectrum (3.25).

The first one comes from the variation of the coefficients $\tilde{\mathcal{C}}_i$ $(i = 1, 2, 3)$, i.e.,

$$\tilde{\mathcal{C}}_i(\tau) = \tilde{\mathcal{C}}_i(\tau_K) - \frac{d\tilde{\mathcal{C}}_i}{dt}\frac{1}{H_K}\ln\frac{\tau}{\tau_K} + \mathcal{O}(\epsilon^2\tilde{\mathcal{C}}_i) . \tag{3.39}$$

We evaluate all the physical variables at the time $\tau_K = -1/(Kc_{sK})$, which corresponds to the moment when the wave number $K = k_1 + k_2 + k_3$ crosses the Hubble radius $Kc_{sK} = a_K H_K$.

The second one follows from the correction to the scale factor $a \simeq -1/(H\tau)$, i.e.,

$$a = -\frac{1}{H_K\tau} - \frac{\epsilon}{H_K\tau} + \frac{\epsilon}{H_K\tau}\ln(\tau/\tau_K) + \mathcal{O}(\epsilon^2) . \tag{3.40}$$

JCAP03(2013)030

Thirdly, the mode function (2.23) is subject to change by taking into account the $\mathcal{O}(\epsilon)$ terms on the r.h.s. of eq. (2.21):

$$u(y) = -\frac{\sqrt{\pi}}{2\sqrt{2}}\frac{1}{\sqrt{\epsilon_s F c_s}}\frac{H}{M_{\rm pl}}\frac{y^{3/2}}{k^{3/2}}\left(1+\frac{1}{2}\epsilon+\frac{1}{2}s\right)e^{i\frac{\pi}{2}(\epsilon+\frac{1}{2}\eta_{sF})}H_\nu^{(1)}[(1+\epsilon+s)y]\,, \tag{3.41}$$

where $y = c_s k/(aH)$, $\nu = 3/2+\epsilon+\eta_{sF}/2+s/2$, and $H_\nu^{(1)}(x)$ is the Hankel function of the first kind. In the large-scale limit ($y \to 0$) the Hankel function behaves as $H_\nu^{(1)}(x) \to -i/[\sin(\pi\nu)\Gamma(1-\nu)](x/2)^{-\nu}$ and hence the mode function approaches

$$u(0) = \frac{i}{2k^{3/2}}\frac{1}{\sqrt{(\epsilon_s F)_k\, c_{sk}}}\frac{H_k}{M_{\rm pl}}\frac{1}{k^{3/2}}\left[1-(\gamma_2+1)\epsilon-\frac{\gamma_2}{2}\eta_{sF}-\left(\frac{\gamma_2}{2}+1\right)s\right]e^{i\frac{\pi}{2}(\epsilon+\frac{1}{2}\eta_{sF})}\,, \tag{3.42}$$

where $\gamma_2 = \gamma_1 - 2 + \ln 2 \simeq -0.7296\ldots$ and $\gamma_1 = 0.5772\ldots$ is the Euler-Mascheroni constant. Note that the quantities with the subscript k in eq. (3.42) are evaluated at $c_{sk}k/(a_k H_k) = 1$. For the wave number k_i there is the running from k_i to K, as

$$\frac{1}{\sqrt{(\epsilon_s F)_{k_i}\, c_{sk_i}}}\frac{H_{k_i}}{M_{\rm pl}} = \frac{1}{\sqrt{(\epsilon_s F)_K\, c_{sK}}}\frac{H_K}{M_{\rm pl}}\left[1-\left(\epsilon+\frac{1}{2}\eta_{sF}+\frac{1}{2}s\right)\ln\frac{k_i}{K}\right]+\mathcal{O}(\epsilon^2)\,. \tag{3.43}$$

Writing the correction to the leading order solution (2.23) as $\Delta u^*(\tau, k)$, it follows that

$$\begin{aligned}\Delta u^*(\tau,k_i) = &-\frac{1}{2k_i^{3/2}\sqrt{(\epsilon_s F)_K\, c_{sK}}}\frac{H_K}{M_{\rm pl}}e^{-i\frac{\pi}{2}(\epsilon+\frac{1}{2}\eta_{sF})}e^{-ix}\Bigg[(\epsilon+s)(x-i)+isx^2\\ &+\left\{\left(\epsilon+\frac{1}{2}\eta_{sF}+\frac{1}{2}s\right)(i-x)-isx^2\right\}\ln\frac{\tau}{\tau_K}\\ &+\sqrt{\frac{\pi}{2}}e^{ix}\left(\epsilon+\frac{1}{2}\eta_{sF}+\frac{1}{2}s\right)x^{3/2}\left(\frac{dH_\nu^{(1)*}}{d\nu}\right)_{\nu=3/2}\Bigg],\end{aligned} \tag{3.44}$$

$$\begin{aligned}\frac{d}{d\tau}\Delta u^*(\tau,k_i) = &\frac{1}{2k_i^{3/2}\sqrt{(\epsilon_s F)_K\, c_{sK}}}\frac{H_K}{M_{\rm pl}}e^{-i\frac{\pi}{2}(\epsilon+\frac{1}{2}\eta_{sF})}k_i c_{sK}\,e^{-ix}\Bigg[x(sx-i\epsilon)\\ &+\left(\epsilon+\frac{1}{2}\eta_{sF}+\frac{1}{2}s\right)\left(\frac{i}{x}-1\right)+i\left(\epsilon+\frac{1}{2}\eta_{sF}-\frac{3}{2}s+isx\right)x\ln\frac{\tau}{\tau_K}\\ &+\sqrt{\frac{\pi}{2}}e^{ix}\left(\epsilon+\frac{1}{2}\eta_{sF}+\frac{1}{2}s\right)\frac{d}{dx}\left(x^{3/2}\left(\frac{dH_\nu^{(1)*}}{d\nu}\right)_{\nu=3/2}\right)\Bigg],\end{aligned} \tag{3.45}$$

where $x \equiv -k_i c_{sK}\,\tau$.

In appendix A we give the explicit forms of corrections to the first three terms in eq. (3.25). Each correction can be expressed as

$$\Delta\mathcal{A}_{\mathcal{R}}^{(1)} = \left(\frac{c_s^2}{4\epsilon_s F}\tilde{\mathcal{C}}_1^{\rm lead}\right)_K\delta Q_1\,,\ \Delta\mathcal{A}_{\mathcal{R}}^{(2)} = -\left(\frac{1}{4\epsilon_s F}\tilde{\mathcal{C}}_2^{\rm lead}\right)_K\delta Q_2\,,\ \Delta\mathcal{A}_{\mathcal{R}}^{(3)} = \left(\frac{3c_s^2 H}{4\epsilon_s F M_{\rm pl}}\tilde{\mathcal{C}}_3^{\rm lead}\right)_K\delta Q_3\,, \tag{3.46}$$

where δQ_1, δQ_2, and δQ_3 are the $\mathcal{O}(\epsilon)$ terms derived by summing up the contributions (A.1)–(A.5), (A.6)–(A.9), and (A.10)–(A.13), respectively. On using eqs. (3.30)–(3.32), it fol-

JCAP03(2013)030

lows that

$$\begin{aligned}\Delta\mathcal{A}_{\mathcal{R}} &= \Delta\mathcal{A}_{\mathcal{R}}^{(1)} + \Delta\mathcal{A}_{\mathcal{R}}^{(2)} + \Delta\mathcal{A}_{\mathcal{R}}^{(3)} \\ &= -\frac{1}{4}\left(\frac{1}{c_s^2} - 1 - \frac{2}{c_s^2}\frac{\delta\mathcal{C}_7}{\epsilon_s}\right)(3\delta Q_1 + \delta Q_2) + \frac{3}{4}\left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma} + 4\frac{\delta\mathcal{C}_6}{\epsilon_s} - \frac{4}{c_s^2}\frac{\delta\mathcal{C}_7}{\epsilon_s}\right)\delta Q_3\,.\end{aligned} \tag{3.47}$$

The explicit forms of $3\delta Q_1 + \delta Q_2$ and δQ_3 are

$$\begin{aligned}3\delta Q_1 + \delta Q_2 = &\\ &\Bigg[-2(7+2\gamma_1+6\gamma_2)\epsilon + 3(1-2\gamma_1-2\gamma_2)\eta_{sF} - (21-14\gamma_1+6\gamma_2)s - 2(1-2\gamma_1)\tilde{\eta}_1 \\ &- 2(2\epsilon+\eta_{sF}+s)\ln\frac{k_1k_2k_3}{K^3}\Bigg]\times\frac{1}{K}\sum_{i>j}k_i^2k_j^2 + \Bigg[2(3+\gamma_1+3\gamma_2)\epsilon - 3(1-\gamma_1-\gamma_2)\eta_{sF} \\ &+ \left(17 - \frac{35}{2}\gamma_1 + 3\gamma_2\right)s + 2(1-\gamma_1)\tilde{\eta}_1 + (2\epsilon+\eta_{sF}+s)\ln\frac{k_1k_2k_3}{K^3}\Bigg]\frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3 \\ &+ 3(2\gamma_1-1)s\left[\frac{1}{K^3}\sum_{i\neq j}k_i^2k_j^4 + \frac{2}{K^3}\sum_{i>j}k_i^3k_j^3 - 3\frac{(k_1k_2k_3)^2}{K^3}\right] \\ &- \frac{1}{4}\left[22\epsilon + 9\eta_{sF} + 2\tilde{\eta}_1 + (11+4\gamma_1)s\right]k_1k_2k_3 - \left(\frac{1}{2}\epsilon + \frac{3}{4}\eta_{sF} - \frac{1}{2}\tilde{\eta}_1 - \frac{1}{4}s\right)\sum_{i\neq j}k_ik_j^2 \\ &+ \Bigg[\frac{1}{2}(3+\gamma_1+3\gamma_2)\epsilon - \frac{3}{4}(1-\gamma_1-\gamma_2)\eta_{sF} + \frac{1}{2}(1-\gamma_1)\tilde{\eta}_1 + \frac{1}{4}(13-\gamma_1+3\gamma_2)s \\ &+ \frac{1}{4}(2\epsilon+\eta_{sF}+s)\ln\frac{k_1k_2k_3}{K^3}\Bigg]\sum_i k_i^3 - \frac{1}{2}(1+\gamma_1)s\frac{1}{K}\sum_i k_i^4 - \frac{1}{2}\gamma_1 s\frac{1}{K^2}\sum_{i\neq j}k_ik_j^4 \\ &+ \frac{3}{2}(2\epsilon+\eta_{sF}+s)\left(2\sum_i k_i^3 + 2\sum_{i\neq j}k_ik_j^2 - \mathcal{V}\right),\end{aligned} \tag{3.48}$$

$$\begin{aligned}\delta Q_3 = &\\ &\Bigg[-2(2+2\gamma_1+3\gamma_2)\epsilon + 3\left(\frac{3}{2} - \gamma_1 - \gamma_2\right)\eta_{sF} + \left(3\gamma_1 - 3\gamma_2 - \frac{29}{2}\right)s + (2\gamma_1-3)\tilde{\eta}_3 \\ &- (2\epsilon+\eta_{sF}+s)\ln\frac{k_1k_2k_3}{K^3}\Bigg]\frac{(k_1k_2k_3)^2}{K^3} \\ &+ \left(\epsilon + \frac{1}{2}\eta_{sF} + \frac{1}{2}s\right)\left(\frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3 - \frac{2}{K}\sum_{i>j}k_i^2k_j^2 + \mathcal{U}\right),\end{aligned} \tag{3.49}$$

JCAP03(2013)030

where $\tilde{\eta}_i = (d\tilde{\mathcal{C}}_i^{\rm lead}/dt)/(H\tilde{\mathcal{C}}_i^{\rm lead})$, and

$$\begin{aligned}
\mathcal{V} &\equiv \mathcal{M} - (k_2^3+k_3^3){\rm Re}\left[\int_0^\infty dx_1 \frac{e^{-iKx_1/k_1}}{x_1}\right] - \frac{k_2^2k_3^2}{k_1}\mathcal{G} - \frac{1}{3}\mathcal{N} + {\rm perm.} \\
&= -k_1{\rm Re}\left[\int_0^\infty dx_1 \frac{1}{x_1}\left(k_2^2+k_3^2+ik_2k_3\frac{k_2+k_3}{k_1}x_1\right)e^{-i\frac{k_2+k_3}{k_1}x_1}\frac{dh^*(x_1)}{dx_1}\right] \\
&\quad - (k_2^3+k_3^3){\rm Re}\left[\int_0^\infty dx_1 \frac{e^{-iKx_1/k_1}}{x_1}\right] \\
&\quad + \frac{k_1}{6}\sum_i k_i^2\,{\rm Re}\left[\int_0^\infty dx_1 \frac{1}{x_1^2}e^{-ix_1\frac{k_2+k_3}{k_1}}\left(1+i\frac{k_2+k_3}{k_1}x_1 - \frac{k_2k_3}{k_1^2}x_1^2\right)h^*(x_1)\right] \\
&\quad - \frac{k_2^2k_3^2}{k_1}{\rm Re}\left[\int_0^\infty dx_1\, h^*(x_1)e^{-i\frac{k_2+k_3}{k_1}x_1}\right] + {\rm perm.}\,, \qquad (3.50)\\
\mathcal{U} &\equiv \frac{k_2^2k_3^2}{k_1}\,{\rm Re}\left[\int_0^\infty dx_1\, h^*(x_1)\left(1-i\frac{k_2+k_3}{k_1}x_1\right)e^{-i\frac{k_2+k_3}{k_1}x_1}\right] + {\rm perm.}\,, \qquad (3.51)\\
h(x) &\equiv \sqrt{\frac{\pi}{2}}\,x^{3/2}\left[\frac{dH_\nu^{(1)}(x)}{d\nu}\right]_{\nu=3/2} \\
&= -2ie^{ix} + ie^{-ix}(1+ix)\left[{\rm Ci}(2x) + i\,{\rm Si}(2x)\right] - i\pi\sin x + i\pi x\cos x\,. \qquad (3.52)
\end{aligned}$$

The definition of $\mathcal{M}$, $\mathcal{G}$, and $\mathcal{N}$ is given in appendix A. In eq. (3.50), we have used the relation between the variables $x_K = -Kc_{sK}\tau$ and $x_i = -k_ic_{sK}\tau$ $(i=1,2,3)$, as $x_K = (K/k_i)x_i$ (without summation over i). The variable $x_K = -Kc_{sK}\tau$ is related to $x_i = -k_ic_{sK}\tau$ $(i=1,2,3)$, as $x_K = (K/k_i)x_i$. The symbol "perm." stands for cyclic permutations with respect to k_1, k_2, and k_3. In eq. (3.48) we also used the relation $\tilde{\eta}_2 = \tilde{\eta}_1 + 2s$ to eliminate $\tilde{\eta}_2$ (which follows from $\tilde{\mathcal{C}}_2^{\rm lead} = -(c_s^2/3)\tilde{\mathcal{C}}_1^{\rm lead}$). In appendix B we evaluate the values of $\mathcal{V}$ and $\mathcal{U}$ as functions of $r_2 \equiv k_2/k_1$ and $r_3 \equiv k_3/k_1$.

The total bispectrum $\mathcal{A}_\mathcal{R}$ is the sum of eqs. (3.25) and (3.47), which can be written as

$$\mathcal{A}_\mathcal{R} = \mathcal{A}_\mathcal{R}^{\rm lead} + \mathcal{A}_\mathcal{R}^{\rm corre}\,, \qquad (3.53)$$

where

$$\begin{aligned}
\mathcal{A}_\mathcal{R}^{\rm lead} &= \left[\frac{1}{4}\left(1-\frac{1}{c_s^2}\right) + \frac{1}{2c_s^2}\frac{\delta\mathcal{C}_7}{\epsilon_s}\right](3S_1 - S_2) \qquad (3.54)\\
&\quad + \left[\frac{3}{2}\left(\frac{1}{c_s^2}-1\right) - \frac{3\lambda}{\Sigma} + \frac{6\delta\mathcal{C}_6}{\epsilon_s} - \frac{6}{c_s^2}\frac{\delta\mathcal{C}_7}{\epsilon_s}\right]S_3\,, \\
\mathcal{A}_\mathcal{R}^{\rm corre} &= \frac{1}{4c_s^2}\left[\delta\mathcal{C}_1 + (2\epsilon + 7\eta_{sF} - 5\eta_7)\frac{\delta\mathcal{C}_7}{\epsilon_s} + 3\delta\mathcal{C}_8\right]S_1 \qquad (3.55)\\
&\quad + \frac{1}{4c_s^2}\left[\delta\mathcal{C}_2 + (2\epsilon - \eta_{sF} - \eta_7 + 4s)\frac{\delta\mathcal{C}_7}{\epsilon_s} - \delta\mathcal{C}_8\right]S_2 \\
&\quad + \left[\frac{3}{2}\delta\mathcal{C}_3 + (3\eta_{sF} - \eta_6 - 4s)\frac{\delta\mathcal{C}_6}{\epsilon_s} - \frac{1}{c_s^2}(3\eta_{sF} - 2\eta_7 - s)\frac{\delta\mathcal{C}_7}{\epsilon_s}\right]S_3 - \frac{1}{4c_s^2}(\epsilon_s - 3\delta\mathcal{C}_8)S_4 \\
&\quad - \frac{1}{4}\left(\frac{1}{c_s^2} - 1 - \frac{2}{c_s^2}\frac{\delta\mathcal{C}_7}{\epsilon_s}\right)(3\delta Q_1 + \delta Q_2) + \frac{3}{4}\left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma} + 4\frac{\delta\mathcal{C}_6}{\epsilon_s} - \frac{4}{c_s^2}\frac{\delta\mathcal{C}_7}{\epsilon_s}\right)\delta Q_3\,.
\end{aligned}$$

The leading-order bispectrum $\mathcal{A}_\mathcal{R}^{\rm lead}$ (given already in refs. [61, 62]) and the correction $\mathcal{A}_\mathcal{R}^{\rm corre}$ are of the orders of $\mathcal{O}(\epsilon^0)$ and $\mathcal{O}(\epsilon)$, respectively.

JCAP03(2013)030

4 Local, equilateral, and enfolded non-Gaussianities

The non-linear parameter characterizing the strength of non-Gaussianities is defined by

$$f_{\rm NL} = \frac{10}{3}\frac{\mathcal{A}_{\mathcal{R}}}{\sum_{i=1}^{3} k_i^3}\,. \tag{4.1}$$

In the following we estimate $f_{\rm NL}$ for three different shapes of non-Gaussianities.

4.1 Local non-Gaussianities

The local shape corresponds to $k_3 \to 0$ and $k_2 \to k_1 \equiv k$, in which case $f_{\rm NL}^{\rm local} = (5/3)\mathcal{A}_{\mathcal{R}}/k^3$. Since $S_1 = k^3/2$, $S_2 = 3k^3/2 = 3S_1$, and $S_3 = S_4 = 0$, the leading-order bispectrum (3.54) vanishes. In the limit that $k_3 \to 0$ the function $\mathcal{U}$ given by eq. (3.51) approaches $k^3/2$ [56] (see also appendix B), so that the term δQ_3 in eq. (3.49) vanishes. Then the bispectrum (3.55) reduces to

$$\begin{aligned}\mathcal{A}_{\mathcal{R}}^{\rm corre} &= \frac{k^3}{8c_s^2}(\delta\mathcal{C}_1 + 3\delta\mathcal{C}_2) - \frac{1}{4}\left(\frac{1}{c_s^2} - 1\right)(3\delta Q_1 + \delta Q_2) \\ &\quad + \frac{\delta\mathcal{C}_7}{8c_s^2\epsilon_s}\left[4(2\epsilon + \eta_{sF} - 2\eta_7 + 3s)k^3 + 4(3\delta Q_1 + \delta Q_2)\right]\,. \end{aligned}\tag{4.2}$$

Using eqs. (3.33) and (3.34) together with the relations (2.9), (2.17), and (2.22), we have $\delta\mathcal{C}_1 + 3\delta\mathcal{C}_2 = 4\epsilon + 2\eta_{sF} - 6s$. In the limit $k_3 \to 0$ the function $\mathcal{V}$ behaves as $\mathcal{V} \to 20k^3/3$ [56] (see also appendix B), so that eq. (3.48) reduces to $3\delta Q_1 + \delta Q_2 \to (2\epsilon - \eta_{sF} + 5s + 2\tilde{\eta}_1)k^3$. Then eq. (4.2) reads

$$\mathcal{A}_{\mathcal{R}}^{\rm corre} = \frac{k^3}{4}(2\epsilon - \eta_{sF} + 5s + 2\tilde{\eta}_1) + \frac{k^3}{2c_s^2}(\eta_{sF} - 4s - \tilde{\eta}_1) + k^3\frac{\delta\mathcal{C}_7}{c_s^2\epsilon_s}(2\epsilon + 4s + \tilde{\eta}_1 - \eta_7)\,. \tag{4.3}$$

Taking the time-derivatives of $\tilde{\mathcal{C}}_1^{\rm lead}$ and $\mathcal{C}_7$, we obtain the following relation

$$\frac{\delta\mathcal{C}_7}{c_s^2\epsilon_s}(2\epsilon + 4s + \tilde{\eta}_1 - \eta_7) = \frac{1}{2}(\eta_{sF} - \tilde{\eta}_1 - 2s) - \frac{1}{2c_s^2}(\eta_{sF} - 4s - \tilde{\eta}_1)\,. \tag{4.4}$$

Substituting eq. (4.4) into eq. (4.3), it follows that

$$\mathcal{A}_{\mathcal{R}}^{\rm corre} = \frac{k^3}{4}(2\epsilon + \eta_{sF} + s)\,. \tag{4.5}$$

Using the spectral index $n_{\mathcal{R}}$ given in eq. (2.25), the non-linear parameter is expressed as

$$f_{\rm NL}^{\rm local} = \frac{5}{12}(1 - n_{\mathcal{R}})\,. \tag{4.6}$$

This matches with the Maldacena's result [12] derived for a canonical scalar field (see also refs. [50, 53, 56] for the derivation of the same relation in other single field models). Creminelli and Zaldarriaga [47] pointed out that the consistency relation (4.6) should hold for any slow-variation single-field inflation. In fact we have shown that this holds for most general single-field scalar-tensor theories with second-order equations of motion by explicitly computing the slow-variation corrections to the bispectrum (3.25). Since $|f_{\rm NL}^{\rm local}|$ is much smaller than 1 in such models, the observational detection of local non-Gaussianities with $|f_{\rm NL}^{\rm local}| \gtrsim 1$ implies that we need to go beyond the slow-variation single-field scenario.

JCAP03(2013)030

In the limit that $k_3 \to 0$ the shape functions S_6, S_7, and S_8 vanish. This means that the functions $\mathcal{C}_6$, $\mathcal{C}_7$, $\mathcal{C}_8$ on the r.h.s. of eqs. (3.20)–(3.23) do not contribute to the local non-Gaussianities. In fact, we can derive the consistency relation (4.6) by setting $\delta\mathcal{C}_7 = 0$ in eqs. (4.2) and (4.4). In this sense the situation is analogous to that in k-inflation.

Let us consider the not-so squeezed case in which the ratio $r_3 = k_3/k_1$ is non-vanishing, i.e., $0 < r_3 \ll 1$ and $k_1 = k_2$. The leading-order non-linear parameter following from eq. (3.54) is given by

$$f_{\rm NL}^{\rm lead} = \frac{5r_3^2\left[\alpha_1(22+4r_3-4r_3^2-r_3^3)(2+r_3)+2\alpha_2\right]}{3(2+r_3)^3(2+r_3^3)}\,, \tag{4.7}$$

where

$$\alpha_1 = \frac{1}{4}\left(1-\frac{1}{c_s^2}\right)+\frac{1}{2c_s^2}\frac{\delta\mathcal{C}_7}{\epsilon_s}\,, \qquad \alpha_2 = \frac{3}{2}\left(\frac{1}{c_s^2}-1\right)-\frac{3\lambda}{\Sigma}+\frac{6\delta\mathcal{C}_6}{\epsilon_s}-\frac{6}{c_s^2}\frac{\delta\mathcal{C}_7}{\epsilon_s}\,. \tag{4.8}$$

In the regime $r_3 \ll 1$ we have $f_{\rm NL}^{\rm lead} \simeq 5(22\alpha_1+\alpha_2)r_3^2/24$. The values of α_1 and α_2 depend on the models, but for $c_s^2 \ll 1$ they are at most of the order of $1/c_s^2$. In this case the leading-order non-linear parameter can be estimated as

$$\left|f_{\rm NL}^{\rm lead}\right| \approx \frac{r_3^2}{c_s^2}\,. \tag{4.9}$$

Then the transition from the value (4.6) to the value (4.9) occurs at

$$r_3 \approx c_s\sqrt{1-n_{\mathcal{R}}}\,. \tag{4.10}$$

The effect of the term $1/c_s^2$ in eq. (4.9) becomes important for $r_3 > c_s\sqrt{1-n_{\mathcal{R}}}$. When $n_{\mathcal{R}} = 0.96$ this condition translates into $r_3 > 0.2\,c_s$. If $c_s = 0.1$ and $r_3 > 0.1$, for example, the non-linear parameter (4.9) can be larger than the order of 1. However, for $c_s = 0.1$, we also expect the dominant contribution to $f_{\rm NL}$ to come from other shapes (equilateral, orthogonal, etc.)

For the models in which c_s^2 is close to 1, α_1 and α_2 are of the order of unity. Hence the leading-order non-linear parameter can be estimated as $\left|f_{\rm NL}^{\rm lead}\right| \approx r_3^2 \ll 1$ in the regime $r_3 \ll 1$. By increasing the value of r_3 from 0, we can observationally discriminate between the models with $c_s^2 \ll 1$ and $c_s^2 \approx 1$.

4.2 Equilateral non-Gaussianities

The equilateral shape is characterized by $k_1 = k_2 = k_3 \equiv k$, in which case the non-linear parameter is $f_{\rm NL}^{\rm equil} = (10/9)(\mathcal{A}_{\mathcal{R}}/k^3)$. Since $S_1 = 4k^3/3$, $S_2 = 17k^3/6$, and $S_3 = k^3/27$, the bispectrum (3.54) gives the leading-order non-linear parameter

$$\begin{aligned} f_{\rm NL}^{\rm equil,lead} =& \frac{85}{324}\left(1-\frac{1}{c_s^2}\right)-\frac{10}{81}\frac{\lambda}{\Sigma}+\frac{20}{81\epsilon_s}\left[(1+\lambda_{3X})\delta_{G3X}+4(3+2\lambda_{4X})\delta_{G4XX}\right. \\ &\left.+\delta_{G5X}+(5+2\lambda_{5X})\delta_{G5XX}\right]+\frac{65}{162c_s^2\epsilon_s}(\delta_{G3X}+6\delta_{G4XX}+\delta_{G5X}+\delta_{G5XX})\,. \end{aligned} \tag{4.11}$$

If $c_s^2 \ll 1$, then we have $|f_{\rm NL}^{\rm equil,lead}| \gg 1$.

JCAP03(2013)030

In the equilateral limit the functions $\mathcal{V}$ and $\mathcal{U}$ are given by $\mathcal{V} = 15[1 + (1/2)\ln(2/3)]k^3$ and $\mathcal{U} = [6\ln(3/2) - 1]k^3$, respectively (see appendix B). Then the functions $\delta\tilde{Q}_{12} \equiv (3\delta Q_1 + \delta Q_2)/k^3$ and $\delta\tilde{Q}_3 \equiv \delta Q_3/k^3$ reduce to

$$\begin{aligned}\delta\tilde{Q}_{12} &= \epsilon[6 - 78\ln(2/3) - 14\gamma_1]/3 - \eta_{sF}[7\gamma_1 + 26\ln(2/3)]/2 \\ &\quad + s[16 - 39\ln(2/3)]/3 + \tilde{\eta}_1(7\gamma_1 + 20)/6\,, \qquad (4.12)\\ \delta\tilde{Q}_3 &= -\epsilon[10\gamma_1 + 55 + 168\ln(2/3)]/27 - \eta_{sF}[2\gamma_1 + 7 + 28\ln(2/3)]/9 \\ &\quad - 4s[10 + 21\ln(2/3)]/27 + \tilde{\eta}_3(2\gamma_1 - 3)/27. \qquad (4.13)\end{aligned}$$

The correction to $f_{\rm NL}^{\rm equil,lead}$ coming from eq. (4.3) is given by

$$\begin{aligned}f_{\rm NL}^{\rm equil,corre} &= \frac{5}{972c_s^2\epsilon_s}\Big[3\epsilon_s(24\delta\mathcal{C}_1 + 51\delta\mathcal{C}_2 + 4c_s^2\delta\mathcal{C}_3) + 8c_s^2(3\eta_{sF} - 4s - \eta_6)\delta\mathcal{C}_6 \\ &\quad + (450\epsilon + 327\eta_{sF} + 620s - 497\eta_7)\delta\mathcal{C}_7 - 153\epsilon_s\delta\mathcal{C}_8 + 72\epsilon_s^2\Big] \\ &\quad - \frac{5}{18}\left(\frac{1}{c_s^2} - 1 - \frac{2\delta\mathcal{C}_7}{c_s^2\epsilon_s}\right)\delta\tilde{Q}_{12} + \frac{5}{6}\left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma} + \frac{4\delta\mathcal{C}_6}{\epsilon_s} - \frac{4\delta\mathcal{C}_7}{c_s^2\epsilon_s}\right)\delta\tilde{Q}_3. \qquad (4.14)\end{aligned}$$

For the theories in which $f_{\rm NL}^{\rm equil,lead}$ vanishes, the next-order correction $f_{\rm NL}^{\rm equil,corre}$ is the dominant contribution. In the case of a canonical scalar field with the Lagrangian $P = X - V(\phi)$, $G_3 = 0$, $G_4 = 0$, $G_5 = 0$, for example, it follows that $f_{\rm NL}^{\rm equil,lead} = 0$ and $f_{\rm NL}^{\rm equil,corre} = 55\epsilon_s/36 + 5\eta_s/12$.

For the theories with $c_s^2 \neq 1$ the non-linear parameters (4.11) and (4.14) reproduce the results known in literature for specific models of inflation. For example, this is the case for k-inflation [18, 56], k-inflation with the Galileon terms [62, 63], potential-driven Galileon inflation [73], and inflation with a field derivative coupling to the Einstein tensor [53]. Generally we require that $c_s^2 \ll 1$ to realize the large equilateral non-linear parameter.

4.3 Enfolded non-Gaussianities

The enfolded shape is characterized by $k_2 + k_3 = k_1$. Taking the momenta $k_1 = k$ and $k_2 \to k_3 = k/2$, the non-linear parameter[4] is $f_{\rm NL}^{\rm enfold} = 8\mathcal{A}_\mathcal{R}/(3k^3)$. Since $S_1 = 23k^3/64$, $S_2 = 63k^3/64$, and $S_3 = k^3/128$ in this case, the leading-order non-linear parameter is given by

$$\begin{aligned}f_{\rm NL}^{\rm enfold,lead} &= \frac{1}{32}\left(1 - \frac{1}{c_s^2}\right) - \frac{1}{16}\frac{\lambda}{\Sigma} \qquad (4.15)\\ &\quad + \frac{1}{8\epsilon_s}\left[(1 + \lambda_{3X})\delta_{G3X} + 4(3 + 2\lambda_{4X})\delta_{G4XX} + \delta_{G5X} + (5 + 2\lambda_{5X})\delta_{G5XX}\right],\end{aligned}$$

where, unlike the equilateral case, the $\delta\mathcal{C}_7$-dependent term in eq. (3.54) disappears.

[4] At the point $k_1 = k$ and $k_2 \to k_3 = k/2$, the equilateral shape gives no contribution. However, the contribution from the orthogonal shape will be of the same order of the enfolded one, since, by definition, in this case we have $f_{\rm NL}^{\rm ortho} \to -2f_{\rm NL}^{\rm enfold}$.

JCAP03(2013)030

In the enfolded limit one has $\mathcal{V} = [315/64 - \ln(2)/2]k^3$ and $\mathcal{U} = [5/24 + \ln(2)/4]k^3$ (see appendix B). The functions $\delta\tilde{Q}_{12}$ and $\delta\tilde{Q}_3$ are

$$\begin{aligned}\delta\tilde{Q}_{12} &= \epsilon[45 - 24\gamma_1 + 108\ln(2)]/64 - \eta_{sF}[87 + 36\gamma_1 - 108\ln(2)]/128 \\ &\quad + s[333 + 108\ln(2)]/128 + \tilde{\eta}_1(39 + 3\gamma_1)/32\,, \qquad (4.16)\\ \delta\tilde{Q}_3 &= -\epsilon[15\gamma_1 + 17 - 54\ln(2)]/192 - \eta_{sF}[36\gamma_1 - 5 - 108\ln(2)]/768 \\ &\quad - s[109 - 108\ln(2)]/768 + \tilde{\eta}_3(2\gamma_1 - 3)/128\,. \qquad (4.17)\end{aligned}$$

The correction to $f_{\rm NL}^{\rm enfold,lead}$ is

$$\begin{aligned}f_{\rm NL}^{\rm enfold,corre} &= \frac{1}{96c_s^2\epsilon_s}[\epsilon_s(23\delta\mathcal{C}_1 + 63\delta\mathcal{C}_2 + 3c_s^2\delta\mathcal{C}_3) - 2c_s^2(\eta_6 + 4s - 3\eta_{sF})\delta\mathcal{C}_6 \qquad (4.18)\\ &\quad + (172\epsilon + 254s + 92\eta_{sF} - 174\eta_7)\delta\mathcal{C}_7 \\ &\quad + 2\epsilon_s^2] - \frac{2}{3}\left(\frac{1}{c_s^2} - 1 - \frac{2\delta\mathcal{C}_7}{c_s^2\epsilon_s}\right)\delta\tilde{Q}_{12} + 2\left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma} + \frac{4\delta\mathcal{C}_6}{\epsilon_s} - \frac{4\delta\mathcal{C}_7}{c_s^2\epsilon_s}\right)\delta\tilde{Q}_3\,.\end{aligned}$$

For a canonical scalar field we have that $f_{\rm NL}^{\rm enfold,lead} = 0$ and $f_{\rm NL}^{\rm enfold,corre} = 7\epsilon_s/8 + 5\eta_s/12$.

5 Shapes of non-Gaussianities

The leading-order bispectrum (3.16) can be written in terms of the sum of each component, as $\mathcal{A}_{\mathcal{R}} = \sum_{i=1}^{8}\mathcal{A}_{\mathcal{R}}^{(i)}$. Then the function $\mathcal{F}_{\mathcal{R}}(k_1, k_2, k_3)$ in eq. (3.15) is decomposed into eight components

$$\mathcal{F}_{\mathcal{R}}^{(i)} = (2\pi)^4\frac{\mathcal{A}_{\mathcal{R}}^{(i)}}{\prod_{i=1}^{3}k_i^3} = \frac{(2\pi)^4}{(k_1k_2k_3)^3}\mathcal{B}_iS_i\,, \qquad (5.1)$$

where $\mathcal{B}_i$'s are the coefficients appearing in front of each shape function S_i in eq. (3.16), say, $\mathcal{B}_1 = c_s^2\mathcal{C}_1/(4\epsilon_sF)$.

In order to estimate the correlation between two different shapes, we define the following quantity [42]

$$C(\mathcal{F}_{\mathcal{R}}^{(i)}, \mathcal{F}_{\mathcal{R}}^{(j)}) = \frac{\mathcal{I}(\mathcal{F}_{\mathcal{R}}^{(i)}, \mathcal{F}_{\mathcal{R}}^{(j)})}{\sqrt{\mathcal{I}(\mathcal{F}_{\mathcal{R}}^{(i)}, \mathcal{F}_{\mathcal{R}}^{(i)})\,\mathcal{I}(\mathcal{F}_{\mathcal{R}}^{(j)}, \mathcal{F}_{\mathcal{R}}^{(j)})}}\,, \qquad (5.2)$$

where

$$\mathcal{I}(\mathcal{F}_{\mathcal{R}}^{(i)}, \mathcal{F}_{\mathcal{R}}^{(j)}) = \int d\mathcal{V}_k\,\mathcal{F}_{\mathcal{R}}^{(i)}(k_1, k_2, k_3)\mathcal{F}_{\mathcal{R}}^{(j)}(k_1, k_2, k_3)\frac{(k_1k_2k_3)^4}{(k_1 + k_2 + k_3)^3}\,. \qquad (5.3)$$

The integration should be done in the region $0 \le k_1 < \infty$, $0 < k_2/k_1 < 1$, and $1 - k_2/k_1 \le k_3/k_1 \le 1$. Note that the above integral can be expressed in terms of $r_2 = k_2/k_1$ and $r_3 = k_3/k_1$ with the integral of k_1 factorized out. For $|C(\mathcal{F}_{\mathcal{R}}^{(i)}, \mathcal{F}_{\mathcal{R}}^{(j)})|$ close to 1 the correlation is large, whereas for $|C(\mathcal{F}_{\mathcal{R}}^{(i)}, \mathcal{F}_{\mathcal{R}}^{(j)})|$ close to 0 the two shapes are almost orthogonal with a small correlation.

The CMB data analysis of non-Gaussianities has been carried out by using the factorizable shape functions which are written as the sums of monomials of k_1, k_2, and k_3. There are a number of templates $\mathcal{F}_{\mathcal{R}}$ which resemble model predictions of the bispectrum. The templates corresponding to local and equilateral non-Gaussianities are given, respectively, by [37, 41]

$$\mathcal{F}_{\mathcal{R}}^{\rm local}(k_1, k_2, k_3) = (2\pi)^4\left(\frac{3}{10}f_{\rm NL}^{\rm local}\right)\left(\frac{1}{k_1^3k_2^3} + \frac{1}{k_2^3k_3^3} + \frac{1}{k_3^3k_1^3}\right)\,, \qquad (5.4)$$

JCAP03(2013)030

	$\mathcal{F}_{\mathcal{R}}^{\rm equil}$	$\mathcal{F}_{\mathcal{R}}^{\rm ortho}$	$\mathcal{F}_{\mathcal{R}}^{\rm enfold}$	$\mathcal{F}_{\mathcal{R}}^{(3)}$	$\mathcal{F}_{\mathcal{R}}^{(4)}$	$\mathcal{F}_{\mathcal{R}}^{(5)}$	$\mathcal{F}_{\mathcal{R}}^{(7{\rm equil})}$	$\mathcal{F}_{\mathcal{R}}^{(8)}$	$\mathcal{F}_{\mathcal{R}}^{(7{\rm ortho})}$	$3\mathcal{F}_{\mathcal{R}}^{(1)}-\mathcal{F}_{\mathcal{R}}^{(2)}$
$\mathcal{F}_{\mathcal{R}}^{\rm equil}$	1	0.0254062	0.511911	0.936177	-0.998757	-0.994234	0.999892	-0.999994	-0.00693357	0.986954
$\mathcal{F}_{\mathcal{R}}^{\rm ortho}$		1	-0.845755	-0.290742	0.0177139	-0.117961	0.0353534	-0.0277283	0.904843	-0.116557
$\mathcal{F}_{\mathcal{R}}^{\rm enfold}$			1	0.749518	-0.548302	-0.4293	0.503306	-0.509913	-0.781246	0.626939
$\mathcal{F}_{\mathcal{R}}^{(3)}$				1	-0.952469	-0.893224	0.933797	-0.935615	-0.357802	0.980504
$\mathcal{F}_{\mathcal{R}}^{(4)}$					1	0.987653	-0.998384	0.998686	0.056402	-0.993745
$\mathcal{F}_{\mathcal{R}}^{(5)}$						1	-0.994696	0.99437	-0.0995524	-0.964012
$\mathcal{F}_{\mathcal{R}}^{(7{\rm equil})}$							1	-0.999936	0	0.9859
$\mathcal{F}_{\mathcal{R}}^{(8)}$								1	0.00524865	-0.986715
$\mathcal{F}_{\mathcal{R}}^{(7{\rm ortho})}$									1	-0.167335

Table 1. The correlation (5.2) between two different shape functions. $\mathcal{F}_{\mathcal{R}}^{(7{\rm equil})}$ is the normalized shape of $\mathcal{F}_{\mathcal{R}}^{(7)}$, whereas $\mathcal{F}_{\mathcal{R}}^{(7{\rm ortho})}$ is the shape function orthogonal to $\mathcal{F}_{\mathcal{R}}^{(7{\rm equil})}$ (or, equivalently, to $\mathcal{F}_{\mathcal{R}}^{(7)}$).

and

$$\mathcal{F}_{\mathcal{R}}^{\rm equil}(k_1,k_2,k_3)=(2\pi)^4\left(\frac{9}{10}f_{\rm NL}^{\rm equil}\right)\left[-\frac{1}{k_1^3k_2^3}-\frac{1}{k_2^3k_3^3}-\frac{1}{k_3^3k_1^3}-\frac{2}{k_1^2k_2^2k_3^2}+\left(\frac{1}{k_1k_2^2k_3^3}+5\,{\rm perm.}\right)\right]. \tag{5.5}$$

Since the local non-Gaussianities are small in the Horndeski's theories, we do not consider the correlation with the local template.

The orthogonal template, which has a small correlation with the equilateral one, is given by [43]

$$\mathcal{F}_{\mathcal{R}}^{\rm ortho}(k_1,k_2,k_3)=(2\pi)^4\left(\frac{9}{10}f_{\rm NL}^{\rm ortho}\right)\left[-\frac{3}{k_1^3k_2^3}-\frac{3}{k_2^3k_3^3}-\frac{3}{k_3^3k_1^3}-\frac{8}{k_1^2k_2^2k_3^2}+\left(\frac{3}{k_1k_2^2k_3^3}+5\,{\rm perm.}\right)\right]. \tag{5.6}$$

The enfolded template, which is a linear combination of the orthogonal and equilateral templates, is defined by [44]

$$\mathcal{F}_{\mathcal{R}}^{\rm enfold}(k_1,k_2,k_3)=(2\pi)^4\left(\frac{9}{10}f_{\rm NL}^{\rm enf}\right)\left[\frac{1}{k_1^3k_2^3}+\frac{1}{k_2^3k_3^3}+\frac{1}{k_3^3k_1^3}+\frac{3}{k_1^2k_2^2k_3^2}-\left(\frac{1}{k_1k_2^2k_3^3}+5\,{\rm perm.}\right)\right]. \tag{5.7}$$

In table 1 we show the correlation between $\mathcal{F}_{\mathcal{R}}^{(i)}$ $(i=3,4,5,7,8)$ and the three templates given above. Since the correlations between $\mathcal{F}_{\mathcal{R}}^{(i)}$ $(i=4,5,7,8)$ and $\mathcal{F}_{\mathcal{R}}^{\rm equil}$ are close to 1, $\mathcal{F}_{\mathcal{R}}^{(i)}$ $(i=4,5,7,8)$ are well approximated by the equilateral shape. In particular, both $C(\mathcal{F}_{\mathcal{R}}^{(7)},\mathcal{F}_{\mathcal{R}}^{\rm ortho})$ and $C(\mathcal{F}_{\mathcal{R}}^{(8)},\mathcal{F}_{\mathcal{R}}^{\rm ortho})$ are close to 0 with the same level of correlation between $\mathcal{F}_{\mathcal{R}}^{\rm equil}$ and $\mathcal{F}_{\mathcal{R}}^{\rm ortho}$. Hence the shape functions S_7 and S_8 highly mimic the equilateral template. They also vanish in the local limit ($k_3\to 0$) and in the enfolded limit ($k_2+k_3\to k_1$). Note that both $|C(\mathcal{F}_{\mathcal{R}}^{(7)},\mathcal{F}_{\mathcal{R}}^{\rm enfold})|$ and $|C(\mathcal{F}_{\mathcal{R}}^{(8)},\mathcal{F}_{\mathcal{R}}^{\rm enfold})|$ are close to 0.5 (which is similar to the value $|C(\mathcal{F}_{\mathcal{R}}^{\rm equil},\mathcal{F}_{\mathcal{R}}^{\rm enfold})|=0.511911$). Since there is the relation $\mathcal{F}_{\mathcal{R}}^{\rm enfold}=(\mathcal{F}_{\mathcal{R}}^{\rm equil}-\mathcal{F}_{\mathcal{R}}^{\rm ortho})/2$, we have $|C(\mathcal{F}_{\mathcal{R}}^{(i)},\mathcal{F}_{\mathcal{R}}^{\rm enfold})|\simeq 0.5$ for the shape function $\mathcal{F}_{\mathcal{R}}^{(i)}$ very close to the equilateral one.

We recall that the functions $3S_1-S_2$ and S_3 are related with S_7 [see eq. (3.18)]. We can use this property in order to rewrite the leading-order part of the bispectrum in a convenient

JCAP03(2013)030

basis. We introduce the following shape

$$S_7^{\rm equil} = -\frac{12}{13}\, S_7\,, \tag{5.8}$$

whose minus sign has been chosen so that $\mathcal{F}_{\mathcal{R}}^{(7{\rm equil})}$ has a positive high correlation with the equilateral profile. Furthermore, we can analytically show that the following shape is exactly orthogonal to $S_7^{\rm equil}$:

$$S_7^{\rm ortho} = \frac{12}{14-13\beta}\left(\beta S_7 + 3S_1 - S_2\right), \tag{5.9}$$

where

$$\beta = \frac{16}{3}\frac{248041 - 25200\pi^2}{1986713 - 201600\pi^2} = 1.1967996\ldots \tag{5.10}$$

The normalizations of $S_7^{\rm equil}$ and $S_7^{\rm ortho}$ have been done such that, at the equilateral configuration ($k_1 = k_2 = k_3 = k$), we have $S_7^{\rm equil} = k^3 = S_7^{\rm ortho}$. This normalization follows from the standard definition of the previous templates introduced in the literature.

We note here that the leading-order bispectrum (3.54) includes the term $3S_1 - S_2$, so that we also consider the correlation between the combination $3\mathcal{F}_{\mathcal{R}}^{(1)} - \mathcal{F}_{\mathcal{R}}^{(2)}$ and other shapes in table I. The shape $3\mathcal{F}_{\mathcal{R}}^{(1)} - \mathcal{F}_{\mathcal{R}}^{(2)}$ has a high correlation with $\mathcal{F}_{\mathcal{R}}^{\rm equil}$. Compared to $\mathcal{F}_{\mathcal{R}}^{(7)}$ and $\mathcal{F}_{\mathcal{R}}^{(8)}$, however, it is not very close to the equilateral shape. Moreover $\mathcal{F}_{\mathcal{R}}^{(3)}$ has some correlation with the orthogonal shape, i.e., $C(\mathcal{F}_{\mathcal{R}}^{(3)}, \mathcal{F}_{\mathcal{R}}^{\rm ortho}) = -0.290742$.

As we see in table 1, the correlation between $\mathcal{F}_{\mathcal{R}}^{(7{\rm ortho})}$ and $\mathcal{F}_{\mathcal{R}}^{\rm equil}$ is very small. In terms of $S_7^{\rm equil}$ and $S_7^{\rm ortho}$, the functions $3S_1 - S_2$ and S_3 are expressed as

$$3S_1 - S_2 = \frac{13}{12}\,\beta\, S_7^{\rm equil} + \frac{14-13\beta}{12}\, S_7^{\rm ortho}\,, \qquad S_3 = \frac{13}{432}\,(3\beta - 2)\, S_7^{\rm equil} + \frac{14-13\beta}{144}\, S_7^{\rm ortho}\,. \tag{5.11}$$

Using these relations, the leading-order bispectrum (3.54) can be written in terms of the equilateral basis $S_7^{\rm equil}$ and the orthogonal basis $S_7^{\rm ortho}$, as

$$\mathcal{A}_{\mathcal{R}}^{\rm lead} = c_1 S_7^{\rm equil} + c_2 S_7^{\rm ortho}\,, \tag{5.12}$$

where

$$c_1 = \frac{13}{12}\left[\frac{1}{24}\left(1 - \frac{1}{c_s^2}\right)(2+3\beta) + \frac{\lambda}{12\Sigma}(2-3\beta) - \frac{\delta\mathcal{C}_6}{6\epsilon_s}(2-3\beta) + \frac{\delta\mathcal{C}_7}{3\epsilon_s c_s^2}\right], \tag{5.13}$$

$$c_2 = \frac{14-13\beta}{12}\left[\frac{1}{8}\left(1 - \frac{1}{c_s^2}\right) - \frac{\lambda}{4\Sigma} + \frac{\delta\mathcal{C}_6}{2\epsilon_s}\right]. \tag{5.14}$$

The coefficients c_1 and c_2 characterize the magnitudes of the three-point correlation function coming from equilateral and orthogonal contributions, respectively. Finally, we also introduce the enfolded shape function

$$S_7^{\rm enfold} = (S_7^{\rm equil} - S_7^{\rm ortho})/2\,, \tag{5.15}$$

which has a maximum at $k_1 \to 2k_2$, $k_2 \to k_3$. Note that $S_7^{\rm enfold}$ vanishes at the equilateral configuration.

JCAP03(2013)030

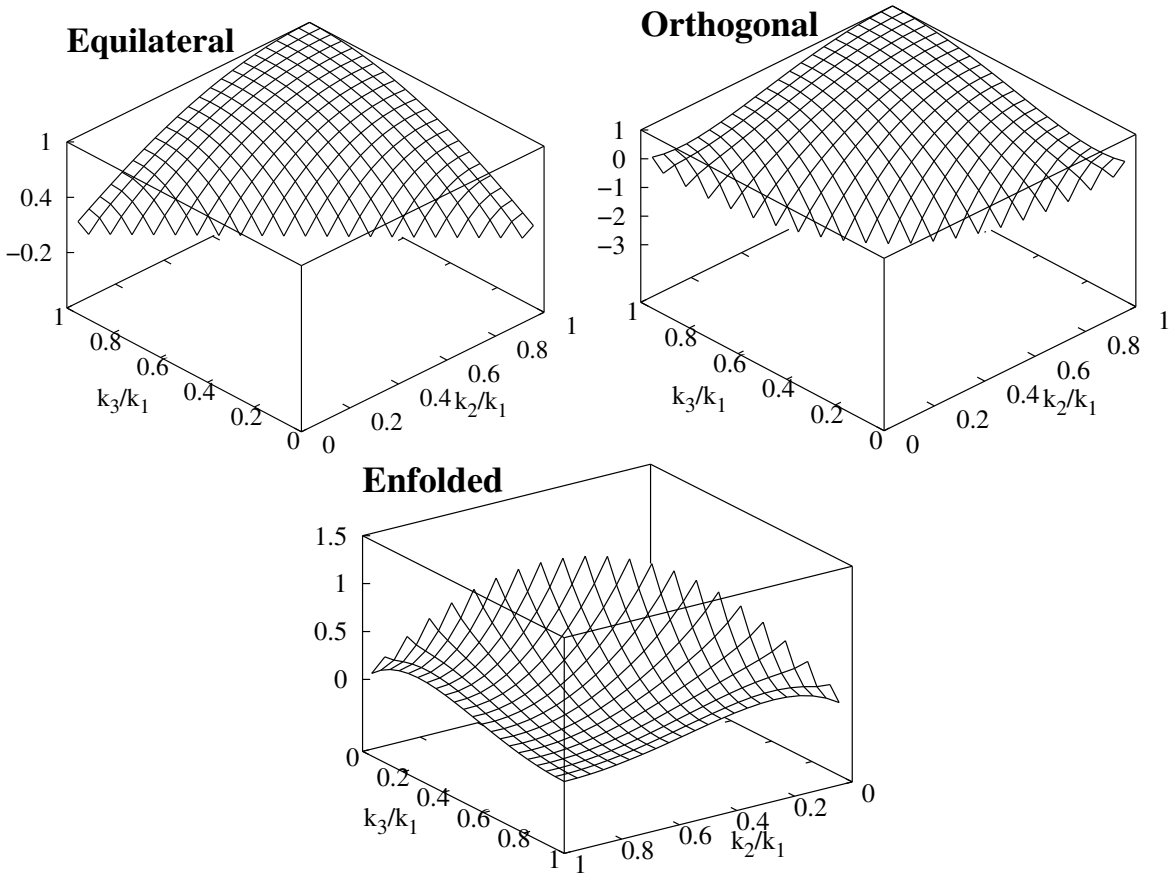

Figure 1. The shape functions $\mathcal{F}_{\mathcal{R}}^{(7\mathrm{equil})}(1,k_2/k_1,k_3/k_1)(k_2/k_1)^2(k_3/k_1)^2$ (top left), $\mathcal{F}_{\mathcal{R}}^{(7\mathrm{ortho})}(1,k_2/k_1,k_3/k_1)(k_2/k_1)^2(k_3/k_1)^2$ (top right), and $\mathcal{F}_{\mathcal{R}}^{(7\mathrm{enfold})}(1,k_2/k_1,k_3/k_1)(k_2/k_1)^2(k_3/k_1)^2$ (bottom).

In figure 1 we plot the three shape functions $\mathcal{F}_{\mathcal{R}}^{(7\mathrm{equil})}$, $\mathcal{F}_{\mathcal{R}}^{(7\mathrm{ortho})}$ and $\mathcal{F}_{\mathcal{R}}^{(7\mathrm{enfold})}$ multiplied by the functions $(k_2/k_1)^2(k_3/k_1)^2$. The correlations of $\mathcal{F}_{\mathcal{R}}^{\mathrm{lead}} = (2\pi)^4\mathcal{A}_{\mathcal{R}}^{\mathrm{lead}}/\prod_{i=1}^3 k_i^3$ with the shape functions $\mathcal{F}_{\mathcal{R}}^{\mathrm{equil}}$, $\mathcal{F}_{\mathcal{R}}^{\mathrm{ortho}}$, and $\mathcal{F}_{\mathcal{R}}^{\mathrm{enfold}}$ can be evaluated as

$$C^{\mathrm{equil}} \equiv C(\mathcal{F}_{\mathcal{R}}^{\mathrm{lead}},\mathcal{F}_{\mathcal{R}}^{\mathrm{equil}}) = \frac{8.25104\left(1.44717\times10^{-2}c_1 - 1.70045\times10^{-4}c_2\right)}{\sqrt{1.42610\times10^{-2}c_1^2 + 4.09480\times10^{-2}c_2^2}}\,, \quad (5.16)$$

$$C^{\mathrm{ortho}} \equiv C(\mathcal{F}_{\mathcal{R}}^{\mathrm{lead}},\mathcal{F}_{\mathcal{R}}^{\mathrm{ortho}}) = \frac{5.12494\left(8.23792\times10^{-4}c_1 + 3.57273\times10^{-2}c_2\right)}{\sqrt{1.42610\times10^{-2}c_1^2 + 4.09480\times10^{-2}c_2^2}}\,, \quad (5.17)$$

$$C^{\mathrm{enfold}} \equiv C(\mathcal{F}_{\mathcal{R}}^{\mathrm{lead}},\mathcal{F}_{\mathcal{R}}^{\mathrm{enfold}}) = \frac{8.80788\left(6.82395\times10^{-3}c_1 - 1.79487\times10^{-2}c_2\right)}{\sqrt{1.42610\times10^{-2}c_1^2 + 4.09480\times10^{-2}c_2^2}}\,, \quad (5.18)$$

which depend on the coefficients c_1 and c_2. In particular, we find $C(\mathcal{F}_{\mathcal{R}}^{(7\mathrm{enfold})},\mathcal{F}_{\mathcal{R}}^{\mathrm{enfold}}) = 0.928621$.

JCAP03(2013)030

6 Shapes of non-Gaussianities in concrete models

Let us study the non-Gaussianities of concrete models of inflation in which the bispectrum (5.12) can be large due to the small scalar propagation speed c_s. As we will see below, there are some models where the orthogonal shape provides an important contribution to the bispectrum.

6.1 Power-law k-inflation

We first consider k-inflation characterized by

$$P(\phi, X) = K(\phi)(-X + X^2)\,, \quad G_3 = 0\,, \quad G_4 = 0\,, \quad G_5 = 0\,, \tag{6.1}$$

where $K(\phi)$ is a function in terms of ϕ. From the background equations (2.5) and (2.6) it follows that

$$K(\phi) = \frac{3M_{\rm pl}^2 H^2}{X(3X-1)}\,, \qquad \frac{\dot{H}}{H^2} = -\frac{3(2X-1)}{3X-1}\,. \tag{6.2}$$

As an example, we study power-law inflation characterized by $a \propto t^{1/\gamma}$ and $H = 1/(\gamma t)$, where γ ($\ll 1$) is constant. Substituting the Hubble parameter into the second of eq. (6.2), we obtain $X = (3-\gamma)/[3(2-\gamma)]$ and $\phi = \phi_0 + \sqrt{(3-\gamma)/[3(2-\gamma)]}\,t$ (ϕ_0 is the initial value of the field). From the first of eq. (6.2) we find that power-law inflation is realized for the choice [54]

$$K(\phi) = \frac{6(2-\gamma)M_{\rm pl}^2}{\gamma^2(\phi-\phi_0)^2}\,. \tag{6.3}$$

In this case c_s^2 and λ/Σ in eq. (5.13) are given by

$$c_s^2 = \frac{\gamma}{3(4-\gamma)}\,, \qquad \frac{\lambda}{\Sigma} = \frac{1}{2}(1-c_s^2)\,. \tag{6.4}$$

For $\gamma \ll 1$ we have that $1/c_s^2 \simeq 12/\gamma \gg 1$.

In this model (dubbed Model A) the leading-order bispectrum (5.12) reduces to

$$\mathcal{A}_{\mathcal{R}}^{\rm lead} = \frac{13}{12}\left[-\frac{2+3\beta}{24c_s^2} + \frac{1}{6} - \frac{1}{24}(2-3\beta)c_s^2\right] S_7^{\rm equil} - \frac{14-13\beta}{96}\left(\frac{1}{c_s^2} - c_s^2\right) S_7^{\rm ortho}\,. \tag{6.5}$$

In the limit $c_s^2 \ll 1$ it follows that $\mathcal{A}_{\mathcal{R}}^{\rm lead} \simeq (-0.252/c_s^2) S_7^{\rm equil} + (0.016/c_s^2) S_7^{\rm ortho}$. Since in this limit the non-linear parameters (4.11) and (4.15) are given by $f_{\rm NL}^{\rm equil,lead} \simeq -85/(324c_s^2)$ and $f_{\rm NL}^{\rm enfold,lead} \simeq -1/(32c_s^2)$ respectively, the WMAP9 year constraint (1.2) of the equilateral shape gives the bound $c_s^2 > 1.2 \times 10^{-3}$ (95 % CL).

The scalar spectral index (2.25) and the tensor-to-scalar ratio (2.27) can be expressed in terms of c_s, as

$$n_{\mathcal{R}} = 1 - \frac{24c_s^2}{1+3c_s^2}\,, \qquad r = \frac{192c_s^3}{1+3c_s^2}\,. \tag{6.6}$$

For the ΛCDM model without the running scalar spectral index, the bounds on n_s and r from the WMAP9 data alone are $n_{\mathcal{R}} = 0.992 \pm 0.019$ (68 % CL) and $r < 0.38$ (95 % CL), respectively. If we combine the WMAP9 data with the measurements of high-l CMB anisotropies, baryon acoustic oscillations, and the Hubble constant, the constraints are $n_{\mathcal{R}} = 0.9636 \pm 0.0084$ (68 % CL) and $r < 0.13$ (95 % CL). If we employ the bound $0.95 < n_{\mathcal{R}} < 1$ then the

JCAP03(2013)030

scalar propagation is constrained to be $c_s^2 < 2.1 \times 10^{-3}$. Since in this case $r < 0.018$ from eq. (6.6), the observational constraint on r is satisfied. Combining the bound of c_s^2 with that of the scalar non-Gaussianity, it follows that $1.2 \times 10^{-3} < c_s^2 < 2.1 \times 10^{-3}$.

In figure 2 we plot the shape function $\mathcal{F}_{\mathcal{R}}^{\rm lead}(1, k_2/k_1, k_3/k_1)(k_2/k_1)^2(k_3/k_1)^2$ for $c_s^2 = 2.0\times10^{-3}$ (labelled as "A" in the figure), where $\mathcal{F}_{\mathcal{R}}^{\rm lead} = (2\pi)^4 \mathcal{A}_{\mathcal{R}}^{\rm lead}/\prod_{i=1}^3 k_i^3$. When $c_s^2 = 2.0\times 10^{-3}$, the correlations (5.16)–(5.18) are $C^{\rm equil} = -0.99474$, $C^{\rm ortho} = 0.06305$, and $C^{\rm enfold} = -0.58511$, respectively. Hence the shape of non-Gaussianities is close to the equilateral one illustrated in figure 1, whose property is is independent of the choice of c_s^2.

6.2 k-inflation with the term $G_3(X)$

We study k-inflation in the presence of the covariant Galileon term $G_3(X)$ characterized by [63, 71]

$$P(X) = -X + \frac{X^2}{2M^4}\,, \qquad G_3(X) = \frac{\mu X}{M^4}\,, \tag{6.7}$$

where M and μ are constants having a dimension of mass. In this model (dubbed Model B) the de Sitter solution is present when $\epsilon = \delta_{PX} + 3\delta_{G3X} = 0$. Using eq. (2.5) as well, we obtain

$$H^2 = \frac{M^4}{18\mu^2}\frac{(1-x)^2}{x}\,, \qquad \frac{\mu}{M_{\rm pl}} = \frac{1-x}{x\sqrt{3(2-x)}}\,, \tag{6.8}$$

where $x = X/M^4$. As long as inflation is realized in the regime $\mu/M_{\rm pl} \ll 1$, x is close to 1. In what follows we replace x for 1 except for the terms including $1-x$.

Along the de Sitter solution we have that $\lambda/\Sigma = 6c_s^2/\epsilon_s$, $\delta\mathcal{C}_6 = \delta\mathcal{C}_7 = \delta_{G3X} = 2(1-x)$. Since $\epsilon_s = 2(1-x)$ and $c_s^2 = (1-x)/6$, the bispectrum (5.12) reads

$$\mathcal{A}_{\mathcal{R}}^{\rm lead} = \frac{13}{12}\left[\frac{1}{4c_s^2}\left(1-\frac{\beta}{2}\right) - \frac{1}{6}(1-3\beta)\right] S_7^{\rm equil} - \frac{14-13\beta}{12}\left(\frac{1}{8c_s^2} - \frac{1}{2}\right) S_7^{\rm ortho}\,. \tag{6.9}$$

If $c_s^2 \ll 1$, we obtain $\mathcal{A}_{\mathcal{R}}^{\rm lead} \simeq (0.109/c_s^2)S_7^{\rm equil} + (0.016/c_s^2)S_7^{\rm ortho}$ together with the non-linear parameters $f_{\rm NL}^{\rm equil,lead} \simeq 5/(36c_s^2)$ and $f_{\rm NL}^{\rm enfold,lead} \simeq -1/(32c_s^2)$. Notice that the sign of $f_{\rm NL}^{\rm equil,lead}$ is opposite to that in Model A. From the WMAP 9 year bound (1.2) it follows that $c_s^2 > 4.3 \times 10^{-4}$ (95 % CL). In Model B the scalar spectral index is $n_{\mathcal{R}} = 1$, whereas r is related to c_s^2 via

$$c_s^2 = \frac{3^{1/3}}{48} r^{2/3}\,. \tag{6.10}$$

The WMAP9 bound $r < 0.38$ gives the constraint $c_s^2 < 1.6 \times 10^{-2}$. Using the severer bound $r < 0.13$, it follows that $c_s^2 < 7.7 \times 10^{-3}$. In both cases there are viable parameter spaces compatible with the constraint from the scalar non-Gaussainity. When $c_s^2 = 2.0 \times 10^{-3}$, for example, we have $C^{\rm equil} = 0.96865$, $C^{\rm ortho} = 0.25272$, and $C^{\rm enfold} = 0.29984$. As we see in figure 2, the shape of non-Gaussianities for $c_s^2 = 2.0 \times 10^{-3}$ is approximately close to the equilateral one.

For potential-driven inflation ($P = X - V(\phi)$) with the term $G_3(X) = \mu X/M^4$ [72], the non-Gaussianities are small because c_s^2 is not much smaller than 1 [73]. In such models, if the Galileon self-interaction dominates over the standard kinetic term even after inflation, there is an instability associated with the appearance of the negative c_s^2 after the field velocity $\dot{\phi}$

JCAP03(2013)030

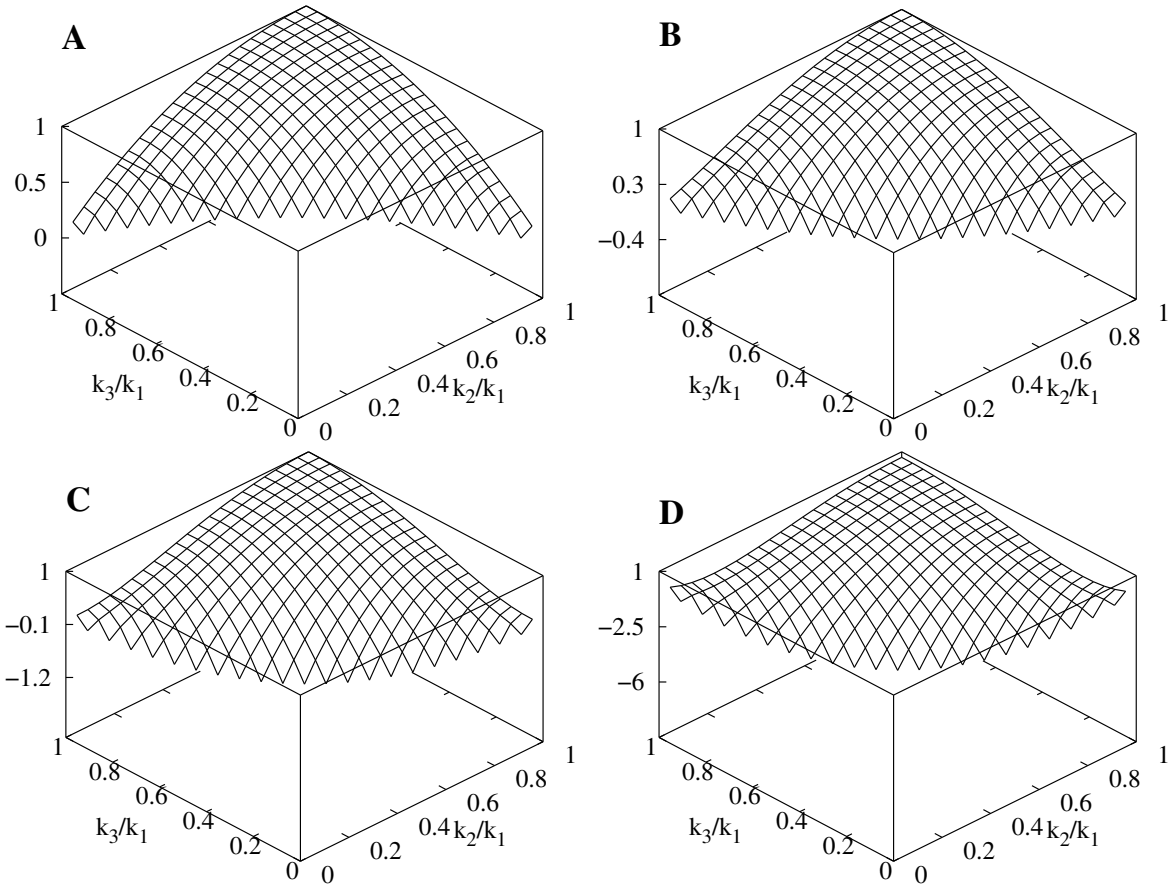

Figure 2. The leading-order shape function $\mathcal{F}_{\mathcal{R}}^{\rm lead}(1, k_2/k_1, k_3/k_1)(k_2/k_1)^2(k_3/k_1)^2$ for $c_s^2 = 2.0 \times 10^{-3}$ normalized to 1 at the equilateral configuration. Each panel corresponds to the models A, B, C, D.

changes its sign [74].[5] In the model B discussed above, reheating needs to occur gravitationally [71], so the situation should be different from that in Galileon inflation driven by a potential with a minimum.

6.3 k-inflation with the term $G_4(X)$

The next model (dubbed Model C) is k-inflation with the covariant Galileon term $G_4(X)$ [62], i.e.,

$$P(X) = -X + \frac{X^2}{2M^4}\,, \qquad G_4(X) = \frac{\mu X^2}{M^7}\,. \tag{6.11}$$

Similar to the case (6.7), there is a de Sitter solution satisfying the conditions

$$H^2 = \frac{M^3}{36\mu}\frac{1-x}{x}\,, \qquad \frac{\mu M}{M_{\rm pl}^2} = \frac{1-x}{6x^2(3-2x)}\,. \tag{6.12}$$

[5] In the framework of the effective field theory the Lagrangian is valid at the energy scale of inflation, but some higher dimensional operators can appear during the reheating stage. There may be a possibility that the Laplacian instability can be avoided by such operators.

JCAP03(2013)030

Inflation occurs in the regime where $x = X/M^4$ is close to 1. Employing the similar approximation to that used previously, we have $\delta\mathcal{C}_6 = 2\delta\mathcal{C}_7 = 12\delta_{G4XX}$, $\epsilon_s = 8\delta_{G4XX}$, $\delta_{G4XX} = (1-x)/3$, $c_s^2 = 2(1-x)/9$, and $\lambda/\Sigma = 1/2$. Hence the bispectrum (5.12) reduces to

$$\mathcal{A}_{\mathcal{R}}^{\rm lead} = \frac{13}{12}\left[\frac{1}{24c_s^2}(4-3\beta) - \frac{1}{12}(4-9\beta)\right] S_7^{\rm equil} - \frac{14-13\beta}{12}\left(\frac{1}{8c_s^2} - \frac{3}{4}\right) S_7^{\rm ortho}\,. \tag{6.13}$$

In the limit $c_s^2 \ll 1$, we have $\mathcal{A}_{\mathcal{R}}^{\rm lead} \simeq (0.018/c_s^2)S_7^{\rm equil} + (0.016/c_s^2)S_7^{\rm ortho}$. The equilateral and orthogonal non-linear parameters in this limit are given by $f_{\rm NL}^{\rm equil,lead} \simeq 25/(648c_s^2)$ and $f_{\rm NL}^{\rm enfold,lead} \simeq -1/(32c_s^2)$, respectively. The tighter constraint on c_s^2 comes from the enfolded bound (1.4) rather than the equilateral bound (1.2), i.e., $c_s^2 > 3.6 \times 10^{-4}$. Note that the scalar spectral index is $n_{\mathcal{R}} = 1$ and that the same relation as eq. (6.10) holds in Model C. Hence the upper bound of c_s^2 coming from the observational constraint of r is the same as that of Model B, i.e., $c_s^2 < 1.6 \times 10^{-2}$ for $r < 0.38$ and $c_s^2 < 7.7 \times 10^{-3}$ for $r = 0.13$.

In figure 2 we plot the shape of non-Gaussianities for $c_s^2 = 2.0 \times 10^{-3}$, in which case the shape is between the equilateral and orthogonal ones shown in figure 1. In fact, we have $C^{\rm equil} = 0.58145$, $C^{\rm ortho} = 0.75322$, and $C^{\rm enfold} = -0.33691$ for $c_s^2 = 2.0 \times 10^{-3}$. The orthogonal contribution tends to be less important for the values of c_s^2 larger than the order of 10^{-2}.

6.4 k-inflation with the term $G_5(X)$

Finally we study the following model (dubbed Model D) [62]

$$P(X) = -X + \frac{X^2}{2M^4}\,, \qquad G_5(X) = \frac{\mu X^2}{M^{10}}\,. \tag{6.14}$$

In this case, for $x = X/M^4$ close to 1, there is a de Sitter solution satisfying the conditions $H^2 = M^4/(6M_{\rm pl}^2)$ and $\mu^2 M^4/M_{\rm pl}^6 = 27(1-x)^2/25$. Since $\delta\mathcal{C}_6 = 3\delta\mathcal{C}_7 = 36(1-x)/5$, $\epsilon_s = 18(1-x)/5$, $c_s^2 = 3(1-x)/10$, and $\lambda/\Sigma = 1/2$, the bispectrum is given by

$$\mathcal{A}_{\mathcal{R}}^{\rm lead} = \frac{13}{12}\left[-\frac{1}{c_s^2}\left(\frac{\beta}{8} - \frac{5}{36}\right) - \frac{1}{2} + \beta\right] S_7^{\rm equil} - \frac{14-13\beta}{12}\left(\frac{1}{8c_s^2} - 1\right) S_7^{\rm ortho}\,. \tag{6.15}$$

In the limit $c_s^2 \ll 1$ we have $\mathcal{A}_{\mathcal{R}}^{\rm lead} \simeq (-0.012/c_s^2)S_7^{\rm equil} + (0.016/c_s^2)S_7^{\rm ortho} = -(0.023/c_s^2)$ $S_7^{\rm enfold} + (0.005/c_s^2)\, S_7^{\rm ortho}$, in which case the sign in front of the shape $S_7^{\rm equil}$ in eq. (6.15) is opposite to those in the models B and C. In the same limit the equilateral and enfolded non-linear parameters are $f_{\rm NL}^{\rm equil,lead} \simeq 5/(972c_s^2)$ and $f_{\rm NL}^{\rm enfold,lead} \simeq -1/(32c_s^2)$, respectively. In this case $f_{\rm NL}^{\rm equil,lead}$ is smaller than $|f_{\rm NL}^{\rm enfold,lead}|$ by one order of magnitude. The WMAP 9 year enfolded bound (1.4) gives the constraint $c_s^2 > 3.6 \times 10^{-4}$. In model D we have that $n_{\mathcal{R}} = 1$ and that the relation between r and c_s^2 is the same as eq. (6.10). Hence the upper bound on c_s^2 is the same as that of Models B and C.

The shape of non-Gaussianities for $c_s^2 = 2.0 \times 10^{-3}$ is plotted in figure 2. When $c_s^2 = 2.0 \times 10^{-3}$ the correlations (5.16)–(5.18) are $C^{\rm equil} = -0.35587$, $C^{\rm ortho} = 0.83546$, and $C^{\rm enfold} = -0.90787$, respectively, in which case the shape has quite high (anti)-correlations with both the orthogonal and enfolded templates. In figure 3 we show the correlations $C^{\rm equil}$, $C^{\rm ortho}$, and $C^{\rm enfold}$ versus c_s^2. For $c_s^2 \gtrsim 0.04$ the correlation with the equilateral template is larger than those with other templates. For $c_s^2 \lesssim 0.04$ the contributions of the orthogonal and enfolded shapes tend to be important. In the limit that $c_s^2 \ll 1$, $|C^{\rm enfold}|$ is largest among other correlations. Model D is an explicit example where the orthogonal (or enfolded) shape provides a significant contribution to the bispectrum.

JCAP03(2013)030

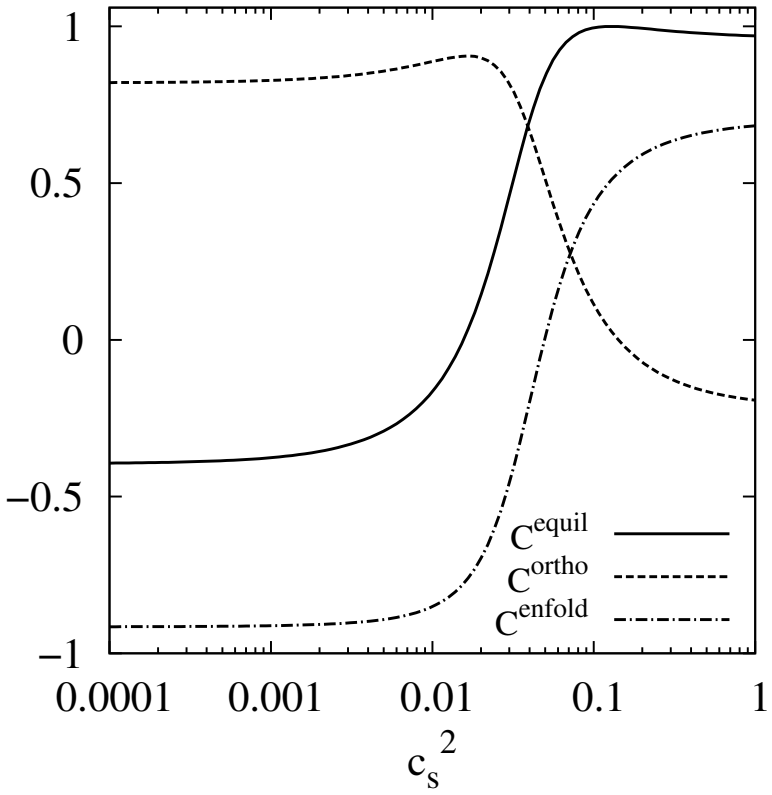

Figure 3. The correlations (5.16)–(5.18) versus c_s^2 for Model D. In the limit that $c_s^2 \ll 1$, $|C^{\rm enfold}|$ and $|C^{\rm ortho}|$ are larger than $|C^{\rm equil}|$. On the other hand, for $c_s^2 \gtrsim 0.04$, $|C^{\rm equil}|$ is largest.

7 Conclusions

In the Horndeski's most general scalar-tensor theories we derived the three-point correlation function of primordial curvature perturbations generated during inflation in the presence of slow-variation corrections to the leading-order bispectrum. Unlike previous works [61, 62], the bispectrum (3.53) is valid for any shape of non-Gaussianities at first order of ϵ.

In the squeezed limit ($k_3 \to 0$, $k_1 \to k_2$) the leading-order bispectrum (3.54) vanishes, so that the correction (3.55) is the dominant contribution to $\mathcal{A}_{\mathcal{R}}$. By using eq. (3.55), we showed that the non-linear parameter in this limit is given by $f_{\rm NL}^{\rm local} = (5/12)(1 - n_{\mathcal{R}})$. This agrees with the result of refs. [12, 47] in which the three-point correlation function was derived by dealing with the long-wavelength curvature perturbation (mode $\boldsymbol{k}_3$) as a classical background. As demonstrated in ref. [47], this result should be valid for any single-field inflation in which the decaying mode of $\mathcal{R}$ is neglected relative to the growing mode. Our direct computation of the three-point correlation function in the presence of all possible slow-variation corrections is another independent proof that the non-Gaussianity consistency relation holds for most general single-field theories with second-order equations of motion.

The result of the local non-Gaussianities shows that $|f_{\rm NL}^{\rm local}|$ is much smaller than 1, e.g., $f_{\rm NL}^{\rm local} = 0.0125$ for $n_{\mathcal{R}} = 0.97$. In the case where the shape of non-Gaussianities is not exactly the squeezed one ($0 < r_3 = k_3/k_1 \ll 1$), the leading-order bispectrum gives the non-linear parameter $|f_{\rm NL}^{\rm lead}| \approx r_3^2/c_s^2$. Hence the leading-order term dominates over the correction for $r_3 > c_s\sqrt{1 - n_{\mathcal{R}}}$. For the models with $c_s^2 \ll 1$ the non-linear parameter can be as large as

JCAP03(2013)030

$|f_{\rm NL}| > 1$ with the growth of r_3. By measuring the shape which is not so squeezed, it should be possible to discriminate the models with different values of c_s^2. The leading-order non-linear parameters in the equilateral and enfolded limits are given by eqs. (4.11) and (4.15), respectively, whose magnitudes can be larger than 1 for the models with $c_s^2 \ll 1$. These results will be useful to constrain concrete models of inflation in future high-precision observations.

We also showed that the leading-order bispectrum can be expressed in terms of the sum of the two bases $S_7^{\rm equil}$ and $S_7^{\rm ortho}$. The shape $S_7^{\rm equil}$ is very highly correlated with the equilateral template (5.5). It also vanishes in both local and enfolded limits. The shape $S_7^{\rm ortho}$, which is defined by (5.9), is exactly orthogonal to $S_7^{\rm equil}$. The coefficients c_1 and c_2 in front of $S_7^{\rm equil}$ and $S_7^{\rm ortho}$ in eq. (5.12) characterize the equilateral and orthogonal contributions, respectively.

In section 6 we presented concrete models in which the orthogonal shape can provide important contributions to the bispectrum. In power-law k-inflation the shape of non-Gaussianities is well approximated by the equilateral type. However, in k-inflation described by the Lagrangian $P(X) = -X + X^2/(2M^4)$ with a number of different Galileon terms like $G_4(X) = \mu X^2/M^7$ and $G_5(X) = \mu X^2/M^{10}$, we found that the orthogonal contribution is crucially important for $c_s^2 \ll 1$. In the presence of the term $G_5(X) = \mu X^2/M^{10}$, the correlations with the orthogonal and enfolded templates in the regime $c_s^2 \ll 1$ are higher than that with the equilateral template.

It will be interesting to see how the observations such as Planck [75] provide the constraints on the scalar non-Gaussianities as well as the scalar spectral index and the tensor-to-scalar ratio. In particular, if future observations confirm the value $|f_{\rm NL}^{\rm local}| > 1$ at more than 95 % CL, this implies that we need to go beyond the slow-variation single-field inflationary scenario (including the Horndeski's theories). The information of other shapes of non-Gaussianities (including the not so squeezed one) will be useful to discriminate between many different models.

Acknowledgments

We are grateful to Eiichiro Komatsu for motivating us to derive the full bispectrum with slow-variation corrections in the Horndeski's theories. We also thank Xingang Chen, Shinji Mukohyama, and Misao Sasaki for useful correspondence and discussions. This work was supported in part by the Grant-in-Aid for Scientific Research Fund of No. 24540286 and Scientific Research on Innovative Areas (No. 21111006).

A Corrections to the bispectrum

Following the calculations of ref. [56] we present the explicit forms of the corrections to the bispectrum (3.25). They come from the first three integrals in the action (3.19). We write each $\mathcal{O}(\epsilon)$ contribution to the bispectrum as $\Delta\mathcal{A}_{\mathcal{R}}^{(i)}$ $(i = 1, 2, 3)$. Note that similar calculations were also carried out in ref. [31].

(i) $\int dt\, d^3x\, a^3 \tilde{\mathcal{C}}_1 M_{\rm pl}^2 \mathcal{R}\dot{\mathcal{R}}^2$

- (a) The correction from the variation of $\tilde{\mathcal{C}}_1$ in eq. (3.39)

$$\Delta\mathcal{A}_{\mathcal{R}}^{(1)} \supset -\left(\frac{c_s^2}{4\epsilon_s F}\frac{1}{H}\frac{d\tilde{\mathcal{C}}_1^{\rm lead}}{dt}\right)_K \left[(1-2\gamma_1)\frac{1}{K}\sum_{i>j} k_i^2 k_j^2 - (1-\gamma_1)\frac{1}{K^2}\sum_{i\neq j} k_i^2 k_j^3\right]. \quad \text{(A.1)}$$

JCAP03(2013)030

- (b) The correction from the scale factor a in eq. (3.40)

$$\Delta\mathcal{A}^{(1)}_{\mathcal{R}} \supset \left(\frac{c_s^2\,\epsilon}{2\epsilon_s F}\tilde{\mathcal{C}}_1^{\rm lead}\right)_K \left[(1+2\gamma_1)\frac{1}{K}\sum_{i>j}k_i^2k_j^2 - \gamma_1\frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3\right]. \tag{A.2}$$

- (c) The contribution from the correction to $u(0,k_i)$ in eq. (3.42)

$$\Delta\mathcal{A}^{(1)}_{\mathcal{R}} \supset -\left(\frac{c_s^2}{4\epsilon_s F}\tilde{\mathcal{C}}_1^{\rm lead}\right)_K \left[3(1+\gamma_2)\epsilon + \frac{3\gamma_2}{2}\eta_{sF} + 3\left(1+\frac{\gamma_2}{2}\right)s + \left(\epsilon + \frac{\eta_{sF}}{2} + \frac{s}{2}\right)\ln\frac{k_1k_2k_3}{K^3}\right] \times \left(\frac{2}{K}\sum_{i>j}k_i^2k_j^2 - \frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3\right). \tag{A.3}$$

- (d) The contribution from the correction to $u^*(\tau,k_i)$ in eq. (3.44)

$$\Delta\mathcal{A}^{(1)}_{\mathcal{R}} \supset \left(\frac{c_s^2}{4\epsilon_s F}\tilde{\mathcal{C}}_1^{\rm lead}\right)_K \left[3(1-2\gamma_1)s\frac{(k_1k_2k_3)^2}{K^3} - \left\{(1+2\gamma_1)\epsilon + \left(\gamma_1 - \frac{1}{2}\right)\eta_{sF} + \left(\frac{3}{2}+\gamma_1\right)s\right\} \times \frac{1}{K}\sum_{i>j}k_i^2k_j^2 + \left\{\gamma_1\epsilon - \frac{1}{2}(1-\gamma_1)\eta_{sF} + \frac{1}{2}(1+\gamma_1)s\right\}\frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3 + \left(\epsilon + \frac{\eta_{sF}}{2} + \frac{s}{2}\right)\left(\frac{k_2^2k_3^2}{k_1}\mathcal{G} + \text{perm.}\right)\right], \tag{A.4}$$

where

$$\mathcal{G}(k_1,k_2,k_3) \equiv \text{Re}\left[\int_0^\infty dx_1\, h^*(x_1)e^{-\frac{k_2+k_3}{k_1}x_1}\right],$$

and $x_1 = -k_1c_{sK}\tau$.

- (e) The contribution from the correction to $\frac{d}{d\tau}u^*(\tau,k_i)$ in eq. (3.45)

$$\Delta\mathcal{A}^{(1)}_{\mathcal{R}} \supset -\left(\frac{c_s^2}{4\epsilon_s F}\tilde{\mathcal{C}}_1^{\rm lead}\right)_K \left[\{2(1+2\gamma_1)\epsilon + (2\gamma_1-1)(\eta_{sF}-3s)\}\frac{1}{K}\sum_{i>j}k_i^2k_j^2 + (2\epsilon+\eta_{sF}+s)k_1k_2k_3 - \{2\gamma_1\epsilon + (\gamma_1-1)\eta_{sF} + (4-6\gamma_1)s\}\frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3 + (1-2\gamma_1)s\left(\frac{1}{K^3}\sum_{i\neq j}k_i^2k_j^4 + \frac{2}{K^3}\sum_{i>j}k_i^3k_j^3\right) - (2\epsilon+\eta_{sF}+s) \times \left\{\sum_i k_i^3 + \sum_{i\neq j}k_ik_j^2 + \sum_i k_i^3\,\text{Re}\int_0^\infty dx_K\frac{e^{-ix_K}}{x_K} - \frac{1}{2}(\mathcal{M}+\text{perm.})\right\}\right], \tag{A.5}$$

where $x_K = -Kc_{sK}\tau$, and

$$\mathcal{M}(k_1,k_2,k_3) \equiv -k_1\text{Re}\left[\int_0^\infty dx_1\,\frac{1}{x_1}\left(k_2^2+k_3^2+ik_2k_3\frac{k_2+k_3}{k_1}x_1\right)e^{-i\frac{k_2+k_3}{k_1}x_1}\frac{dh^*(x_1)}{dx_1}\right].$$

(ii) $\int dt\,d^3x\,a\,\tilde{\mathcal{C}}_2M_{\rm pl}^2\mathcal{R}(\partial\mathcal{R})^2$

JCAP03(2013)030

- (a) The correction from the variation of $\tilde{\mathcal{C}}_2$

$$\Delta\mathcal{A}_{\mathcal{R}}^{(2)} \supset -\left(\frac{1}{4\epsilon_s F}\frac{1}{H}\frac{d\tilde{\mathcal{C}}_2^{\rm lead}}{dt}\right)_K\left[(1-\gamma_1)\left(\frac{1}{2}\sum_i k_i^3 - \frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3\right) - \frac{1}{2}k_1k_2k_3 + \frac{1-2\gamma_1}{K}\sum_{i>j}k_i^2k_j^2 + \frac{1}{2}\sum_{i\neq j}k_ik_j^2\right]. \tag{A.6}$$

- (b) The correction from the scale factor a

$$\Delta\mathcal{A}_{\mathcal{R}}^{(2)} \supset \left(\frac{\epsilon}{2\epsilon_s F}\tilde{\mathcal{C}}_2^{\rm lead}\right)_K\left[\gamma_1\left(\frac{1}{2}\sum_i k_i^3 - \frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3\right) + \frac{1}{2}k_1k_2k_3 + \frac{1+2\gamma_1}{K}\sum_{i>j}k_i^2k_j^2 - \frac{1}{2}\sum_{i\neq j}k_ik_j^2\right]. \tag{A.7}$$

- (c) The contribution from the correction to $u(0,k_i)$

$$\Delta\mathcal{A}_{\mathcal{R}}^{(2)} \supset -\left(\frac{1}{4\epsilon_s F}\tilde{\mathcal{C}}_2^{\rm lead}\right)_K\left[3(1+\gamma_2)\epsilon + \frac{3\gamma_2}{2}\eta_{sF} + 3\left(1+\frac{\gamma_2}{2}\right)s + \left(\epsilon + \frac{\eta_{sF}}{2} + \frac{s}{2}\right)\ln\frac{k_1k_2k_3}{K^3}\right] \times\left(\frac{1}{2}\sum_i k_i^3 + \frac{2}{K}\sum_{i>j}k_i^2k_j^2 - \frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3\right). \tag{A.8}$$

- (d) The contribution from the correction to $u^*(\tau,k_i)$

$$\begin{aligned}\Delta\mathcal{A}_{\mathcal{R}}^{(2)} \supset & -\left(\frac{1}{8\epsilon_s F}\tilde{\mathcal{C}}_2^{\rm lead}\right)_K\left[\left\{3\gamma_1\epsilon + \frac{3}{2}(\gamma_1-1)\eta_{sF} + \frac{3}{2}(1+\gamma_1)s\right\}\sum_i k_i^3\right. \\ & -3\left(\epsilon+\frac{\eta_{sF}}{2}+\frac{s}{2}\right)\sum_{i\neq j}k_ik_j^2 + \left\{3\epsilon+\frac{3}{2}\eta_{sF}+\left(\frac{5}{2}-2\gamma_1\right)s\right\}k_1k_2k_3 \\ & + \{6(1+2\gamma_1)\epsilon - 3(1-2\gamma_1)\eta_{sF} + (5+6\gamma_1)s\}\frac{1}{K}\sum_{i>j}k_i^2k_j^2 \\ & -\{6\gamma_1\epsilon+3(\gamma_1-1)\eta_{sF}+(1+6\gamma_1)s\}\frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3 - (1+\gamma_1)s\frac{1}{K}\sum_i k_i^4 \\ & \left. -\gamma_1 s\frac{1}{K^2}\sum_{i\neq j}k_ik_j^4 + (2\epsilon+\eta_{sF}+s)(\mathcal{N}+\text{perm.})\right], \end{aligned} \tag{A.9}$$

where

$$\mathcal{N}(k_1,k_2,k_3) \equiv \frac{k_1}{2}\sum_i k_i^2\,{\rm Re}\left[\int_0^\infty dx_1\,\frac{1}{x_1^2}e^{-ix_1\frac{k_2+k_3}{k_1}}\left(-1-i\frac{k_2+k_3}{k_1}x_1+\frac{k_2k_3}{k_1^2}x_1^2\right)h^*(x_1)\right].$$

Note that the coefficient $\tilde{\mathcal{C}}_2^{\rm lead}$ is related to $\tilde{\mathcal{C}}_1^{\rm lead}$ via $\tilde{\mathcal{C}}_2^{\rm lead} = -(c_s^2/3)\tilde{\mathcal{C}}_1^{\rm lead}$.

(iii) $\int dt\,d^3x\,a^3\tilde{\mathcal{C}}_3 M_{\rm pl}\dot{\mathcal{R}}^3$

JCAP03(2013)030

- (a) The correction from the variation of $\tilde{\mathcal{C}}_3$

$$\Delta\mathcal{A}_{\mathcal{R}}^{(3)} \supset \left(\frac{3(2\gamma_1-3)c_s^2}{4\epsilon_s F M_{\rm pl}}\frac{d\tilde{\mathcal{C}}_3^{\rm lead}}{dt}\right)_K \frac{(k_1k_2k_3)^2}{K^3}\,. \tag{A.10}$$

- (b) The correction from the scale factor a

$$\Delta\mathcal{A}_{\mathcal{R}}^{(3)} \supset \left(\frac{3(2\gamma_1-1)c_s^2\,\epsilon\,H}{4\epsilon_s F M_{\rm pl}}\tilde{\mathcal{C}}_3^{\rm lead}\right)_K \frac{(k_1k_2k_3)^2}{K^3}\,. \tag{A.11}$$

- (c) The contribution from the correction to $u(0,k_i)$

$$\Delta\mathcal{A}_{\mathcal{R}}^{(3)} \supset -\left(\frac{3Hc_s^2}{2\epsilon_s F M_{\rm pl}}\tilde{\mathcal{C}}_3^{\rm lead}\right)_K \left[3(1+\gamma_2)\epsilon+\frac{3}{2}\gamma_2\eta_{sF}+3\left(1+\frac{\gamma_2}{2}\right)s + \left(\epsilon+\frac{\eta_{sF}}{2}+\frac{s}{2}\right)\ln\frac{k_1k_2k_3}{K^3}\right]\frac{(k_1k_2k_3)^2}{K^3}. \tag{A.12}$$

- (d) The contribution from the correction to $\frac{d}{d\tau}u^*(\tau,k_i)$

$$\Delta\mathcal{A}_{\mathcal{R}}^{(3)} \supset \left(\frac{3Hc_s^2}{4\epsilon_s F M_{\rm pl}}\tilde{\mathcal{C}}_3^{\rm lead}\right)_K \left[\left\{3(1-2\gamma_1)\epsilon+3\left(\frac{3}{2}-\gamma_1\right)\eta_{sF}+\left(3\gamma_1-\frac{17}{2}\right)s\right\}\frac{(k_1k_2k_3)^2}{K^3} + \left(\epsilon+\frac{\eta_{sF}}{2}+\frac{s}{2}\right)\left\{\left(\frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3-\frac{2}{K}\sum_{i>j}k_i^2k_j^2\right)+\mathcal{U}\right\}\right], \tag{A.13}$$

  where $\mathcal{U}$ is defined in eq. (3.51).

B Expressions for $\mathcal{U}$ and $\mathcal{V}$

We evaluate the integrals $\mathcal{U}$ and $\mathcal{V}$ defined in eqs. (3.51) and (3.50) for general values of $r_2 = k_2/k_1, r_3 = k_3/k_1$, and in three different limits: local shapes ($r_3 \to 0$, $r_2 = 1$), equilateral ($r_3 = 1, r_2 = 1$), and enfolded ($r_3 \to 1/2$, $r_2 \to 1/2$) shapes. In doing so, we need to use the following relations

$$\begin{aligned} h^*(x) &= 2\sin(x)+[\sin(x)-x\cos(x)]\mathrm{Ci}(2x)-[\cos(x)+x\sin(x)]\mathrm{Si}(2x) \\ &\quad +i\left\{2\cos(x)+\pi\sin(x)-\pi x\cos(x)-[\cos(x)+x\sin(x)]\mathrm{Ci}(2x)\right. \\ &\quad \left.-[\sin(x)-x\cos(x)]\mathrm{Si}(2x)\right\}, \end{aligned} \tag{B.1}$$

$$\begin{aligned} \frac{dh^*(x)}{dx} &= \cos(x)-\sin(x)/x+x\sin(x)\mathrm{Ci}(2x)-x\cos(x)\mathrm{Si}(2x) \\ &\quad +i\left[\pi x\sin(x)-\sin(x)-\cos(x)/x-x\cos(x)\mathrm{Ci}(2x)-x\sin(x)\mathrm{Si}(2x)\right]. \end{aligned} \tag{B.2}$$

The $\mathcal{U}$ integral

Let us first evaluate the integral $\mathcal{U}$. We employ a few different procedures of regularizations, but they lead to the same final results. One possibility is to solve directly the integral, as

$$\mathcal{U} = \int_0^\infty f_{\mathcal{U}}(x_1)\,dx_1 + \text{perm.} = \lim_{y\to\infty}\int_0^y f_{\mathcal{U}}(x_1)\,dx_1 + \text{perm.}\,, \tag{B.3}$$

JCAP03(2013)030

where $f_{\mathcal{U}}$ is the integrand in eq. (3.51). We can either neglect the terms which rapidly oscillate around 0, or we can, equivalently, shift y in the complex domain for the oscillating terms like $e^{\pm iky} \to e^{\pm ik\rho^2(1\pm i/\rho)} \to 0$ (Method I). Another possible method (Method II) consists of solving the following limit

$$\mathcal{U} = \int_0^\infty f_{\mathcal{U}}(x_1)\, dx_1 + \text{perm.} = \lim_{c\to 0} \int_0^\infty e^{-cx_1}\, f_{\mathcal{U}}(x_1)\, dx_1 + \text{perm.}\,. \tag{B.4}$$

We need to check whether both methods lead to the same results, and we can — at least for some values of r_2, r_3 this proves to be possible — check the result against numerical integrations.

By employing the Method I mentioned above, the result of the integration is

$$\begin{aligned}
\mathcal{U}(r_2,r_3) = k_1^3 &\left[\frac{4r_2^2r_3^3\left(3r_2^2-6r_2+r_3^2+3\right)}{(r_2-r_3-1)^3\,(r_2+r_3-1)^3} - \frac{2r_2^2r_3^2}{(r_2-r_3+1)^3}\right]\ln 2 \\
&+ \frac{k_1^3}{(r_2+r_3+1)^2}\left\{\frac{[(r_2+1)^3-6r_3^2\,(r_2+1)+r_3^3]r_2^2}{(r_2-r_3+1)^2}\right. \\
&+ \frac{r_3^2[r_2^3+3r_3r_2^2+3(r_3^2-2)r_2+r_3^3-6r_3+1]r_2^2}{(r_2+r_3-1)^2} \\
&+ \frac{r_3^2[r_2^3-6\,(r_3+1)\,r_2^2+(r_3+1)^3]}{(1-r_2+r_3)^2} + \frac{2r_3^2\,(r_2+r_3+1)^2r_2^2}{(r_2-r_3+1)^3}\,\ln\left(\frac{r_2+r_3+1}{r_3}\right) \\
&\left.+ \frac{2r_3^2\,(r_2+r_3+1)^2r_2^2}{(1-r_2+r_3)^3}\,\ln\left(\frac{r_2+r_3+1}{r_2}\right) + \frac{2r_3^2\,(r_2+r_3+1)^2r_2^2}{(r_2+r_3-1)^3}\,\ln(r_2+r_3+1)\right\},
\end{aligned} \tag{B.5}$$

where $\mathcal{U}(r_1,r_2) = \mathcal{U}(r_2,r_1)$, that is $\mathcal{U}$ is symmetric under the exchange $r_1 \leftrightarrow r_2$.

Let us consider this general expression in several different cases. Some of these cases look — only apparently — singular: this behavior takes place as the triangle of the momenta $\boldsymbol{k}_i$ degenerates into a line.

- *Equilateral case*, $r_2 = r_3 = 1$. In this case we find

$$\mathcal{U}(1,1) = \mathcal{U}_{\text{equil}} = [6\ln(3/2) - 1]\, k_1^3 \approx 1.43279\, k_1^3\,. \tag{B.6}$$

  Since there is no apparent singular behavior for $\mathcal{U}$, this result can also be confirmed numerically.[6]

- *Local case*, $r_2 = 1$, $r_3 \to 0$ (or $r_2 \to 0$, $r_3 = 1$). In this case, the limit exists and gives

$$\lim_{r_3\to 0} \mathcal{U}(1,r_3) = \mathcal{U}_{\text{local}} = \frac{1}{2}k_1^3\,. \tag{B.7}$$

  The result in the local case matches with that derived by Chen *et al.* [56].

- Singular line, $r_3 = \lim_{\epsilon\to 0}(1 - r_2 + \epsilon)$, with $0 < r_2 < 1$. In this case we find

$$\lim_{\epsilon\to 0}\mathcal{U} = \frac{r_2\{(r_2-1)r_2[20\,(r_2-1)\,r_2+9]+6\}(r_2-1)-6(r_2-1)^3\ln(1-r_2)+6r_2^3\ln r_2}{24\,(r_2-1)\,r_2}\,k_1^3\,. \tag{B.8}$$

  The limits $r_2 \to 0, 1$ give again the local result, as expected.

[6] Mathematica, working in high precision, returns this same numerical value. The same value can also be found by applying the Method II mentioned above.

JCAP03(2013)030

- *Enfolded case*, $r_2 = 1/2$, $r_3 = \lim_{\epsilon\to 0}(1/2+\epsilon)$. This is a particular case of the previous one. Then we find

$$\lim_{\epsilon\to 0}\mathcal{U} = \mathcal{U}_{\rm enfold} = \frac{1}{24}\,(5+6\ln 2)k_1^3 \approx 0.38162\,k_1^3\,. \tag{B.9}$$

- Singular line, $r_3 = \lim_{\epsilon\to 0}(r_2 - 1 + \epsilon)$, with $r_2 > 1$. In this case we obtain

$$\lim_{\epsilon\to 0}\mathcal{U} = \frac{(r_2-1)\left\{r_2[r_2\left(6r_2^2-9r_2+29\right)-40]+20\right\}+6r_2^3\left[(r_2-1)^3\ln\left(\frac{r_2}{r_2-1}\right)+\ln r_2\right]}{24\,(r_2-1)\,r_2}k_1^3. \tag{B.10}$$

  In the limit $r_2 \to 1^+$, this reproduces the local limit result, as expected.

- Singular line, $r_3 = \lim_{\epsilon\to 0}(1 + r_2 - \epsilon)$. In this case we find

$$\lim_{\epsilon\to 0}\mathcal{U} = \frac{r_2\left\{r_2\left\{r_2[3r_2(2r_2+5)+38]+15\right\}+6\right\}+6r_2^3(r_2+1)^3\ln\left(\frac{1}{r_2}+1\right)+6(r_2+1)^3\ln(r_2+1)}{24r_2\,(r_2+1)}k_1^3\,, \tag{B.11}$$

  which recovers the local limit as $r_2 = \epsilon \to 0$.

Therefore, we have shown that the physical limits are all finite.

The $\mathcal{V}$ integral

The next step is to compute the integral (3.50). It is convenient to solve the integral by studying the limit

$$\mathcal{V} = \lim_{\varepsilon\to 0}\lim_{y\to\infty}\int_\varepsilon^y f_{\mathcal{V}}(x_1)dx_1 + \text{perm.}\,, \tag{B.12}$$

where $f_{\mathcal{V}}$ is the integrand in eq. (3.50). As for the $y \to \infty$ limit, we set the rapidly oscillating functions to vanish by regularizing $e^{\pm iky}$ as in the Method I of the integral $\mathcal{U}$. After taking

JCAP03(2013)030

the limit $\varepsilon \to 0$, we finally obtain

$$
\begin{aligned}
\mathcal{V} = & \\
& \frac{k_1^3}{3(r_2-r_3-1)(r_2+r_3-1)(r_2+r_3+1)}[3r_2^6+6r_3r_2^5+r_2^4(r_3^2-9r_3-3) \\
& + r_3r_2^3(6r_3^2-13r_3+3)+3r_3^4r_2^2+3-15r_3^3r_2^2-4r_3^2r_2^2+3r_2^2(r_3-1) \\
& - 12r_3^5(r_2+1)-r_2r_3(19r_3^3+15r_3^2+13r_3+9)-7r_3^6+3r_3^4+6r_3^3+r_3^2+6r_3] \\
& + \frac{k_1^3}{6(r_2-r_3+1)(r_2+r_3+1)}(7r_2^5+5r_3r_2^4+19r_2^4-6r_3^2r_2^3+5r_3r_2^3+14r_2^3-6r_3^3r_2^2+6r_3^2r_2^2 \\
& + 4r_3r_2^2+14r_2^2-r_3^3r_2+6r_3^2r_2+5r_3r_2+19r_2+r_3^5-r_3^4-6r_3^3-6r_3^2+5r_3+7-r_3^4r_2) \\
& - \frac{k_1^3\ln 2}{3(r_2-r_3-1)^2(r_2+r_3-1)^2}\,r_3^3\,(5r_2^4-20r_2^3-6r_3^2r_2^2+22r_2^2+4r_3^2r_2-20r_2+r_3^4-6r_3^2+5) \\
& + \frac{k_1^3\ln(r_2+r_3+1)}{6\,(r_2+r_3-1)^2}(r_2^5+2r_3r_2^4-2r_2^4-3r_3^2r_2^3-2r_3r_2^3-3r_2^3-3r_3^3r_2^2+8r_3^2r_2^2-8r_3r_2^2+3r_2^2 \\
& + 2r_3^4r_2-2r_3^3r_2-8r_3^2r_2-2r_3r_2+2r_2+r_3^5-2r_3^4-3r_3^3+3r_3^2+2r_3-1) \\
& - \frac{k_1^3\ln\left(\frac{r_2+r_3+1}{r_2}\right)}{6\,(r_2-r_3-1)^2}(r_2^5-2r_3r_2^4-2r_2^4-3r_3^2r_2^3+2r_3r_2^3-3r_2^3+3r_3^3r_2^2+8r_3^2r_2^2+8r_3r_2^2+3r_2^2 \\
& + 2r_3^4r_2+2r_3^3r_2-8r_3^2r_2+2r_3r_2+2r_2-r_3^5-2r_3^4+3r_3^3+3r_3^2-2r_3-1) \\
& + \frac{k_1^3\ln\left(\frac{r_2+r_3+1}{2r_3}\right)}{6\,(r_2-r_3+1)^2}(r_2^5-2r_3r_2^4+2r_2^4-3r_3^2r_2^3-2r_3r_2^3-3r_2^3+3r_3^3r_2^2-8r_3^2r_2^2+8r_3r_2^2-3r_2^2 \\
& + 2r_3^4r_2-2r_3^3r_2-8r_3^2r_2-2r_3r_2+2r_2-r_3^5+2r_3^4+3r_3^3-3r_3^2-2r_3+1)\,, \qquad \text{(B.13)}
\end{aligned}
$$

where $\mathcal{V}(r_1, r_2) = \mathcal{V}(r_2, r_1)$. Let us now analyze this expression, on the lines/points of physical interest.

- *Equilateral* case, $r_2 = r_3 = 1$. In this case we find

$$\mathcal{V}(1,1) = \mathcal{V}_{\rm equil} = \frac{15}{2}\,[2+\ln(2/3)]\,k_1^3 \approx 11.959\,k_1^3\,. \qquad \text{(B.14)}$$

- *Local* case, $r_2 = 1$, $r_3 \to 0$ (or $r_2 \to 0$, $r_3 = 1$). In this case, the limit exists and gives

$$\lim_{r_3\to 0^+} \mathcal{V}(1, r_3) = \mathcal{V}_{\rm local} = \frac{20}{3}k_1^3\,, \qquad \text{(B.15)}$$

  where this value of $\mathcal{V}_{\rm local}$ matches with the one derived by Chen et al. [56].

- Singular line, $r_3 = \lim_{\epsilon\to 0}(1 - r_2 + \epsilon)$, with $0 < r_2 < 1$. In this case we obtain

$$\lim_{\epsilon\to 0}\mathcal{V} = \frac{\{(r_2-1)r_2[11(r_2-1)r_2-105]-86\}r_2(1-r_2)+6(r_2-1)^3\ln(1-r_2)-6r_2^3\ln r_2}{12\,(r_2-1)\,r_2}k_1^3\,. \qquad \text{(B.16)}$$

  The limits $r_2 \to 0, 1$ again give the result (B.15).

- *Enfolded* case, $r_2 = 1/2$, $r_3 = \lim_{\epsilon\to 0}(1/2 + \epsilon)$. This is a particular case of the previous one. Then we find

$$\lim_{\epsilon\to 0}\mathcal{V} = \mathcal{V}_{\rm enfold} = \left(\frac{315}{64} - \frac{1}{2}\ln 2\right)k_1^3 \approx 4.5753\,k_1^3\,. \qquad \text{(B.17)}$$

JCAP03(2013)030

- Singular line, $r_3 = \lim_{\epsilon\to 0}(r_2 - 1 + \epsilon)$, with $r_2 > 1$. In this case we find

$$\lim_{\epsilon\to 0}\mathcal{V} = \frac{(r_2-1)\left[\{r_2[r_2(86r_2-105)+94]+22\}r_2-6(r_2-1)^2 r_2^3\ln\left(\frac{r_2}{r_2-1}\right)-11\right]-6r_2^3\ln r_2}{12\,(r_2-1)\,r_2}\,k_1^3\,. \tag{B.18}$$

  Taking the limit $r_2 \to 1^+$, we recover the result (B.15).

- Singular line, $r_3 = \lim_{\epsilon\to 0}(1 + r_2 - \epsilon)$. In this case we obtain

$$\lim_{\epsilon\to 0}\mathcal{V} = \frac{r_2\big(r_2\{r_2[r_2(86r_2+239)+295]+239\}+86\big)-6r_2^3(r_2+1)^3\ln\left(\frac{1}{r_2}+1\right)-6(r_2+1)^3\ln(r_2+1)}{12r_2\,(r_2+1)}k_1^3\,, \tag{B.19}$$

  which again reproduces the value (B.15) as $r_2 = \epsilon \to 0$.

Therefore, the integral $\mathcal{V}$ remains finite in the physical parameter space of r_2 and r_3.

References

[1] A.A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, *Phys. Lett.* **B 91** (1980) 99 [INSPIRE];
D. Kazanas, *Dynamics of the Universe and Spontaneous Symmetry Breaking*, *Astrophys. J.* **241** (1980) L59 [INSPIRE];
K. Sato, *First-order phase transition of a vacuum and the expansion of the Universe*, *Mon. Not. R. Astron. Soc.* **195** (1981) 467; *Cosmological baryon-number domain structure and the first order phase transition of a vacuum*, *Phys. Rev. Lett.* **B 99** (1981) 66;
A.H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, *Phys. Rev.* **D 23** (1981) 347 [INSPIRE].

[2] V.F. Mukhanov and G. Chibisov, *Quantum Fluctuation and Nonsingular Universe. (In Russian)*, *JETP Lett.* **33** (1981) 532 [INSPIRE];
A.H. Guth and S. Pi, *Fluctuations in the New Inflationary Universe*, *Phys. Rev. Lett.* **49** (1982) 1110 [INSPIRE];
S. Hawking, *The Development of Irregularities in a Single Bubble Inflationary Universe*, *Phys. Lett.* **B 115** (1982) 295 [INSPIRE];
A.A. Starobinsky, *Dynamics of Phase Transition in the New Inflationary Universe Scenario and Generation of Perturbations*, *Phys. Lett.* **B 117** (1982) 175 [INSPIRE].;
J.M. Bardeen, P.J. Steinhardt and M.S. Turner, *Spontaneous Creation of Almost Scale - Free Density Perturbations in an Inflationary Universe*, *Phys. Rev.* **D 28** (1983) 679 [INSPIRE].

[3] G.F. Smoot et al., *Structure in the COBE differential microwave radiometer first year maps* , *Astrophys. J.* **L 396** (1992) 1.

[4] WMAP collaboration, D. Spergel et al., *First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Determination of cosmological parameters*, *Astrophys. J. Suppl.* **148** (2003) 175 [astro-ph/0302209] [INSPIRE].

[5] G. Hinshaw, D. Larson, E. Komatsu, D. Spergel, C. Bennett et al., *Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results*, arXiv:1212.5226 [INSPIRE].

JCAP03(2013)030

[6] K. Enqvist and M.S. Sloth, *Adiabatic CMB perturbations in pre - big bang string cosmology*, *Nucl. Phys.* **B 626** (2002) 395 [hep-ph/0109214] [INSPIRE];
D.H. Lyth and D. Wands, *Generating the curvature perturbation without an inflaton*, *Phys. Lett.* **B 524** (2002) 5 [hep-ph/0110002] [INSPIRE];
T. Moroi and T. Takahashi, *Effects of cosmological moduli fields on cosmic microwave background*, *Phys. Lett.* **B 522** (2001) 215 [*Erratum ibid.* **B 539** (2002) 303] [hep-ph/0110096] [INSPIRE].

[7] D. Salopek and J. Bond, *Nonlinear evolution of long wavelength metric fluctuations in inflationary models*, *Phys. Rev.* **D 42** (1990) 3936 [INSPIRE].

[8] A. Gangui, F. Lucchin, S. Matarrese and S. Mollerach, *The Three point correlation function of the cosmic microwave background in inflationary models*, *Astrophys. J.* **430** (1994) 447 [astro-ph/9312033] [INSPIRE].

[9] L. Verde, L.-M. Wang, A. Heavens and M. Kamionkowski, *Large scale structure, the cosmic microwave background and primordial non-Gaussianity*, *Mon. Not. Roy. Astron. Soc.* **313** (2000) L141 [astro-ph/9906301] [INSPIRE].

[10] E. Komatsu and D.N. Spergel, *Acoustic signatures in the primary microwave background bispectrum*, *Phys. Rev.* **D 63** (2001) 063002 [astro-ph/0005036] [INSPIRE].

[11] N. Bartolo, S. Matarrese and A. Riotto, *NonGaussianity from inflation*, *Phys. Rev.* **D 65** (2002) 103505 [hep-ph/0112261] [INSPIRE];
N. Bartolo, E. Komatsu, S. Matarrese and A. Riotto, *Non-Gaussianity from inflation: Theory and observations*, *Phys. Rept.* **402** (2004) 103 [astro-ph/0406398] [INSPIRE].

[12] J.M. Maldacena, *Non-Gaussian features of primordial fluctuations in single field inflationary models*, *JHEP* **05** (2003) 013 [astro-ph/0210603] [INSPIRE].

[13] P. Creminelli, *On non-Gaussianities in single-field inflation*, *JCAP* **10** (2003) 003 [astro-ph/0306122] [INSPIRE].

[14] G. Rigopoulos and E. Shellard, *The separate universe approach and the evolution of nonlinear superhorizon cosmological perturbations*, *Phys. Rev.* **D 68** (2003) 123518 [astro-ph/0306620] [INSPIRE]; *Non-linear inflationary perturbations*, *JCAP* **10** (2005) 006 [astro-ph/0405185] [INSPIRE].

[15] N. Arkani-Hamed, P. Creminelli, S. Mukohyama and M. Zaldarriaga, *Ghost inflation*, *JCAP* **04** (2004) 001 [hep-th/0312100] [INSPIRE].

[16] M. Alishahiha, E. Silverstein and D. Tong, *DBI in the sky*, *Phys. Rev.* **D 70** (2004) 123505 [hep-th/0404084] [INSPIRE].

[17] N. Bartolo, S. Matarrese and A. Riotto, *On nonGaussianity in the curvaton scenario*, *Phys. Rev.* **D 69** (2004) 043503 [hep-ph/0309033] [INSPIRE];
M. Sasaki, J. Valiviita and D. Wands, *Non-Gaussianity of the primordial perturbation in the curvaton model*, *Phys. Rev.* **D 74** (2006) 103003 [astro-ph/0607627] [INSPIRE];
K.A. Malik and D.H. Lyth, *A numerical study of non-Gaussianity in the curvaton scenario*, *JCAP* **09** (2006) 008 [astro-ph/0604387] [INSPIRE].

[18] D. Seery and J.E. Lidsey, *Primordial non-Gaussianities in single field inflation*, *JCAP* **06** (2005) 003 [astro-ph/0503692] [INSPIRE].

[19] D. Seery and J.E. Lidsey, *Primordial non-Gaussianities from multiple-field inflation*, *JCAP* **09** (2005) 011 [astro-ph/0506056] [INSPIRE].

[20] D.H. Lyth and Y. Rodriguez, *The Inflationary prediction for primordial non-Gaussianity*, *Phys. Rev. Lett.* **95** (2005) 121302 [astro-ph/0504045] [INSPIRE]; *Non-Gaussianity from the second-order cosmological perturbation*, *Phys. Rev.* **D 71** (2005) 123508 [astro-ph/0502578] [INSPIRE];

JCAP03(2013)030

D.H. Lyth, K.A. Malik and M. Sasaki, *A General proof of the conservation of the curvature perturbation*, *JCAP* **05** (2005) 004 [astro-ph/0411220] [INSPIRE].

[21] K. Enqvist and A. Vaihkonen, *Non-Gaussian perturbations in hybrid inflation*, *JCAP* **09** (2004) 006 [hep-ph/0405103] [INSPIRE];
B.A. Bassett, S. Tsujikawa and D. Wands, *Inflation dynamics and reheating*, *Rev. Mod. Phys.* **78** (2006) 537 [astro-ph/0507632] [INSPIRE];
F. Vernizzi and D. Wands, *Non-Gaussianities in two-field inflation*, *JCAP* **05** (2006) 019 [astro-ph/0603799] [INSPIRE];
T. Battefeld and R. Easther, *Non-Gaussianities in Multi-field Inflation*, *JCAP* **03** (2007) 020 [astro-ph/0610296] [INSPIRE];
S. Yokoyama, T. Suyama and T. Tanaka, *Primordial Non-Gaussianity in Multi-Scalar Slow-Roll Inflation*, *JCAP* **07** (2007) 013 [arXiv:0705.3178] [INSPIRE];
F. Arroja, S. Mizuno and K. Koyama, *Non-Gaussianity from the bispectrum in general multiple field inflation*, *JCAP* **08** (2008) 015 [arXiv:0806.0619] [INSPIRE];
C.T. Byrnes, K.-Y. Choi and L.M. Hall, *Conditions for large non-Gaussianity in two-field slow-roll inflation*, *JCAP* **10** (2008) 008 [arXiv:0807.1101] [INSPIRE];
C.T. Byrnes and G. Tasinato, *Non-Gaussianity beyond slow roll in multi-field inflation*, *JCAP* **08** (2009) 016 [arXiv:0906.0767] [INSPIRE];
H.R. Cogollo, Y. Rodriguez and C.A. Valenzuela-Toledo, *On the Issue of the zeta Series Convergence and Loop Corrections in the Generation of Observable Primordial Non-Gaussianity in Slow-Roll Inflation. Part I: The Bispectrum*, *JCAP* **08** (2008) 029 [arXiv:0806.1546] [INSPIRE];
C.T. Byrnes and K.-Y. Choi, *Review of local non-Gaussianity from multi-field inflation*, *Adv. Astron.* **2010** (2010) 724525 [arXiv:1002.3110] [INSPIRE];
Y. Rodriguez and C.A. Valenzuela-Toledo, *On the Issue of the zeta Series Convergence and Loop Corrections in the Generation of Observable Primordial Non-Gaussianity in Slow-Roll Inflation. Part 2. The Trispectrum*, *Phys. Rev.* **D 81** (2010) 023531 [arXiv:0811.4092] [INSPIRE];
S. Renaux-Petel, S. Mizuno and K. Koyama, *Primordial fluctuations and non-Gaussianities from multifield DBI Galileon inflation*, *JCAP* **11** (2011) 042 [arXiv:1108.0305] [INSPIRE];
J. Elliston, D.J. Mulryne, D. Seery and R. Tavakol, *Evolution of f_{NL} to the adiabatic limit*, *JCAP* **11** (2011) 005 [arXiv:1106.2153] [INSPIRE].

[22] X. Chen, R. Easther and E.A. Lim, *Generation and Characterization of Large Non-Gaussianities in Single Field Inflation*, *JCAP* **04** (2008) 010 [arXiv:0801.3295] [INSPIRE]; *Generation and Characterization of Large Non-Gaussianities in Single Field Inflation*, *JCAP* **04** (2008) 010 [arXiv:0801.3295] [INSPIRE].

[23] N. Barnaby and J.M. Cline, *Large NonGaussianity from Nonlocal Inflation*, *JCAP* **07** (2007) 017 [arXiv:0704.3426] [INSPIRE]; *Predictions for NonGaussianity from Nonlocal Inflation*, *JCAP* **06** (2008) 030 [arXiv:0802.3218] [INSPIRE].

[24] K. Koyama, S. Mizuno, F. Vernizzi and D. Wands, *Non-Gaussianities from ekpyrotic collapse with multiple fields*, *JCAP* **11** (2007) 024 [arXiv:0708.4321] [INSPIRE];
E.I. Buchbinder, J. Khoury and B.A. Ovrut, *Non-Gaussianities in new ekpyrotic cosmology*, *Phys. Rev. Lett.* **100** (2008) 171302 [arXiv:0710.5172] [INSPIRE].

[25] T. Suyama and M. Yamaguchi, *Non-Gaussianity in the modulated reheating scenario*, *Phys. Rev.* **D 77** (2008) 023505 [arXiv:0709.2545] [INSPIRE];
K. Ichikawa, T. Suyama, T. Takahashi and M. Yamaguchi, *Non-Gaussianity, Spectral Index and Tensor Modes in Mixed Inflaton and Curvaton Models*, *Phys. Rev.* **D 78** (2008) 023513 [arXiv:0802.4138] [INSPIRE];
T. Suyama, T. Takahashi, M. Yamaguchi and S. Yokoyama, *On Classification of Models of Large Local-Type Non-Gaussianity*, *JCAP* **12** (2010) 030 [arXiv:1009.1979] [INSPIRE].

JCAP03(2013)030

[26] D. Langlois, S. Renaux-Petel, D.A. Steer and T. Tanaka, *Primordial fluctuations and non-Gaussianities in multi-field DBI inflation*, *Phys. Rev. Lett.* **101** (2008) 061301 [arXiv:0804.3139] [INSPIRE]; *Primordial perturbations and non-Gaussianities in DBI and general multi-field inflation*, *Phys. Rev.* **D 78** (2008) 063523 [arXiv:0806.0336] [INSPIRE].

[27] M. Sasaki, *Multi-brid inflation and non-Gaussianity*, *Prog. Theor. Phys.* **120** (2008) 159 [arXiv:0805.0974] [INSPIRE];
A. Naruko and M. Sasaki, *Large non-Gaussianity from multi-brid inflation*, *Prog. Theor. Phys.* **121** (2009) 193 [arXiv:0807.0180] [INSPIRE].

[28] J. Khoury and F. Piazza, *Rapidly-Varying Speed of Sound, Scale Invariance and Non-Gaussian Signatures*, *JCAP* **07** (2009) 026 [arXiv:0811.3633] [INSPIRE].

[29] Y.-i. Takamizu, S. Mukohyama, M. Sasaki and Y. Tanaka, *Non-Gaussianity of superhorizon curvature perturbations beyond δ N formalism*, *JCAP* **06** (2010) 019 [arXiv:1004.1870] [INSPIRE].

[30] N. Bartolo, M. Fasiello, S. Matarrese and A. Riotto, *Large non-Gaussianities in the Effective Field Theory Approach to Single-Field Inflation: the Bispectrum*, *JCAP* **08** (2010) 008 [arXiv:1004.0893] [INSPIRE]; *Large non-Gaussianities in the Effective Field Theory Approach to Single-Field Inflation: the Trispectrum*, *JCAP* **09** (2010) 035 [arXiv:1006.5411] [INSPIRE]; *Tilt and Running of Cosmological Observables in Generalized Single-Field Inflation*, *JCAP* **12** (2010) 026 [arXiv:1010.3993] [INSPIRE];
M. Fasiello, *Effective Field Theory for Inflation*, arXiv:1106.2189 [INSPIRE].

[31] C. Burrage, R.H. Ribeiro and D. Seery, *Large slow-roll corrections to the bispectrum of noncanonical inflation*, *JCAP* **07** (2011) 032 [arXiv:1103.4126] [INSPIRE];
R.H. Ribeiro and D. Seery, *Decoding the bispectrum of single-field inflation*, *JCAP* **10** (2011) 027 [arXiv:1108.3839] [INSPIRE].

[32] M. Libanov, S. Mironov and V. Rubakov, *Non-Gaussianity of scalar perturbations generated by conformal mechanisms*, *Phys. Rev.* **D 84** (2011) 083502 [arXiv:1105.6230] [INSPIRE].

[33] N.S. Sugiyama, E. Komatsu and T. Futamase, *Non-Gaussianity Consistency Relation for Multi-field Inflation*, *Phys. Rev. Lett.* **106** (2011) 251301 [arXiv:1101.3636] [INSPIRE].

[34] R.H. Ribeiro, *Inflationary signatures of single-field models beyond slow-roll*, *JCAP* **05** (2012) 037 [arXiv:1202.4453] [INSPIRE].

[35] E. Komatsu, D.N. Spergel and B.D. Wandelt, *Measuring primordial non-Gaussianity in the cosmic microwave background*, *Astrophys. J.* **634** (2005) 14 [astro-ph/0305189] [INSPIRE].

[36] D. Babich, P. Creminelli and M. Zaldarriaga, *The Shape of non-Gaussianities*, *JCAP* **08** (2004) 009 [astro-ph/0405356] [INSPIRE].

[37] P. Creminelli, A. Nicolis, L. Senatore, M. Tegmark and M. Zaldarriaga, *Limits on non-Gaussianities from wmap data*, *JCAP* **05** (2006) 004 [astro-ph/0509029] [INSPIRE].

[38] M. Liguori, F. Hansen, E. Komatsu, S. Matarrese and A. Riotto, *Testing primordial non-Gaussianity in CMB anisotropies*, *Phys. Rev.* **D 73** (2006) 043505 [astro-ph/0509098] [INSPIRE].

[39] K.M. Smith and M. Zaldarriaga, *Algorithms for bispectra: Forecasting, optimal analysis and simulation*, *Mon. Not. Roy. Astron. Soc.* **417** (2011) 2 [astro-ph/0612571] [INSPIRE].

[40] A.P. Yadav, E. Komatsu and B.D. Wandelt, *Fast Estimator of Primordial Non-Gaussianity from Temperature and Polarization Anisotropies in the Cosmic Microwave Background*, *Astrophys. J.* **664** (2007) 680 [astro-ph/0701921] [INSPIRE].

[41] P. Creminelli, L. Senatore and M. Zaldarriaga, *Estimators for local non-Gaussianities*, *JCAP* **03** (2007) 019 [astro-ph/0606001] [INSPIRE].

JCAP03(2013)030

[42] J. Fergusson and E. Shellard, *The shape of primordial non-Gaussianity and the CMB bispectrum*, *Phys. Rev.* **D 80** (2009) 043510 [arXiv:0812.3413] [INSPIRE].

[43] L. Senatore, K.M. Smith and M. Zaldarriaga, *Non-Gaussianities in Single Field Inflation and their Optimal Limits from the WMAP 5-year Data*, *JCAP* **01** (2010) 028 [arXiv:0905.3746] [INSPIRE].

[44] P.D. Meerburg, J.P. van der Schaar and P.S. Corasaniti, *Signatures of Initial State Modifications on Bispectrum Statistics*, *JCAP* **05** (2009) 018 [arXiv:0901.4044] [INSPIRE].

[45] P. Creminelli, G. D'Amico, M. Musso, J. Norena and E. Trincherini, *Galilean symmetry in the effective theory of inflation: new shapes of non-Gaussianity*, *JCAP* **02** (2011) 006 [arXiv:1011.3004] [INSPIRE].

[46] C. Bennett, D. Larson, J. Weiland, N. Jarosik, G. Hinshaw et al., *Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results*, arXiv:1212.5225 [INSPIRE].

[47] P. Creminelli and M. Zaldarriaga, *Single field consistency relation for the 3-point function*, *JCAP* **10** (2004) 006 [astro-ph/0407059] [INSPIRE].

[48] F. Arroja, A.E. Romano and M. Sasaki, *Large and strong scale dependent bispectrum in single field inflation from a sharp feature in the mass*, *Phys. Rev.* **D 84** (2011) 123503 [arXiv:1106.5384] [INSPIRE];
P. Adshead, C. Dvorkin, W. Hu and E.A. Lim, *Non-Gaussianity from Step Features in the Inflationary Potential*, *Phys. Rev.* **D 85** (2012) 023531 [arXiv:1110.3050] [INSPIRE].

[49] M.H. Namjoo, H. Firouzjahi and M. Sasaki, *Violation of non-Gaussianity consistency relation in a single field inflationary model*, *Europhys. Lett.* **101** (2013) 39001 [arXiv:1210.3692] [INSPIRE].

[50] C. Cheung, A.L. Fitzpatrick, J. Kaplan and L. Senatore, *On the consistency relation of the 3-point function in single field inflation*, *JCAP* **02** (2008) 021 [arXiv:0709.0295] [INSPIRE].

[51] J. Ganc and E. Komatsu, *A new method for calculating the primordial bispectrum in the squeezed limit*, *JCAP* **12** (2010) 009 [arXiv:1006.5457] [INSPIRE].

[52] S. Renaux-Petel, *On the squeezed limit of the bispectrum in general single field inflation*, *JCAP* **10** (2010) 020 [arXiv:1008.0260] [INSPIRE].

[53] C. Germani and Y. Watanabe, *UV-protected (Natural) Inflation: Primordial Fluctuations and non-Gaussian Features*, *JCAP* **07** (2011) 031 [*Addendum ibid.* **1107** (2011) A01] [arXiv:1106.0502] [INSPIRE].

[54] C. Armendariz-Picon, T. Damour and V.F. Mukhanov, *k-inflation*, *Phys. Lett.* **B 458** (1999) 209 [hep-th/9904075] [INSPIRE].

[55] J. Garriga and V.F. Mukhanov, *Perturbations in k-inflation*, *Phys. Lett.* **B 458** (1999) 219 [hep-th/9904176] [INSPIRE].

[56] X. Chen, M.-x. Huang, S. Kachru and G. Shiu, *Observational signatures and non-Gaussianities of general single field inflation*, *JCAP* **01** (2007) 002 [hep-th/0605045] [INSPIRE].

[57] G.W. Horndeski, *Second-order scalar-tensor field equations in a four-dimensional space*, *Int. J. Theor. Phys.* **10** (1974) 363 [INSPIRE].

[58] C. Deffayet, X. Gao, D. Steer and G. Zahariade, *From k-essence to generalised Galileons*, *Phys. Rev.* **D 84** (2011) 064039 [arXiv:1103.3260] [INSPIRE].

[59] C. Charmousis, E.J. Copeland, A. Padilla and P.M. Saffin, *General second order scalar-tensor theory, self tuning and the Fab Four*, *Phys. Rev. Lett.* **108** (2012) 051101 [arXiv:1106.2000] [INSPIRE].

JCAP03(2013)030

[60] T. Kobayashi, M. Yamaguchi and J. Yokoyama, *Generalized G-inflation: Inflation with the most general second-order field equations*, *Prog. Theor. Phys.* **126** (2011) 511 [arXiv:1105.5723] [INSPIRE].

[61] X. Gao and D.A. Steer, *Inflation and primordial non-Gaussianities of 'generalized Galileons'*, *JCAP* **12** (2011) 019 [arXiv:1107.2642] [INSPIRE].

[62] A. De Felice and S. Tsujikawa, *Inflationary non-Gaussianities in the most general second-order scalar-tensor theories*, *Phys. Rev.* **D 84** (2011) 083504 [arXiv:1107.3917] [INSPIRE].

[63] S. Mizuno and K. Koyama, *Primordial non-Gaussianity from the DBI Galileons*, *Phys. Rev.* **D 82** (2010) 103518 [arXiv:1009.0677] [INSPIRE].

[64] A. De Felice and S. Tsujikawa, *Primordial non-Gaussianities in general modified gravitational models of inflation*, *JCAP* **04** (2011) 029 [arXiv:1103.1172] [INSPIRE].

[65] T. Kobayashi, M. Yamaguchi and J. Yokoyama, *Primordial non-Gaussianity from G-inflation*, *Phys. Rev.* **D 83** (2011) 103524 [arXiv:1103.1740] [INSPIRE].

[66] X. Gao, T. Kobayashi, M. Yamaguchi and J. Yokoyama, *Primordial non-Gaussianities of gravitational waves in the most general single-field inflation model*, *Phys. Rev. Lett.* **107** (2011) 211301 [arXiv:1108.3513] [INSPIRE];
X. Gao, T. Kobayashi, M. Shiraishi, M. Yamaguchi, J. Yokoyama et al., *Full bispectra from primordial scalar and tensor perturbations in the most general single-field inflation model*, arXiv:1207.0588 [INSPIRE].

[67] R.L. Arnowitt, S. Deser and C.W. Misner, *Canonical variables for general relativity*, *Phys. Rev.* **117** (1960) 1595 [INSPIRE].

[68] A. De Felice and S. Tsujikawa, *Conditions for the cosmological viability of the most general scalar-tensor theories and their applications to extended Galileon dark energy models*, *JCAP* **02** (2012) 007 [arXiv:1110.3878] [INSPIRE].

[69] S. Renaux-Petel, *On the redundancy of operators and the bispectrum in the most general second-order scalar-tensor theory*, *JCAP* **02** (2012) 020 [arXiv:1107.5020] [INSPIRE].

[70] L. Senatore, K.M. Smith and M. Zaldarriaga, *Non-Gaussianities in Single Field Inflation and their Optimal Limits from the WMAP 5-year Data*, *JCAP* **01** (2010) 028 [arXiv:0905.3746] [INSPIRE].

[71] T. Kobayashi, M. Yamaguchi and J. Yokoyama, *G-inflation: Inflation driven by the Galileon field*, *Phys. Rev. Lett.* **105** (2010) 231302 [arXiv:1008.0603] [INSPIRE].

[72] K. Kamada, T. Kobayashi, M. Yamaguchi and J. Yokoyama, *Higgs G-inflation*, *Phys. Rev.* **D 83** (2011) 083515 [arXiv:1012.4238] [INSPIRE].

[73] A. De Felice, S. Tsujikawa, J. Elliston and R. Tavakol, *Chaotic inflation in modified gravitational theories*, *JCAP* **08** (2011) 021 [arXiv:1105.4685] [INSPIRE].

[74] J. Ohashi and S. Tsujikawa, *Potential-driven Galileon inflation*, *JCAP* **10** (2012) 035 [arXiv:1207.4879] [INSPIRE].

[75] PLANCK collaboration, *The Scientific programme of planck*, astro-ph/0604069 [INSPIRE].

JCAP03(2013)030