Density perturbations in generalized Einstein scenarios and constraints on nonminimal couplings from the cosmic microwave background

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(Received 9 February 2004; published 29 June 2004)

We study cosmological perturbations in generalized Einstein scenarios and show the equivalence of inflationary observables both in the Jordan frame and the Einstein frame. In particular the consistency relation relating the tensor-to-scalar ratio with the spectral index of tensor perturbations coincides with the one in Einstein gravity, which leads to the same likelihood results in terms of inflationary observables. We apply this formalism to nonminimally coupled chaotic inflationary scenarios with the potential $V=c\phi^p$ and place constraints on the strength of the nonminimal couplings using a compilation of the latest observational data. In the case of the quadratic potential $(p=2)$, the nonminimal coupling is constrained to be $\xi>-7.0\times10^{-3}$ for negative ξ from the 1σ observational contour bound. Although the quartic potential $(p=4)$ is under strong observational pressure for $\xi=0$, this property is relaxed by taking into account negative nonminimal couplings. We find that inflationary observables are within the 1σ contour bound as long as $\xi<-1.7\times10^{-3}$. We also show that the $p\geqslant6$ cases are disfavored even in the presence of nonminimal couplings.

DOI: 10.1103/PhysRevD.69.123523 PACS number(s): 98.80.Cq

I. INTRODUCTION

The inflationary paradigm has been the backbone of high-energy cosmology over the past 20 years [1]. The striking feature of the inflationary cosmology is that it predicts nearly scale-invariant, Gaussian, adiabatic density perturbations in its simplest form. This prediction shows an excellent agreement with all existing and accumulated data within observational errors. In particular the recent measurement of the Wilkinson Microwave Anisotropy Probe (WMAP) [2] provided the high-precision dataset from which inflationary models can be seriously constrained [3–8].

The prediction of inflationary observables exhibits some difference depending on the models of inflation. It is important to pick up this slight difference in order to discriminate between a host of inflationary scenarios. Conventionally inflationary models can be classified in three classes [9]—"large-field," "small-field," and "hybrid" models—depending upon the shape of the inflaton potential. The large field model is characterized by the potential

$$V(\phi)=c\phi^p, \tag{1.1}$$

which includes only one free parameter for a fixed value of p. Therefore it is not generally difficult to place strong constraints on the potential (1.1) compared to small-field and hybrid models that involve additional model parameters. In fact it was recently shown in Refs. [4–8] that the quartic potential $(p=4)$ is under a strong observational pressure due to the deviation of a scale-invariant spectrum in addition to a high tensor-to-scalar ratio. This situation does not change much in the context of the Randall-Sundrum II braneworld scenario [10,11]. Note that the quadratic potential $(p=2)$ is within observational contour bounds due to the flatness of the potential relative to the quartic case.

When we try to construct models of inflation based on particle physics, we do not need to restrict ourselves to the standard Einstein gravity. For example, low-energy effective string theory gives rise to a coupling between the scalar curvature and the dilaton field, which leads to an inflationary solution in the string frame [12]. The Jordan-Brans-Dicke (JBD)-like theories can be viewed as the low-energy limit of superstring theory if the Brans-Dicke scalar plays the role of the dilaton [13,14]. In this sense the proposal of the extended inflation scenario [15] stimulated a further study of more generic classes of inflation models in non-Einstein theories [16,17], in spite of the fact that the first version of the extended inflation resulted in failure due to the graceful exit problem [18].

From the viewpoint of quantum field theory in curved spacetime, the nonminimal couplings naturally arise due to their own nontrivial renormalization-group flows. The ultraviolet fixed point of these flows is often divergent, implying that the nonminimal couplings can be important in the early universe. In this respect Futamase and Maeda [19] studied the effect of nonminimal couplings on the dynamics of chaotic inflation and estimated the strength of the coupling ξ from the requirement of a sufficient inflation. While $|\xi|$ is required to be much smaller than unity in the quadratic potential, such a constraint is absent in the quadratic potential for negative ξ. Fakir and Unruh [20] showed that the fine-tuning problem of the self-coupling c in Eq. (1.1) is relaxed for large negative values of ξ by evaluating the amplitude of scalar metric perturbations. A number of authors further investigated the spectra of scalar and tensor perturbations generated in inflation [21–28] and particle productions in reheating [29].[1] See, e.g., Refs. [31] for other interesting aspects of nonminimally coupled scalar fields.

In this work we do not restrict ourselves to the Fakir and Unruh scenario, but will place constraints on the strength of nonminimal coupling for the general potential (1.1) by using a compilation of recent observational datasets. We shall provide a general formalism for scalar and tensor perturbations in generalized Einstein theories including dilaton gravity, JBD theory, and a nonminimally coupled scalar field. This analysis explicitly shows the equivalence of inflationary observables in both the Jordan frame and the Einstein frame, which results in the fact that a separate likelihood analysis in terms of observational quantities is not needed compared to the Einstein gravity. Making use of the two-dimensional observational constraints of the scalar spectral index $n_{\rm S}$ and the tensor-to-scalar ratio R, we shall carry out a detailed analysis about the constraint on the nonminimally coupled inflaton field for the potential (1.1). This provides us an interesting possibility to place strong constraints on the strength of nonminimal couplings from the recent high-precision observations. In fact we will show that even small nonminimal couplings with $|\xi| \ll 1$ can alter the standard prediction of the Einstein gravity.

II. GENERAL FORMALISM FOR PERTURBATION SPECTRA

We start with a generalized action [32]

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{1}{2}F(\phi)R - \frac{1}{2}\omega(\phi)(\nabla\phi)^2 - V(\phi)\right], \tag{2.1}$$

where R is the Ricci scalar. $F(\phi)$, $\omega(\phi)$ and $V(\phi)$ are general functions of a scalar field ϕ. The action (2.1) includes a variety of gravity theories such as the Einstein gravity, scalar tensor theories, and low-energy effective string theories. For example, we have $F(\phi)=1/\kappa^2$ and $\omega(\phi)=1$ in the Einstein gravity, $F(\phi)=(1-\xi\kappa^2\phi^2)/\kappa^2$ and $\omega(\phi)=1$ for a nonminimally coupled scalar field, $F(\phi)=e^{-\phi}$ and $\omega(\phi)=-e^{-\phi}$ for low-energy effective string theories. Hereafter we basically use the unit $\kappa^2 \equiv 8\pi/m_{\rm pl}^2 = 1$, but we restore the Planck mass $m_{\rm pl}$ when it is needed.

In a flat Friedmann-Lemaitre-Robertson-Walker (FLRW) background with a scale factor a, the background equations are

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{6F}(\omega\dot{\phi}^2 + 2V - 6H\dot{F}), \tag{2.2}$$

$$\dot{H} = \frac{1}{2F}(-\omega\dot{\phi}^2 + H\dot{F} - \ddot{F}), \tag{2.3}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{1}{2\omega}(\omega_{\phi}\dot{\phi}^2 - F_{\phi}R + 2V_{\phi}) = 0, \tag{2.4}$$

where H is the Hubble expansion rate and a dot denotes the derivative with respect to a cosmic time t.

A. Perturbations in Jordan frame

We consider a general perturbed metric for scalar perturbations,

$$ds^2 = -(1+2A)dt^2 + 2a(t)B_{,i}dx^i dt + a^2(t)[(1+2\varphi)\delta_{ij} + 2E_{,i,j}]dx^i dx^j, \tag{2.5}$$

where a comma denotes a flat-space coordinate derivative. It is convenient to introduce comoving curvature perturbations $\mathcal{R}$, defined by

$$\mathcal{R} = \varphi - \frac{H}{\dot{\phi}}\delta\phi, \tag{2.6}$$

where $\delta\phi$ is the perturbation of the field ϕ. The equation of motion for the Lagrangian (2.1) was derived in Refs. [32] and is simply given by

$$\frac{1}{a^3 Q_{\rm S}}(a^3 Q_{\rm S}\dot{\mathcal{R}})^{\bullet} + \frac{k^2}{a^2}\mathcal{R} = 0, \quad \text{with} \quad Q_{\rm S} = \frac{\omega\dot{\phi}^2 + 3\dot{F}^2/2F}{(H+\dot{F}/2F)^2}, \tag{2.7}$$

where k is a comoving wave number. Note that a similar form of equation is derived even in the presence of more complicated terms such as the Gauss-Bonnet term and the $(\nabla\phi)^4$ term [33]. If we neglect the contribution of the decaying mode, the curvature perturbation is conserved in the large-scale limit $(k \to 0)$. Introducing new variables, $z = a\sqrt{Q_{\rm S}}$ and $v = a\mathcal{R}$, the above equation reduces to

$$v'' + (k^2 - z''/z)v = 0, \tag{2.8}$$

where a prime denotes a derivative with respect to a conformal time $\eta = \int a^{-1}dt$.

The gravity term z''/z can be written as

$$\frac{z''}{z} = (aH)^2\left[(1+\delta_{\rm S})(2+\delta_{\rm S}+\epsilon) + \frac{\delta_{\rm S}'}{aH}\right], \tag{2.9}$$

where

$$\epsilon = \frac{\dot{H}}{H^2}, \quad \delta_{\rm S} = \frac{\dot{Q}_{\rm S}}{2HQ_{\rm S}}. \tag{2.10}$$

In the context of slow-roll inflation, it is a good approximation to neglect the variations of ϵ and $\delta_{\rm S}$. Since $\eta = -1/[(1+\epsilon)aH]$ in this case, we have

$$\frac{z''}{z} = \frac{\gamma_{\rm S}}{\eta^2}, \quad \text{with} \quad \gamma_{\rm S} = \frac{(1+\delta_{\rm S})(2+\delta_{\rm S}+\epsilon)}{(1+\epsilon)^2}. \tag{2.11}$$

[1]The presence of negative nonminimal couplings also leads to the strong variation of curvature perturbations in the context of multi-field inflation. See Refs. [30] for details.

Then the solution for Eq. (2.8) is expressed by the combination of the Hankel functions:

$$v=(\sqrt{\pi|\eta|}/2)[c_1 H_{\nu_S}^{(1)}(k|\eta|)+c_2 H_{\nu_S}^{(2)}(k|\eta|)], \quad (2.12)$$

where $\nu_S \equiv \sqrt{\gamma_S+1/4}$. We choose the coefficients to be $c_1=0$ and $c_2=1$, so that positive frequency solutions in a Minkowski vacuum are recovered in an asymptotic past. Making use of the relation $H_{\nu_S}^{(2)}(k|\eta|)\to(i/\pi)\Gamma(\nu_S)(k|\eta|/2)^{-\nu_S}$ for long-wavelength perturbations $(k\to 0)$, one gets the spectrum of curvature perturbations, $\mathcal{P}_S \equiv (k^3/2\pi^2)|\mathcal{R}|^2$, as

$$\mathcal{P}_S=\frac{1}{Q_S}\left(\frac{H}{2\pi}\right)^2\left(\frac{1}{aH|\eta|}\right)^2\left(\frac{\Gamma(\nu_S)}{\Gamma(3/2)}\right)^2\left(\frac{k|\eta|}{2}\right)^{3-2\nu_S} \equiv A_S^2\left(\frac{k|\eta|}{2}\right)^{3-2\nu_S}. \quad (2.13)$$

When $\nu=0$, we have an additional $\ln(k|\eta|)$ factor [33].

Then the spectral index, $n_S \equiv 1+d\ln\mathcal{P}_S/d\ln k$, is given by

$$n_S-1=3-2\nu_S=3-\sqrt{4\gamma_S+1}, \quad (2.14)$$

where γ_S is given in Eq. (2.11).

Let us next consider the spectrum of tensor perturbations, h_{ij}. Since h_i^j satisfies the same form of equation as in Eq. (2.7) with replacement $Q_S \to Q_T = F$, we get the power spectrum to be

$$\mathcal{P}_T=\frac{8}{Q_T}\left(\frac{H}{2\pi}\right)^2\left(\frac{1}{aH|\eta|}\right)^2\left(\frac{\Gamma(\nu_T)}{\Gamma(3/2)}\right)^2\left(\frac{k|\eta|}{2}\right)^{3-2\nu_T} \equiv A_T^2\left(\frac{k|\eta|}{2}\right)^{3-2\nu_T}, \quad (2.15)$$

where

$$\nu_T=\sqrt{\gamma_T+1/4}, \quad \text{with}$$

$$\gamma_T=\frac{(1+\delta_T)(2+\delta_T+\epsilon)}{(1+\epsilon)^2} \quad \text{with} \quad \delta_T=\frac{\dot{Q}_T}{2HQ_T}. \quad (2.16)$$

Note that we take into account the polarization states of gravitational waves. The spectral index of tensor perturbations is given by

$$n_T=3-\sqrt{4\gamma_T+1}. \quad (2.17)$$

The difference of scalar and tensor perturbations comes from the fact that Q_S differs from Q_T.

The tensor-to-scalar ratio is defined as

$$R\equiv\frac{A_T^2}{A_S^2}=8\frac{Q_S}{Q_T}\left(\frac{\Gamma(\nu_T)}{\Gamma(\nu_S)}\right)^2=8\frac{\omega\dot{\phi}^2+3\dot{F}^2/2F}{F(H+\dot{F}/2F)^2}\left(\frac{\Gamma(\nu_T)}{\Gamma(\nu_S)}\right)^2. \quad (2.18)$$

B. Einstein frame and the equivalence of perturbation spectra

The discussion in the previous subsection corresponds to the analysis in the Jordan frame. It is not obvious whether the same property holds in the Einstein frame. Let us make the conformal transformation for the action (2.1),

$$\hat{g}_{\mu\nu}=\Omega g_{\mu\nu}, \quad \text{with} \quad \Omega=F. \quad (2.19)$$

Then we get the action in the Einstein frame [34,35],

$$\mathcal{S}_E=\int d^4\hat{x}\sqrt{-\hat{g}}\left[\frac{1}{2}\hat{R}-\left\{\frac{3}{4}\left(\frac{F_\phi}{F}\right)^2+\frac{\omega}{2F}\right\}(\hat{\nabla}\phi)^2-\hat{V}(\phi)\right],$$

$$\text{with} \quad \hat{V}(\phi)=\frac{V(\phi)}{F^2}. \quad (2.20)$$

If we introduce a new scalar field $\hat{\phi}$, so that

$$d\hat{\phi}\equiv G(\phi)d\phi, \quad \text{with} \quad G(\phi)=\sqrt{\frac{3}{2}\left(\frac{F_\phi}{F}\right)^2+\frac{\omega}{F}}, \quad (2.21)$$

the action (2.20) can be written in the canonical form

$$\mathcal{S}_E=\int d^4\hat{x}\sqrt{-\hat{g}}\left[\frac{1}{2}\hat{R}-\frac{1}{2}(\nabla\hat{\phi})^2-\hat{V}(\phi)\right]. \quad (2.22)$$

We shall consider a perturbed metric in the Einstein frame,

$$d\hat{s}^2=\Omega ds^2 \quad (2.23)$$

$$=-(1+2\hat{A})d\hat{t}^2+2\hat{a}(\hat{t})\hat{B}_{,i}d\hat{x}^i d\hat{t} + \hat{a}^2(\hat{t})[(1+2\hat{\varphi})\delta_{ij}+2\hat{E}_{,i,j}]d\hat{x}^i d\hat{x}^j, \quad (2.24)$$

and decompose the conformal factor into the background and the perturbed part as

$$\Omega(\mathbf{x},t)=\bar{\Omega}(t)\left(1+\frac{\delta\Omega(\mathbf{x},t)}{\bar{\Omega}(t)}\right). \quad (2.25)$$

In what follows we drop a "bar" when we express $\bar{\Omega}(t)$. Then we get the following relations:

$$\hat{a}=a\sqrt{\Omega}, \quad d\hat{t}=\sqrt{\Omega}dt, \quad \hat{H}=\frac{1}{\sqrt{\Omega}}\left(H+\frac{\dot{\Omega}}{2\Omega}\right),$$

$$\hat{A}=A+\frac{\delta\Omega}{2\Omega}, \quad \hat{\varphi}=\varphi+\frac{\delta\Omega}{2\Omega}. \quad (2.26)$$

Making use of these relations, it is easy to show that curvature perturbations in the Einstein frame exactly coincide with those in the Jordan frame:

$$\hat{\mathcal{R}} \equiv \hat{\varphi} - \frac{\hat{H}}{d\hat{\phi}/d\hat{t}}\delta\hat{\phi} \tag{2.27}$$

$$= \varphi - \frac{H}{\dot{\phi}}\delta\phi = \mathcal{R}. \tag{2.28}$$

Since tensor perturbations are also invariant under a conformal transformation, we have the following relations:

$$\hat{\mathcal{P}}_{\mathrm{S}} = \mathcal{P}_{\mathrm{S}}, \quad \hat{\mathcal{P}}_{\mathrm{T}} = \mathcal{P}_{\mathrm{T}}. \tag{2.29}$$

Introducing the following quantities:

$$\hat{\gamma}_{\mathrm{S}} = \frac{(1+\hat{\delta}_{\mathrm{S}})(2+\hat{\delta}_{\mathrm{S}}+\hat{\epsilon})}{(1+\hat{\epsilon})^2}, \quad \hat{\epsilon} = \frac{d\hat{H}/d\hat{t}}{\hat{H}^2},$$

$$\hat{\delta}_{\mathrm{S}} = \frac{d\hat{Q}_{\mathrm{S}}/d\hat{t}}{2\hat{H}\hat{Q}_{\mathrm{S}}}, \quad \hat{Q}_{\mathrm{S}} = \left(\frac{d\hat{\phi}/d\hat{t}}{\hat{H}}\right)^2 = Q_{\mathrm{S}}/F, \tag{2.30}$$

the spectral index of scalar perturbations in the Einstein frame is given by

$$\hat{n}_{\mathrm{S}} - 1 = 3 - \sqrt{4\hat{\gamma}_{\mathrm{S}}+1}. \tag{2.31}$$

The two quantities $\hat{\epsilon}$ and $\hat{\delta}_{\mathrm{S}}$ can be written as

$$\hat{\epsilon} = \frac{\epsilon-\beta}{1+\beta} + \frac{\dot{\beta}}{H(1+\beta)^2}, \quad \hat{\delta}_{\mathrm{S}} = \frac{\delta-\beta}{1+\beta}, \quad \text{with} \quad \beta = \frac{\dot{F}}{2HF}. \tag{2.32}$$

When the variation of β is negligible ($\dot{\beta} \simeq 0$), which is valid in the context of slow-roll inflation, one can easily show that $\hat{\gamma}_{\mathrm{S}} = \gamma_{\mathrm{S}}$. Therefore the spectral index (2.31) in the Einstein frame coincides with the one in the Jordan frame ($\hat{n}_{\mathrm{S}} = n_{\mathrm{S}}$). The spectral index of tensor perturbations is also the same in both frames ($\hat{n}_{\mathrm{T}} = n_{\mathrm{T}}$), and is simply given as

$$\hat{n}_{\mathrm{T}} = 3 - \sqrt{4\hat{\gamma}_{\mathrm{T}}+1}, \quad \text{with} \quad \hat{\gamma}_{\mathrm{T}} = \frac{2+\hat{\epsilon}}{(1+\hat{\epsilon})^2}, \tag{2.33}$$

where we used $\hat{Q}_{\mathrm{T}} = Q_{\mathrm{T}}/F = 1$ and $\hat{\delta}_{\mathrm{T}} = 0$.

The tensor-to-scalar ratio is unchanged by a conformal transformation and is given by

$$\hat{R} = \frac{\hat{A}_{\mathrm{T}}^2}{\hat{A}_{\mathrm{S}}^2} = 8\frac{\hat{Q}_{\mathrm{S}}}{\hat{Q}_{\mathrm{T}}}\left(\frac{\Gamma(\hat{\nu}_{\mathrm{T}})}{\Gamma(\hat{\nu}_{\mathrm{S}})}\right)^2 = 8\frac{Q_{\mathrm{S}}}{Q_{\mathrm{T}}}\left(\frac{\Gamma(\nu_{\mathrm{T}})}{\Gamma(\nu_{\mathrm{S}})}\right)^2 = R, \tag{2.34}$$

where we used $\hat{Q}_{\mathrm{S}} = Q_{\mathrm{S}}/F$, $\hat{Q}_{\mathrm{T}} = Q_{\mathrm{T}}/F$, $\hat{\nu}_{\mathrm{S}} = \nu_{\mathrm{S}}$, and $\hat{\nu}_{\mathrm{T}} = \nu_{\mathrm{T}}$. Making use of the background equation, $2d\hat{H}/d\hat{t} = -(d\hat{\phi}/d\hat{t})^2$, we get

$$\hat{R} = 8\left(\frac{d\hat{\phi}/d\hat{t}}{\hat{H}}\right)^2\left(\frac{\Gamma(\hat{\nu}_{\mathrm{T}})}{\Gamma(\hat{\nu}_{\mathrm{S}})}\right)^2 = -16\frac{d\hat{H}/d\hat{t}}{\hat{H}^2}\left(\frac{\Gamma(\hat{\nu}_{\mathrm{T}})}{\Gamma(\hat{\nu}_{\mathrm{S}})}\right)^2. \tag{2.35}$$

Hereafter we shall drop a "hat" for the quantities $\hat{n}_{\mathrm{S}}$, $\hat{n}_{\mathrm{T}}$, $\hat{R}$.

C. Slow-roll analysis

If the slow-roll approximation is employed ($|\epsilon| \ll 1$, $|\delta_{\mathrm{S}}| \ll 1$, $|\delta_{\mathrm{T}}| \ll 1$, and $|\beta| \ll 1$), we get the following relations:

$$n_{\mathrm{S}} = 1 + 2\epsilon - 2\delta_{\mathrm{S}} = 1 + 2\hat{\epsilon} - 2\hat{\delta}_{\mathrm{S}}, \tag{2.36}$$

$$n_{\mathrm{T}} = 2\epsilon - 2\delta_{\mathrm{T}} = 2\hat{\epsilon}, \tag{2.37}$$

$$R = -8n_{\mathrm{T}}. \tag{2.38}$$

Here we used $\Gamma(\nu_{\mathrm{S}}) \simeq \Gamma(\nu_{\mathrm{T}}) \simeq \Gamma(3/2)$, since the spectra of scalar and tensor perturbations are close to scale invariant. The consistency relation (2.38) is the same as in the case of the standard Einstein gravity.[2] This means that a separate likelihood analysis of observational data is not needed even for the generalized action (2.1), when we vary four observational quantities n_{S}, n_{T}, R, and A_{S}^2. This situation is similar to the perturbations in Randall-Sundrum II braneworld scenario in which the same consistency relation (2.38) holds [37]. Thus we can exploit the observational constraints on the values of n_{S} and R which were recently derived in Ref. [11].

On the other hand, when we constrain the inflation models which belong to the generalized action (2.1), the difference is seen compared to the Einstein gravity. This comes from the fact that the potential in the Einstein frame includes the conformal factor $1/F^2$ and that the field $\hat{\phi}$ is different from the inflaton ϕ [see Eq. (2.21)]. Under the slow-roll approximation, the background equations in the Einstein frame can be written as

$$3\hat{H}^2 \simeq \kappa^2\hat{V}, \quad 3\hat{H}\frac{d\hat{\phi}}{d\hat{t}} \simeq -\hat{V}_{\hat{\phi}}. \tag{2.39}$$

Then $\hat{\epsilon}$ and $\hat{\delta}_{\mathrm{S}}$ are written in terms of the slope of the potential

$$\hat{\epsilon} \simeq -\frac{1}{2\kappa^2}\left(\frac{\hat{V}_{\hat{\phi}}}{\hat{V}}\right)^2, \quad \hat{\delta}_{\mathrm{S}} \simeq \frac{1}{\kappa^2}\left[\left(\frac{\hat{V}_{\hat{\phi}}}{\hat{V}}\right)^2 - \frac{\hat{V}_{\hat{\phi}\hat{\phi}}}{\hat{V}}\right]. \tag{2.40}$$

Then we can evaluate the theoretical values n_{S}, n_{T}, R and constrain model parameters by comparing with observational data.

[2] Note that the consistency relation is different in the context of multifield inflation due to the correlation between adiabatic and isocurvature perturbations. See Refs. [36] for details.

III. NONMINIMALLY COUPLED INFLATON FIELD

In this section we wish to apply the formalism in the previous section to a nonminimally coupled inflaton field. In this case we have

$$F(\phi)=(1-\xi\kappa^2\phi^2)/\kappa^2, \quad \omega(\phi)=1, \tag{3.1}$$

for the action (2.1). Here we used the notation where the conformal coupling corresponds to $\xi=1/6$, which is the same notation as Futamase and Maeda [19] but different from Fakir and Unruh [20].

A. Background equations

In the case (3.1) the background equations (2.2)–(2.4) are written in the form

$$H^2=\frac{\kappa^2}{6(1-\xi\kappa^2\phi^2)}[\dot{\phi}^2+2V+12H\xi\phi\dot{\phi}], \tag{3.2}$$

$$\ddot{\phi}+3H\dot{\phi}-\frac{\xi(1-6\xi)\kappa^2\phi\dot{\phi}^2}{1-(1-6\xi)\xi\kappa^2\phi^2}+\frac{4\xi\kappa^2\phi V+(1-\xi\kappa^2\phi^2)V_\phi}{1-(1-6\xi)\xi\kappa^2\phi^2}=0, \tag{3.3}$$

where we used $R=6(2H^2+\dot{H})$.

Hereafter we shall consider the large-field potential (1.1). Under the slow-roll approximations, $|\ddot{\phi}|\ll|3H\dot{\phi}|$ and $|3H\dot{\phi}|\ll|V_\phi|$, one has

$$3H\dot{\phi}\simeq-\frac{c\phi^{p-1}[p+\psi(4-p)]}{1-(1-6\xi)\psi}, \tag{3.4}$$

$$H^2\simeq\frac{\kappa^2 c\phi^p}{3(1-\psi)}\left[1-\frac{2\xi\{p+\psi(4-p)\}}{1-(1-6\xi)\psi}\right], \tag{3.5}$$

where

$$\psi\equiv\xi\kappa^2\phi^2. \tag{3.6}$$

We get the following equation from Eqs. (3.4) and (3.5):

$$\frac{\dot{\psi}}{H}=-\frac{2\xi(\psi-1)[(p-4)\psi-p]}{(2\xi p-2\xi-1)\psi+1-2\xi p}. \tag{3.7}$$

The number of e folds is defined as

$$N\equiv\int_t^{t_f}Hdt=\int_\psi^{\psi_f}\frac{H}{\dot{\psi}}d\psi, \tag{3.8}$$

where the subscript "f" denotes the values at the end of inflation. Making use of Eq. (3.7), we have

$$N=-\frac{1}{4\xi}\ln\left|\frac{(p-4)\psi_f-p}{(p-4)\psi-p}\right|^{(3\xi p-2)/(p-4)}\left|\frac{\psi_f-1}{\psi-1}\right|^{\xi}, \quad \text{for} \quad p\neq 4, \tag{3.9}$$

$$N=-\frac{1-6\xi}{8\xi}(\psi_f-\psi)-\frac{1}{4}\ln\left|\frac{\psi_f-1}{\psi-1}\right|, \quad \text{for} \quad p=4. \tag{3.10}$$

Later we shall use this relation to express ψ in terms of N.

B. Potential in the Einstein frame

The potential in the Einstein frame and the effective gravitational constant are given, respectively, by

$$\hat{V}=\frac{c\phi^p}{(1-\xi\kappa^2\phi^2)^2}, \quad G_{\rm eff}=\frac{G}{1-\xi\kappa^2\phi^2}. \tag{3.11}$$

When ξ is positive, we need the condition $\phi^2<\phi_c^2\equiv m_{\rm pl}^2/(8\pi\xi)$ in order to reproduce the present value of the gravitational constant. Futamase and Maeda obtained the constraint $\xi\lesssim10^{-3}$ from the requirement that the initial value of the inflaton ($\phi_i\sim5m_{\rm pl}$) is smaller than ϕ_c [19]. They also pointed out that this constraint becomes more stringent when ϕ_i is larger.

We do not have the singularities of the potential $\hat{V}$ and the effective gravitational constant for negative values of ξ. Futamase and Maeda found that the negative values of ξ are generically allowed for $p=4$, while the nonminimal coupling is constrained to be $|\xi|\lesssim10^{-3}$ for $p=2$ from the requirement of a sufficient inflation. In particular Fakir and Unruh [20] showed that the fine tuning of the coupling constant c is avoided for $p=4$ by considering large negative values of ξ satisfying $|\xi|\gg1$. Hereafter we shall concentrate on the case of negative ξ for the general potential (1.1). We wish to constrain the strength of the nonminimal couplings by exploiting latest observational data. While Komatsu and Futamase [25,26] focused on large negative nonminimal couplings for $p=4$, we shall consider general values of ξ with the generic potential (1.1).

In Fig. 1 we plot the potential $\hat{V}$ in the Einstein frame for $p=2$, $p=4$, and $p=6$ when ξ is negative. From Eq. (3.11) the potential has a local maximum at

$$\phi_M=\sqrt{\frac{p}{8\pi(4-p)|\xi|}}m_{\rm pl}, \tag{3.12}$$

as long as $p<4$. Therefore we have $\phi_M=m_{\rm pl}/\sqrt{8\pi|\xi|}$ for $p=2$. Naively we expect that ϕ_M is required to be larger than $3m_{\rm pl}$ in order to lead to a sufficient number of e folds ($N\gtrsim60$), which gives $|\xi|\lesssim4.4\times10^{-3}$. Note, however, that one can get a large number of e folds if the initial value of ϕ is close to ϕ_M. Therefore the nonminimal couplings satisfying $|\xi|\gtrsim4.4\times10^{-3}$ may not be excluded.[3] In this work we will place complete constraints on the strength of ξ by comparing the perturbation spectra with observations.

When $p=4$ the potential $\hat{V}$ monotonically increases toward a constant value, $\hat{V}\to c/\xi^2\kappa^4$, as $\phi\to\infty$. Since the po-

[3]In Ref. [27] it was shown that the constraint on ξ is relaxed by considering topological inflation.

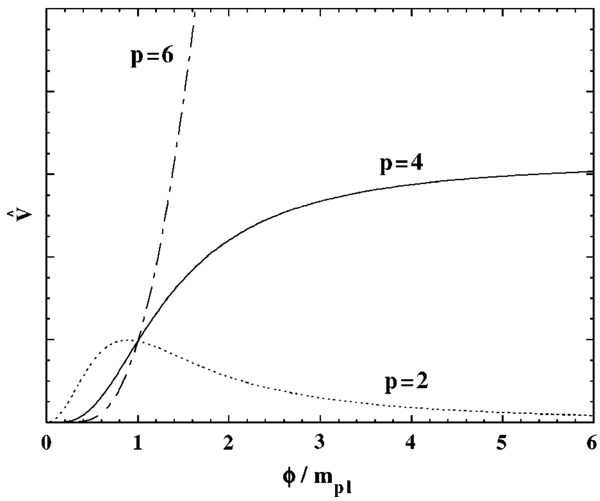

FIG. 1. The potential of the inflaton in the Einstein frame with $\xi=-0.05$ for $p=2$, $p=4$, and $p=6$. It has a local maximum at $\phi_{\rm M}=m_{\rm pl}/\sqrt{8\pi|\xi|}$ for $p=2$. When $p=4$ the potential approaches a constant value $\hat{V}=c/\xi^2\kappa^4$ as $\phi\to\infty$.

tential becomes flatter by taking into account the negative nonminimal couplings, the amount of inflation gets larger compared to the case of $\xi=0$. For $|\psi|\gg 1$, we obtain the following background solution from Eqs. (3.4) and (3.5):

$$\phi=\phi_0-\frac{4\sqrt{c}}{(1-6\xi)\sqrt{-3\xi\kappa^2}}t,$$

$$a=a_0\ \exp\left[\sqrt{\frac{c}{-3\xi}}\left(\phi_0 t-\frac{2\sqrt{c}}{(1-6\xi)\sqrt{-3\xi\kappa^2}}t^2\right)\right], \tag{3.13}$$

where ϕ_0 and a_0 are constants. This means that the universe expands quasiexponentially for large negative values of ξ.

In the case of $p>4$, the steepness of the potential $\hat{V}$ is relaxed by taking into account negative ξ. If $|\psi|$ is much smaller than unity, the effect of the nonminimal couplings helps to lead to a larger number of e folds. When $|\psi|\gg 1$, we have the following analytic solution:

$$\phi=\left[\frac{(p-2)(p-4)\sqrt{c}}{12\sqrt{-\xi(p-1)}}t+\phi_0\right]^{2/(2-p)},$$

$$a=a_0\left[t-\frac{6\xi\phi_0}{c(4-p)}\right]^{4(p-1)/(p-2)(p-4)}. \tag{3.14}$$

This explicitly shows that the field ϕ decreases with time for $p>4$. We also find that the solution (3.14) does not correspond to an inflationary solution for $p>5+\sqrt{13}$.

The slow-roll parameter in the Einstein frame is given as

$$|\hat{\epsilon}|=\frac{1}{2\kappa^2 G^2(\phi)}\left(\frac{\hat{V}_\phi}{\hat{V}}\right)^2=\frac{\xi}{2\psi}\frac{[p+(4-p)\psi]^2}{1-(1-6\xi)\psi}, \tag{3.15}$$

where we used $G^2(\phi)=[1-(1-6\xi)\psi]/(1-\psi)^2$. Therefore one has $|\hat{\epsilon}|=(p-4)^2/12$ for $|\psi|\gg 1$ and $|\xi|\gg 1$, which means that $|\hat{\epsilon}|$ is larger than unity for $p>4+2\sqrt{3}$. This again shows that inflationary solutions are not obtained for $p\gtrsim 8$. Since we cannot keep the $1/G^2(\phi)$ term to be small in Eq. (3.15) for $|\psi|\gg 1$, the slow-roll parameter exceeds unity unless p is small. Hereafter we shall mainly investigate the cases of $p=2$, $p=4$, and $p=6$. This is sufficient to understand what happens for the perturbation spectra in the presence of the nonminimal couplings.

C. Perturbation spectra and the tensor-to-scalar ratio

We are now in the stage to evaluate the spectra of perturbations for the nonminimally coupled inflaton field. Making use of the results (2.36) and (2.38) with slow-roll parameters (2.40), we get the values of $n_{\rm S}$ and R as

$$n_{\rm S}-1=\frac{\xi}{\psi}\frac{p+(4-p)\psi}{1-(1-6\xi)\psi}\left[-3p+(3p-8)\psi+\frac{2(1-6\xi)\psi(1-\psi)}{1-(1-6\xi)\psi}+2\frac{(1-\psi)^2p(p-1)+4\psi\{1+p+(1-p)\psi\}}{p+(4-p)\psi}\right], \tag{3.16}$$

$$R=\frac{8\xi}{\psi}\frac{[p+(4-p)\psi]^2}{1-(1-6\xi)\psi}, \tag{3.17}$$

which are written in terms of the functions of ξ and ψ for a fixed value of p.

The end of inflation is characterized by $|\hat{\epsilon}|=1$, thereby yielding

$$\psi_{\rm f}=\frac{1-\xi p(4-p)-\sqrt{(1-2p\xi)(1-6p\xi)}}{\xi(4-p)^2+2(1-6\xi)}, \tag{3.18}$$

which we choose the negative sign of $\psi_{\rm f}$, since we are considering the case of $\xi<0$. When $|\xi|\ll 1$, Eqs. (3.9) and (3.10) give the following relation:

$$|(p-4)\psi-p|\simeq|(p-4)\psi_{\rm f}-p|\exp[-2\xi N(p-4)],\quad \text{for}\quad p\neq 4, \tag{3.19}$$

$$\psi\simeq\psi_{\rm f}+\frac{8\xi}{1-6\xi}N,\quad \text{for}\quad p=4. \tag{3.20}$$

Making use of Eqs. (3.19) and (3.20), one can express $n_{\rm S}$ and R in terms of ξ and N. Fixing the e fold at the cosmologically relevant scale $N=55$, $n_{\rm S}$ and R are the function of ξ only. Therefore we can constrain the strength of the nonminimal couplings by comparing the theoretical predictions (3.16) and (3.17) with the observational data.

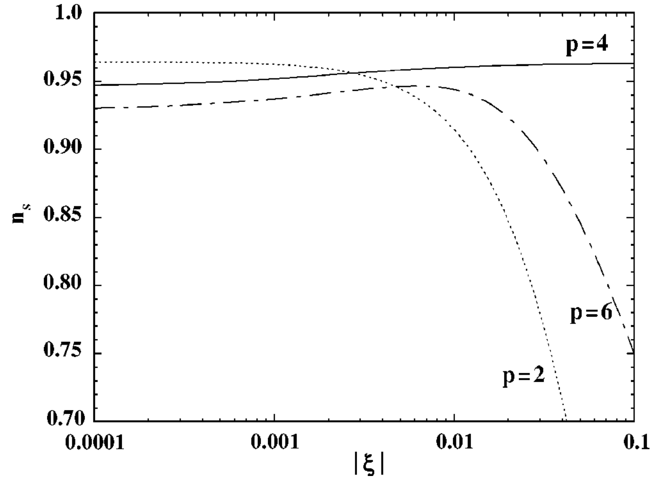

FIG. 2. The spectral index n_S as a function of $|\xi|$ for $p=2$, $p=4$, and $p=6$. Note that we are considering negative values of ξ. See the text for the interpretation of this figure.

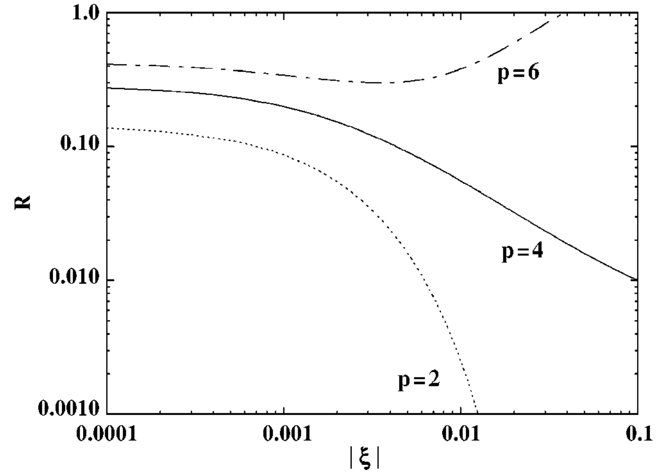

FIG. 3. The tensor-to-scalar ratio R as a function of $|\xi|$ for $p=2$, $p=4$, and $p=6$ with negative ξ.

Before proceeding the constraint on ξ, we shall investigate the effect of nonminimal couplings under the approximations of $|\xi|\ll 1$ and $|\psi|\ll 1$. In this case we have

$$n_S-1\simeq-\frac{\xi p(p+2)}{\psi}\left[1-\frac{p^2-8p+8}{p(p+2)}\psi\right], \quad (3.21)$$

$$R\simeq\frac{8p^2\xi}{\psi}\left(1+\frac{8-p}{p}\psi\right). \quad (3.22)$$

In what follows we shall investigate the cases of $p=2$, $p=4$, and $p=6$ separately.

1. Case of p=2

In this case one has the following relation from Eqs. (3.18) and (3.19):

$$\psi=\left|\frac{3}{2}-\frac{1}{2}\sqrt{\frac{1-12\xi}{1-4\xi}}\right|e^{4\xi N}-1\simeq e^{4\xi N}-1, \quad (3.23)$$

which is valid for $|\xi|\ll 1$. Then Eqs. (3.21) and (3.22) reduce to

$$n_S-1\simeq-4\xi\frac{e^{4\xi N}+1}{e^{4\xi N}-1}\simeq-\frac{2}{N}\left(1+\frac{4}{3}\xi^2N^2\right), \quad (3.24)$$

$$R\simeq 32\xi\frac{3e^{4\xi N}-2}{e^{4\xi N}-1}\simeq\frac{8}{N}(1+10\xi N). \quad (3.25)$$

In the minimally coupled case ($\xi=0$), we have $n_S=1-2/N$ and $R=8/N$, corresponding to $n_S=0.964$ and $R=0.145$ for $N=55$. If we take into account negative nonminimal couplings, we find the increase of $|n_S-1|$ and the decrease of R compared to the case of $\xi=0$. This behavior is clearly seen in Figs. 2 and 3 that are obtained without using the approximation above. The rapid decrease of R for $|\xi|\gtrsim 10^{-3}$ reflects the fact that ϕ approaches the value ϕ_M for larger $|\xi|$, which works to decrease $|\hat{\epsilon}|$ toward zero. On the other hand, the second derivative of the potential gets larger on the right-hand side of $\hat{\delta}_S$ with the growth of $|\xi|$. This is the reason why n_S departs from 1 for larger $|\xi|$ in spite of the fact that $\hat{\epsilon}$ gets smaller toward zero.

2. Case of p=4

In the case of the quartic potential, Eqs. (3.18) and (3.20) give the following relation:

$$\psi=\frac{1-\sqrt{(1-8\xi)(1-24\xi)}+16\xi N}{2(1-6\xi)}. \quad (3.26)$$

Note that this is valid for general values of ξ, since the second term on the right-hand side of Eq. (3.10) is always subdominant relative to the first term. When the condition, $|\xi|\ll 1$, is satisfied, one has $\psi\simeq 8\xi N$ from Eq. (3.26). If $|\psi|$ is smaller than of order unity, we find

$$n_S-1\simeq-\frac{24\xi}{\psi}\left(1+\frac{1}{3}\psi\right)\simeq-\frac{3}{N}\left(1+\frac{8}{3}\xi N\right), \quad (3.27)$$

$$R\simeq\frac{128\xi}{\psi}(1+\psi)\simeq\frac{16}{N}(1+8\xi N). \quad (3.28)$$

Then we have $n_S=1-3/N$ and $R=16/N$ for $\xi\to 0$, corresponding to $n_S=0.945$ and $R=0.291$ for $N=55$. Inclusion of the nonminimal couplings leads to the decrease of both $|n_S-1|$ and R, as seen in Figs. 2 and 3. This comes from the fact that the potential becomes flatter in the presence of negative nonminimal couplings.

It is worth mentioning the case of large negative nonminimal couplings ($|\xi|\gg 1$). Since $\hat{\epsilon}\simeq 8\xi/(1-6\xi)\psi^2$ and $\hat{V}_{\hat{\phi}\hat{\phi}}/\kappa^2\hat{V}\simeq-8\xi/(1-6\xi)\psi$ in this case, we find

$$n_S-1\simeq-\frac{16\xi}{1-6\xi}\frac{1}{\psi}, \quad R\simeq\frac{-128\xi}{1-6\xi}\frac{1}{\psi^2}. \quad (3.29)$$

From Eq. (3.26) one obtains $\psi \simeq -4N/3$ for $|\xi| \gg 1$. Then we get the following results:

$$n_S - 1 \simeq -2/N, \quad R \simeq 12/N^2. \tag{3.30}$$

Note that the spectral index n_S is the same as in the minimally coupled case with the quadratic potential ($\xi=0$ and $p=2$). One has $n_S=0.964$ and $R=0.00397$ for $N=55$.[4] Therefore the Fakir and Unruh scenario with $|\xi| \gg 1$ predicts a much smaller value of R compared to the minimally coupled case.

3. Case of p=6

For $p=6$ and $|\xi| \ll 1$, one obtains the following relation from Eq. (3.19):

$$\psi = 3 - |3-\psi_f| e^{-4\xi N}, \tag{3.31}$$

where ψ_f is approximately given as $\psi_f \simeq 18\xi$ by Eq. (3.18). Then we find $\psi \simeq 12\xi N$ and

$$n_S - 1 \simeq -\frac{48\xi}{\psi}\left(1+\frac{1}{12}\psi\right) \simeq -\frac{4}{N}(1+\xi N), \tag{3.32}$$

$$R \simeq \frac{288\xi}{\psi}\left(1+\frac{1}{3}\psi\right) \simeq -\frac{24}{N}(1+4\xi N). \tag{3.33}$$

This indicates that the negative nonminimal couplings lead to the decrease of $|n_S-1|$ and R for $|\xi| \ll 1$.

From Figs. 2 and 3 we find that $|n_S-1|$ and R begin to increase for $|\xi| \gtrsim 10^{-2}$. This can be understood as follows. When $p \neq 4$ and $|\psi|$ is larger than of order unity, we obtain

$$n_S - 1 \simeq \frac{4\xi}{1-6\xi}(p-4)^2, \tag{3.34}$$

$$R \simeq \frac{-16\xi}{1-6\xi}(p-4)^2, \tag{3.35}$$

which are independent of N. In the case of $p=6$, this yields $n_S-1=16\xi/(1-6\xi)$ and $R=-64\xi/(1-6\xi)$. Therefore both $|n_S-1|$ and R grow with the increase of $|\xi|$. The asymptotic values correspond to $n_S-1 \to -8/3$ and $R \to 32/3$ as $|\xi| \to \infty$.

D. Observational constraints on nonminimal couplings

Lets us now place observational constraints on the strength of the nonminimal couplings. As shown in Sec. II, the inflationary observables P_S, R, n_S, and n_T are equivalent both in the Jordan frame and the Einstein frame. This correspondence indicates that a separate likelihood analysis of observational data is not required compared to the Einstein gravity. Recently one of the present authors carried out a likelihood analysis [11] in the context of braneworld inflation using a compilation of data including WMAP [38–40], the 2 Degree Field System (2dF) [41], and latest Sloan Digital Sky Survey (SDSS) galaxy redshift surveys [42]. Since the same correspondence holds for inflationary observables in this case as well, we can exploit the observational constraints derived in Ref. [11]. Note that we used the COSMOMC (Cosmological Monte Carlo) code [43] with the CAMB program [44], and varied four inflationary variables in addition to four cosmological parameters ($\Omega_b h^2$, $\Omega_c h^2$, $Z=e^{-2\tau}$, H_0).

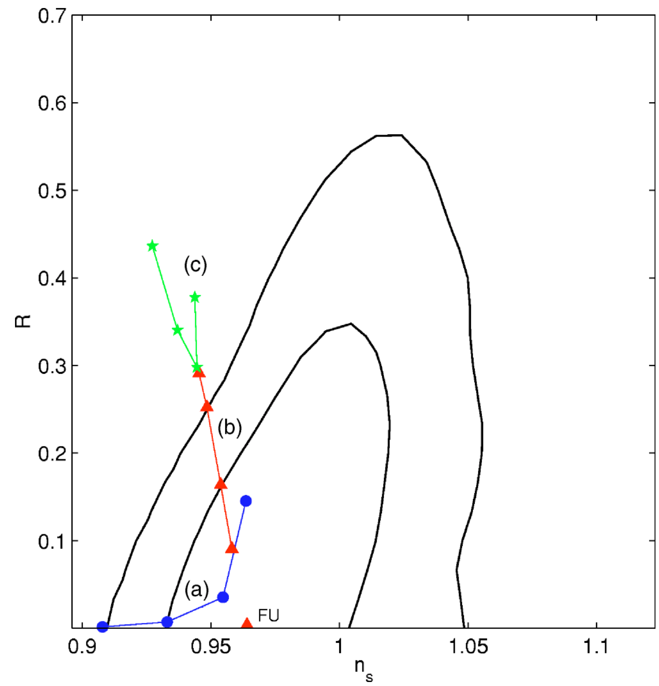

FIG. 4. 2D posterior constraints in the n_S-R plane with the 1σ and 2σ contour bounds. We also show the theoretical predictions for (a) $p=2$, (b) $p=4$, and (c) $p=6$ with a fixed e fold, $N=55$. Each case corresponds to, from top to bottom, (a) $\xi=0,-0.003,-0.007,-0.011$, and (b) $\xi=0,-0.0003,-0.0017,-0.005$. The point denoted by "FU" is the Fakir and Unruh scenario with $|\xi| \gg 1$. The plot (c) shows the cases of $\xi=0,-0.001,-0.0035$ from the left top to bottom, and another point corresponds to $\xi=-0.01$.

In Fig. 4 we plot the 2D posterior constraints in the n_S-R plane and also show the 1σ and 2σ contour bounds. In the previous subsection we obtained n_S and R in terms of the function of ξ by fixing the e fold to be $N=55$. Therefore one can constrain the strength of nonminimal couplings by plotting theoretical predictions of n_S and R in the same figure.

1. Case of p=2

In the case of the quadratic potential, the theoretical point is within the 1σ contour bound for $\xi=0$, as is seen in Fig. 4. Taking into account the negative nonminimal couplings leads to the decrease of both n_S and R (see Figs. 2 and 3). From Fig. 4 we obtain the observational constraint on negative nonminimal couplings:

$$\xi > -7.0 \times 10^{-3} \quad (1\sigma \text{ bound}), \tag{3.36}$$

$$\xi > -1.1 \times 10^{-2} \quad (2\sigma \text{ bound}). \tag{3.37}$$

[4] Komatsu and Futamase [26] obtained the values $n_S=0.97$ and $R=0.002$, since they chose the e fold $N=70$.

If ξ is less than of order 10^{-2}, the curvature of the potential around $\phi=\phi_{\rm M}$ is too steep to generate a nearly scale-invariant spectrum.

2. Case of p=4

It is now well known that the quartic potential is under a strong observational pressure in the minimally coupled case [4–8]. In fact the $\xi=0$ point is outside of the 2σ contour bound in Fig. 4. In the presence of the negative nonminimal couplings, we have the increase of $n_{\rm S}$ and the decrease of R, which is favored observationally. Figure 4 indicates that ξ is constrained to be

$$\xi<-1.7\times10^{-3} \quad (1\sigma \ \text{bound}), \tag{3.38}$$

$$\xi<-3.0\times10^{-4} \quad (2\sigma \ \text{bound}). \tag{3.39}$$

Thus nonminimal couplings of order $\xi=-10^{-3}$ make it possible to generate observational preferred power spectra. In Fig. 4 we also plot the theoretical point in the limit of $|\xi|\to\infty$ (denoted by ''FU''). This corresponds to the values given in Eq. (3.30) which is deep inside the 1σ contour bound. Thus the Fakir and Unruh scenario with $|\xi|\gg1$ is favored observationally relative to the minimally coupled case. In addition this scenario can relax the fine tuning problem of the coupling constant c [20].

3. Case of p=6

The $p=6$ case is far away from the 2σ bound for $\xi=0$. Negative nonminimal couplings lead to the increase of $n_{\rm S}$ and the decrease of R when $|\xi|$ is much smaller than unity. However this behavior is altered with the growth of $|\xi|$, as we showed in the previous section. The tensor-to-scalar ratio R is minimum around $\xi=-3.5\times10^{-3}$, whose point is outside of the 2σ contour bound. Since one has the decrease of $n_{\rm S}$ and the increase of R for $\xi<-3.5\times10^{-3}$, this regime is also away from the 2σ bound. Therefore the $p=6$ case is disfavored observationally even in the presence of nonminimal couplings. This situation does not change for $p>6$, since the theoretical points tend to be away from the observationally allowed region for larger p. In fact we numerically checked that the $p=8$ case is outside of the 3σ bound for any values of ξ.

IV. CONCLUSIONS AND DISCUSSIONS

In this work we studied cosmological perturbations in generalized gravity theories based on the action (2.1). We showed that curvature perturbations in the Jordan frame coincide with those in the Einstein frame. Since tensor perturbations are also invariant under a conformal transformation, the inflationary observables ($n_{\rm S}$, $n_{\rm T}$, R, and $A_{\rm S}$) are the same in both frames. This property indicates that the same likelihood can be employed as for the standard Einstein gravity. This is similar to what happens for the Randall-Sundrum II braneworld scenario in which the degeneracy of the consistency relation does not explicitly give rise to the signature of the braneworld [37], although the constraints of model parameters in terms of underlying potentials are different [11]. Remarkably the consistency relation (2.38) holds even for the generalized action (2.1) that includes dilaton gravity, JBD theory, and a nonminimally coupled scalar field.

We then apply our general formalism to the nonminimally coupled inflaton field with potential (1.1). Our main aim is to place strong constraints on the strength of nonminimal couplings using the latest observational data including WMAP, the 2dF, and SDSS galaxy redshift surveys. We focused on the case of the negative nonminimal couplings, since the positive coupling ξ was already severely constrained from the requirement of a sufficient inflation [19] (ξ is at least smaller than 10^{-3} and is even much smaller depending on the initial condition of the inflaton).

For the quadratic potential ($p=2$), inclusion of the negative nonminimal couplings leads to the decrease of the spectral index $n_{\rm S}$ and the tensor-to-scalar ratio R (see Figs. 2 and 3). While the minimally coupled case ($\xi=0$) is within the 1σ contour bound, the theoretical points of larger $|\xi|$ tend to be away from the observational bounds due to the departure from the scale invariance of the spectral index (see Fig. 4). We found the constraints $\xi>-7.0\times10^{-3}$ at the 1σ level and $\xi>-1.1\times10^{-2}$ at the 2σ level.

The quartic potential ($p=4$) suffers from a strong observational pressure for $\xi=0$, since the $\xi=0$ case is outside of the 2σ contour bound. However, this situation is easily improved by taking into account negative nonminimal couplings, as seen in Fig. 4. The strength of the coupling is constrained to be $\xi<-1.7\times10^{-3}$ at the 1σ level and $\xi<-3.0\times10^{-4}$ at the 2σ level. Note that the Fakir and Unruh scenario with large negative ξ ($|\xi|\gg1$) is deep inside the 1σ bound, thus preferred observationally. We also found that the $p\geq6$ cases are outside of the 2σ bound even in the presence of negative nonminimal couplings (see Fig. 4).

While we mainly concentrated on slow-roll inflation, the formula (2.31), (2.33), and (2.35) can be used in more general theories if the terms $\dot{\epsilon}$, $\dot{\delta}$, and $\dot{\beta}$ vanish. Actually this happens for the dilaton gravity ($F=e^{-\phi}$ and $\omega=-e^{-\phi}$) with an exponential potential. Let us consider a negative exponential potential, $\hat{V}=-V_0\exp(-\sqrt{2/\alpha}\hat{\phi})$, which appears in the Ekpyrotic scenario [45]. Note that this potential is the one in the Einstein frame and the dilaton ϕ is related with the separation of two parallel branes $\hat{\phi}$ through the relation $\phi=-\sqrt{2}\hat{\phi}$ [46]. In this case the background evolution is characterized by

$$\hat{H}=\frac{\alpha}{\hat{t}}, \quad \frac{d\hat{\phi}}{d\hat{t}}=-\frac{\sqrt{2\alpha}}{\hat{t}} \tag{4.1}$$

in the Einstein frame and

$$H=-\frac{\sqrt{\alpha}}{t}, \quad \phi=-\frac{2\sqrt{\alpha}}{1-\sqrt{\alpha}}\ln[-(1-\sqrt{\alpha})t] \tag{4.2}$$

in the string frame [46]. Then we have $\epsilon=1/\sqrt{\alpha}$, $\hat{\epsilon}=-1/\alpha$, $\delta_{\rm S}=\delta_{\rm T}=\beta=-1/(1-\sqrt{\alpha})$, and $\hat{\delta}_{\rm S}=\hat{\delta}_{\rm T}=0$, which are all constant. Therefore Eqs. (2.31), (2.33), and

(2.35) can be *exactly* employed in spite of the fact that the background evolution is not slow roll. The spectral indices n_S, n_T, and the tensor-to-scalar ratio R are invariant under a conformal transformation, and simply given by

$$\hat{n}_S = n_S = 1 + \frac{2}{1-\alpha}, \quad \hat{n}_T = n_T = \frac{2}{1-\alpha}, \quad \hat{R} = R = \frac{16}{\alpha}, \tag{4.3}$$

which are highly blue-tilted spectra for $0 < \alpha \ll 1$ [47]. Note that these are the spectra generated during the contracting phase and may be affected by the physics around the bounce [48]. We simply presented this example in order to show the validity of the formula (2.31), (2.33), and (2.35) rather than working on the detailed evolution of perturbations in the bouncing cosmology. We note that the formula (2.31), (2.33), and (2.35) are automatically valid in slow-roll inflation, since the variation of the terms ϵ, δ, and β are negligibly small.

There exist other generalized gravity theories where the function F in the action (2.1) depends upon not only ϕ but also the Ricci scalar R. The R^2 inflationary scenario proposed by Starobinsky [49] belongs to this class, in which the spectrum of density perturbations was derived in Refs. [50–52]. In particular Hwang and Noh [52] showed that the spectra of both scalar and tensor perturbations are invariant under a conformal transformation as in our action (2.1). These facts imply that the degeneracy of the consistency relation persists in a wide variety of gravity theories including the higher-curvature gravity theory and the Randall-Sundrum II braneworld.

ACKNOWLEDGMENTS

We are grateful to Sam Leach for providing the latest SDSS code and for his kind help in implementing the likelihood analysis. We also thank Antony Lewis and David Parkinson for their support in technical details of numerics. The research of S.T. was financially supported by JSPS (No. 04942). S.T. thanks all members of IUCAA for their warm hospitality during which this work was completed and especially to Rita Sinha for her kind support in numerics.

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