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Phantom field dynamics in loop quantum cosmology

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We consider a dynamical system of phantom scalar field under exponential potential in the background of loop quantum cosmology. In our analysis, there is neither stable node nor repeller unstable node but only two saddle points, hence no big rip singularity. Physical solutions always possess potential energy greater than the magnitude of the negative kinetic energy. We found that the universe bounces after accelerating even in the domination of the phantom field. After bouncing, the universe finally enters the oscillatory regime.

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I. INTRODUCTION

Recently, present accelerating expansion of the universe has been confirmed with observations via cosmic microwave background anisotropies [1,2], large scale galaxy surveys [3], and type Ia supernovae [4,5]. However, the problem is that the acceleration can not be understood in standard cosmology. This motivates many groups of cosmologists to find out the answers. Proposals to explain this acceleration made previously could be, in general, categorized into three ways of approach [6]. In the first approach, in order to achieve acceleration, we need some form of scalar fluid called *dark energy* with equation of state $p = w\rho$, where $w < -1/3$. Various types of models in this category have been proposed and classified (for a recent review, see Refs. [7,8]). The other two ways are that accelerating expansion is an effect of backreaction of cosmological perturbations [9] or late acceleration is an effect of modification in the action of general relativity. This modified gravity approach includes braneworld models (for review, see [10]). Until today, there has not yet been a true satisfactory explanation of the present acceleration expansion.

Considering dark energy models, a previous first-year WMAP data analysis combined with 2dF galaxy survey and SN-Ia data and even a previous SN-Ia analysis alone favor $w < -1$ more than cosmological constant or quintessence [11,12]. A precise observational data analysis (combining CMB, Hubble Space Telescope, type Ia Supernovae, and 2dF data sets) allows the equation of state $p = w\rho$ with a constant w value between -1.38 and -0.82 at the 95% confident level [13]. The recent WMAP three-year results combined with Supernova Legacy Survey (SNLS) data when assuming flat universe yields $-1.06 < w < -0.90$. However, without assumption of flat universe but only combined WMAP, large scale structure and supernova data implies a strong constraint, $w = -1.06^{+0.13}_{-0.08}$ [14]. While assuming a flat universe, the first result from ESSENCE Supernova Survey Ia combined with SuperNova Legacy Survey Ia gives a constraint of $w = -1.07 \pm 0.09$ [15]. Interpretation of various data brings about a possibility that dark energy could be in a form of phantom field—a fluid with $w < -1$ (which violates dominant energy condition, $\rho \geq |p|$) rather than quintessence field [16–18]. The phantom equation of state $p < -\rho$ can be attained by the negative kinetic energy term of the phantom field. However, there are some types of braneworld model [19] as well as Brans-Dicke scalar-tensor theory [20] and gravitational theory with higher derivatives of scalar field [21] that can also yield phantom energy. There has been investigation on dynamical properties of the phantom field in the standard Friedmann-Robertson-Walker (FRW) background with exponential and inverse power law potentials by [22–25] and with other forms of potential by [25–27]. These studies describe fates of the phantom dominated universe with different steepness of the potentials.

A problem for phantom field dark energy in standard FRW cosmology is that it leads to singularity. Fluid with w less than -1 can end up with future singularity called the big rip [28], which is of type I singularity according to classification by [29,30]. The big rip singularity corresponds to $a \to \infty$, $\rho \to \infty$, and $|p| \to \infty$ at finite time $t \to t_s$ in the future. Choosing a particular class of potential for the phantom field enables us to avoid future singularity. However, the avoidance does not cover general classes of potential [26]. In addition, an alternative model, in which two scalar fields appear with inverse power law and exponential potentials, can as well avoid the big rip singularity [31]. The higher-order string curvature correction terms can also show the possibility that the big rip singularity can be absent [32].

Since the phantom dominated FRW universe possesses a singularity problem as stated above, in this work, instead of using standard FRW cosmology, the fundamental background theory in which we are interested is loop quantum gravity (LQG). This theory is a nonperturbative type of quantization of gravity and is background independent

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[33,34]. It has been applied in a cosmological context as seen in various literature where it is known as loop quantum cosmology (LQC) (for a review, see Ref. [35]). The effective loop quantum modifies the standard Friedmann equation by adding a correction term $-\rho^2/\rho_{\rm lc}$ into the Friedmann equation [36–40]. When this term becomes dominant, the universe begins to bounce and then expands backwards. LQG can resolve the singularity problem in various situations [34,37,41,42]. However, derivation of the modified term is under a condition that there is no matter potential otherwise; in the presence of a potential, quantum correction would be more complicated [43]. A nice feature of LQC is avoidance of the future singularity from the correction quadratic term $-\rho^2/\rho_{\rm lc}$ in the modified LQC Friedmann equation [44] as well as the singularity avoidance at the semiclassical regime [45]. The early-universe inflation has also been studied in the context of LQC at the semiclassical limit [40,46–50]. We aim to investigate the dynamics of the phantom field and its late time behavior in the loop quantum cosmological context, and to check if the loop quantum effect could remove big rip singularity from the phantom dominated universe. The study could also reveal some other interesting features of the model.

We organize this article as follows: in Sec. II, we introduce the LQC Friedmann equation; after that we briefly present relevant features of the phantom scalar field in Sec. III. Section IV contains dynamical analysis of the phantom field in LQC background with exponential potential. The potential is a simplest case due to constancy of its steepness variable λ. Two real fixed points are found in this section. Stability analysis yields that both fixed points are saddle points. Numerical results and analysis of solutions can be seen in Sec. V where we give conditions for physical solutions. Finally, the conclusion is in Sec. VI.

II. LOOP QUANTUM COSMOLOGY

LQC naturally gives rise to the inflationary phase of the early universe with graceful exit; however, the same mechanism leads to a prediction that present-day acceleration must be very small [46]. At late time and at large scale, the semiclassical approximation in LQC formalisms can be validly used [51]. The effective Friedmann equation can be obtained by using an effective Hamiltonian with loop quantum modifications [38,44,52]:

$$\mathcal{C}_{\rm eff} = -\frac{3M_{\rm P}^2}{\gamma^2\bar{\mu}^2}a\sin^2(\bar{\mu}\mathfrak{c}) + \mathcal{C}_{\rm m}. \tag{1}$$

The effective constraint (1) is valid for the isotropic model, and if there is scalar field, the field must be a free, massless scalar field. Equation (1), when including field potential, must have some additional correction terms [43]. In this scenario, Hamilton's equation is

$$\dot{\mathfrak{p}} = \{\mathfrak{p}, \mathcal{C}_{\rm eff}\} = -\frac{\gamma}{3M_{\rm P}^2}\frac{\partial\mathcal{C}_{\rm eff}}{\partial\mathfrak{c}}, \tag{2}$$

where $\mathfrak{c}$ and $\mathfrak{p}$ are, respectively, conjugate connection and triad satisfying $\{\mathfrak{c}, \mathfrak{p}\} = \gamma/3M_{\rm P}^2$. The dot symbol denotes time derivative. These are two variables in the simplified phase space structure under FRW symmetries [35]. Here $M_{\rm P}^2 = (8\pi G)^{-1}$ is the square of reduced Planck mass, G is Newton's gravitational constant, and γ is the Barbero-Immirzi dimensionless parameter. There are relations between the two variables to scale factor as $\mathfrak{p} = a^2$ and $\mathfrak{c} = \gamma\dot{a}$. The parameter $\bar{\mu}$ is inferred as kinematical length of the square loop since its order of magnitude is similar to that of length. The area of the loop is given by the minimum eigenvalue of the LQG area operator. $\mathcal{C}_{\rm m}$ is the corresponding matter Hamiltonian. Using Eq. (2) with the constraint from realization that the loop quantum correction of the effective Hamiltonian $\mathcal{C}_{\rm eff}$ is small at a large scale, $\mathcal{C}_{\rm eff} \approx 0$ [35,38,39,44], one can obtain the (effective) modified Friedmann equation in a flat universe:

$$H^2 = \frac{\rho_{\rm t}}{3M_{\rm P}^2}\left(1 - \frac{\rho_{\rm t}}{\rho_{\rm lc}}\right), \tag{3}$$

where $\rho_{\rm lc} = \sqrt{3}/(16\pi\gamma^3 G^2\hbar)$ is the critical loop quantum density, $\hbar$ is the Planck constant, and $\rho_{\rm t}$ is the total density.

III. PHANTOM SCALAR FIELD

The energy density ρ and the pressure p of the phantom field contain a negative kinetic term. They are given as [16]

$$\rho = -\tfrac{1}{2}\dot{\phi}^2 + V(\phi), \tag{4}$$

$$p = -\tfrac{1}{2}\dot{\phi}^2 - V(\phi). \tag{5}$$

The conservation law is

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{6}$$

Using Eqs. (4)–(6), we obtain the Klein-Gordon equation:

$$\ddot{\phi} + 3H\dot{\phi} - V' = 0, \tag{7}$$

where $V' \equiv {\rm d}V/{\rm d}\phi$ and the negative sign comes from the negative kinetic terms. The phantom equation of state is therefore given by

$$w = \frac{p}{\rho} = \frac{\dot{\phi}^2 + 2V}{\dot{\phi}^2 - 2V}. \tag{8}$$

From Eq. (8), when the field is slowly rolling, as long as the approximation, $\dot{\phi}^2 \sim 0$ holds, the approximated value of w is -1. When the bound, $\dot{\phi}^2 < 2V$ holds, w is always less than -1.

As mentioned previously in Secs. I and II, there has not yet been a derivation of the effective LQC Friedmann equation consistent with a presence of potential. Even though, the Friedmann background is valid only in the absence of the field potential, however, investigation of a phantom field evolving under a potential is a challenged task. Here we also neglect the loop quantum correction effect in the classical expression of Eqs. (4) and (5) (see Refs. [43,53] for discussion).

IV. DYNAMICAL ANALYSIS

Differentiating Eq. (3) and using the fluid Eq. (6), we obtain

$$\dot{H} = -\frac{(\rho + p)}{2M_{\rm P}^2}\left(1 - \frac{2\rho}{\rho_{\rm lc}}\right). \tag{9}$$

Equations (3), (6), and (9), in domination of the phantom field, become

$$H^2 = \frac{1}{3M_{\rm P}^2}\left(-\frac{\dot{\phi}^2}{2} + V\right)\left(1 - \frac{\rho}{\rho_{\rm lc}}\right), \tag{10}$$

$$\dot{\rho} = -3H\rho\left(1 + \frac{\dot{\phi}^2 + 2V}{\dot{\phi}^2 - 2V}\right), \tag{11}$$

$$\dot{H} = \frac{\dot{\phi}^2}{2M_{\rm P}^2}\left(1 - \frac{2\rho}{\rho_{\rm lc}}\right). \tag{12}$$

We define dimensionless variables following the style of [54]

$$X \equiv \frac{\dot{\phi}}{\sqrt{6}M_{\rm P}H}, \qquad Y \equiv \frac{\sqrt{V}}{\sqrt{3}M_{\rm P}H}, \qquad Z \equiv \frac{\rho}{\rho_{\rm lc}}, \tag{13}$$

$$\lambda \equiv -\frac{M_{\rm P}V'}{V}, \qquad \Gamma \equiv \frac{VV''}{(V')^2}, \qquad \frac{\rm d}{{\rm d}N} \equiv \frac{1}{H}\frac{\rm d}{{\rm d}t}, \tag{14}$$

where $N \equiv \ln a$ is the e-folding number. Using new variables in Eqs. (8) and (10), the equation of state is rewritten as[1]

$$w = \frac{X^2 + Y^2}{X^2 - Y^2}, \tag{15}$$

where $|X| \neq |Y|$ and the Friedmann constraint is reexpressed as

$$(-X^2 + Y^2)(1 - Z) = 1. \tag{16}$$

Clearly, if $|X| \neq |Y|$, following Eq. (16), then $Z \neq 1$. Using the new defined variables above, Eq. (12) becomes

$$\frac{\dot{H}}{H^2} = 3X^2(1 - 2Z). \tag{17}$$

The acceleration condition,

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 > 0, \tag{18}$$

in expression of the new variables, is therefore

$$3X^2(2Z - 1) < 1. \tag{19}$$

Divided by Eq. (16), the acceleration condition under the constraint is

$$\frac{3}{1 - (Y^2/X^2)}\left(\frac{1 - 2Z}{1 - Z}\right) < 1, \tag{20}$$

where the conditions $|X| \neq |Y|$ and $Z \neq 1$ must hold. As we consider $Z = \rho/\rho_{\rm lc}$ with $\rho = -(\dot{\phi}^2/2) + V$, we can write

$$\frac{\rho_{\rm lc}Z}{3M_{\rm P}^2H^2} = -X^2 + Y^2. \tag{21}$$

With the condition $|X| \neq |Y|$, clearly from Eq. (21), we have one additional condition, $Z \neq 0$.

A. Autonomous system

Differential equations in the autonomous system are

$$\frac{{\rm d}X}{{\rm d}N} = -3X - \sqrt{\frac{3}{2}}\lambda Y^2 - 3X^3(1 - 2Z), \tag{22}$$

$$\frac{{\rm d}Y}{{\rm d}N} = -\sqrt{\frac{3}{2}}\lambda XY - 3X^2Y(1 - 2Z), \tag{23}$$

$$\frac{{\rm d}Z}{{\rm d}N} = -3Z\left(1 + \frac{X^2 + Y^2}{X^2 - Y^2}\right), \tag{24}$$

$$\frac{{\rm d}\lambda}{{\rm d}N} = -\sqrt{6}(\Gamma - 1)\lambda^2 X. \tag{25}$$

Here we will apply the exponential potential,

$$V(\phi) = V_0 \exp\left(-\frac{\lambda}{M_{\rm P}}\phi\right), \tag{26}$$

to this system. The potential is known to yield power-law inflation in standard cosmology with a canonical scalar field. Its slow-roll parameters are related as $\epsilon = \eta/2 = 1/P$, where $\lambda = \sqrt{2/P}$ and $P > 1$ [55,56]. Although the potential has been applied to the quintessence scalar field with tracking behavior in standard cosmology [57], the quintessence field cannot dominate the universe due to constancy of the ratio between densities of matter and quintessence field (see the discussion in Ref. [7]). In the case of a phantom field in standard cosmology under this potential, a stable node is a scalar-field dominated solution with the equation of state, $w = -1 - \lambda^2/3$ [24,27,58]. In our LQC phantom domination context, from Eq. (25), we can see that for the exponential potential, $\Gamma = 1$. This yields a trivial value of ${\rm d}\lambda/{\rm d}N$ and therefore λ is a nonzero constant; otherwise the potential is flat.

[1]The relation $\Omega_\phi = \rho/3H^2M_{\rm P}^2 = -X^2 + Y^2 = 1$ cannot be applied here since it is valid only for standard cosmology with flat geometry.

TABLE I. Properties of fixed points of phantom field dynamics in LQC background under the exponential potential.

Name	X	Y	Z	Existence	Stability	w	Acceleration
(a)	$-\frac{\lambda}{\sqrt{6}}$	$\sqrt{1+\frac{\lambda^2}{6}}$	0	All λ	Saddle point for all λ	$-1-\frac{\lambda^2}{3}$	For all λ (i.e. $\lambda^2 > -2$)
(b)	$-\frac{\lambda}{\sqrt{6}}$	$-\sqrt{1+\frac{\lambda^2}{6}}$	0	All λ	Saddle point for all λ	$-1-\frac{\lambda^2}{3}$	For all λ (i.e. $\lambda^2 > -2$)

B. Fixed points

Let $f \equiv \mathrm{d}X/\mathrm{d}N$, $g \equiv \mathrm{d}Y/\mathrm{d}N$, and $h \equiv \mathrm{d}Z/\mathrm{d}N$. We can find fixed points of the autonomous system under condition

$$(f, g, h)\,|_{(X_c, Y_c, Z_c)} = 0. \tag{27}$$

There are two real fixed points of this system:[2]

- Point (a): $$\left(\frac{-\lambda}{\sqrt{6}}, \sqrt{1+\frac{\lambda^2}{6}}, 0\right), \tag{28}$$
- Point (b): $$\left(\frac{-\lambda}{\sqrt{6}}, -\sqrt{1+\frac{\lambda^2}{6}}, 0\right). \tag{29}$$

C. Stability analysis

Suppose that there is a small perturbation δX, δY, and δZ about the fixed point (X_c, Y_c, Z_c), i.e.,

$$X = X_c + \delta X, \qquad Y = Y_c + \delta Y, \qquad Z = Z_c + \delta Z. \tag{30}$$

From Eqs. (22)–(24), neglecting higher-order terms in the perturbations, we obtain first-order differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}N}\begin{pmatrix}\delta X\\ \delta Y\\ \delta Z\end{pmatrix} = \mathcal{M}\begin{pmatrix}\delta X\\ \delta Y\\ \delta Z\end{pmatrix}. \tag{31}$$

The matrix $\mathcal{M}$ defined at a fixed point (X_c, Y_c, Z_c) is given by

$$\mathcal{M} = \begin{pmatrix}\frac{\partial f}{\partial X} & \frac{\partial f}{\partial Y} & \frac{\partial f}{\partial Z}\\ \frac{\partial g}{\partial X} & \frac{\partial g}{\partial Y} & \frac{\partial g}{\partial Z}\\ \frac{\partial h}{\partial X} & \frac{\partial h}{\partial Y} & \frac{\partial h}{\partial Z}\end{pmatrix}_{(X=X_c, Y=Y_c, Z=Z_c)}. \tag{32}$$

We find eigenvalues of the matrix $\mathcal{M}$ for each fixed point:

(1) At point (a):

$$\mu_1 = \lambda^2, \qquad \mu_2 = -\lambda^2, \qquad \mu_3 = -3 - \frac{\lambda^2}{2}. \tag{33}$$

(2) At point (b):

$$\mu_1 = \lambda^2, \qquad \mu_2 = -\lambda^2, \qquad \mu_3 = -3 - \frac{\lambda^2}{2}. \tag{34}$$

From the above analysis, each point possesses eigenvalues with opposite signs; therefore both point (a) and point (b) are saddle. Results from our analysis are concluded in Table I. Location of the points depends only on λ and the points exist for all values of λ. Both points correspond to the equation of state $-1-\lambda^2/3$, that is to say, it has a phantom equation of state for all values of $\lambda \neq 0$. Since there is not any attractor in the system, a phase trajectory is very sensitive to initial conditions given to the system. The stable node (the big rip) of the standard general relativistic case in the presence of a phantom field and a barotropic fluid disappears here (see [23]).

V. NUMERICAL RESULTS

Numerical results from the autonomous set (22)–(24) are presented in Figs. 1 and 2 where we set $\lambda = 1$. Locations of the two saddle points are: point (a) ($X_c = -0.408\,25$, $Y_c = 1.0801$, $Z_c = 0$) and point (b) ($X_c = -0.408\,25$, $Y_c = -1.0801$, $Z_c = 0$), which match our analytical results. In Fig. 3, we present a trajectory solution of a phantom field evolving in standard cosmological background for comparing with the trajectories in Fig. 2 when including loop quantum effects. The standard case has only two simple trajectories corresponding to a constraint $-X^2 + Y^2 = 1$. This is attained when taking the classical limit, $Z = 0$. In the loop quantum case, since there is not any stable node and the solutions are sensitive to initial conditions, we need to classify solutions according to each domain region separated by separatrices $|X| = |Y|$, $Z = 0$, and $Z = 1$, so that we can analyze them separately. Note that the condition, $Z > 0$ must hold for physical solutions since the density cannot be negative or zero, i.e. $\rho > 0$. The blue lines and red lines in Figs. 1 and 2 are solutions in the region $Z < 0$ hence are not physical and will no longer be of our interest. From now on, we consider only the region $Z > 0$. In regions with $|X| > |Y|$, the solutions therein are green lines (hereafter classified as class I). The other regions with $|Y| > |X|$ contain solutions seen as black lines (classified as class II). Note that all solutions cannot cross the separatrices due to conditions in Eqs. (16), (20), and (21).

[2]The other two imaginary fixed points $(i, 0, 0)$ and $(-i, 0, 0)$ also exist. However, they are not interesting here since we do not consider the model that includes a complex scalar field.

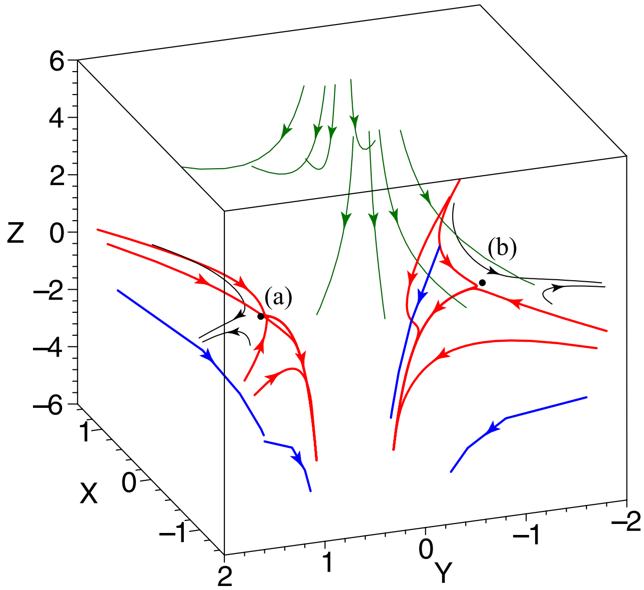

FIG. 1 (color online). Three-dimensional phase space of X, Y, and Z. The saddle points (a) $(-0.40825, 1.0801, 0)$ and (b) $(-0.40825, -1.0801, 0)$ appear in the figure. λ is set to 1. In region $Z < 0$, the solutions (red and blue lines) are nonphysical. In this region, $Z \to -\infty$ when $(X, Y) \to (0, 0)$. The green lines (class I) are in region $|X| > |Y|$ and $Z > 1$ but they are also nonphysical since they correspond to imaginary H values. The only set of physical solutions (class II) is presented with black lines. They are in region $|Y| > |X|$ and range from $0 < Z < 1$. This is the region above (a) and (b) of which H takes real value. There are separatrices $|X| = |Y|$, $Z = 0$, and $Z = 1$ in the system (see Sec. V B).

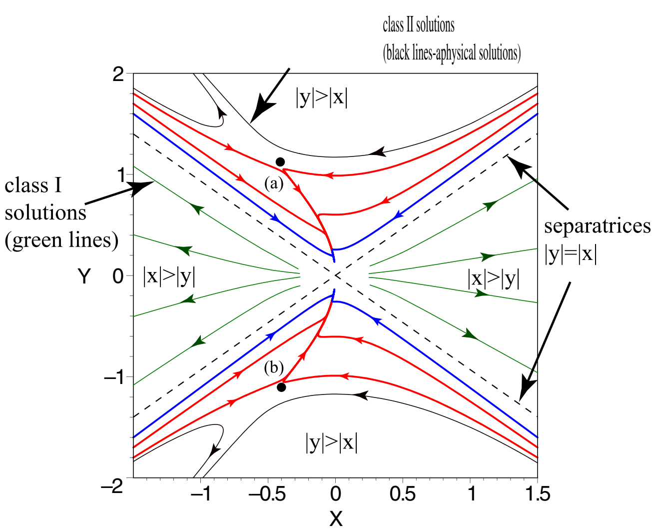

FIG. 2 (color online). Phase space of the kinetic part X and potential part Y (top view). The saddle points (a) $(-0.40825, 1.0801)$ and (b) $(-0.40825, -1.0801)$ are shown here. The blue lines and red lines are in the region $Z < 0$ which is nonphysical. Green lines are of class I solutions which yield imaginary H. Only class II solutions shown as black lines are physical with a real H value.

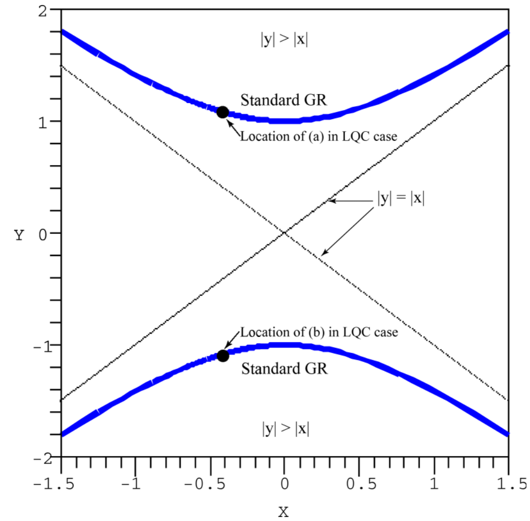

FIG. 3 (color online). Phase space of the kinetic part X and potential part Y in standard general relativistic case. The location of points (a) and (b) in Fig. 2 is on the trajectory solutions here. This plot shows the dynamics of a phantom field in standard cosmological background without any other fluids. In the presence of a barotropic fluid with any equation of state, points (a) and (b) correspond to the big rip [23,25].

A. Class I solutions

Consider the Friedmann equation (10); the Hubble parameter H takes real value only if

$$\frac{1}{3M_{\rm P}^2}\left(-\frac{\dot{\phi}^2}{2} + V\right)\left(1 - \frac{\rho}{\rho_{\rm lc}}\right) \geq 0. \tag{35}$$

Divided by H^2 on both sides, the expression above becomes

$$(-X^2 + Y^2)(1 - Z) \geq 0. \tag{36}$$

It is clear from (36) that, in order to obtain a real value of H, class I solutions (green line) must obey both conditions $|X| > |Y|$ and $Z > 1$ together. However, when imposing $|X| > |Y|$ to Eq. (21), we obtain $Z < 0$ instead. This contradicts the required range $Z > 1$. Therefore this class of solutions does not possess any real values of H and hence not physical solutions.

B. Class II solutions

Proceeding with the same analysis done for class I, we found that in order for H to be real, class II solutions must obey both $|Y| > |X|$ and $0 < Z < 1$ together. Moreover, when imposing $|Y| > |X|$ into Eq. (21), we obtain $Z > 0$. Therefore, as we combine both results, it can be concluded that class II solutions can possess real H value in the region $|Y| > |X|$ and $0 < Z < 1$, i.e. $0 < \rho < \rho_{\rm lc}$. The bound is slightly different from the case of canonical scalar field in LQC (see Ref. [59]) of which the bound is $0 \leq \rho \leq \rho_{\rm lc}$. Class II is therefore the only class of physical solutions.

For class II solutions, we consider another set of autonomous equations from which the evolution of cosmological variables is conveniently obtained by using the numerical approach. In the new autonomous set, instead of using N, which could decrease after the bounce from LQC effect, time is taken as an independent variable. We define the new variable as

$$\dot{\phi} = S. \tag{37}$$

Equations (7) and (12) are therefore rewritten as

$$\dot{H} = \frac{S^2}{2M_{\rm P}^2}\left[1 - \frac{2}{\rho_{\rm lc}}\left(-\frac{S^2}{2} + V(\phi)\right)\right], \tag{38}$$

$$\dot{S} = -3HS + V'. \tag{39}$$

Equations (37)–(39) form another closed autonomous system. Numerical integrations from the new system yield the result plotted in Figs. 4 and 5 in which the set values are $\lambda = 1$, $\rho_{\rm lc} = 1.5$, $V_0 = 1$, and $M_{\rm P} = 2$. From Eq. (3) the slope of H with respect to ρ, $\mathrm{d}H/\mathrm{d}\rho$, is flat when $\rho = \rho_{\rm lc}/2$ [59]. Another fact is

$$\left(\frac{\mathrm{d}^2 H}{\mathrm{d}\rho^2}\right)_{\rho=\rho_{\rm lc}/2} = \frac{-2}{M_{\rm P}\sqrt{3\rho_{\rm lc}^3}} < 0, \tag{40}$$

hence, as $\rho = \rho_{\rm lc}/2$, H takes the maximum value, $H_{\max} = \sqrt{\rho_{\rm lc}/12M_{\rm P}^2}$. This result is valid in the LQC scenario regardless of types of fluid. In Figs. 4 and 5, with set parameters given above, as $\rho = \rho_{\rm lc}/2 = 0.75$, H is maximum, $H_{\max} = 0.176\,78$. When $H \approx 0$, i.e. ρ is approximately $\rho_{\rm lc} = 1.5$, the expansion halts and then bounces. At this bouncing point, the dynamics enters the loop quantum regime which is a quantum gravity limit. Beyond the bounce, H turns negative, i.e. contracting of scale factor. The universe undergoes accelerating contraction until reaching $H_{\min}$. After that it contracts, decelerating until

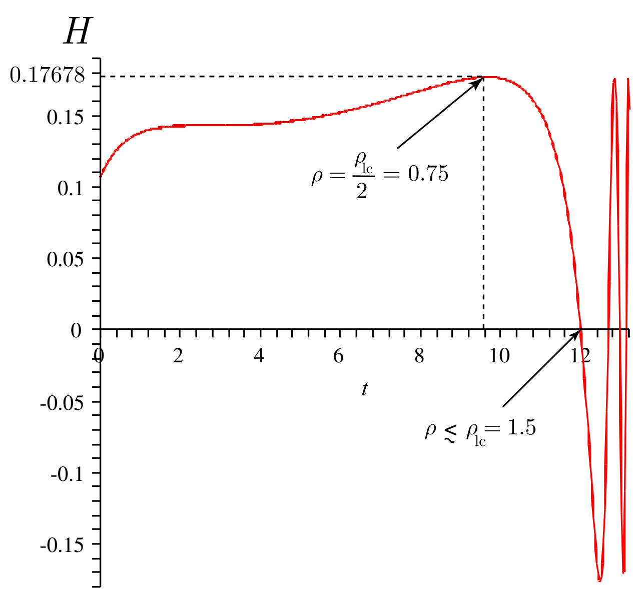

FIG. 4 (color online). Evolution of H with time of a class II solution. Set values are $\lambda = 1$, $\rho_{\rm lc} = 1.5$, $V_0 = 1$, and $M_{\rm P} = 2$. The universe undergoes acceleration from the beginning until reaching a turning point at $\rho = \rho_{\rm lc}/2 = 0.75$, where $H = H_{\max} = 0.176\,78$. Beyond this point, the universe expands with deceleration until halting ($H = 0$) at $\rho \approx \rho_{\rm lc} = 1.5$. After halting, it undergoes contraction until H bounces. The oscillating in H goes on forever.

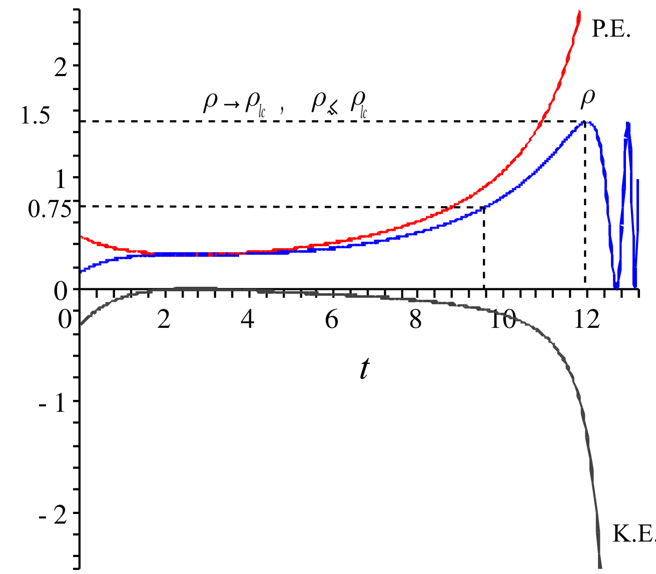

FIG. 5 (color online). Time evolution of potential energy density (P.E.), kinetic energy density (K.E.), and ρ = K.E. + P.E. of the field for a class II solution. K.E. is always negative and, at late time, it goes to $-\infty$ while P.E. is always positive. ρ is maximum when $\rho \approx \rho_{\rm lc} = 1.5$. Other features are discussed as in Fig. 4.

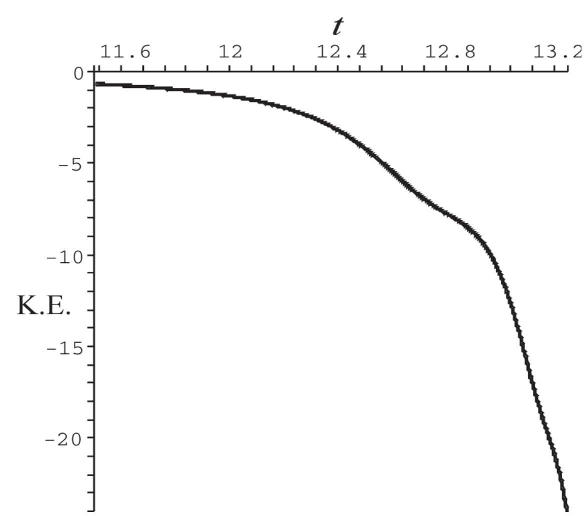

FIG. 6. Oscillation in kinetic energy density (K.E.) that contributes to oscillation in ρ. This is a zoomed-in portion of Fig. 5.

bouncing at $H \approx 0$. The universe goes on faster bouncing forward and backward. The faster bounce in H is an effect from the faster bounce in ρ as illustrated in Fig. 5 where the red line represents potential energy density $V(\phi)$, the black line represents kinetic energy density $-\dot{\phi}^2/2$, and the blue line is total energy density ρ. Oscillation in ρ is from oscillation in the field speed $\dot{\phi}$ and therefore oscillation in K.E. as shown in Fig. 6. This hence contributes to oscillation in ρ. The negative magnitude of kinetic energy density becomes larger and larger as the field rolling faster and faster up the potential. The exponential potential energy density therefore becomes larger and larger. This results in oscillation of ρ and affects in oscillation of H about the bounce $H = 0$. With a different approach, recently a similar result in H oscillation is also obtained by Naskar and Ward [60].

VI. CONCLUSION

A dynamical system of phantom canonical scalar field evolving in a background of loop quantum cosmology is considered and analyzed in this work. The exponential potential is used in this system. The dynamical analysis of the autonomous system renders two real fixed points $(-\lambda/\sqrt{6}, \sqrt{1 + \lambda^2/6}, 0)$ and $(-\lambda/\sqrt{6}, -\sqrt{1 + \lambda^2/6}, 0)$, both of which are saddle points corresponding to an equation of state, $w = -1 - \lambda^2/3$. Note that, in the case of standard cosmology, the fixed point $(X_c, Y_c) = (-\lambda/\sqrt{6}, \sqrt{1 + \lambda^2/6})$ is the big rip attractor with the same equation of state, $w = -1 - \lambda^2/3$ [24]. Because of the absence of a stable node, the late time behavior depends on the initial conditions given. Therefore we do numerical plots to investigate solutions of the system and then classify the solutions. Separatrix conditions $|X| \neq |Y|$, $Z \neq 1$, and $Z \neq 0$ arise from the equation of state (15), Friedmann constraint (16), and definition of Z in Eq. (21). At first, we consider solutions in region $Z > 0$, i.e. $\rho > 0$ for physical solutions. Second, within this $Z > 0$ region, we classify solutions into class I and class II. Solutions in region $|X| > |Y|$ and $Z > 1$ are of class I. However, in order to obtain a real value of H in class I, Z must be negative which contradicts $Z > 1$. Therefore the class I solutions are nonphysical. The class II set is identified by $|Y| > |X|$ and $0 < Z < 1$. It is the only set of physical solutions since it yields a real value of H. In the class II set, the universe undergoes an accelerating expansion from the beginning until $\rho = \rho_{\rm lc}/2$, where $H = H_{\max} = \sqrt{\rho_{\rm lc}/12M_{\rm P}^2}$. After that the universe expands, decelerating until it bounces, i.e. stops expansion $H \approx 0$ at $\rho \approx \rho_{\rm lc}$. At the bounce the universe enters the quantum gravity regime. Contraction with backward acceleration happens right after the bounce; however, the contraction does not go on forever. When the universe reaches a minimum value of negative H, the contraction decelerates, i.e. contracts slower and slower down. The universe, after undergoing contraction to a minimum spatial size, bounces again and starts to expand, accelerating. Our numerical results yield that oscillation in H becomes faster as time passes.

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