

Coupled phantom field in loop quantum cosmology

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A model of phantom scalar field dark energy under exponential potential coupling to barotropic dark matter fluid in loop quantum cosmology is addressed here. We derive a closed-autonomous system for cosmological dynamics in this scenario. It is found that LQC can yield a bounce in scale factor even in presence of the phantom field. The greater decaying from dark matter to dark phantom energy results in greater energy storing in the phantom field. This results in further turning point of the field. Greater coupling also delays bouncing time. In the case of phantom decaying, oscillation in phantom density makes small oscillation in the increasing matter density.

1. INTRODUCTION

There has recently been evidence of present accelerating expansion of the universe from cosmic microwave background (CMB) anisotropies, large scale galaxy surveys and type Ia supernovae [1-3]. Dark energy (DE) in form of either cosmological constant or scalar field matter is a candidate answer to the acceleration expansion which could not be explained in the regime of standard big bang cosmology [4]. DE possesses equation of state $p = w\rho$ with $w < -1/3$ enabling it to give repulsive gravity and therefore accelerate the universe. Combination of observational data analysis of CMB, Hubble Space Telescope, type Ia Supernovae and 2dF datasets allows constant w value between -1.38 and -0.82 at the 95 % of confident level [5]. Meanwhile, assuming flat universe, the analysis result, $-1.06 < w < -0.90$ has been reported by [6] using WMAP three-year results combined with Supernova Legacy Survey (SNLS) data. Without assumption of flat universe, mean value of w is -1.06 (within a range of -1.14 to -0.93). Most recent data (flat geometry assumption) from ESSENCE Supernova Survey Ia combined with SuperNova Legacy Survey Ia gives a constraint of $w = -1.07 \pm 0.09$ [7]. Observations above show a possibility that a fluid with $w < -1$ could be allowed in the universe [8]. This type of cosmological fluid is dubbed "phantom". Conventionally Phantom behavior arises from having negative kinetic energy term.

Dynamical properties of the phantom field in the standard FRW cosmology were studied before. However the scenario encounters singularity problems at late time [9]. While investigation of phantom in standard cosmological model is still ongoing, there is an alternative approach in order to resolve the singularity problem by considering Loop Quantum Cosmology (LQC) background instead of standard general relativistic background [9-10]. Loop Quantum Gravity-LQG is a non-perturbative type of quantization of gravity and is background-independent [11-12]. LQG provides cosmological background evolution for LQC. Effect from loop quantum modification gives an extra correction term $-\rho^2/\rho_c$ into the standard Friedmann equation [13-14]. This term, when dominant at late time, causes bouncing of expansion hence solving future singularity problem [15-16]. Recently, a general dynamics of scalar field including phantom scalar field

coupled to barotropic fluid has been investigated in standard cosmological background. In this scenario, the scaling solution of the coupled phantom field is always unstable and it can not yield the observed value $\Omega_\phi \approx 0.7$ [17].

Here, in this letter, we will investigate a case of coupled phantom field in LQC background alternative to the standard cosmology case. In Section 2, we introduce framework of cosmological equations before consider dynamical autonomous equations in Section 3. We show some numerical results in Section 4 where the coupling strength is adjusted and compared. Conclusion and comments are in Section 5.

2 COSMOLOGICAL EQUATIONS

2.1 Loop Quantum Cosmology

The effective Friedmann equation from LQC is given as [18]

$$H^2 = \frac{\rho}{3M_p^2} \left(1 - \frac{\rho}{\rho_c} \right), \quad (1)$$

where H is Hubble constant, M_p is reduced Planck mass, ρ is density of cosmic fluid, $\rho_c = \sqrt{3}/(16\pi\epsilon^3 G^2 \hbar)$. The parameter ϵ is Barbero-Immirzi dimensionless parameter and G is Newton's gravitational constant.

2.2 Phantom Scalar Field

Nature of the phantom field can be extracted from action,

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} (\nabla\phi)^2 - V(\phi) \right], \quad (2)$$

which, with variational principle, yields

$$\rho_\phi = -\frac{1}{2} \dot{\phi}^2 + V(\phi), \quad (3)$$

and

$$p_\phi = -\frac{1}{2} \dot{\phi}^2 - V(\phi). \quad (4)$$

The phantom field possesses equation of state,

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 + 2V(\phi)}{\dot{\phi}^2 - 2V(\phi)}. \quad (5)$$

When the field is slowly rolling, the approximate value of w is -1. As long as the approximation, $\dot{\phi}^2 \ll 0$ or the bound, $\dot{\phi}^2 < 2V$ holds, w is always less than -1. In our scenario, the universe contains two components which are barotropic fluid with equation of state $p_m = \rho_m w_m$ and phantom scalar field fluid. The total energy density is $\rho = \rho_m + \rho_\phi$.

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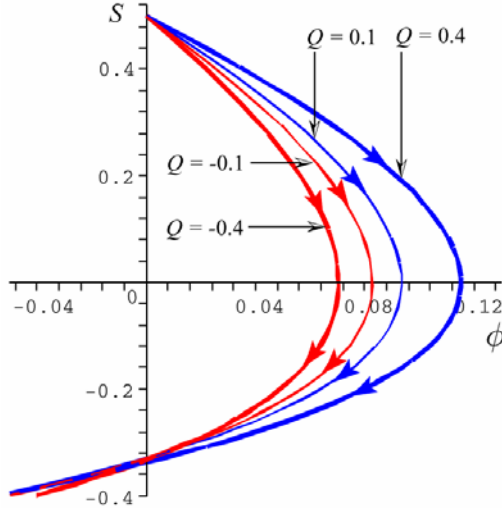


FIGURE 1. Phase portrait of $S(t)$ versus ϕ for $Q = -0.4, -0.1, 0.1$ and 0.4 from left to right. All trajectories have the same initial conditions $S(0) = 0.5, \phi(0) = 0$.

2.3 Coupled Scalar Field

Here we consider both components coupling to each other. Fluid equations for couple scalar fields proposed by [19] assuming flat FRW universe are

$$\dot{\rho}_\phi + 3H(1+w_\phi)\rho_\phi = -Q\rho_m\dot{\phi}, \quad (6)$$

and

$$\dot{\rho}_m + 3H(1+w_m)\rho_m = +Q\rho_m\dot{\phi}. \quad (7)$$

These fluid equations contain a constant coupling Q between dark matter (the barotropic fluid) and dark energy (the phantom scalar field) as in [20]. Though Eqs. (6) and (7) are derived in FRW background, the LQC effective Friedmann equation, Eq. (1) is also obtained under flat and maximally symmetries. Discrete quantum effect of LQG shows up at high energy regimes. Therefore, Eqs. (6) and (7) can be used in the consideration. Total action for matter and phantom scalar field is [19]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R + p(X, \phi) \right] + S_m(\phi) \quad (8)$$

Assuming scaling solution of the dark energy, therefore the pressure is written as in [17, 19]

$$p(X, \phi) = -X - c \exp(-\lambda\phi/M_p^2), \quad (9)$$

where X is the kinetic term, $-g^{ab}\partial_a\phi\partial_b\phi/2$ of the Lagrangian density (9) and (2). The second term on the right of Eq. (9) is exponential potential, $V(\phi) = c \exp(-\lambda\phi/M_p^2)$ which gives scaling solution for canonical and phantom ordinary scalar field in standard general relativistic cosmology when steepness of the potential, λ is fine tuned as

$$\lambda = Q \frac{1+w_m - \Omega_\phi(w_m - w_\phi)}{\Omega_\phi(w_m - w_\phi)}. \quad (10)$$

The steepness (10) is, in standard cosmological circumstance, constant in the scaling regime due to constancy of w_ϕ and Ω_ϕ [19]. However, in LQC case, there has been a report recently that the scaling solution does not exist for phantom field evolving in LQC [10]. Therefore, in our situation, our spirit to consider constant λ is a motivation from tracking

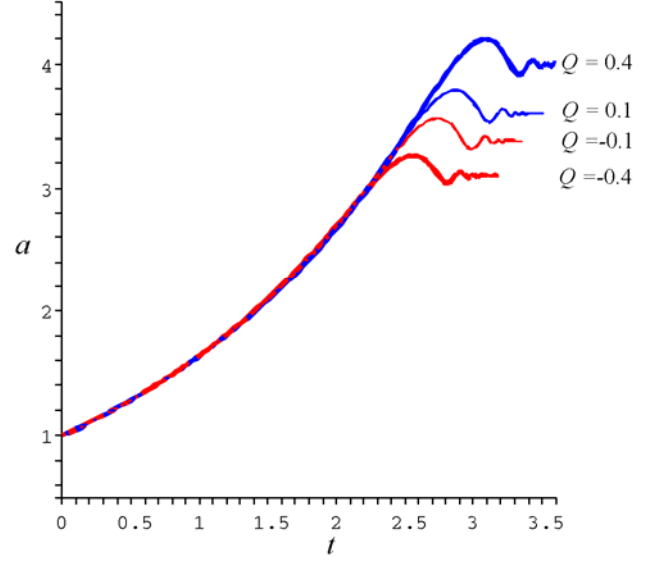


FIGURE 2. Scale factor plotted versus time for $Q = -0.4, -0.1, 0.1$ and 0.4 (from bottom to top).

behavior as in [21, 22], not a motivation from scaling solution as in [19]. The exponential potential is also originated from fundamental physics theories such as higher-order gravity [23] or higher dimensional gravity [24].

3 COSMOLOGICAL DYNAMICS

Time derivative of the effective LQC Friedmann equation LQC (1) is

$$\dot{H} = -\frac{(\rho+p)}{2M_p^2} \left(1 - \frac{2\rho}{\rho_c} \right), \quad (11.1)$$

$$= -\frac{[(1+w_\phi)\rho_\phi + (1+w_m)\rho_m]}{2M_p^2} \left[1 - \frac{2}{\rho_c} (\rho_\phi + \rho_m) \right], \quad (11.2)$$

$$= -\frac{[-S^2 + (1+w_m)\rho_m]}{2M_p^2} \left[1 - \frac{2}{\rho_c} \left(-\frac{S^2}{2} + ce^{-\lambda\phi/M_p^2} + \rho_m \right) \right]. \quad (11.3)$$

In above equations we define new variable

$$\dot{\phi} \equiv S. \quad (12)$$

The coupled fluid equations (6) and (7) are re-expressed in term of S as

$$\dot{S} = -3HS + \frac{dV}{d\phi} + Q\rho_m, \quad (13)$$

$$\dot{S} = -3HS + \frac{dV}{d\phi} + Q\rho_m \quad (14)$$

The Eqs. (11.3), (12), (13) and (14) form a closed autonomous set of four equations. The variables here are ρ_m, S, ϕ and H . The autonomous set recovers standard GR cosmology in the limit $\rho_c \rightarrow \infty$. This GR limit affects only the equation involving H . From the above autonomous set, one can do a qualitative analysis with numerical integration similar to [25]. Another approach of analysis is to consider a quantitative analysis [26].

4. NUMERICAL SOLUTIONS

Here we present some numerical solution for a positive and negative coupling between the phantom and barotropic fluid. The solutions presented here are physically valid solutions corresponding to Class II solution as characterized in

[10]. For nonminimally coupled scalar field, in Einstein frame, the coupling Q lies in a range $-1/\sqrt{6} < Q < 1/\sqrt{6}$ (see [4]). Here we set $Q = -0.4, -0.1, 0.1$ and 0.4 which lie in the range. Effect of the coupling can be seen from Eqs. (6) and (7). Negative Q enhances decay rate of scalar field to matter while giving higher matter creation rate. On the other hand, positive Q yields opposite result. Greater magnitude of $Q < 0$ gives higher decay rate of the field to matter. Greater magnitude of $Q > 0$ will result in higher production rate of field from matter.

4.1 Phase Portrait

The greater Q value results in greater value of the field turning point (see S -intercept in both figures.). The kinetic term $S(t)$ turns negative at the turning points corresponding to the field rolls down and then stops before rolling up the hill of exponential potential. When Q is greater, the field can fall down further, therefore gaining more total energy. The result agrees with the prediction of Eqs. (6) and (7).

4.2 Scale Factor

From Figure 2, the bounce in scale factor occurs later for greater Q value of which the phantom field production rate is higher. The field has more phantom energy to accelerate the universe in counteracting the effect of loop quantum (the bounce). For less positive Q , the phantom production rate is smaller, and for negative Q , the phantom decays therefore it has less energy for accelerating the expansion in counteracting with the loop quantum effect. This makes the bounce occurs sooner.

4.3 Energy Density

Time evolutions of energy density of the matter and the phantom field are presented in Figs. 3 and 4. If $Q > 0$, the matter decays to phantom. This reduces density of matter. While for $Q < 0$, the matter gains its density from decaying of phantom field. In Fig. 3 there is a bounce of phantom density before undergoing oscillation. For a non-coupled case, it has recently been reported that the phantom density also undergoes expansion [10]. As seen in Figs. 4(a) and 4(b), the oscillation in phantom density of the phantom decay case ($Q < 0$) affects in small oscillation in matter density while for the case matter decay ($Q > 0$), the matter density is reduced for stronger coupling.

5. CONCLUSIONS AND COMMENTS

In this letter, we have derived an autonomous system of a loop quantum cosmological equation in presence of phantom scalar field coupling to barotropic matter fluid. We choose constant coupling Q between matter and the phantom field to positive and negative values and check numerically the effect of Q values on (1) phase portrait, (2) scale factor and (3) energy density of phantom field and matter. We found that field value tends to roll up the hill of potential due to phantom nature. With greater Q , the field can fall down on the potential further. This increases total energy of the field. For canonical scalar field either standard or phantom, LQC yields a bounce. The bounce is useful since it is able to avoid Big Bang singularity in the early universe [13]. Here our numerical result shows a bouncing in scale factor at late time. This is a Type I singularity avoidance even in presence of phantom energy. The greater coupling results in more and more phantom density. Greater phantom effect therefore delays the bounce, which is LQC effect, to later time. In the case of matter decay to phantom ($Q < 0$), oscillation in phantom energy density does

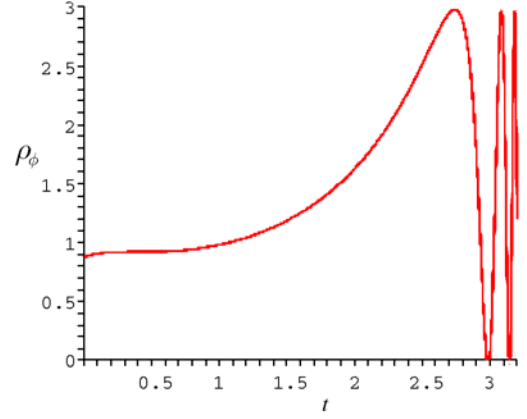


FIGURE 3. Phantom field density plotted versus time for $Q = -0.1$. The other values of Q also yield bouncing and oscillation.

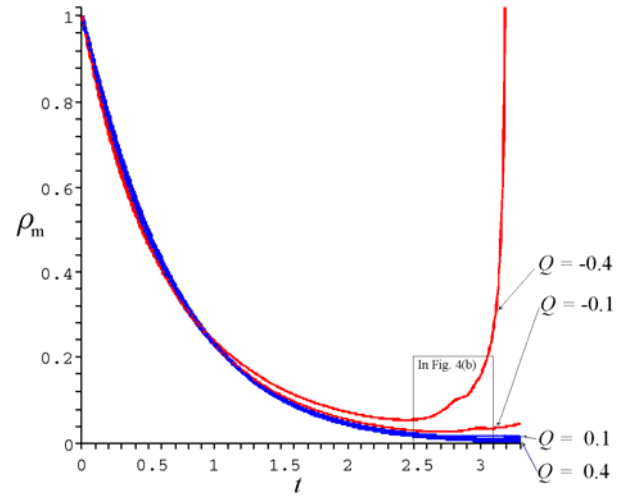


FIGURE 4(a). Matter density plotted versus time for $Q = -0.4, -0.1, 0.1$ and 0.4 (from top to bottom).

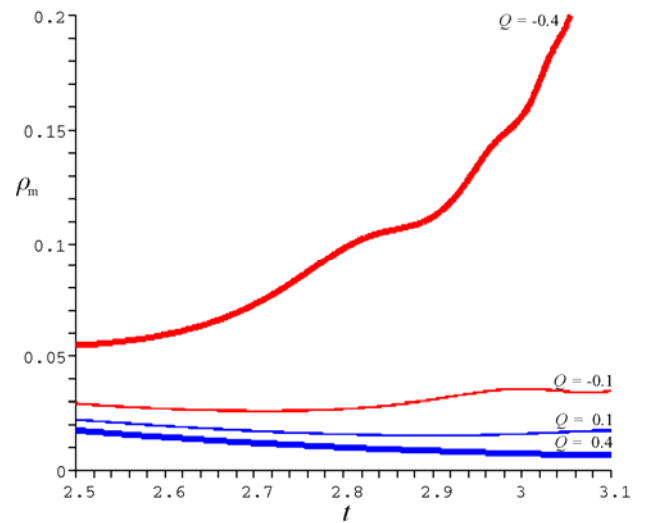


FIGURE 4(b). Zoom-in portion of Figure 4(a). The phantom field decays to matter at highest rate for $Q = -0.4$ (top line). Oscillation in matter density due to oscillation in the phantom field density is seen clearly here.

not affect matter density. On the other hand, when $Q > 0$, phantom decays to matter, oscillation in phantom density results in oscillation in the increasing matter density.

This work considers only the effects of sign and magnitude of the coupling constant to qualitative dynamics and evolution of the system. Studies of field dependent effects of coupling $Q(\phi)$ in some scalar-tensor theory of gravity and investigation of an evolution of effective equation of state could also yield further interesting features of the model. Quantitative dynamical analysis of the model under different types of potential is also motivated for future work. Although this work is to propose a way to resolve Type I future singularity, frequency function of the oscillation in scale factor and phantom density is still unknown and there might be a possibility that it might leads to infinite frequency of oscillation which is another new singularity.

ACKNOWLEDGMENTS

This work was presented as an invited talk at the SIAM Physics Congress 2007. B.G. expresses his gratitude to Thiraphat Vilaithong and the committee of the Thai Physics Society for an invitation to the congress. This work is supported by a TRF-CHE Research Career Development Grant of the Thailand Research Fund and the Royal Thai Government's Commission on Higher Education.

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