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Effect structure in physics and hints to economics equation of state

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Abstract. We point out that, there is effect structure in physical laws which is noticed from both phenomenological and postulate points of view. Here we consider effect structure in equations of state (EoS). Effect of one variable to the other variables are expressed as a diagram. Formal criteria to judge a status of EoS of an empirical relation is proposed here for classifying the EoS into classes. The framework shades light on testing of existence of an EoS in economics system. Demand-side EoS in perfect-competitive market is proposed employing hints given by the criteria. The demand-side EoS turns to be new class of EoS.

1. Introduction

Considering laws of physics written as a constraint surface embedded in physical or abstract spaces and considering how they are formulated, they can be classified into two types. These are the equations of motion (EoM) derived from Hamilton's principle and the others are equation of state (EoS) empirically constructed in thermostatics (known as equilibrium thermodynamics). The EoS is a two-dimensional constraint surface in three-dimensional thermodynamics space. Albeit expressed with mathematical clarity, these laws still have deeper physical insight which is not yet described in the equation. It is variables' role of being the cause to other variables and role of being affected by the others. In an EoM, for instance, Newton's law, force is the cause and the effect is on the velocity. Similarly in Ohm's law, potential difference is the cause and the effect is on charge quantity moving through an area. The physical properties are inertial mass and electrical resistivity. Similarities of the two laws give no surprise to similarity of mechanical vibration and RLC circuits. In EoS, unlike Newton's or Ohm's laws, there are three variables, temperature, T with extensive coordinate, X and intensive coordinate, Y. In microeconomics, demand quantity and price can be both cause and effect. Demand quantity can affect price and price can reversely affect demand quantity. These examples of cause and effect are related by *truly endogenous function* which must not be composite function of independent variables. Truly endogenous effects in some EoS are discussed so that one can classify EoS into two classes. The effect structure is expressed diagramatically, namely *the effect structure diagram of an EoS*. It addresses how an EoS should look like if we are to propose a new EoS empirically from a system of consideration. Knowing these aspects could hint how to construct other EoS, awaiting for empirical verifications. We propose a formal way in describing classes of EoS and rules for

testing EoS status of an empirical relation. New EoS for a system of information-symmetric homogenous consumers in a perfect competitive market of a good is proposed to exist.
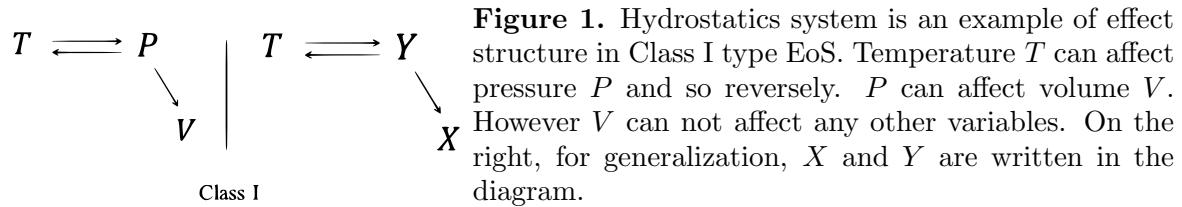

Figure 1. Hydrostatics system is an example of effect structure in Class I type EoS. Temperature T can affect pressure P and so reversely. P can affect volume V. However V can not affect any other variables. On the right, for generalization, X and Y are written in the diagram.
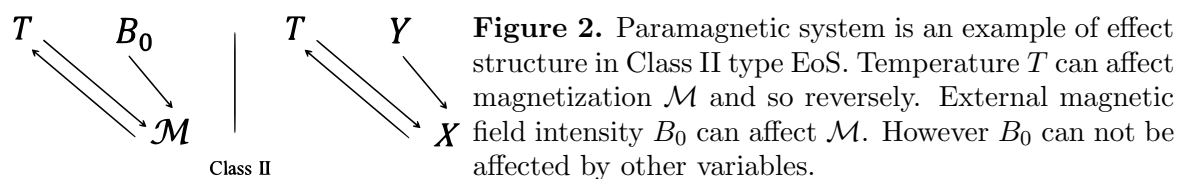

Figure 2. Paramagnetic system is an example of effect structure in Class II type EoS. Temperature T can affect magnetization $\mathcal{M}$ and so reversely. External magnetic field intensity B_0 can affect $\mathcal{M}$. However B_0 can not be affected by other variables.

2. Effect Structure of Equation of State

In effect structure diagram of an EoS, there are arrows linking each variables. These arrows represent truly endogenous effect between two variables. Patterns of the diagram are found in two classes as in figure 1 and figure 2. Typically EoS are speculated to be in form of

$$X = k\left(\frac{T}{Y}\right) \qquad \text{(Class I)} \qquad \text{and} \qquad X = k\left(\frac{Y}{T}\right) \qquad \text{(Class II)} \tag{1}$$

where k is an arbitrary magnification factor to combined result of Y and T. EoS is not necessary written as equation (1). For example, EoS of simple solid can be expressed differently. Thus how it is written has nothing to do with how the diagram should look like. Aspects noticed are taken as criteria for status of an EoS, $g(X, Y, T) = 0$. These are,

(i) Number of the arrows are three.

(ii) There is at least one arrow pointing $Y \to X$.

(iii) Only T and(or) Y can take external (outside of (X, Y, T)) influence or exogenous shock[1].

3. Demand-Side Equation of State

Albeit, not as mainstream econophysics or financial physics[2], significant proposals were made for thermodynamics formulation of economics in determining the relation of price and demand quantity with concept of utility maximization. For instance, in [1], analogies are: utility to internal energy, wealth to Helmholtz potential, price to chemical potential, quantities of commodity to particle number, quantity surplus to TS (product of temperature and entropy). In [2], initial setup is that the difference in macroeconomics production yield and cost of production

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[1] One might argue that X (e.g. volume) can change in equation that related Y and T, i.e. $Y = Y(T(X)) = Y \circ T(X)$ or $T = T(Y(X)) = T \circ Y(X)$ such that changing in X looks like having external effect on X. However in reality, one can not change value of X unless with truly endogenous effect of Y and(or) of T.

[2] Complexities of social physics arises from fundamental facts that physical agents have function (duty) to react in obeying physical laws but intelligent agents have both duty (to react as dictated by social laws) and rights (or options or choices) to react or not to.

is analogous to economics surplus that increases the capital. The capital is interpreted as internal energy. In [3], utility is analogous to entropy. Mechanical pair is commodity and marginal rate of substitution (MRS). In strictly deriving of thermodynamics by Carathéodory's axioms (see e.g. [4, 5]), distinction of empirical facts (e.g. pressure, volume) and theoretical entities (e.g. temperature, heat) must be considered. Empirical temperature is defined with mechanical pair so that the EoS is constructed. Internal energy, U and the first law is defined purely empirically. Hence existence of the EoS is the beginning step in strictly deriving of thermodynamics. We adopt the Carathéodory approach and use it in building the foundation of thermodynamics formulation of economics. In [6], an economics EoS was proposed and was disproved by the fact that the proposed EoS has only one degree of freedom. Thus it is not a constraint surface which is a crucial nature of an EoS. Although price, Pr and T share theoretical similarities that they are both abstract and they identify thermal and price equilibriums, however Pr can not be regarded as temperature. Thermal equilibrium is different from price equilibrium. As in thermal equilibrium, the system is one sole sector. Energy is exchanged amongst molecules in closed system therefore U is conserved. In price equilibrium, there are two sectors, i.e. consumers and producers. Commodities are exchanged with other assets (e.g. money) and neither of commodities nor money are conserved in each sector. Here unlike [6], we do not consider market as a system, but instead we consider group of consumers as a system and construct a demand-side EoS.

3.1. Equilibriums
If liquidity asset (e.g. money) is considered as energy, transfers of liquidity asset is hence an expense of the buyers determined by mechanical pair, Pr (as intensive coordinate Y) and demand quantity Q^{d} (as extensive coordinate X). Money at amount of $Pr\,dQ^{\mathrm{d}}$ is transferred out of the demand-side system in a transaction so that more amount of utility is attained. If there is an economic quasi-static *thermal process*, i.e. process allowing changes in Pr and Q^{d}, following Carathéodory's axioms, the process can take the system from an arbitrary state to another arbitrary state given by

$$f(Pr_1, Q_1^{\mathrm{d}}) = \varphi_1 \qquad \text{and} \qquad f(Pr_2, Q_2^{\mathrm{d}}) = \varphi_2. \tag{2}$$

The two different states shall be identified with average personal wealth, φ which is analogous to T.

3.2. Average Personal Wealth, Price, Total Wealth and the First Law
Average personal wealth, φ is average of total wealth of all types, $\mathcal{W}$ of group of N consumers, i.e.

$$\varphi = \frac{\mathcal{W}}{N}, \tag{3}$$

thus it is intensive. This is similar to monatomic ideal gas that $T = 2U/(3Nk_{\mathrm{B}})$. There are not as many fundamental units in economics as in physics. In economics, there are units of value (e.g. Dollars), number of goods and number of buyers only. Hence there is no such constant k_{B} here to balance the equation. Price, Pr is, in perfect competitive market, a marginal revenue, MR (increment of revenue for a unit of good sale) which is also equal to marginal cost of production, MC (increment of cost for a unit of good produced). Price is an influence per unit of good, thus the same spirit as pressure, i.e. force per unit area. The total wealth of all types, $\mathcal{W}$ plays the same role as internal energy, U, i.e. for demand-side economics adiabatic process, the first law should read

$$d\mathcal{W} = \delta W_{\mathrm{DM}} \equiv -Pr\,dQ^{\mathrm{d}}. \tag{4}$$

The demand-side economics work is $\delta W_{\mathrm{DM}} \equiv -Pr\,dQ^{\mathrm{d}}$. Transaction reduces total wealth of the whole group of consumers by a magnitude of $d\mathcal{W}$.

3.3. Effect Structure in the EoS

Following concepts and criteria in Sec. 2, we propose an EoS,

$$g(Q^{\mathrm{d}}, Pr, \varphi) = 0 \qquad \text{written as} \qquad Q^{\mathrm{d}} = \tilde{Q}^{\mathrm{d}}(Pr, \varphi) \qquad (5)$$

where $\tilde{f}$ denotes truly endogenous function. In perfect competitive market, it is known that Q^{d} is likely to be inversely proportional to Pr but proportional to φ. Diagram in figure 3 bases on this known fact and it obeys all criterial rules given in Sec. 2. Notice that effect of externality can appear only on Pr and on φ as required. The diagram is distinct from the Class I and II hence is categorized as Class III.
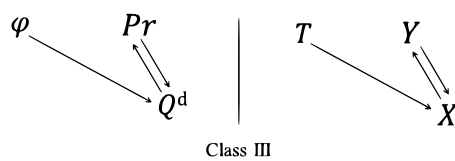

Figure 3. Effect structure diagram of a group of consumers in perfect competitive market. It is considered as Class III type EoS. Average personal wealth φ can affect demand quantity Q^{d}. Price Pr can affect Q^{d} and so reversely. φ can only affect Q^{d}.

4. Conclusion

We point out in this work that there are effect structures of variables in thermodynamics. We consider the effect structure in EoS and propose a formal way to standardize the status of an EoS. We classify the EoS into a classes of diagram. EoS for a group of consumers in perfect-competitive market is proposed and tested with the criteria. The new economic EoS satisfies the criteria thus can be considered as Class III EoS. Total wealth is considered as analogous to internal energy. In our setting, demand-side economics work term is product of price to the change in demand quantity. Demand-side coordinates are demand quantity as extensive coordinate, price as intensive coordinate and average personal wealth as demand-side economics temperature. Further work is to construct the EoS employing economics assumption or econometrics methods.

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