COSMOLOGICAL DYNAMICS OF NON-MINIMAL DERIVATIVE COUPLING FIELDS

CANDRASYAH MUHAMMAD

A Thesis Submitted to the Graduate School of Naresuan University
in Partial Fulfillment of the Requirements
for the Master of Science Degree in Theoretical Physics
July 2018

This thesis entitled "Cosmological Dynamics of Non-minimal Derivative Coupling Fields" submitted by Candrasyah Muhammad in partial fulfillment of the requirements for the Master of Science in Theoretical Physics is hereby approved.

. . . . .Chair
(Asst.Prof. Seckson Sukhasena, Ph.D.)

. . . . .Committee
(Assoc.Prof. Burin Gumjudpai , Ph.D.)

. . . . .Committee
(Asst.Prof. Pichet Vanichchapongjaroen, Ph.D.)

. . . . .Committee
(Itzadah Thongkool, Ph.D.)

Approved

. . . .
(Associate Professor Paisarn Muneesawang, Ph.D.)
Acting Dean of the Graduate School

July 2018

ACKNOWLEDGEMENT

I would like to thank my advisor Assoc.Prof.Dr. Burin Gumjudpai, for the guidance, motivation and interesting stories. I also thank all my lecturers at IF for their inspiration that encourage me to do theoretical physics. I thank all IF family (students, staff, visitors) for their kindness and supports, my Indonesian fellows at Phitsanulok who helped me to adjust my life in Thailand. Last but not least, I thank my mother, my father and my sisters.

Candrasyah Muhammad

July 2018

Title	Cosmological Dynamics of Non-minimal Derivative Coupling Fields
Author	Candrasyah Muhammad
Advisor	Assoc.Prof. Burin Gumjudpai, Ph.D.
Academic Paper	Thesis M.S. in Theoretical Physics, Naresuan University, 2018
Keywords	Non-Minimal Derivative Coupling, Palatini formalism, Dynamical System Inflation

ABSTRACT

We study the cosmological dynamics of Non-Minimal Derivative Coupling (NMDC) scalar field to Einstein tensor. Equation of motions obtained from both metric and Palatini formalism are investigated. The acceleration conditions for exponential potential are derived. We plotted the phase portraits for GR, metric NMDC and Palatini NMDC cases. It is shown that Palatini NMDC effect restrict the attractor into the smaller accelartion region. As the e-folding number growing, metric NMDC model enhance the expansion rate and Palatini NMDC model surpress the expansion rate.

CONTENTS

LIST OF FIGURES

Figure **Page**

CHAPTER I

INTRODUCTION

1.1 Background and motivation

Physicists believe that our universe is in the accelerated expansion. This view is strongly supported by astrophysical observations such as, redshift of Supernovae Type Ia measurement [1, 2, 3]. In very early universe, inflation is believed to be happened. This idea was introduced by Guth [4] and supported by recent observations, e.g. cosmic microwave background (CMB) anisotropies [5], WMAP [6]. Dynamical scalar field such as inflaton or quintessence [7], k-essence [8] or by effect of modified gravity such as scalar-tensor theories [9, 10, 11] is suggested to be responsible with cosmic inflation at the early universe and late time acceleration expansion.

In order to explain early universe and late-time acceleration, many scalar-tensor theories has been proposed and studied. Some of these theories has non minimal coupling (NMC) of scalar field to gravity sector such as $f(\phi, \phi_{,\mu}, \phi_{,\mu\nu}, ...)$. The extension of NMC models named as Non-Minimal Derivative Coupling (NMDC) contain coupling in the form of $f(\phi, \partial_\mu \phi)$. We focused on NMDC term in which Einstein tensor which couples to kinetic part of scalar field as $G_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$ [12, 13, 14, 15, 16, 17, 18, 19]. In this thesis, NMDC Cosmology is studied using so-called Palatini formalism. Palatini formalism employ the variation with respect to metric and connection field, which is treated as independent variables.

We study the cosmological dynamics of Palatini NMDC model with the exponential potential by plotting the phase portrait of the dynamical systems. Phase portraits for metric NMDC and GR cases are also plotted as comparison.

1.2 Objectives

- Derive acceleration condition of NMDC model both in metric and Palatini formalism.
- Introduce autonomous systems of the models.

- Present dynamical phase portraits of the models with acceleration region.

1.3 Framework

- FLRW universe
- NMDC model
- metric and Palatini formalism
- Phase portraits

1.4 Unit and notation

Units with $c = \hbar = k_B = 1$

$[G] = \left[M_{\mathrm{P}}^{-2}\right]$

$[\text{length}] = [\text{time}] = [\mathrm{L}]$

$[\text{mass}] = [\text{energy}] = [\mathrm{M}]$

$[\text{energy density: } \rho] = [\text{pressure: } p] = \left[\mathrm{ML}^{-3}\right] = \left[\mathrm{L}^{-4}\right]$

$[g_{\mu\nu}] = [\text{dimensionless}]$

$[\Gamma^{\mu}_{\nu\kappa}] = \left[\mathrm{L}^{-1}\right]$

$[R^{\mu}_{\nu\kappa\sigma}] = [R_{\mu\nu}] = [R] = \left[\mathrm{L}^{-2}\right]$

$[\phi] = [\mathrm{M}]$

$[\kappa] = \left[\mathrm{M}^{-2}\right]$

$[V(\phi)] = \left[\mathrm{M}^{4}\right]$

$[H] = [\mathrm{M}]$

$[G] = \left[\mathrm{M}^{-2}\right]$

Notation

G : Newton's gravitational constant ($G = 6.67 \times 10^{-8}\text{cm}^3\text{g}^{-1}\text{sec}^{-2}$)

M_P : Reduced Plank mass $\left(M_\text{P} = (8\pi G)^{-1/2} = 2.4357 \times 10^{18}\text{GeV}\right)$

a : Scale factor of the universe (where $a_0 = 1$ at the present time)

t : The cosmic time

$\dot{x}$: Derivative of x variable with respect to t

x' : Derivative of x variable with respect to $N \equiv \ln a$

H : Hubble parameter $\left(H \equiv \frac{\dot{a}}{a}\right)$

ρ : Energy density

p : Pressure

w : Equation of state parameter $\left(w = \frac{P}{\rho}\right)$

w_eff : Effective equation of state parameter

R : Ricci scalar

$g_{\mu\nu}$: Metric tensor

$\Gamma^\mu_{\nu\kappa}$: Connection field

$G_{\mu\nu}(g)$: Einstein tensor in metric formalism

$G_{\mu\nu}(\Gamma)$: Einstein tensor in Palatini formalism

ϕ : Scalar field

$V(\phi)$: Scalar field potential

CHAPTER II

STANDARD MODEL OF COSMOLOGY

2.1 General relativity

Our current understanding of the fundamental gravity interaction is based on Einstein's Field Equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{2.1}$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ is called Einsten Tensor. $g_{\mu\nu}$ is metric tensor. $R_{\mu\nu}$ is Riemann tensor. R is Ricci scalar. $T_{\mu\nu}$ is stress-energy tensor.

Einstein's field equation can be derived from the variational principle of Einstein-Hilbert action

$$S_{EH} = \int \mathrm{d}^4 x \sqrt{-g} R \ , \tag{2.2}$$

where g is determinant of metric $g_{\mu\nu}$.

2.2 Friedmann-Lemaitre-Robertson-Walker universe

The solutions of Einstein's field equation are in the form of metric $g_{\mu\nu}$. One of the solution is named as Friedmann-Lemaitre-Robertson-Walker (FLRW) metric, which is consistent with our homogeneous, isotropic and non-static universe.

$$\mathrm{d}s^2 = -c^2 \mathrm{d}t^2 + a^2(t) \Big[\frac{\mathrm{d}r^2}{1 - kr^2} + r^2 (\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \Big] \tag{2.3}$$

where k is the curvature parameter characterizing types of curvature and taking values of $+1, 0, -1$.

Energy-momentum tensor for perfect fluid is given by

$$T_{\mu\nu} = \Big(\rho + \frac{p}{c^2} \Big) u_\mu u_\nu + g_{\mu\nu} p. \tag{2.4}$$

Combining (2.4), (2.1), and (2.3), one can obtain

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \Big(\rho + 3\frac{p}{c^2} \Big) \tag{2.5}$$

which is known as the acceleration equation, and

$$H^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2}. \tag{2.6}$$

with Hubble's parameter $H = \dot{a}/a$, dubbed as Friedmann equation.

By taking time-derivative of Friedmann equation (2.6) and substituting to the acceleration equation (2.5), we can derive fluid equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0. \tag{2.7}$$

2.3 Inflationary universe

Cosmologists suggests inflation theory to solve horizon and flatness problem [4]. Inflation said to be occured, whenever

$$\ddot{a} > 0 \ , \tag{2.8}$$

in other words, from (2.5)

$$p < -\frac{\rho}{3} \ , \tag{2.9}$$

from this section on, we will employ the notation $c = 1$. It has been suggested that inflation was driven by a single scalar field $\phi(t)$. Named as inflaton field, and its energy and pressure is given by

$$\rho_\phi = \frac{1}{2}\varepsilon\dot{\phi}^2 + V(\phi) \tag{2.10}$$

$$p_\phi = \frac{1}{2}\varepsilon\dot{\phi}^2 - V(\phi) \tag{2.11}$$

where $V(\phi)$ is scalar potential and $\varepsilon \pm 1$.

Subtituting (2.10) and (2.11) to Friedmann equation (2.6) in the flat space ($k = 0$) will obtain the scalar dominated Friedmann equation

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\varepsilon\dot{\phi}^2 + V(\phi)\right) \ . \tag{2.12}$$

Klein-Gordon equation as the equation of motion of the scalar field can be obtained from fluid equation (2.7) in terms of (2.10) and (2.11)

$$\varepsilon\ddot{\phi} + 3H\varepsilon\dot{\phi} + V_{,\phi} = 0 \tag{2.13}$$

And the acceleration condition (2.9) become

$$\varepsilon\dot{\phi}^2 < V \ , \tag{2.14}$$

named as slow-roll condition, because physically it means that the ϕ change very slowly.

By the virtue of (2.14), Friedman equation (2.12) and Klein-Gordon equation can be approximated as

$$H^2 \simeq \frac{8\pi G}{3} V \tag{2.15}$$

$$\dot{\phi} \simeq -\frac{V_{,\phi}}{3H} \tag{2.16}$$

CHAPTER III

PALATINI FORMALISM IN GENERAL RELATIVITY

In many textbooks [20, 21, 22], the common way to derive Einstein field equation (2.1) from Einstein-Hilbert action (2.2), is to do the variational principle of the action with respect to metric $\delta g^{\mu\nu}$, with the known relation between metric tensor $g_{\mu\nu}$ and $\Gamma^{\gamma}_{\mu\nu}$. We refer this way as *metric formalism*. Another approach is to vary the action (2.2) with respect to both metric $\delta g^{\mu\nu}$ and $\delta\Gamma^{\alpha}_{\beta\gamma}$ connection. In other words, we treat metric and connection as independent variables. Assuming that we do not know the relation between metric and connection, in order to reveal later after doing variation. We refer this second approach as *Palatini formalism*.

3.1 Motivation from classical mechanics : coordinate and momentum treated as independent variables

In the particle mechanics, in order to find the equation of motion, one can extremize the integral action

$$I = \int L\left(x, \dot{x}, t\right) dt \ . \tag{3.1}$$

Variation $\delta I = 0$ with respect to $\delta x(t)$ resulting Euler-Lagrange equation

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = 0 \ . \tag{3.2}$$

Moreover, there is another interesting way to do the variation. First we express the Lagrangian in terms of coordinates and momenta $L\left(x, \dot{x}, t\right) = p\dot{x} - H\left(p, x, t\right)$. But then, assume we do not know the nature of p and x (i.e. we abandon any relations such as $p = \partial L/\partial \dot{x}$). Hence p and x can be savely treated as independent variables, and vary the action with respect to both variables.

$$\begin{aligned}
\delta I &= 0 \\
&= \int dt \left[\frac{\partial (p\dot{x})}{\partial x}\delta x - \frac{\partial H}{\partial x}\delta x + \frac{\partial (p\dot{x})}{\partial p}\delta p - \frac{\partial H}{\partial p}\delta p\right] \\
&= \int dt \left[p\frac{\partial}{\partial x}\left(\frac{dx}{dt}\right)\delta x - \frac{\partial H}{\partial x}\delta x + \left(\dot{x} - \frac{\partial H}{\partial p}\right)\delta p\right] \\
&= \int dt \left[\frac{d}{dt}\left(p\delta x\right) - \dot{p}\delta x - \frac{\partial H}{\partial x}\delta x + \left(\dot{x} - \frac{\partial H}{\partial p}\right)\delta p\right]
\end{aligned}$$

$$= \cancelto{0}{p\delta x} + \int dt \left[\left(-\dot{p} - \frac{\partial H}{\partial x} \right) \delta x + \left(\dot{x} - \frac{\partial H}{\partial p} \right) \delta p \right] . \tag{3.3}$$

For vanishing coefficents of δx and δp respectively, we have

$$\dot{p} = -\frac{\partial H}{\partial x} \tag{3.4}$$

and

$$\dot{x} = \frac{\partial H}{\partial p} . \tag{3.5}$$

Equations (3.4) and (3.5) are known as Hamitonian equations of motion, which is equivalent to Euler-Lagrange equation (3.2).

We can see that, even we have treated x and p as independent unknown variables, the role of x and p can be revealed later as canonical position and canonical momentum in (3.5) and (3.4). More detailed discussion and another example of motivation of the Palatini approach can be found in MTW book [23].

3.2 Metric approach in general relativity

In the Lagrangian formulation, Einstein field equation (2.1) can be obtained by varying the Einstein Hilbert action gravitational action

$$S_G = \int \mathrm{d}^4x\sqrt{-g}R . \tag{3.6}$$

Where g is determinant of metric $g_{\mu\nu}$, and $\mathrm{d}^4x\sqrt{-g} = \mathrm{d}V$ is volume element. R is Ricci scalar, defined by

$$R = g^{\mu\nu}R_{\mu\nu} . \tag{3.7}$$

$R_{\mu\lambda\nu}$ is a contraction of Riemann tensor, given by

$$\begin{aligned} R_{\mu\nu} &= R^{\lambda}_{\mu\lambda\nu} \\ &= \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\nu\lambda} + \Gamma^\lambda_{\sigma\lambda}\Gamma^\sigma_{\mu\nu} - \Gamma^\lambda_{\sigma\nu}\Gamma^\sigma_{\mu\lambda} , \end{aligned} \tag{3.8}$$

where $\Gamma^\lambda_{\mu\nu}$ is Christoffel connection, described by

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\mu\nu}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}) . \tag{3.9}$$

Equation (3.9) can be obtain by imposing metric compatibility propety of metric, which is covariant derivative of metric equal to zero

$$\nabla_\rho g_{\mu\nu} = 0 . \tag{3.10}$$

The variation of gravitational action become

$$\delta S_G = \int \mathrm{d}^4 x \left[g^{\mu\nu} R_{\mu\nu} \delta\sqrt{-g} + \sqrt{-g} R_{\mu\nu} \delta g^{\mu\nu} + \sqrt{-g} g^{\mu\nu} \delta R_{\mu\nu} \right] . \tag{3.11}$$

With some identities

$$\delta g_{\mu\nu} = -g_{\mu\rho} g_{\sigma\nu} \delta g^{\rho\sigma} \tag{3.12}$$

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g} g_{\mu\nu} \delta g^{\mu\nu} , \tag{3.13}$$

it is preferable to vary $\delta S_g/\delta g^{\mu\nu}$, if one can express the third term of right hand side equation (3.11) in terms of $\delta g^{\mu\nu}$.

$$\begin{aligned} \delta R_{\mu\nu} &= \delta R^{\lambda}_{\mu\lambda\nu} \\ &= \partial_\lambda \delta\Gamma^{\lambda}_{\nu\mu} - \partial_\nu \delta\Gamma^{\lambda}_{\lambda\mu} + \delta\Gamma^{\lambda}_{\lambda\sigma}\Gamma^{\sigma}_{\nu\mu} + \Gamma^{\lambda}_{\lambda\sigma}\delta\Gamma^{\sigma}_{\nu\mu} - \delta\Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\lambda\mu} - \Gamma^{\lambda}_{\nu\sigma}\delta\Gamma^{\sigma}_{\lambda\mu} . \end{aligned} \tag{3.14}$$

The connection $\Gamma^{\lambda}_{\nu\mu}$ is not a tensor, but its variation $\delta\Gamma^{\lambda}_{\nu\mu}$ is. Therefore we can apply covariant derivative to $\delta\Gamma^{\lambda}_{\nu\mu}$

$$\nabla_\lambda \left(\delta\Gamma^{\lambda}_{\nu\mu}\right) = \partial_\lambda \delta\Gamma^{\lambda}_{\nu\mu} + \Gamma^{\lambda}_{\lambda\sigma}\delta\Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\lambda\nu}\delta\Gamma^{\lambda}_{\sigma\mu} - \Gamma^{\sigma}_{\lambda\mu}\delta\Gamma^{\lambda}_{\nu\sigma} . \tag{3.15}$$

Realizing some identical terms in the right hand side of equations (3.14) and (3.15), we can substitute (3.15) into (3.14)

$$\begin{aligned} \delta R_{\mu\nu} &= \nabla_\lambda \left(\delta\Gamma^{\lambda}_{\nu\mu}\right) + \Gamma^{\sigma}_{\lambda\nu}\delta\Gamma^{\lambda}_{\sigma\mu} - \partial_\nu \delta\Gamma^{\lambda}_{\lambda\mu} + \delta\Gamma^{\lambda}_{\lambda\sigma}\Gamma^{\sigma}_{\nu\mu} - \Gamma^{\lambda}_{\nu\sigma}\delta\Gamma^{\sigma}_{\lambda\mu} \\ &= \nabla_\lambda \left(\delta\Gamma^{\lambda}_{\nu\mu}\right) - \nabla_\nu \left(\delta\Gamma^{\lambda}_{\lambda\mu}\right) . \end{aligned} \tag{3.16}$$

Expression (3.16) is called *Palatini identity*, named after Italian mathematician Attilio Palatini. It should be noted that Palatini identity does not direcly imply the Palatini formalism. Palatini identity is true in any torsion-free manifolds (i.e $\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\nu\mu}$) . The third term of equation (3.11) as

$$\begin{aligned} \int \mathrm{d}^4 x \sqrt{-g} g^{\mu\nu} \delta R_{\mu\nu} &= \int \mathrm{d}^4 x \sqrt{-g} g^{\mu\nu} \left[\nabla_\lambda \left(\delta\Gamma^{\lambda}_{\nu\mu}\right) - \nabla_\nu \left(\delta\Gamma^{\lambda}_{\lambda\mu}\right)\right] \\ &= \int \mathrm{d}^4 x \sqrt{-g} \nabla_\sigma \left[g^{\mu\nu}\delta\Gamma^{\sigma}_{\nu\mu} - g^{\mu\sigma}\delta\Gamma^{\lambda}_{\lambda\mu}\right] \\ &= \int \mathrm{d}^4 x \sqrt{-g} \nabla_\sigma V^\sigma \\ &= 0 \end{aligned} \tag{3.17}$$

where metric compatibility (3.10) have been used in the second line of derivation of (3.17). Since $\delta\Gamma^{\lambda}_{\nu\mu}$ is a tensor, defining a new vector field V^σ is possible

$$V^\sigma = g^{\mu\nu}\delta\Gamma^{\sigma}_{\nu\mu} - g^{\mu\sigma}\delta\Gamma^{\lambda}_{\lambda\mu} \tag{3.18}$$

and using Stokes' theorem, integral with respect to volume element of divergence of the vector field is equal to zero (recalling that $\int \mathrm{d}^4x\sqrt{-g} = \int \mathrm{d}V$ is volume element integral). Therefore the third term of (3.11) vanished.

Substitute (3.13) and (3.17) to (3.11)

$$\begin{aligned} \delta S_G &= \int \mathrm{d}^4x \left[R - \frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu} + \sqrt{-g}R_{\mu\nu}\delta g^{\mu\nu}\right] \\ &= \int \mathrm{d}^4x\sqrt{-g}\left[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right]\delta g^{\mu\nu} \ , \end{aligned} \tag{3.19}$$

varying the gravitational action with respect to $\delta S_G/\delta g^{\mu\nu} = 0$, we obtain the Einstein field equation in the vacuum

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0 \ . \tag{3.20}$$

Equation (3.19) shown that varying Einstein-Hilbert action can be done only with respect to $\delta g^{\mu\nu}$, using the relation between metric and connection (3.10) or (3.9).

3.3 Palatini approach in general relativity

Palatini approach, actually was invented by Einstein in 1925. Einstein mistakenly thought that it was Palatini's idea. Probably because Palatini's paper was written in Italian [24]. In the Palatini approach, any explicit relation between metric and connection such as metric compatibility (3.10) is not assumed. In another word, metric and connection are treated as independent variables. From the variation (3.11), $R_{\mu\nu}$ is solely depend on $\Gamma^\lambda_{\mu\nu}$, the first and second term in (3.11) is same with the metric approach, and the variation of the third term become

$$\begin{aligned} \int \mathrm{d}^4x\sqrt{-g}g^{\mu\nu}\delta R_{\mu\nu} &= \int \mathrm{d}^4x\sqrt{-g}g^{\mu\nu}\left[\nabla_\lambda\left(\delta\Gamma^\lambda_{\nu\mu}\right) - \nabla_\nu\left(\delta\Gamma^\lambda_{\lambda\mu}\right)\right] \\ &= \int \mathrm{d}^4x\sqrt{-g}\nabla_\sigma\left[g^{\mu\nu}\delta\Gamma^\sigma_{\nu\mu} - g^{\mu\sigma}\delta\Gamma^\lambda_{\lambda\mu}\right] \\ &\quad - \int \mathrm{d}^4x\sqrt{-g}\left[\left(\nabla_\sigma g^{\mu\nu}\right)\delta\Gamma^\sigma_{\nu\mu} - \left(\nabla_\sigma g^{\mu\sigma}\right)\delta\Gamma^\lambda_{\lambda\mu}\right] \\ &= \int \mathrm{d}^4x\sqrt{-g}\left[\left(\nabla_\sigma g^{\mu\sigma}\right)\delta^\nu_\lambda - \left(\nabla_\lambda g^{\mu\nu}\right)\right]\delta\Gamma^\lambda_{\nu\mu} \ . \end{aligned} \tag{3.21}$$

Now the total variation of gravitational action is given by

$$\delta S_G = \int \mathrm{d}^4x\sqrt{-g}\left[\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\delta g^{\mu\nu} + \left(\nabla_\sigma g^{\mu\rho}\delta^\nu_\lambda - \nabla_\lambda g^{\mu\nu}\right)\delta\Gamma^\lambda_{\nu\mu}\right] \ . \tag{3.22}$$

Variation $\delta S_G/\delta g^{\mu\nu} = 0$ will lead to Einstein equation in the vacuum (3.20). Moreover variation $\delta S_G/\delta\Gamma^\lambda_{\mu\nu} = 0$ lead to another equation. Since the torsion-free manifold is assumed $\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$, $\nabla_\lambda g^{\mu\nu} = 0$ is the only solution of variation [25, 26]. Hence the metric compatibility (3.10) is recovered

$$\nabla_\rho g_{\mu\nu} = 0 \ .$$

And the definition of Christoffel symbol (3.9) came after that

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\mu\nu} (\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}) \ .$$

The explicit relation between metric and connection can be recovered, by treating both metric and connection as independent variables.

CHAPTER IV

NON-MINIMAL DERIVATIVE COUPLING COSMOLOGY

Cosmological scalar field models might be able to describe inflation and accelerating expansion of the universe. Motivated from Brans-Dicke models, it is possible to have a model which has a non minimal coupling (Non-Minimal Coupling) between Ricci scalar and function of scalar field, $f(\phi)R$ [27]. One of the extension of NMC is called NMDC (Non-Minimal Derivative Coupling) where the function $f(\phi)$ not only depend on the scalar field but also its derivative $f(\phi, \phi_{,\mu})$. NMDC firstly studied by Amendola in 1993 [28]. In 2000, Capozziello, Lambiase and Schmidt investigated NMDC in the form $R^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ [29]. They found that de Sitter spacetime as an attractor solution. NMDC with two separated coupling terms ξ and η such as in $-\frac{1}{2}\xi R\phi^{-2}\partial_\mu\phi\phi^{,\mu}$ and $-\frac{1}{2}\eta R_{\mu\nu}\phi^{-2}\phi^{,\mu}\phi_{,\mu}$ studied by Granda in 2000 [30].

Consider NMDC models with two separated couplings κ_1 and κ_2 such as $\kappa_1 R\phi_{,\mu}\phi^{,\mu}$ and $\kappa_2 R_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$. Setting $\kappa \equiv -2\kappa_1 = \kappa_2$, derivative of the scalar field coupled to the Einstein tensor $\kappa G_{\mu\nu}\phi^{\mu}\phi^{\nu}$ [12, 13, 14, 15, 16, 17]. In this thesis, we consider NMDC model with the action [12]

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\Big[\frac{R}{8\pi G_\mathrm{N}} - [\varepsilon g_{\mu\nu} + \kappa G_{\mu\nu}]\phi^{,\mu}\phi^{,\nu} - 2V(\phi)\Big]. \tag{4.1}$$

The model is known as subclass of the Horndeski action (with $G_5 = \phi\kappa/2$) [31]. This model was studied by Kaewkhao and Gumjudpai [19] using Palatini formalism. A different class of NMDC model was studied using Palatini formalism before by Luo, Wu, and Yu [32].

4.1 Metric NMDC cosmology

From the action (4.1), the scalar field equation of motion in slow-rolling regime given as follow [18]

$$\ddot{\phi}\left(\varepsilon - 3\kappa H^2\right) + 3H\dot{\phi}\left(\varepsilon - 2\kappa\dot{H} - 3\kappa H^2\right) + V_{,\phi} \simeq 0$$
$$\ddot{\phi}\left(\varepsilon - 3\kappa H^2\right) + 3H\dot{\phi}\left(\varepsilon - 3\kappa H^2\right) + V_{,\phi} \simeq 0, \tag{4.2}$$

where approximation during inflationary epoch, $|\ddot{H}/H| \ll |\dot{H}| \ll |H^2|$ was used. At the late-time $\ddot{\phi} \approx 0$, the trajectory from (4.2) is given by

$$\dot{\phi} \simeq -\frac{V_{,\phi}}{3H\left(\varepsilon - 3\kappa H^2\right)} \ . \tag{4.3}$$

Consider single scalar field species during inflationary epoch,

$$\rho_{\rm tot} \equiv \rho_\phi = \frac{\varepsilon\dot{\phi}^2}{2} + V \ , \tag{4.4}$$

$$p_{\rm tot} \equiv p_\phi = \frac{\varepsilon\dot{\phi}^2}{2} + V \ , \tag{4.5}$$

$$w_{\rm tot} = \frac{p_{\rm tot}}{\rho_{\rm tot}} = \frac{p_\phi}{\rho_\phi} \ , \tag{4.6}$$

Hence modified Friedmann equation from (4.1) read as [18]

$$H^2 = \frac{1}{3M_{\rm P}^2}\left[\frac{\dot{\phi}^2}{2}\left(\varepsilon - 9\kappa H^2\right) + V\right] \ . \tag{4.7}$$

Take its derivative with respect to time, during inflationary epoch we have

$$\begin{aligned} 2H\dot{H} &\simeq \frac{1}{3M_{\rm P}^2}\left(\varepsilon\dot{\phi}\ddot{\phi} - 9\kappa\dot{\phi}\ddot{\phi}H^2 - 9\kappa\dot{\phi}^2 H\dot{H} + V_{,\phi}\dot{\phi}\right) \\ \dot{H} &\simeq \frac{1}{6HM_{\rm P}^2}\left(\varepsilon\dot{\phi}\ddot{\phi} - 9\kappa\dot{\phi}\ddot{\phi}H^2 - 9\kappa\dot{\phi}^2 H\dot{H} + V_{,\phi}\dot{\phi}\right) \ . \end{aligned} \tag{4.8}$$

4.2 Palatini NMDC cosmology

In 2016 Kaewkhao and Gumjudpai studied a NMDC model using Palatini formalism [19] as

$$S_{\rm Palatini} = \frac{M_{\rm P}^2 c^4}{2}\int {\rm d}^4x\,\sqrt{-g}\Big\{\tilde{R}(\Gamma) - \Big[\varepsilon g_{\mu\nu} + \kappa\tilde{G}_{\mu\nu}(\Gamma)\Big]\phi^{;\mu}\phi^{;\nu} - V(\phi)\Big\}, \tag{4.9}$$

where $\tilde{G}_{\mu\nu}(\Gamma) = \tilde{R}_{\mu\nu}(\Gamma) - \frac{1}{2}\tilde{R}(\Gamma)$; $\tilde{R}(\Gamma) = g^{\mu\nu}\tilde{R}_{\mu\nu}(\Gamma)$ and Ricci tensor is defined by connection field $\tilde{R}_{\mu\nu}(\Gamma) = \tilde{R}^{\lambda}_{\mu\lambda\nu}(\Gamma) = \partial_\lambda\Gamma^\lambda_{\mu\nu} - \partial_\nu\Gamma^\lambda_{\mu\lambda} + \Gamma^\lambda_{\sigma\lambda}\Gamma^\sigma_{\mu\nu} - \Gamma^\lambda_{\sigma\nu}\Gamma^\sigma_{\mu\lambda}$. Tilde symbol implies dependency on connection field (Γ) without any known explicit relation with the metric. Action (4.9) allows two dynamical fields, metric field g and connection field Γ, to vary. The modified Klein-Gordon field equation of Palatini NMDC hence is given by

$$\begin{aligned} \ddot{\phi}\left(\varepsilon - (9/2)\kappa\dot{H} - (15/2)\kappa H^2\right) + 3H\dot{\phi}\left(\varepsilon - 4\kappa\dot{H}\right) + V_{,\phi} &\simeq 0 \\ \ddot{\phi}\left(\varepsilon - (15/2)\kappa H^2\right) + 3H\dot{\phi}\varepsilon + V_{,\phi} &\simeq 0 \ . \end{aligned} \tag{4.10}$$

whereas during inflationary epoch, $|4\dot{H}\kappa| \ll 1$; $|9\kappa\dot{H}/2| \ll 1$. At $\ddot{\phi} \approx 0$, the late time trajectory is given by

$$\dot{\phi} = -\frac{V_{,\phi}}{3H\varepsilon} \ . \tag{4.11}$$

Hence the Palatini NMDC Friedmann equation is given by [19]

$$\begin{aligned} H^2 &\simeq \frac{\rho_{\rm tot}}{3M_{\rm P}^2}\left[1 + \frac{3}{2}\frac{\kappa\dot{\phi}^2}{M_{\rm P}^2}\left(1 + w_{\rm eff}\right)\right] \\ &\simeq \frac{1}{3M_{\rm P}^2}\left[\rho_{\rm tot} + \frac{3}{2}\frac{\kappa\dot{\phi}^2}{M_{\rm P}^2}\left(\rho_{\rm tot} + p_{\rm tot}\right)\right] \\ &\simeq \frac{1}{3M_{\rm P}^2}\left[\rho_{\phi} + \frac{3}{2}\frac{\kappa\dot{\phi}^2}{M_{\rm P}^2}\left(\rho_{\phi} + p_{\phi}\right)\right] \\ &\simeq \frac{1}{3M_{\rm P}^2}\left[\frac{\varepsilon\dot{\phi}^2}{2} + V + \frac{3}{2}\frac{\kappa\varepsilon\dot{\phi}^4}{M_{\rm P}^2}\right] \end{aligned} \tag{4.12}$$

Hence we have

$$\begin{aligned} H^2 \simeq &\quad \frac{1}{3M_{\rm P}^2}\left[\rho_{\phi} + \frac{3\kappa\dot{\phi}^2}{2M_{\rm P}^2}(\rho_{\phi} + p_{\phi})\right] \\ 2H\dot{H} \simeq &\quad \frac{1}{3M_{\rm P}^2}\left[\dot{\rho_{\phi}} + \frac{3\kappa}{2M_{\rm P}^2}\left(2\dot{\phi}\ddot{\phi}\right)(\rho_{\phi} + p_{\phi}) + \frac{3\kappa\dot{\phi}^2}{2M_{\rm P}^2}(\dot{\rho_{\phi}} + \dot{p_{\phi}})\right] \\ \dot{H} \simeq &\quad \frac{1}{2M_{\rm P}^2}\left[\frac{\dot{\rho_{\phi}}}{3H} + \frac{\kappa\dot{\phi}\ddot{\phi}}{HM_{\rm P}^2}(\rho_{\phi} + p_{\phi}) + \frac{3\kappa\dot{\phi}^2}{2M_{\rm P}^2}\left(\frac{\dot{\rho_{\phi}}}{3H} + \frac{\dot{p_{\phi}}}{3H}\right)\right] \\ \simeq &\quad \frac{1}{2M_{\rm P}^2}\left[-(\rho_{\phi} + p_{\phi}) + \frac{\kappa\dot{\phi}\ddot{\phi}}{HM_{\rm P}^2}(\rho_{\phi} + p_{\phi}) + \frac{3\kappa\dot{\phi}^2}{2M_{\rm P}^2}\left(-(\rho_{\phi} + p_{\phi}) + \frac{\dot{p_{\phi}}}{3H}\right)\right] \\ \simeq &\quad \frac{1}{2M_{\rm P}^2}\left[-\left(\varepsilon\dot{\phi}^2\right) + \frac{\kappa\dot{\phi}\ddot{\phi}}{HM_{\rm P}^2}\left(\varepsilon\dot{\phi}^2\right) + \frac{3\kappa\dot{\phi}^2}{2M_{\rm P}^2}\left(-\left(\varepsilon\dot{\phi}^2\right) + \frac{\varepsilon\dot{\phi}\ddot{\phi} - V_{,\phi}\,\dot{\phi}}{3H}\right)\right] \\ \simeq &\quad \frac{1}{2M_{\rm P}^2}\left[-\varepsilon\dot{\phi}^2 + \frac{\kappa\varepsilon\dot{\phi}^3\ddot{\phi}}{HM_{\rm P}^2} - \frac{3\varepsilon\kappa\dot{\phi}^4}{2M_{\rm P}^2} + \frac{\kappa\varepsilon\dot{\phi}^3\ddot{\phi}}{2HM_{\rm P}^2} - \frac{\kappa\dot{\phi}^3V_{,\phi}}{2HM_{\rm P}^2}\right] \\ \dot{H} \simeq &\quad -\frac{\varepsilon\dot{\phi}^2}{2M_{\rm P}^2} + \frac{3\kappa\varepsilon\dot{\phi}^3\ddot{\phi}}{4HM_{\rm P}^4} - \frac{3\varepsilon\kappa\dot{\phi}^4}{4M_{\rm P}^4} - \frac{\kappa\dot{\phi}^3V_{,\phi}}{4HM_{\rm P}^4} \ . \end{aligned} \tag{4.13}$$

CHAPTER V

ACCELERATION CONDITIONS IN EXPONENTIAL POTENTIAL

Previous works [19, 33] studied the effect of Palatini NMDC in chaotic inflation. $V \approx \phi^2$ and $V \approx \phi^4$ cases was considered. $V \approx \phi^2$ is likely viable to surpass Planck 2015 constraint, however with positive κ would have superluminal speed. In this thesis, we study the exponential potential $V = V_0 e^{-\lambda\phi/M_{\rm P}}$.

In the standard GR case, the acceleration condition is given by (2.14)

$$\begin{aligned} \varepsilon\dot{\phi}^2 &< V \\ &< V_0 e^{-\lambda\phi/M_{\rm P}} \,. \end{aligned} \tag{5.1}$$

5.1 Acceleration condition of metric NMDC

As in (2.8), acceleration condition require $\ddot{a}/a \equiv \dot{H} + H^2 > 0$, by the virtue of (4.7) and (4.8) we have

$$\begin{aligned} \frac{\ddot{a}}{a} &= \dot{H} + H^2 \\ &\simeq \frac{\varepsilon\ddot{\phi}\dot{\phi}}{6HM_{\rm P}^2} - \frac{9\kappa\dot{\phi}\ddot{\phi}H}{6M_{\rm P}^2} - \frac{9\kappa\dot{\phi}^2\dot{H}}{6M_{\rm P}^2} + \frac{V_{,\phi}\,\dot{\phi}}{6HM_{\rm P}^2} + \frac{\varepsilon\dot{\phi}^2}{6M_{\rm P}^2} - \frac{9\kappa\dot{\phi}^2H^2}{6M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2} \\ &\simeq \frac{\dot{\phi}^2}{6M_{\rm P}^2}\left(\varepsilon - 9\kappa H^2 - 9\kappa\dot{H}\right) + \frac{\dot{\phi}\ddot{\phi}}{6M_{\rm P}^2}\left(\frac{\varepsilon}{H} - 9\kappa H\right) + \frac{V_{,\phi}\,\dot{\phi}}{6HM_{\rm P}^2} + \frac{V}{3M_{\rm P}^2} \\ &\simeq \frac{\dot{\phi}^2}{6M_{\rm P}^2}\left(\varepsilon - 9\kappa H^2\right) + \frac{\dot{\phi}\ddot{\phi}}{6M_{\rm P}^2H}\left(\varepsilon - 9\kappa H^2\right) + \frac{V_{,\phi}\,\dot{\phi}}{6HM_{\rm P}^2} + \frac{V}{3M_{\rm P}^2}, \end{aligned} \tag{5.2}$$

where we have used slow-roll approximation $|\dot{H}| \ll |H^2|$.

The idea is to find the analog expression with $\varepsilon\dot{\phi}^2 < V$ in GR case (2.8). In order to do so, we need to write (5.2) in terms of $\dot{\phi}^2$ and V only. First we can substitute the field equation (4.2) for $\ddot{\phi}$ into (5.2)

$$\begin{aligned} \frac{\ddot{a}}{a} &\simeq \frac{\dot{\phi}^2}{6M_{\rm P}^2}\left(\varepsilon - 9\kappa H^2\right) + \frac{\dot{\phi}}{6M_{\rm P}^2H}\left(\frac{-V_{,\phi} - 3H\dot{\phi}\left(\varepsilon - 3\kappa H^2\right)}{\left(\varepsilon - 3\kappa H^2\right)}\right)\left(\varepsilon - 9\kappa H^2\right) \\ &\quad + \frac{V_{,\phi}\dot{\phi}}{6HM_{\rm P}^2} + \frac{V}{3M_{\rm P}^2} \\ &\simeq \frac{\dot{\phi}^2}{6M_{\rm P}^2}\left(\varepsilon - 9\kappa H^2\right) - \frac{V_{,\phi}\dot{\phi}}{6HM_{\rm P}^2}\frac{\left(\varepsilon - 9\kappa H^2\right)}{\left(\varepsilon - 3\kappa H^2\right)} - \frac{3\dot{\phi}^2}{6M_{\rm P}^2}\left(\varepsilon - 9\kappa H^2\right) \end{aligned}$$

$$
\begin{aligned}
&+\frac{V_{,\phi}\dot{\phi}}{6HM_{\rm P}^2}+\frac{V}{3M_{\rm P}^2}\\
\simeq\ & -\frac{\dot{\phi}^2}{3M_{\rm P}^2}\left(\varepsilon-9\kappa H^2\right)-\frac{V_{,\phi}\dot{\phi}}{6HM_{\rm P}^2}\left(1-\frac{(\varepsilon-9\kappa H^2)}{(\varepsilon-3\kappa H^2)}\right)+\frac{V}{3M_{\rm P}^2}\ ,
\end{aligned}
\tag{5.3}
$$

hence only $\dot{\phi}^2$ term is needed, so $\dot{\phi}$ can be approximated using late time trajectory (4.3)

$$
\begin{aligned}
\frac{\ddot{a}}{a} &\simeq -\frac{\dot{\phi}^2}{3M_{\rm P}^2}\left(\varepsilon-9\kappa H^2\right)+\frac{V}{3M_{\rm P}^2}\\
&\quad -\frac{V_{,\phi}}{6HM_{\rm P}^2}\left(\frac{-V_{,\phi}}{3H(\varepsilon-3\kappa H^2)}\right)\left(1-\frac{(\varepsilon-9\kappa H^2)}{(\varepsilon-3\kappa H^2)}\right)\\
&\simeq -\frac{\dot{\phi}^2}{3M_{\rm P}^2}\left(\varepsilon-9\kappa H^2\right)+\frac{V}{3M_{\rm P}^2}\\
&\quad +\frac{V_{,\phi}^2}{18H^2M_{\rm P}^2(\varepsilon-3\kappa H^2)}\left(1-\frac{(\varepsilon-9\kappa H^2)}{(\varepsilon-3\kappa H^2)}\right)\ .
\end{aligned}
\tag{5.4}
$$

Slow roll approximation $\dot{\phi}^2 \ll V$ is applied for Friedmann equation (4.7)

$$
H^2 \approx \frac{V}{3M_{\rm P}^2}\ , \tag{5.5}
$$

and substitute (5.5) in into (5.4)

$$
\frac{\ddot{a}}{a} \simeq -\frac{\dot{\phi}^2}{3M_{\rm P}^2}\left(\varepsilon-\frac{3\kappa V}{M_{\rm P}^2}\right)+\frac{V}{3M_{\rm P}^2}+\frac{V_{,\phi}^2}{6V(\varepsilon-\kappa V/M_{\rm P}^2)}\left(1-\frac{(\varepsilon-3\kappa V/M_{\rm P}^2)}{(\varepsilon-\kappa V/M_{\rm P}^2)}\right)\ . \tag{5.6}
$$

Given $|\kappa V| \ll |M_{\rm P}^2|$ to avoid super-Planckian regime. Therefore binomial approximation is possible, which is $(1+x)^a \approx 1+ax$ for $x \ll 1$. Hence,

$$
\begin{aligned}
\left(1-\frac{(\varepsilon-3\kappa V/M_{\rm P}^2)}{(\varepsilon-\kappa V/M_{\rm P}^2)}\right) &\approx 1-\left(\varepsilon-3\kappa V/M_{\rm P}^2\right)\left(\varepsilon+\kappa V/M_{\rm P}^2\right)\\
&\approx 1-\left(\underbrace{\varepsilon^2}_{1}-\frac{3\varepsilon\kappa V}{M_{\rm P}^2}+\frac{\varepsilon\kappa V}{M_{\rm P}^2}-\underbrace{\frac{3\kappa^2V^2}{M_{\rm P}^4}}_{\approx 0}\right)\\
&\approx \frac{2\varepsilon\kappa V}{M_{\rm P}^2}\ .
\end{aligned}
\tag{5.7}
$$

Equation (5.6) become

$$
\frac{\ddot{a}}{a} \simeq -\frac{\dot{\phi}^2}{3M_{\rm P}^2}\left(\varepsilon-\frac{\kappa V}{M_{\rm P}^2}\right)+\frac{V}{3M_{\rm P}^2}+\frac{V_{,\phi}^2\varepsilon\kappa}{3M_{\rm P}^2(\varepsilon-\kappa V/M_{\rm P}^2)}\ . \tag{5.8}
$$

For acceleration condition $\ddot{a}/a > 0$, we have

$$
\frac{\dot{\phi}^2}{3M_{\rm P}^2}\left(\varepsilon-\frac{3\kappa V}{M_{\rm P}^2}\right) < \frac{V}{3M_{\rm P}^2}+\frac{V_{,\phi}^2\varepsilon\kappa}{3M_{\rm P}^2(\varepsilon-\kappa V/M_{\rm P}^2)}
$$

$$\left(\varepsilon - \frac{3\kappa V}{M_{\rm P}^2}\right)\dot{\phi}^2 \;<\; V\left(1 + \frac{V_{,\phi}^2\varepsilon\kappa}{V(\varepsilon - \kappa V/M_{\rm P}^2)}\right) \;. \tag{5.9}$$

In which for GR case, $\varepsilon\dot{\phi}^2 < V$ is recovered by taking $\kappa = 0$.

Applying exponential potential,

$$V = V_0 e^{-\lambda\phi/M_{\rm P}} \;, \tag{5.10}$$

acceleration condition for exponential potential in metric NMDC cosmology is obtained as follow,

$$\left(\varepsilon - \frac{3\kappa V_0}{M_{\rm P}^2}e^{-\lambda\phi/M_{\rm P}}\right)\dot{\phi}^2 \;<\; V_0 e^{-\lambda\phi/M_{\rm P}}\left(1 + \frac{\varepsilon\kappa\lambda^2 V_0 e^{-\lambda\phi/M_{\rm P}}}{M_{\rm P}^2(\varepsilon - \kappa V_0 e^{-\lambda\phi/M_{\rm P}}/M_{\rm P}^2)}\right) . \tag{5.11}$$

5.2 Acceleration condition of Palatini NMDC

Same as before, the acceleration conditon for Palatini NMDC cosmology is calculated from $\ddot{a}/a \equiv \dot{H} + H^2 > 0$, with the help of (4.12) and (4.13)

$$\begin{aligned} \frac{\ddot{a}}{a} &= \dot{H} + H^2 \\ &\simeq -\frac{\varepsilon\dot{\phi}^2}{2M_{\rm P}^2} + \frac{3\kappa\varepsilon\dot{\phi}^3\ddot{\phi}}{4HM_{\rm P}^4} - \underbrace{\frac{3\varepsilon\kappa\dot{\phi}^4}{4M_{\rm P}^4}}_{\approx 0} - \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4HM_{\rm P}^4} \\ &\quad + \frac{\varepsilon\dot{\phi}^2}{6M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2} + \underbrace{\frac{3\kappa\varepsilon\dot{\phi}^4}{6M_{\rm P}^4}}_{\approx 0} \\ &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2} + \frac{3\kappa\varepsilon\dot{\phi}^3}{4HM_{\rm P}^4}\left[\ddot{\phi} - \frac{V_{,\phi}}{3\varepsilon}\right] , \end{aligned} \tag{5.12}$$

where terms involving ϕ^4 is neglible. Palatini NMDC Klein-Gordon equation (4.10) now can be substituted into (5.12)

$$\begin{aligned} \frac{\ddot{a}}{a} &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2} + \frac{3\kappa\varepsilon\dot{\phi}^3}{4HM_{\rm P}^4}\left[\left(\frac{-V_{,\phi} - 3H\dot{\phi}\varepsilon}{\varepsilon - (15/2)\kappa H^2}\right) - \frac{V_{,\phi}}{3\varepsilon}\right] \\ &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V(\phi)}{3M_{\rm P}^2} + \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4HM_{\rm P}^4}\left[\left(\frac{-3\varepsilon}{\varepsilon - (15/2)\kappa H^2}\right) - 1\right] \\ &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2} + \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4HM_{\rm P}^4}\left[\frac{-3\varepsilon - \varepsilon + (15/2)\kappa H^2}{\varepsilon - (15/2)\kappa H^2}\right] \\ &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2} - \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4HM_{\rm P}^4}\left[\frac{4\varepsilon - (15/2)\kappa H^2}{\varepsilon - (15/2)\kappa H^2}\right] . \end{aligned} \tag{5.13}$$

Approximation for $H^2 = V/(3M_{\rm P}^2)$ and $H = M_{\rm P}^{-1}\sqrt{V/3}$ now can be applied,

$$\frac{\ddot{a}}{a} \simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2} - \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4M_{\rm P}^3}\sqrt{\frac{3}{V}}\left[\frac{4\varepsilon - (5/2)\kappa V/M_{\rm P}^2}{\varepsilon - (5/2)\kappa V/M_{\rm P}^2}\right] . \tag{5.14}$$

In the sub-Planckian range $|\kappa| \ll |M_{\rm P}^2/V|$ [19] binomial approximation is allowed, hence

$$\begin{aligned} \frac{4\varepsilon - (5/2)\kappa V/M_{\rm P}^2}{\varepsilon - (5/2)\kappa V/M_{\rm P}^2} &\approx \left(4\varepsilon - \frac{5\kappa V}{2M_{\rm P}^2}\right)\left(\varepsilon + \frac{5\kappa V}{2M_{\rm P}^2}\right) \\ &\approx 4 - \frac{5\kappa\varepsilon V}{2M_{\rm P}^2} + \frac{10\kappa\varepsilon V}{M_{\rm P}^2} + \cancel{\frac{25\kappa^2 V^2}{4M_{\rm P}^4}}^{\approx 0} \\ &\approx 4 + \frac{15\kappa\varepsilon V}{2M_{\rm P}^2}\ . \end{aligned} \tag{5.15}$$

Therefore (5.14) read as

$$\begin{aligned} \frac{\ddot{a}}{a} &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2} - \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4M_{\rm P}^3}\sqrt{\frac{3}{V}}\left[4 + \frac{15\kappa\varepsilon V}{2M_{\rm P}^2}\right] \\ &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2} - \frac{\kappa\dot{\phi}^3 V_{,\phi}}{M_{\rm P}^3}\sqrt{\frac{3}{V}}\left[1 + \frac{15\kappa\varepsilon V}{8M_{\rm P}^2}\right] \\ &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2}\left(1 - \frac{3\sqrt{3}\kappa\dot{\phi}^3 V_{,\phi}}{M_{\rm P} V\sqrt{V}}\left[1 + \frac{15\kappa\varepsilon V}{8M_{\rm P}^2}\right]\right)\ . \end{aligned} \tag{5.16}$$

For the late time $\ddot{\phi} \approx 0$, the Palatini NMDC Klein-Gordon equation can be written as

$$\begin{aligned} \dot{\phi} &\approx \frac{-V_{,\phi}}{3H\varepsilon} \\ &\approx \frac{-V_{,\phi}}{3\varepsilon} M_{\rm P}\sqrt{\frac{3}{V}} \\ &\approx \frac{-V_{,\phi} M_{\rm P}}{\varepsilon\sqrt{3V}}\ . \end{aligned} \tag{5.17}$$

Hence

$$\begin{aligned} \frac{\ddot{a}}{a} &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2}\left(1 - \frac{3\sqrt{3}\kappa\dot{\phi}^3 V_{,\phi}}{M_{\rm P} V\sqrt{V}}\left[1 + \frac{15\kappa\varepsilon V}{8M_{\rm P}^2}\right]\right) \\ &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2}\left(1 - \kappa\frac{3\sqrt{3}}{M_{\rm P}}\frac{V_{,\phi}}{V\sqrt{V}}\left\{\frac{-V_{,\phi}M_{\rm P}}{\varepsilon\sqrt{3V}}\right\}^3\left[1 + \frac{15\kappa\varepsilon V}{8M_{\rm P}^2}\right]\right) \\ &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2}\left(1 + \frac{\kappa}{\varepsilon}\frac{M_{\rm P}^2 V_{,\phi}^4}{V^3}\left[1 + \frac{15\kappa\varepsilon V}{8M_{\rm P}^2}\right]\right) \\ &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2}\left(1 + \frac{\kappa}{\varepsilon}\frac{V_{,\phi}}{M_{\rm P}}\left\{\frac{M_{\rm P}V_{,\phi}}{V}\right\}^3\left[1 + \frac{15\kappa\varepsilon V}{8M_{\rm P}^2}\right]\right) \\ &\simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2}\left(1 + \frac{\kappa}{\varepsilon}\frac{V_{,\phi}}{M_{\rm P}}\left\{\sqrt{2\epsilon_{V,GR}}\right\}^3\left[1 + \frac{15\kappa\varepsilon V}{8M_{\rm P}^2}\right]\right)\ , \end{aligned} \tag{5.18}$$

we have used the definition of slow-roll parameter $\epsilon_{V,GR}$,

$$\epsilon_{V,GR} = \frac{M_{\rm P}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2\ . \tag{5.19}$$

Now the acceleration condition for Palatini NMDC cosmology is given by

$$
\begin{aligned}
0 &< \frac{\ddot{a}}{a} \\
&< \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V}{3M_{\rm P}^2}\left(1 + \frac{\kappa}{\varepsilon}\frac{V_{,\phi}}{M_{\rm P}}\left\{\sqrt{2\epsilon_{V,GR}}\right\}^3\left[1 + \frac{15\kappa\varepsilon V}{8M_{\rm P}^2}\right]\right) \\
\varepsilon\dot{\phi}^2 &< V\left(1 + \frac{\kappa}{\varepsilon}\frac{V_{,\phi}}{M_{\rm P}}\left\{\sqrt{2\epsilon_{V,GR}}\right\}^3\left[1 + \frac{15\kappa\varepsilon V}{8M_{\rm P}^2}\right]\right) \ . \qquad (5.20)
\end{aligned}
$$

It should be noticed that in the (5.20), GR case $\varepsilon\dot{\phi}^2 < V$ can be recovered by taking $\kappa = 0$.

For exponential potential, we have

$$
\begin{aligned}
V &= V_0 e^{-\lambda\phi/M_{\rm P}} \\
V_{,\phi} &= -\frac{\lambda V_0}{M_{\rm P}} e^{-\lambda\phi/M_{\rm P}} \\
\sqrt{2\epsilon_{V,GR}} &= \frac{M_{\rm P} V_{,\phi}}{V} \\
&= -\lambda \ . \qquad (5.21)
\end{aligned}
$$

The acceleration condition for exponential potential in Palatini NMDC now read as

$$
\varepsilon\dot{\phi}^2 < V_0 e^{-\lambda\phi/M_{\rm P}}\left(1 + \frac{\kappa}{\varepsilon}\frac{\lambda^4}{M_{\rm P}^2}\left[1 + \frac{15\kappa\varepsilon}{8M_{\rm P}^2}V_0 e^{-\lambda\phi/M_{\rm P}}\right]\right) \ . \qquad (5.22)
$$

CHAPTER VI

COSMOLOGICAL DYNAMICS FOR NMDC FIELDS

6.1 Dynamical system in a nutshell

Mostly the dynamics of physical world around us can be described by some differential equations. Dynamical system is a powerful mathematical sucjects to study a set of n differential equations associated with n dynamical variables,

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1, x_2, x_3, ..., x_n, t) \\
\dot{x}_2 &= f_2(x_1, x_2, x_3, ..., x_n, t) \\
\dot{x}_3 &= f_3(x_1, x_2, x_3, ..., x_n, t) \\
&\cdots \\
\dot{x}_n &= f_n(x_1, x_2, x_3, ..., x_n, t), \qquad (6.1)
\end{aligned}$$

if all the functions f does not explicitly depend on time, these equation is called n-dimensional *Autonomous system*. However, we can always transform non-autonomous system to autonomous system by increasing dimensionality. Define new variable $x_{n+1} \equiv t$, we have

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1, x_2, x_3, ..., x_n, x_{n+1}) \\
\dot{x}_2 &= f_2(x_1, x_2, x_3, ..., x_n, x_{n+1}) \\
\dot{x}_3 &= f_3(x_1, x_2, x_3, ..., x_n, x_{n+1}) \\
&\cdots \\
\dot{x}_n &= f_n(x_1, x_2, x_3, ..., x_n, x_{n+1}) \\
\dot{x}_{n+1} &= 1 \equiv f_{n+1} \; . \qquad (6.2)
\end{aligned}$$

That is the reason why autonomous system is very important subject in the dynamical system, albeit our nature sometimes appear as explicitly time-dependent differential equations [34, 35].

Some second order differential equations can be described by a 2-dimensional autonomous system. For example, consider damped harmonic oscillator equation

$$m\ddot{x} + b\dot{x} + kx = 0 \; . \qquad (6.3)$$

Define new variable $y \equiv \dot{x}$, we have

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -\frac{k}{m}x - \frac{b}{m}y \ . \end{aligned} \tag{6.4}$$

Also define new vector $\mathbf{v} = (x, y)$ we can write (6.4) in the form of matrix

$$\dot{\mathbf{v}} = A\mathbf{v} \tag{6.5}$$

where

$$A = \begin{pmatrix} 0 & 1 \\ -k/m & -b/m \end{pmatrix} \tag{6.6}$$

.

Dynamical system allow us to study the behaviour of the system without find the exact solution of the differential equations. One can gain a qualitative understanding by plotting so-called *phase portrait*. Phase portrait is a collection of phase flow between $\dot{\mathbf{x}}$ and $\mathbf{x}$.

Any $\mathbf{x}_0$ which satisfy $\dot{\mathbf{x}}_0 = 0$ is called fixed points. The qualitative behaviour of the phase portrait can be examined by looking how may fixed points are there and how is the characteristic of the fixed points.

6.2 Autonomous system for metric NMDC model

For metric NMDC model, the system is governed by Klein-Gordon equation (4.2) and Friedmann equation (4.7),

$$\begin{gathered} \ddot{\phi}\left(\varepsilon - 3\kappa H^2\right) + 3H\dot{\phi}\left(\varepsilon - 3\kappa H^2\right) + V_{,\phi} \simeq 0 \ , \\ H^2 \simeq \frac{1}{3M_{\rm P}^2}\left[\frac{\dot{\phi}^2}{2}\left(\varepsilon - 9\kappa H^2\right) + V\right] \ . \end{gathered}$$

Defining a new variable $\psi \equiv \dot{\phi}$ with some approximations, a 3-dimensional autonomous system can be described as follows

$$\dot{\phi} = \psi \tag{6.7}$$

$$\dot{\psi} \simeq \frac{-V_{,\phi} - 3H\psi(\varepsilon - 3\kappa H^2)}{\varepsilon - 3\kappa H^2} \tag{6.8}$$

$$\dot{H} \simeq \frac{V_{,\phi}\psi}{6M_{\rm P}^2 H} \tag{6.9}$$

Consistently, we can recover the GR case by taking $\kappa = 0$.

6.3 Autonomous system for Palatini NMDC cosmology

In the Palatini NMDC model, Klein-Gordon equation (4.10) and Friedmann equation(4.12) is described by

$$\ddot{\phi}\left(\varepsilon - (15/2)\kappa H^2\right) + 3H\dot{\phi}\varepsilon + V_{,\phi} \simeq 0\ ,$$
$$H^2 \simeq \frac{1}{3M_{\rm P}^2}\left[\frac{\varepsilon\dot{\phi}^2}{2} + V + \frac{3}{2}\frac{\kappa\varepsilon\dot{\phi}^4}{M_{\rm P}^2}\right],$$

A 3-dimensional autonomous system can be defined

$$\dot{\phi} = \psi \tag{6.10}$$
$$\dot{\psi} \simeq \frac{-V_{\phi} - 3H\psi\varepsilon}{\varepsilon - (15/2)\kappa H^2} \tag{6.11}$$
$$\dot{H} \simeq \frac{V_{,\phi}\psi}{6M_{\rm P}^2 H}\ . \tag{6.12}$$

It should be noticed also that GR case can be recovered by taking $\kappa = 0$.

CHAPTER VII

RESULTS AND DISCUSSION

In this chapter we try to investigate qualitatively the dynamics of the field ϕ according to two NMDC models : metric and Palatini, compared to the standard GR model. In doing such, we plot the phase portrait between $\psi = \dot{\phi}$ over ϕ. In another word we try to examine the qualitative difference between equations (6.8) and (6.11).

Earlier studies with chaotic potential [19, 33] with postive κ resulting the superluminal speed, albeit it can surpass Planck 2015 constraint. Therefore in this work we use negative κ.

7.1 Phase portrait for standard GR

In the GR case, the dynamics of the field is governed by Klein-Gordon equation (2.13) written in the autonomous form,

$$\begin{aligned} \dot{\phi} &= \psi \\ \dot{\psi} &\simeq \frac{-V_{,\phi} - 3\varepsilon H\psi}{\varepsilon} \end{aligned}$$

where $\dot{\psi} \equiv \ddot{\phi}$. And the acceleration expansion region in exponential potential is determined by acceleration condition, has been derived in (5.1)

$$\varepsilon\dot{\phi}^2 < V_0 e^{-\lambda\phi/M_{\mathrm{P}}} \ .$$

Setting and scaling $\varepsilon = 1$, $M_{\mathrm{P}} = 1$, $V = 0.1$ and $\lambda = 1$, the phase portrait for GR case is shown in figure 1, the shaded area is accelerated region, and the red line is trajectory from a random starting point. Figure 1 showed that there is only an attractor solution inside the acceleration region.

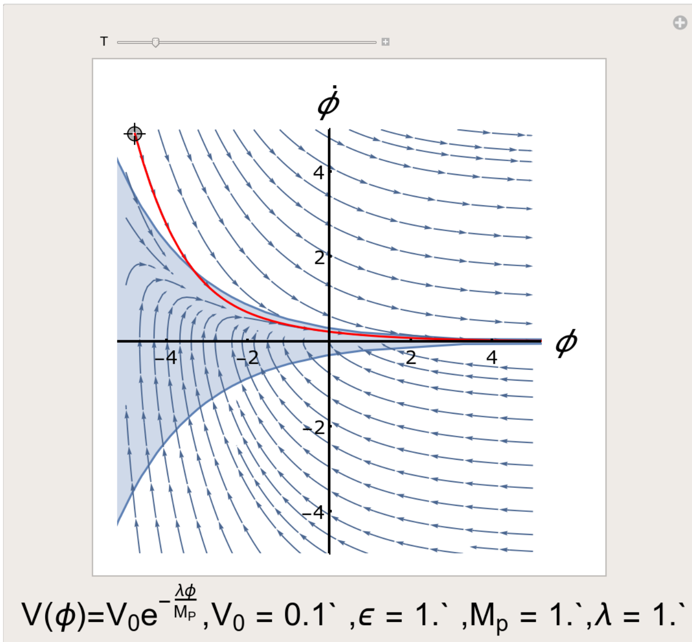

Figure 1 Phase portrait for exponential potential in standard general relativity with acceleration region

7.2 Phase portrait for metric NMDC model

In the metric NMDC model, the modified Klein-Gordon equation (4.2) written in the autonomous for, as in (6.7) and (6.8)

$$\begin{aligned} \dot{\phi} &= \psi \\ \dot{\psi} &\simeq \frac{-V_{,\phi} - 3H\psi(\varepsilon - 3\kappa H^2)}{\varepsilon - 3\kappa H^2} \, , \end{aligned}$$

with the acceleration for exponential potential has been derived in (5.11)

$$\left(\varepsilon - \frac{3\kappa V_0}{M_{\rm P}^2} e^{-\lambda\phi/M_{\rm P}}\right)\dot{\phi}^2 \;<\; V_0 e^{-\lambda\phi/M_{\rm P}}\left(1 + \frac{\varepsilon\kappa\lambda^2 V_0 e^{-\lambda\phi/M_{\rm P}}}{M_{\rm P}^2(\varepsilon - \kappa V_0 e^{-\lambda\phi/M_{\rm P}}/M_{\rm P}^2)}\right) \; .$$

We use the same setting and scaling with GR case, $\varepsilon = 1$, $M_{\rm P} = 1$, $V = 0.1$ and $\lambda = 1$. We use negative κ, which is $\kappa = -1$. Phase portrait for metric case is shown in figure 2, we can see that metric NMDC give the narrower accelerator, meanwhile the attractor is still in the acceleration region.

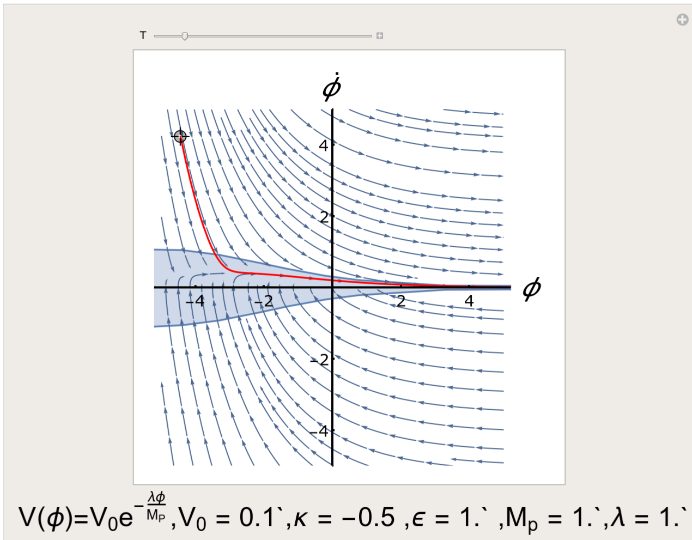

Figure 2 Phase portrait for exponential potential in metric NMDC with acceleration region

7.3 Phase portrait to Palatini NMDC model

For Palatini NMDC case, the autonomous system is given in the (6.10) and (6.11)

$$\begin{aligned} \dot{\phi} &= \psi \\ \dot{\psi} &\simeq \frac{-V_\phi - 3H\psi\varepsilon}{\varepsilon - (15/2)\kappa H^2}, \end{aligned}$$

and the acceleration region, given in (5.22)

$$\varepsilon\dot{\phi}^2 < V_0 e^{-\lambda\phi/M_\mathrm{P}}\left(1 + \frac{\kappa}{\varepsilon}\frac{\lambda^4}{M_\mathrm{P}^2}\left[1 + \frac{15\kappa\varepsilon}{8M_\mathrm{P}^2}V_0 e^{-\lambda\phi/M_\mathrm{P}}\right]\right) .$$

Using$\varepsilon = 1$, $M_\mathrm{P} = 1$, $V = 0.1$, $\lambda = 1$ and $\kappa = -1$, we plot the phase portrait, as in figure 3. We can see the acceleration region appeared to have boundaries, and the attractor is pulled into the smaller acceleration region.

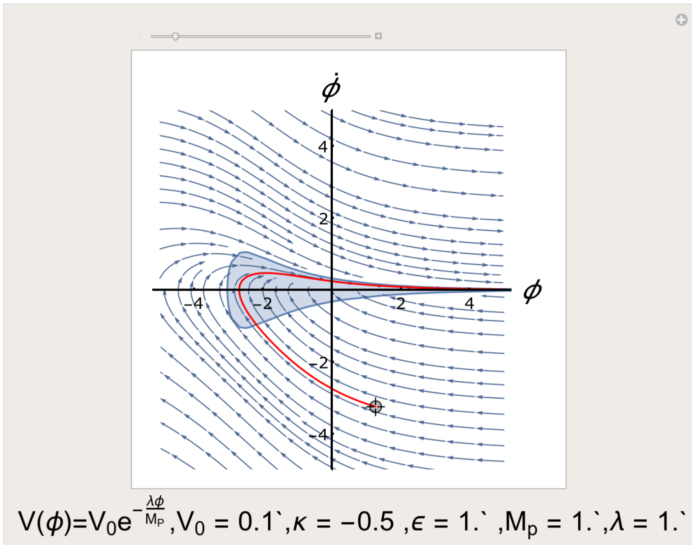

Figure 3 Phase portrait for exponential potential in Palatini NMDC with acceleration region

7.4 Evolution of H over e-folding number N

For three cases, we plot the evolution of H as the e-folding number N growing. It can be seen from Fig. 4 that metric NMDC effect enhances expansion rate compared to the GR case, meanwhile the Palatini NMDC effect surpresses the expansion rate compared to the GR case.

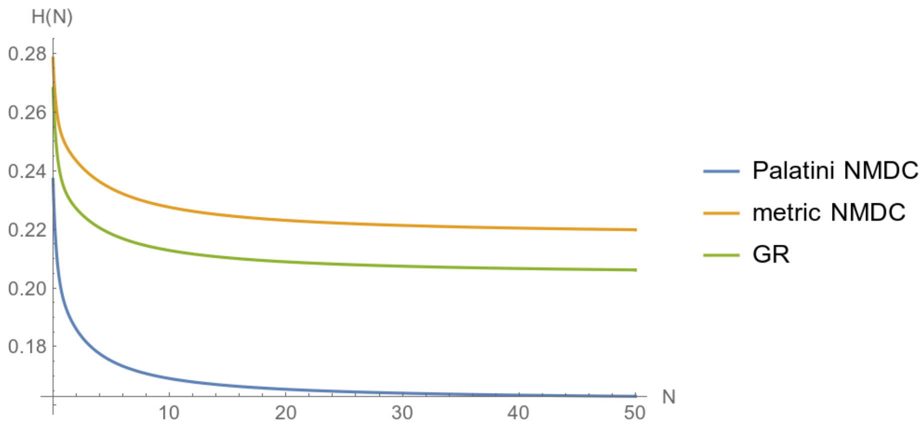

Figure 4 H is plotted against N for GR, metric NMDC and Palatini NMDC cases.

CHAPTER VIII

CONCLUSION

In this thesis we study the dynamics of scalar field in a cosmological model which has non-minimal derivative coupling (NMDC) terms between field and curvature. An NMDC model studied from two different approaches, metric and Palatini which resulting different equation of motions. For exponential potential, the acceleration condition is derived, and the dynamics of the fields is studied qualitatively by plotting the phase portrait for different cases. Three cases only has one attractor and metric NMDC give narrower acceleration region compared to the GR case. Palatini NMDC effect restrict the attractor into the even smaller region compared to GR and metric NMDC cases. As the e-folding number growing, figure 4 shown that H is enhanced by metric NMDC effect and surpressed by Palatini NMDC effect. Full dynamical analysis is awaiting in the future study.

BIBLIOGRAPHY

BIBLIOGRAPHY

[1] Adam G Riess, Alexei V Filippenko, Peter Challis, Alejandro Clocchiatti, Alan Diercks, Peter M Garnavich, Ron L Gilliland, Craig J Hogan, Saurabh Jha, Robert P Kirshner, et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal, 116* (3): 1009, 1998.

[2] Saul Perlmutter, G Aldering, M Della Valle, S Deustua, RS Ellis, S Fabbro, A Fruchter, G Goldhaber, DE Groom, IM Hook, et al. Discovery of a supernova explosion at half the age of the universe. *Nature, 391* 6662): 51–54, 1998.

[3] Saul Perlmutter, G Aldering, G Goldhaber, RA Knop, P Nugent, PG Castro, S Deustua, S Fabbro, A Goobar, DE Groom, et al. Measurements of ω and λ from 42 high-redshift supernovae. *The Astrophysical Journal, 517* (2): 565, 1999.

[4] Alan H. Guth. The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems. *Physical Review D, 23*: 347–356, 1981.

[5] Eiichiro Komatsu, KM Smith, J Dunkley, CL Bennett, B Gold, G Hinshaw, N Jarosik, D Larson, MR Nolta, L Page, et al. Seven-year wilkinson microwave anisotropy probe (WMAP*) observations: cosmological interpretation. *The Astrophysical Journal Supplement Series, 192* (2): 18, 2011.

[6] Max Tegmark, Michael A Strauss, Michael R Blanton, Kevork Abazajian, Scott Dodelson, Havard Sandvik, Xiaomin Wang, David H Weinberg, Idit Zehavi, Neta A Bahcall, et al. Cosmological parameters from SDSS and WMAP. *Physical Review D, 69* (10): 103501, 2004.

[7] Bharat Ratra and Philip JE Peebles. Cosmological consequences of a rolling homogeneous scalar field. *Physical Review D, 37* (12): 3406, 1988.

[8] Takeshi Chiba, Takahiro Okabe, and Masahide Yamaguchi. Kinetically driven quintessence. *Physical Review D, 62* (2): 023511, 2000.

[9] Antonio De Felice and Shinji Tsujikawa. f(R) Theories. *Living Reviews in Relativity, 13* (1): 3, 2010.

[10] Edmund J Copeland, Mohammad Sami, and Shinji Tsujikawa. Dynamics of dark energy. *International Journal of Modern Physics D, 15* (11): 1753–1935, 2006.

[11] Shin'ichi Nojiri and Sergei D Odintsov. Unified cosmic history in modified gravity: from f(R) theory to lorentz non-invariant models. *Physics Reports, 505* (2): 59–144, 2011.

[12] Sergey V Sushkov. Exact cosmological solutions with nonminimal derivative coupling. *Physical Review D, 80* (10): 103505, 2009.

[13] Emmanuel N Saridakis and Sergey V Sushkov. Quintessence and phantom cosmology with nonminimal derivative coupling. *Physical Review D, 81* (8): 083510, 2010.

[14] Cristiano Germani and Alex Kehagias. New model of inflation with nonminimal derivative coupling of standard model higgs boson to gravity. *Physical Review Letters, 105* (1): 011302, 2010.

[15] Sergey V Sushkov. Realistic cosmological scenario with nonminimal kinetic coupling. *Physical Review D, 85* (12): 123520, 2012.

[16] Shinji Tsujikawa. Observational tests of inflation with a field derivative coupling to gravity. *Physical Review D, 85* (8): 083518, 2012.

[17] H Mohseni Sadjadi. Super-acceleration in a nonminimal derivative coupling model. *Physical Review D, 83* (10): 107301, 2011.

[18] Burin Gumjudpai and Phongsaphat Rangdee. Non-minimal derivative coupling gravity in cosmology. *General Relativity and Gravitation, 47*(11):140, 2015.

[19] Narakorn Kaewkhao and Burin Gumjudpai. Cosmology of non-minimal derivative coupling to gravity in palatini formalism and its chaotic inflation. *Physics of the Dark Universe, 20*: 20 – 27, 2018.

[20] Sean Carroll. *Spacetime and Geometry: An Introduction to General Relativity.* Benjamin Cummings, 2003.

[21] Lewis Ryder. *Introduction to General Relativity.* Cambridge University Press, 2009

[22] Jayan V Narlikar. *An Introduction to Relativity.* Cambridge University Press, 2010.

[23] Charles Misner, Kip Thorne, and John Wheeler. *Gravitation.* W H Freeman and Company, 1973.

[24] M Ferraris, M Francaviglia, and C Reina. Variational formulation of general relativity from 1915 to 1925 "Palatini's method" discovered by Einstein in 1925. *General Relativity and Gravitation, 14* (3) 243–254, 1981.

[25] Ray D'Inverno. *Introducing Einstein's Relativity.* Clarendon Press, 1992.

[26] Alberto Vecchiato. *Variational Approach to Gravity Field Theories.* Springer, 2017.

[27] Carl Brans and Robert H Dicke. Mach's principle and a relativistic theory of gravitation. *Physical Review, 124* (3): 925, 1961.

[28] Luca Amendola. Cosmology with nonminimal derivative couplings. *Physics Letters B, 301* (2-3):175–182,1993.

[29] S Capozziello,G Lambiase, and HJ Schmidt. Nonminimal derivative couplings and inflation in generalized theories of gravity. *Annalen der Physics, 9*:39-48, 2000.

[30] LN Granda. Non-minimal Kinetic coupling to gravity and accelerated expansion. *Journal of Cosmology and Astroparticle Physics, 2010* (1007): 006, 2010.

[31] Gregory Walter Horndeski. Second-order scalar-tensor field equations in a four-dimensional space. *International Journal of Theoretical Physics, 10* (6): 363–384, 1974.

[32] X Luo, P Wu and H Yu. Non-minimal derivatively coupled quintessence in the Palatini formalism. *Astrophysics and Space Science, 2014* (250):831-837, 2014.

[33] Somphoach Saichaemchan and Burin Gumjudpai. Non-minimal derivative coupling in Palatini cosmology: acceleration in chaotic inflation potential. *J. Phys. Conf. Ser., 901* (1):012010, 2017.

[34] Morris W Hirsch, Stephen Smale, and Robert L Devaney. *Differential Equations, Dynamical Systems, and An Introduction to Chaos.* Academic Press, 2012.

[35] Dominic William Jordan and Peter Smith. *Nonlinear Ordinary Differential Equations: an Introduction to Dynamical Systems*, volume 2. Oxford University Press, 1999.

BIOGRAPHY

Name-Surname	Candrasyah Muhammad
Date of Birth	20th June 1991
Place of Birth	Malang, Indonesia
Address	Jalan Laksda Adi Sucipto Gang Bakti 48, Malang, Indonesia

Education Background

2015	Bachelor of Science in Physics, Maulana Malik Ibrahim State Islamic University of Malang, Indonesia