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Cosmological constraints for an Eddington-Born-Infeld field

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We consider the Eddington-Born-Infeld (EBI) model here without assuming any cosmological constant. The EBI scalar field is supposed to play a role of both dark matter and dark energy. Different eras in cosmology are reconstructed for the model. A comparison is drawn with the ΛCDM model using Supernova Type Ia data, WMAP7 and baryon acoustic oscillations data. It seems that the EBI field in this form does not give a good fit to observational data in comparison to the ΛCDM model.

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I. INTRODUCTION

The cosmic acceleration is now considered to be one of the frontier quests of fundamental physics. Confirmed by observations [1,2], understanding of acceleration is yet to be satisfied in the regime of standard general relativity (GR). Attempts to explain the acceleration could be performed by adding extra components of fluid into the energy-momentum tensor part of the Einstein field equation. This extra component, dubbed dark energy, gives negative pressure so that it is able to drive the acceleration, see e.g., references in Ref. [3] for reviews. At smaller scales, a problem of an extra attractive gravity in galaxies and galaxy clusters shows up. Effects of extra gravity such as the flat galactic rotational curve, gravitational lensing, bulk velocity and structure formation are explained with dark matter [4]. On the observation side, the simplest model of dark matter and dark energy—the ΛCDM model—is favored; however, it suffers from fine-tuning problem. At present, the nature of dark sectors is still unknown. There is another more radical way of acquiring acceleration—to modify the gravity term in the action (see Ref. [5] for recent reviews).

It is possible that dark energy and dark matter are only a single entity which could effectively have different behaviors at early and late times. This unified scenario is considered in the Chaplygin gas model [6]. The Chaplygin gas model however cannot satisfy the cosmic microwave background (CMB) power spectrum and structure formation [7]. Another idea of unifying dark sectors was proposed recently by Banados [8,9]. The model, called Eddington-Born-Infeld (EBI) gravity, can account for both dark matter and dark energy components without additional degrees of freedom in the energy-momentum tensor. In the model, Einstein gravity couples to Born-Infeld theory giving rise to a bi-metric theory. The second metric of the theory is generated from the Born-Infeld Christoffel symbol, $C^{\rho}_{\mu\nu}$, which is solely responsible for dark sectors. The theory predicts a dust-like effective equation of state at large scales, while at late times it behaves like a cosmological constant. The theory can also accommodate flat galactic disk-rotational curves. The model is motivated by a combination of Eddington's idea of purely affine theory of gravity without using a metric [10], Born-Infeld-Einstein action [11], and the idea of magnetic spin symmetry breaking in the presence of an external magnetic field. Considering that the topological manifold is invariant under a full diffeomorphism group of transformation, Riemannian manifolds are invariant under a smaller class of subgroups of metric isometries. The Eddington action which is diffeomorphism invariant hence is considered as an unbroken state theory [12]. Moreover, it is a ghost-free theory. Introducing $g_{\mu\nu} \neq 0$ to the gravitational theory would break this symmetry, similar to having an external magnetic field applied to a random spin system. The external magnetic field also breaks the symmetry of the spin system. If we let the metric couple to the Eddington action, the result is the EBI action. In the action, there is the Einstein-Hilbert part and the EBI part (see Refs. [8,9,13] for a more detailed discussion). In the context of an anisotropic universe with the Bianchi type I model, at late times, the EBI gravity effectively behaves like Einstein-Hilbert cosmology plus a cosmological constant. The EBI term is stable at the dark matter phase but also gives rise to anisotropic pressure and the perturbation decays oscillatory in time which differs from the standard exponential decay case [14]. Considering the dark energy phase, the Born-Infeld as dark energy is not stable and it produces a very strong integrated Sachs-Wolfe effect on large scales. This suggests that for the model to be viable, a cosmological constant is needed in the action [15]. When adding a cosmological constant into the model, the model still predicts too large CMB fluctuations compared to WMAP5 data. However, while restricting the EBI field as dark matter, the EBI model is a best fit with the ΛCDM prediction [16]. The idea that the Eddington action is a starting point for GR is pursued further when considering a Born-Infeld part and a cosmological constant but without having the Einstein-Hilbert term. The idea was investigated in the Palatini formulation to include matter fields.

For homogeneous and isotropic space-time, the model presents a nonsingular cosmology at early times as well as the nonsingular collapse of compact objects [17–20]. In such a scenario, the Poisson equation is modified and the Jean length is equal to the fundamental length of the theory. Also the critical mass for a black hole to form is equal to the fundamental mass of the theory [21,22].

In this paper, we consider the EBI model without a cosmological constant as originally proposed in Ref. [9]. In fact, we believe that introducing a cosmological constant would make this model less attractive, as the model was introduced as a way to explain both dark energy and dark matter at the same time. In other words, adding a cosmological constant by hand would mean that the model achieves only half of the original goals it was introduced for. We reemphasize the inviability of the original EBI model by fitting it with WMAP7, baryon acoustic oscillations (BAO) and Supernova Type Ia data. Compared to the study of Ref. [16], where the authors studied the growth of structure for these models (they studied the evolution of the cosmological perturbations during radiation and matter domination), we perform a study of the background and look for the constraints on it coming from the most recent data. In Sec. II, we briefly describe the EBI model as a bi-metric theory and its cosmology. The equations of motion are described in Sec. III. We consider the cosmological era in Sec. IV and numerical results are shown in Sec. V. We conclude in Sec. VI.

II. EDDINGTON-BORN-INFELD COSMOLOGY

In the EBI model studied here the action has three variables, the metric $g_{\mu\nu}$, the Born-Infeld connection $C^\rho{}_{\mu\nu}$ and the matter field Ψ. The EBI action is

$$S[g_{\mu\nu}, C^\rho{}_{\mu\nu}, \Psi] = \frac{1}{16\pi G}\int \mathrm{d}^4x\left(\sqrt{|g_{\mu\nu}|}R + \frac{2}{\alpha l^2}\sqrt{|g_{\mu\nu} - l^2 K_{\mu\nu}|}\right) + \int \mathrm{d}^4x \mathcal{L}_{\mathrm{m}}(\Psi, g_{\mu\nu}). \quad (1)$$

The action above has two extra constants, the length scale l which resembles the dimension of a length or $1/\sqrt{R}$, and α, which is a dimensionless parameter. The Born-Infeld Ricci tensor $K_{\mu\nu}$ is symmetric under interchanging μ and ν which is a result of symmetric properties of the Born-Infeld connection $C^\rho{}_{\mu\nu}$. As in standard GR,

$$K_{\mu\nu} \equiv K^\rho{}_{\mu\rho\nu}, \quad (2)$$

where

$$K^\rho{}_{\mu\alpha\nu} = C^\rho{}_{\mu\nu,\alpha} + C^\rho{}_{\sigma\alpha}C^\sigma{}_{\mu\nu} - C^\rho{}_{\mu\alpha,\nu} - C^\rho{}_{\sigma\nu}C^\sigma{}_{\mu\alpha}. \quad (3)$$

The conventional matter fields are included in the Lagrangian $\mathcal{L}_{\mathrm{m}}$. Since the two dynamical fields $g_{\mu\nu}$ and $C^\rho{}_{\mu\nu}$ are independent, the Born-Infeld connection can effectively be expressed in terms of a new symmetric metric $q_{\mu\nu}(x)$,

$$C^\rho{}_{\mu\nu} = \frac{1}{2}q^{\rho\sigma}(q_{\sigma\nu,\mu} + q_{\mu\sigma,\nu} - q_{\mu\nu,\sigma}), \quad (4)$$

giving a version of bi-metric theory. As in the standard case, for the new metric the covariant derivative vanishes,

$$D_\rho q_{\mu\nu} = 0, \quad (5)$$

where the covariant derivative is performed under the Born-Infeld connection, i.e.,

$$D_\rho q_{\mu\nu} \equiv \partial_\rho q_{\mu\nu} - C^\sigma{}_{\mu\rho}q_{\sigma\nu} - C^\sigma{}_{\nu\rho}q_{\sigma\mu}. \quad (6)$$

Varying the action (1) with respect to two dynamical fields, the metric $g_{\mu\nu}$ and the connection $C^\alpha_{\mu\nu}$, yields the following equations of motion,

$$G_{\mu\nu} = \sqrt{\frac{|g_{\mu\nu} - l^2K_{(\mu\nu)}|}{|g_{\mu\nu}|}}g_{\mu\rho}\left(\frac{1}{g - l^2K}\right)^{\rho\sigma}g_{\sigma\nu} + 8\pi G T^{\mathrm{m}}_{\mu\nu}. \quad (7)$$

Defining

$$\sqrt{q}q^{\mu\nu} \equiv -\frac{1}{\alpha}\sqrt{|g_{\mu\nu} - l^2K_{\mu\nu}|}\left(\frac{1}{g - l^2K}\right)^{\mu\nu}, \quad (8)$$

hence (7) can be written as

$$G_{\mu\nu} = -\frac{1}{l^2}\sqrt{\frac{|q_{\mu\nu}|}{|g_{\mu\nu}|}}g_{\mu\alpha}q^{\alpha\beta}g_{\beta\nu} + 8\pi G T^{\mathrm{m}}_{\mu\nu}. \quad (9)$$

Varying the action with respect to the connection $C^\alpha_{\mu\nu}$, one can find $D_\rho(\sqrt{q}q^{\mu\nu}) = 0$. Taking the determinant of (8) then we obtain

$$K_{\mu\nu} = \frac{1}{l^2}(g_{\mu\nu} + \alpha q_{\mu\nu}). \quad (10)$$

The first term in Eq. (9) is a modification of the Born-Infeld part. $T^{\mathrm{m}}_{\mu\nu}$ is the matter field energy-momentum tensor. These results agree with the ones first shown in Ref. [9]. The two metrics $g_{\mu\nu}$ and $q_{\mu\nu}$ possess homogeneity and isotropy with flat spatial curvature,

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a(t)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2), \quad (11)$$

$$q_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -X(t)^2\mathrm{d}t^2 + Y(t)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2), \quad (12)$$

$g_{tt} = -1$, due to the gauge freedom in time. Here $X(t)$ is the time rescaling of the metric $q_{\mu\nu}$ whereas $a(t)$ and $Y(t)$ behave like scale factors in $g_{\mu\nu}$ and $q_{\mu\nu}$, respectively. The $a(t_0)$ is set to 1 so that $H_0 = \dot{a}(t_0)$ as in Ref. [9].

III. THE EQUATIONS OF MOTION

Applying the metric ansatz Eqs. (11) and (12) to the equations of motion (9) and (10), we obtain first order equations, which are

$$H^2 = \frac{1}{3l^2}\left(\frac{Y^3}{a^3}\right)\frac{1}{X} + \frac{8\pi G}{3}(\varrho_m + \varrho_r), \tag{13}$$

$$\frac{d}{dt}\left(\frac{Y^3}{X}\right) = 3XY^3\left(\frac{a^2}{Y^2}\right)H, \tag{14}$$

$$\left(\frac{\dot{Y}}{Y}\right)^2 = \frac{X^2}{3l^2}\left(\alpha - \frac{1}{2X^2} + \frac{3}{2}\frac{a^2}{Y^2}\right). \tag{15}$$

It should be noted that l has dimensions of length (M^{-1}), whereas Y has dimensions of a. Finally, X is dimensionless.

From Eq. (13), we can introduce an energy density as

$$\varrho_X \equiv \frac{1}{8\pi l^2 G}\frac{Y^3}{Xa^3}, \tag{16}$$

and by taking the derivative of the Friedmann equation, one can find an effective pressure for this dark component as

$$p_X = w_X \varrho_X, \tag{17}$$

where

$$w_X = \frac{1}{3}\frac{X'}{X} - \frac{Y'}{Y}, \tag{18}$$

and a prime denotes differentiation with respect to $N = \ln a$.

Let us now introduce the variable

$$\Omega_X \equiv \frac{8\pi G\varrho_X}{3H^2} = \frac{1}{3l^2}\frac{Y^3}{H^2Xa^3}. \tag{19}$$

In terms of this variable, the Friedmann equation can be written as

$$1 = \Omega_X + \Omega_m + \Omega_r, \tag{20}$$

where we have defined, as usual,

$$\Omega_m \equiv \frac{8\pi G\varrho_m}{3H^2}, \quad \text{and} \quad \Omega_r \equiv \frac{8\pi G\varrho_r}{3H^2}, \tag{21}$$

and have assumed $\rho_m \propto a^{-3}$, $\rho_r \propto a^{-4}$.

We will demand $\Omega_X \geq 0$, as Ω_X represents an effective matter density; otherwise $\Omega_{m,r}$ could assume values larger than unity. From Eq. (21), we find

$$\Omega_m' + \frac{2H'}{H}\Omega_m + 3\Omega_m = 0, \tag{22}$$

$$\Omega_r' + \frac{2H'}{H}\Omega_r + 4\Omega_r = 0. \tag{23}$$

Equation (14) can then be rewritten as

$$\Omega_X' + 3\Omega_X + \frac{2H'}{H}\Omega_X = \left(\frac{3X^4}{l^4H^4}\right)^{1/3}\Omega_X^{1/3}, \tag{24}$$

and Eq. (15) can be rewritten as

$$H^2\left[1 + \frac{2}{3}\frac{H'}{H} + \frac{1}{3}\left(\frac{X'}{X} + \frac{\Omega_X'}{\Omega_X}\right)\right]^2 = \frac{1}{3l^2}\left[-\frac{1}{2} + \alpha X^2 + \frac{1}{2}\left(\frac{3X^4}{l^4H^4}\right)^{1/3}\Omega_X^{-2/3}\right]. \tag{25}$$

Therefore, we also need an equation for H. This can be found by differentiating the Friedmann equation as

$$\Omega_X' + \Omega_m' + \Omega_r' = 0,$$

or

$$\Omega_X' = \frac{2H'}{H}(1 - \Omega_X) + 3(1 - \Omega_X) + \Omega_r. \tag{26}$$

Therefore the dynamical autonomous equations can be written as

$$\Omega_X' = \frac{2H'}{H}(1 - \Omega_X) + 3(1 - \Omega_X) + \Omega_r, \tag{27}$$

$$\Omega_r' = -\frac{2H'}{H}\Omega_r - 4\Omega_r, \tag{28}$$

$$\Omega_X' = -3\Omega_X - \frac{2H'}{H}\Omega_X + \left[\frac{3X^4}{K^4(H/H_0)^4}\right]^{1/3}\Omega_X^{1/3}, \tag{29}$$

$$\frac{H^2}{H_0^2}\left[1 + \frac{2}{3}\frac{H'}{H} + \frac{1}{3}\left(\frac{X'}{X} + \frac{\Omega_X'}{\Omega_X}\right)\right]^2 = \frac{1}{3K^2}\left[\alpha X^2 - \frac{1}{2} + \frac{1}{2}\left[\frac{3X^4}{K^4(H/H_0)^4}\right]^{1/3}\Omega_X^{-2/3}\right], \tag{30}$$

where we have introduced the dimensionless variable $K^2 \equiv H_0^2 l^2$. This shows that the present value of H can be reabsorbed into the free parameter K. In terms of these variables we find

$$w_X = -1 - \frac{2}{3}\frac{H'}{H} - \frac{\Omega_X'}{3\Omega_X}. \tag{31}$$

IV. COSMOLOGICAL ERAS

Let us consider the different eras in the cosmological history. We can distinguish the following cases.

(1) Radiation era: We can set $\Omega_r = 1$ and $\Omega_X = 0$. This fixes $\Omega_m = 0$. All the equations of motion are satisfied if

$$\frac{H'}{H} = -2, \quad \text{which implies } H = \frac{1}{2t}, \tag{32}$$

as expected.

(2) Matter era: Now we have two options:

(a) We can assume $\Omega_m = 1$ and $\Omega_r = 0$. In this case $\Omega_X = 0$. This implies that we are considering dark matter as an extra matter component (inside ρ_m) and not the X dark component. In this case the equations of motion are satisfied if

$$\frac{H'}{H} = -\frac{3}{2}, \quad \text{that is } H = \frac{2}{3t}, \tag{33}$$

as expected.

(b) Now we assume that the dominant dark component behaves as dark matter, whereas $\Omega_m \to \Omega_b$, that is, the matter component reduces to the baryon component and we suppose it is not the dominant one. In this case we need to impose $\Omega_X = 1$ and $\Omega_r = 0$. Since we still want that $H'/H = -3/2$, the equations of motion cannot be solved at the same time. Therefore this case shows that if the X-component gives an effective dark matter contribution in the past, it cannot be along a fixed point solution. However, there could be a transient solution from $\Omega_X = 0$ and $\Omega_X = 1$ which could still mimic a dark matter component.

(3) Dark energy era: In this case we set $\Omega_m = 0 = \Omega_r$ together with $\Omega_X = 1$. We look for a de Sitter solution that is $H' = 0$. The equations of motion imply

$$\frac{X^2}{H_{\mathrm{dS}}^2 K^2} = 3. \tag{34}$$

Therefore, $X = X_{\mathrm{dS}} =$ constant. Then for a de Sitter solution we find

$$H_{\mathrm{dS}} = \frac{|X_{\mathrm{dS}}/K|}{\sqrt{3}}. \tag{35}$$

Furthermore, the equations of motion give

$$X_{\mathrm{dS}}^2 = \frac{1}{1-\alpha}, \tag{36}$$

which implies that $\alpha < 1$. We can also write

$$H_{\mathrm{dS}}^2 = \frac{1}{3K^2(1-\alpha)}. \tag{37}$$

(4) Dark energy for the case $\alpha > 1$: Let us consider the case when, at very late times, $X/H = \lambda \approx$ constant, $\Omega_X \approx 1$, $\Omega_r \approx 0 \approx \Omega_m$, but still $H'/H \to$ constant, as well as $X'/X \to$ constant. Then, by neglecting any constant term with respect to the X term, we find the following two conditions which need to be satisfied,

$$\frac{3^{1/3}}{\lambda^{4/3}K^{4/3}} - \frac{2X'}{X} - 3 = 0, \tag{38}$$

$$\frac{\sqrt{\alpha}}{\sqrt{3}\lambda K} - \frac{X'}{X} - 1 = 0, \tag{39}$$

which imply

$$\alpha = \frac{3(3^{1/3} - \lambda^{4/3}K^{4/3})^2}{4\lambda^{2/3}K^{2/3}}. \tag{40}$$

This solution is not a de Sitter solution, as in fact we find

$$w_X \to -\frac{1}{3^{2/3}\lambda^{4/3}K^{4/3}} \neq -1. \tag{41}$$

V. NUMERICAL DISCUSSION

Let us consider a numerical solution of Eqs. (27)–(30). Rewriting Eq. (30) as

$$1 + \frac{2}{3}\frac{H'}{H} + \frac{1}{3}\left(\frac{X'}{X} + \frac{\Omega_X'}{\Omega_X}\right) = \frac{1}{\sqrt{3}Hl}\sqrt{\alpha X^2 - \frac{1}{2} + \frac{1}{2}\left(\frac{3X^4}{l^4H^4}\right)^{1/3}\Omega_X^{-2/3}}, \tag{42}$$

and by allowing the constant l (or K) to also take negative values (but $K \neq 0$), then we recover both the branches of Eq. (30). Notice that, if $K < 0$, Eq. (35) implies $X_{\mathrm{dS}} < 0$, for $\alpha < 1$. This further implies that the two branches, on their de Sitter solution, will differ by the sign of the final value of X.

A. Initial conditions

Let us solve the equations of motion from a given redshift ($z = z_i \gg 1$), such that at $z = z_i$ the universe is in the radiation era. We will set the initial condition for the Hubble parameter, during the radiation era, as the one given by GR, namely

$$H_i = H_i^{(\mathrm{GR})} = \sqrt{\Omega_{r,0}e^{-4N_i} + \Omega_{m,0}^{(\mathrm{GR})}e^{-3N_i} + (1 - \Omega_{m,0}^{(\mathrm{GR})} - \Omega_{r,0})}. \tag{43}$$

In what follows, we will fix the value of $\Omega_{r,i}$, during the radiation era, at $N = N_i \equiv -\log_{10}(1 + 1.76 \times 10^5)$, such that $\Omega_r(N = 0) = \Omega_{r,0}$ which will be set equal to a fixed value. In this model, we have five parameters, $\Omega_{m,0}$, X_i, $\Omega_{X,i}$, α, K. However, we will fix the initial condition for $\Omega_{X,i}$ by requiring the condition $\Omega_{K,0} = 1 - \Omega_{m,0} - \Omega_{r,0}$ to hold. Finally, the four parameters, $\Omega_{m,0}$, X_i, α, K, will be considered to be free. In particular, since the X-component is supposed to explain both dark matter and dark energy, we will set the following range $0 < \Omega_{m,0} \leq 0.4$. We run the other parameters to change over a large range, $-200 < X_i < 200$, $-10 < \alpha < 10$, and $-15 < K < 15$. Notice that in this parameter range, the system does not have a ΛCDM limit; therefore one expects deviations from the concordance model. Since this model has been introduced to explain dark energy and dark matter at the same time, this no-ΛCDM limit is in fact well motivated.

B. Results

We have calculated the total χ^2 for this model by using WMAP7 data (the background constraints on the two CMB shift parameters [23]), the BAO (Sloan Digital Sky Survey Data Release 7) data (two points) [24], and Supernova Type Ia (constitution data) [25], following the same method followed in Ref. [26]. The minimum for the χ^2 is located at

$$\Omega_{m,0} = 0.250078, \quad K = 8.629636, \quad \alpha = 2.760611, \\ X_i = 77.73029, \quad \text{where } \chi^2 = \chi^2_{\rm min} = 484.505, \nabla \tag{44}$$

where we have also fixed $\Omega_{r,i} = 0.999\,827$ and $\Omega_{X,i} = 1.144\,32 \times 10^{-6}$ for the reasons already explained above. Trying to set priors on $\Omega_{m,0}$ like $\Omega_{m.0} = \Omega_{b,0}$ (i.e., fixing the scalar field to be the main source of dark matter) leads to much larger values for χ^2. Furthermore, data tend to prefer clearly the $\alpha > 1$ case, as for $0 < \alpha < 1$, the χ^2 increases.

Nonetheless, the minimum value for $\chi^2_{\rm min}$ is still much larger than ΛCDM's value ($\chi^2_{\Lambda\rm CDM} \approx 469$). The χ^2 for ΛCDM has two free degrees of freedom (Ω_{m0}, Λ) that we can vary. Therefore according to the χ^2-probability distribution, at 95% confidence level, ΛCDM rules out those models, at 2σ, whose fit to the same data will lead to $\chi^2 - \chi^2_{\Lambda\rm CDM} > 5.99$. However, the models discussed here have $\chi^2 = 484.5$, so that $\chi^2 - \chi^2_{\Lambda\rm CDM} = 15.5$, which implies that these models are excluded at 2σ. This large difference implies the model under consideration does not fit the data, already at 2σ, as well as ΛCDM. This is tantamount to saying that the ΛCDM cosmological evolution rules out this class of models. Since the χ^2 for the model studied here is higher than the ΛCDM one, we can deduce that data do not support well the evolution of the effective equation of state plotted in Fig. 1. It should also be pointed out that for the parameters for which $\chi^2 = \chi^2_{\rm min}$, the scalar field, although it has in the past $w_X \approx 0$, it is anyhow a subdominant dark matter component (since on the minimum-χ^2 solution, the dust-like dark matter contributes up to $\Omega_{m,0} \approx 0.25$). This implies that the scalar field starts dominating the evolution of the universe only at late times, that is, it contributes to the dynamics essentially only as a dark energy field. But it is a dark energy field which, at early times, is quite different from a cosmological constant: this may be part of the reason why, in this case, the model cannot fit the data well. One option would be adding a bare cosmological constant (as also proposed in Ref. [16]), but in this case the model loses part of the interest as it would stop being an attractive dark energy model. Furthermore the evolution tends to lead to a fast transition of the effective equation of state parameter. This may also contribute to a worse fit to the data compared to ΛCDM.

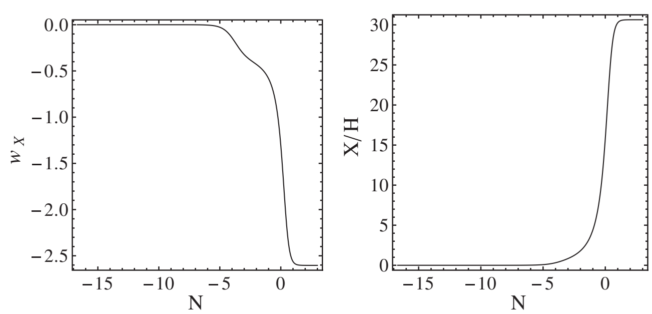

FIG. 1. Effective equation of state for the scalar field (left panel), and plot of the ratio X/H (right panel). At early times the EBI scalar field behaves as a dark matter component ($w_X \approx 0$); then, at late times, it drives the evolution of the universe. This plot shows the evolution for the parameters which minimize the χ^2 given in (44). Notice that since $\alpha > 1$, the final state is not de Sitter ($w_X < -1$); rather it tends to the solution characterized by $H'/H \to$ constant (Fig. 2) and $X/H \to$ constant (right panel).

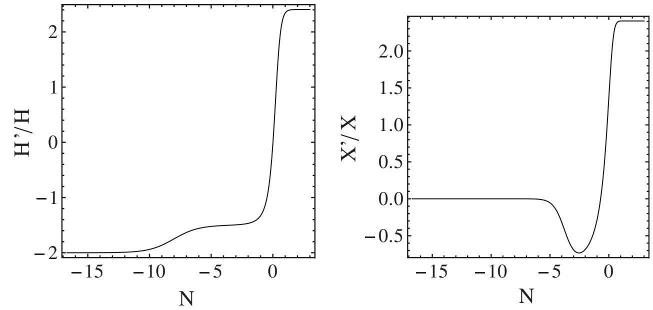

FIG. 2. Evolution for $H'/H = \dot{H}/H^2$ (left panel), and for the variable X'/X (right panel). The evolution, starting from radiation domination, passing through matter domination, at late times, tends to a super-accelerating final state on the minimum-χ^2 solution.

It should be noted that negative values for any of X_i, α, and K leads to very large values for χ^2 (typically larger than 1000), giving a bad fit to the data.

VI. CONCLUSIONS

We have studied the EBI scalar field which was proposed to model both dark matter and dark energy at the same time. We have solved the equations of motion and studied the behavior of the background at different times: at early times, indeed the scalar field behaves as a dark matter component with equation of state parameter $w_X \approx 0$. Only at late times, the field can lead the dynamics of the universe to an accelerated regime, which depending on the parameters of the model, is described by either a de Sitter solution, or a rather different dynamic described by $H/X \to$ constant and $H'/H \to$ constant (where a prime denotes differentiation with respect to the $N = \ln a$) (Fig. 2).

The fact that, at early times, the scalar field behaves as a dust component can in principle alleviate the problem of finding a dark matter component, as indeed the nature of dark matter and dark energy would have the same explanation.

In order to see whether this model is viable or not, we studied the cosmological constraints that its dynamics have to pass when considering WMAP7 data, BAO and Supernova Type Ia. For this goal, we have calculated the χ^2 as a function of four free parameters, that is, K, α (two theoretical dimensionless parameters of the model) together with $\Omega_{m,0}$ (which states how much of an extra standard dust component is needed), and X_i, the initial value for the time-rescaling component of the Born-Infeld metric.

We have found that the model cannot give a good fit to the data (compared to ΛCDM), and hence the model cannot be considered viable. We have proved this statement by constraining the background. This approach differs from the one followed in Ref. [16], where the authors studied the evolution of the cosmological perturbations in order to constrain the growth of structures. In particular, we have used only the constraints on the background coming from the WMAP7 data. Instead in Ref. [16], the authors considered constraints only on the perturbations power spectrum. It is possible, as also suggested in Ref. [16], that introducing a cosmological constant would improve the fit, but, on the other hand, the model would partially lose its original motivation of explaining at the same time both dark energy and dark matter. In particular, data prefer the non-de Sitter solution, preferring a fast transition to values for $w_X < -1$. In this model, at early times, a cosmological constant is absent from the beginning, as the scalar field initially (and up to very recently) behaves as a dark matter component.

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