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Greybody factor for black string in dRGT massive gravity

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Abstract The greybody factor from the black string in the de Rham–Gabadadze–Tolley (dRGT) massive gravity theory is investigated in this study. The dRGT massive gravity theory is one of the modified gravity theories used in explaining the current acceleration in the expansion of the universe. Through the use of cylindrical symmetry, black strings in dRGT massive gravity are shown to exist. When quantum effects are taken into account, black strings can emit thermal radiation, called Hawking radiation. The Hawking radiation at spatial infinity differs from that at the source by the so-called greybody factor. In this paper, we examine the rigorous bounds on the greybody factors from the dRGT black strings. The results show that the greybody factor crucially depends on the shape of the potential, which is characterised by the model parameters. The results agree with ones in quantum mechanics; the higher the potential, the harder it is for the waves to penetrate, and also the lower the bound for the rigorous bounds.

1 Introduction

Based on cosmological observations, our universe is expanding with an acceleration [1,2]. However, the explanation for this phenomenon remains unclear. Many authors propose the existence of exotic matter called dark energy to explain this observed cosmic acceleration. On the other hand, some authors modify gravity without dark energy. One of the modifications of gravity is to give mass to the graviton. The de Rham–Gabadadze–Tolley (dRGT) models [3,4] are successful and viable models of massive gravity. Reviews of the theory of massive gravity can be found in [5,6]. For spherical symmetry, the black hole solutions have also been found, and

their thermodynamics properties extensively investigated [7–34].

When quantum effects are taken into account, black holes can emit thermal radiation called Hawking radiation [35]. The original Hawking radiation emitted from a black hole is blackbody radiation. Due to the curvature of spacetime, the Hawking radiation is modified, while propagating to spatial infinity. The radiation at spatial infinity differs from that at the emitter by the so-called greybody factor. There are various methods to find the greybody factors, such as the matching technique and the WKB approximation [36–44]. Another interesting method is to bound the greybody factor from below [45–55].

Besides the solution with spherical symmetry, the solution to the Einstein field equation in the case of cylindrical symmetry has also been investigated and is known as the black string solution [56,57]. This solution can be achieved by introducing the cosmological constant into the Einstein field equation. The charge and the rotating black string solutions can also be found [58]. The quasinormal modes [59] and the greybody factor of the black string have been investigated [60].

As is well known, the dRGT massive gravity theory can provide a more general solution than the Schwarzschild-dS/AdS. Therefore, it is possible to obtain the cylindrical solution in the dRGT massive gravity theory [61]. The rotating solutions and their thermodynamic properties are also investigated [62]. The quasinormal mode for the dRGT black string solution have been investigated as well [63], while the greybody factor has not been investigated yet. In the present work, the rigorous bounds on the greybody factor from the dRGT black strings are examined.

This paper is organised as follows. In Sect. 2, the background of the dRGT black string is presented. The horizon structures are analysed in Sect. 3. The equation of motion of the massless scalar field emitted from a dRGT black hole and

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the gravitational potential which modifies the scalar field are derived in Sect. 4. The rigorous bounds on the greybody factors are calculated in Sect. 5, and the conclusions are given in Sect. 6.

2 dRGT black string background

In this section, dRGT massive gravity theory, including how the black string solution can be obtained, is roughly reviewed. The main concept in the modification of the general relativity in dRGT massive gravity is the addition of a suitable graviton mass to General Relativity (GR), of which the action can be written as [3,4]

$$S = \int d^4x \sqrt{-g} \, \frac{1}{2} \left[R(g) + m_g^2 \, \mathcal{U}(g, f) \right], \tag{1}$$

where R is the Ricci scalar, $\mathcal{U}$ is a potential term used in characterising the behaviour of the mass term of graviton, and m_g is the parameter interpreted as the graviton mass. The suitable form of the potential $\mathcal{U}$ in four-dimensional spacetime is given by

$$\mathcal{U}(g, \phi^a) = \mathcal{U}_2 + \alpha_3 \mathcal{U}_3 + \alpha_4 \mathcal{U}_4, \tag{2}$$
$$\mathcal{U}_2 \equiv [\mathcal{K}]^2 - [\mathcal{K}^2], \tag{3}$$
$$\mathcal{U}_3 \equiv [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3], \tag{4}$$
$$\mathcal{U}_4 \equiv [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 8[\mathcal{K}][\mathcal{K}^3]$$
$$+ 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4], \tag{5}$$

where α_3 and α_4 are dimensionless free parameters of the theory. The quantity $[\mathcal{K}]$ denotes the trace of the metric $\mathcal{K}_\nu^\mu$, defined by

$$\mathcal{K}_\nu^\mu = \delta_\nu^\mu - \sqrt{g^{\mu\rho} f_{\rho\nu}}, \tag{6}$$

where $[\mathcal{K}^n] = (\mathcal{K}^n)_\mu^\mu$ and $(\mathcal{K}^n)_\nu^\mu = \mathcal{K}_{\rho_2}^\mu \mathcal{K}_{\rho_3}^{\rho_2} \ldots \mathcal{K}_{\rho_n}^{\rho_{(n-1)}} \mathcal{K}_\nu^{\rho_n}$ for $n \geq 2$. It is important to note that the potential terms include the non-dynamical metric $f_{\mu\nu}$ called the fiducial metric or the reference metric. The form of the solution of the physical metric $g_{\mu\nu}$ significantly depends on the form of the fiducial metric [64–66]. The equation of motion corresponding to the above action can be written as

$$G_{\mu\nu} + m_g^2 X_{\mu\nu} = 0, \tag{7}$$

where

$$X_{\mu\nu} = \mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu} - \alpha \left(\mathcal{K}_{\mu\nu}^2 - \mathcal{K}\mathcal{K}_{\mu\nu} + \frac{\mathcal{U}_2}{2} g_{\mu\nu} \right)$$
$$+ 3\beta \left(\mathcal{K}_{\mu\nu}^3 - \mathcal{K}\mathcal{K}_{\mu\nu}^2 + \frac{\mathcal{U}_2}{2} \mathcal{K}_{\mu\nu} - \frac{\mathcal{U}_3}{6} g_{\mu\nu} \right), \tag{8}$$
$$\alpha_3 = \frac{\alpha - 1}{3}, \quad \alpha_4 = \frac{\beta}{4} + \frac{1 - \alpha}{12}. \tag{9}$$

Due to the existence of the Bianchi identity, the tensor $X_{\mu\nu}$ obeys the covariant conservation as

$$\nabla^\mu X_{\mu\nu} = 0. \tag{10}$$

By imposing static and cylindrical symmetry, a general form of the black string solution (physical metric) can be written as [61]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\varphi^2 + \alpha_g^2 dz^2), \tag{11}$$

where α_g is a constant. By choosing the form of the fiducial metric as

$$f_{\mu\nu} = \text{diag}(0, 0, h_0^2, h_0^2), \tag{12}$$

where h_0 is a constant, the function $f(r)$ in the physical metric can be written as [61]

$$f(r) = c_2 r^2 - \frac{4M}{r} - c_1 r + c_0 \tag{13}$$

and $d\Omega^2 = d\varphi^2 + \alpha_g^2 dz^2$, $M = \bar{M}/\alpha_g$, where $\bar{M}$ is the ADM mass per unit length in the z direction. The parameters above can be written in terms of the original parameters as

$$c_2 = m_g^2 (1 + \alpha + \beta), \tag{14a}$$
$$c_1 = m_g^2 h_0 (1 + 2\alpha + 3\beta), \tag{14b}$$
$$c_0 = m_g^2 h_0^2 (\alpha + 3\beta). \tag{14c}$$

The solution in Eq. (11), including the function f in Eq. (13), is an exact black string solution in dRGT massive gravity which, in the limit $c_2 = \alpha_g^2$ and $c_0 = c_1 = 0$, naturally goes over to Lemos' black string in GR with cosmological constant [56,57]. In particular, it incorporates the cosmological constant term (c_2 term) naturally in terms of the graviton mass. Moreover, this solution also provides a global monopole term (c_0 term) and another non-linear scale term (c_1 term).

It is important to note that the strong coupling scale of the dRGT massive gravity theory is $\Lambda_3^{-1} = (m_g^2 M_{Pl})^{1/3} \sim 10^3$ km $\ll r_V \sim 10^{16}$ km, so that we do not have to worry about the strong coupling issue in dRGT massive gravity for a system of scale below Λ_3 (or of a length scale beyond $\sim 10^3$ km), where r_V is the Vainshtein radius characterised by the non-linear scale of the massive gravity theory [61].

One can see that the horizon structure depends on the sign of c_2. If $c_2 > 0$, corresponding to the anti-de Sitter-like solution, the maximum number of horizons is three. If $c_2 < 0$, corresponding to the de Sitter-like solution, the maximum number of horizons is two. This behaviour is explicitly shown in the next section.

3 Horizon structure

In order to investigate the structure of the horizons for the solution in Eq. (11), where f is in Eq. (13), one has to find the number of possible extremum points. As a result, this depends on the asymptotic behaviour of the solution. For the asymptotic dS solution, $c_2 < 0$, the solution becomes the dS black string for the large-r limit, while the solution becomes the AdS black string for the large-r limit of the asymptotic AdS solution, $c_2 > 0$. As a result, one can find the conditions to obtain one positive real maximum of f for the asymptotic dS solution. For the asymptotic AdS solution, one can find the conditions to have one positive real maximum and one negative real minimum f. We will investigate this behaviour separately in the following subsection.

It is important to note that by choosing the fiducial metric as $h_0 = 0$, the solution becomes AdS/dS black string solution. This is not surprising since the potential term becomes a constant.

3.1 Asymptotic dS solution

For the asymptotic dS solution, $c_2 < 0$, one can find conditions for having two horizons by solving $f' = 0$ to obtain a real positive value of the radius, r, by

$$r_{ex} = (\sqrt{3} - 1)\left(\frac{M}{-c_2}\right)^{1/3}. \qquad (15)$$

Note that to guarantee this existence, we choose the condition on c_1 as $c_1 = 6(Mc_2^2)^{1/3}$. As a result, f at the extremum can be written as

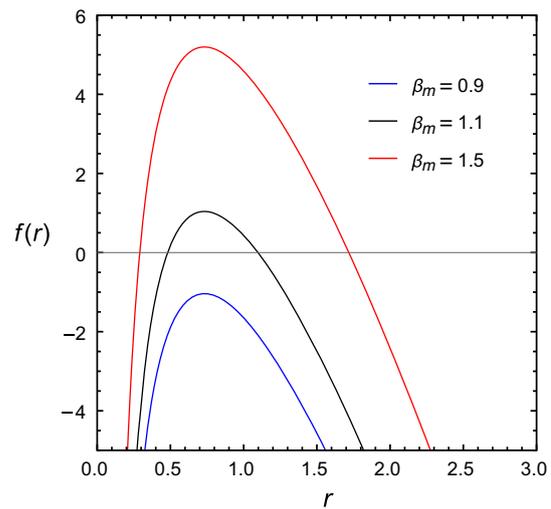$$f(r_{ex}) = c_0 + 6\sqrt{3}c_2\left(-\frac{M}{c_2}\right)^{2/3}. \qquad (16)$$

In order to have two horizons, $f(r_{ex})$ must be positive. Therefore, let us define the parameter $\beta_m > 1$ for having two horizons:

$$c_0 = -\beta_m 6\sqrt{3}c_2\left(-\frac{M}{c_2}\right)^{2/3}. \qquad (17)$$

By substituting this parameter into f, and then finding the solution of $f = 0$ for r, one obtains two horizons as follows:

$$r_1 = \frac{2(c_2^2 M)^{1/3}}{c_2}\left(1 + \sqrt{X}\cos\left(\frac{\cos^{-1} Y}{3} + \frac{\pi}{3}\right)\right), \qquad (18)$$

$$r_2 = \frac{2(c_2^2 M)^{1/3}}{c_2}\left(1 - \sqrt{X}\cos\left(\frac{\cos^{-1} Y}{3}\right)\right), \qquad (19)$$

Fig. 1 Plot of $f(r)$ using different values of β_m, with $M = 1$ and $c_2 = -1$

where

$$X = 4 + 2\sqrt{3}\beta_m \quad \text{and} \quad Y = -\frac{3\sqrt{3}\beta_m + 5}{\sqrt{2}\left(\sqrt{3}\beta_m + 2\right)^{3/2}}. \qquad (20)$$

One can see that we now have two parameters, c_2 and β_m, controlling the behaviour of the horizons. The parameter c_2 controls the strength of the graviton mass or the cosmological constant, while β_m controls the existence of the horizons. For $0 < \beta_m < 1$ there are no horizons, while for $\beta_m > 1$ there are two horizons. Two such horizons become closer and closer when β_m approaches 1, and thus the two horizons merge at $\beta_m = 1$, as shown in Fig. 1.

It is useful to emphasise here that our choice, $c_1 = 6(Mc_2^2)^{1/3}$, provides only a class of conditions characterising the existence of the horizons. It is not valid in general. For example, for $c_0 = 0$, corresponding to $\beta_m = 0$, it is still possible to find the parameter space for c_2 and c_1 so as to have two horizons. Even though this choice and the set of parameters (c_2, β_m) provide a loss of generality of parameter space, it provides us with a qualitative way to analyse the effects of the horizon structure on the potential and the greybody factor. This will be explicitly shown in Sects. 4 and 5.

It is important to note that the existence of parameters, c_1 and c_0, is characterised by the structure of the dRGT massive gravity theory, which provides an additional part to the usual dS black string solution [56,57]. From Eq. (13), one can see that without these parameters ($c_2 < 0$, $c_1 = 0$, $c_0 = 0$), it is not possible to have a horizon since f is always negative; therefore, it is not possible to investigate the thermodynamics of the black string or find the greybody factor for the dS black string solution. This is a crucial issue for the dRGT massive

gravity black string solution, and we will investigate this issue in the next section.

3.2 Asymptotic AdS solution

For the asymptotic AdS solution, $c_2 > 0$, one can use the same strategy as in the previous subsection, in finding two extrema when $f' = 0$. As a result, these two extrema can be written as

$$r_{ex1} = \left(\frac{M}{c_2}\right)^{1/3}, \tag{21}$$

$$r_{ex2} = (1 + \sqrt{3})\left(\frac{M}{c_2}\right)^{1/3}. \tag{22}$$

Following the same step, the function f at the extrema can be written as

$$f(r_{ex1}) = c_0 - 6\sqrt{3}c_2\left(\frac{M}{c_2}\right)^{2/3}, \tag{23}$$

$$f(r_{ex2}) = c_0 - 9c_2\left(\frac{M}{c_2}\right)^{2/3}. \tag{24}$$

In order to see the structure of the horizons, let us define a parameter to parametrise the existence of three horizons:

$$c_0 = \beta_m 6\sqrt{3}c_2\left(\frac{M}{c_2}\right)^{2/3}, \tag{25}$$

where the condition for having three horizons is

$$\frac{\sqrt{3}}{2} < \beta_m < 1. \tag{26}$$

By substituting these parameters into f, and then finding the solution of $f = 0$ for r, one obtains three horizons as follows:

$$r_1 = \frac{2(c_2^2 M)^{1/3}}{c_2}\left(1 - \sqrt{x}\sin\left(\frac{\cos^{-1} y}{3} + \frac{\pi}{6}\right)\right), \tag{27}$$

$$r_2 = \frac{2(c_2^2 M)^{1/3}}{c_2}\left(1 - \sqrt{x}\cos\left(\frac{\cos^{-1} y}{3} + \frac{\pi}{3}\right)\right), \tag{28}$$

$$r_3 = \frac{2(c_2^2 M)^{1/3}}{c_2}\left(1 + \sqrt{x}\cos\left(\frac{\cos^{-1} y}{3}\right)\right), \tag{29}$$

where

$$x = 4 - 2\sqrt{3}\beta_m \quad \text{and} \quad y = \frac{5 - 3\sqrt{3}\beta_m}{\sqrt{2}\left(2 - \sqrt{3}\beta_m\right)^{3/2}}. \tag{30}$$

As we have analysed in the previous subsection, we recover the usual AdS black string solution by setting $c_2 > 0$ and $c_0 = c_1 = 0$. In this case, it is found that there exists only one horizon. Therefore, the crucial difference is characterised by the existence of c_1 and c_0, which are now re-parametrised by only one parameter β_m. As we have seen in Fig. 2, one can obtain three horizons for $\sqrt{3}/2 < \beta_m < 1$. For $\beta_m = \sqrt{3}/2$, the first and the second horizons are merged, while when
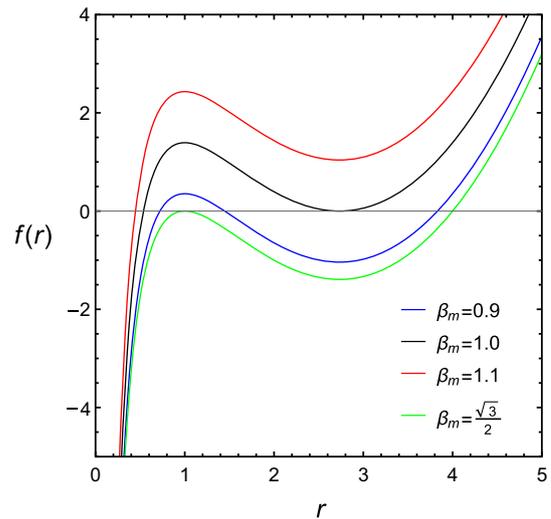
Fig. 2 Plot of $f(r)$ using different values of β_m, with $M = 1$ and $c_2 = 1$

$\beta_m = 1$, the second and the third horizons are merged, with two horizons for these two specific cases. Finally, one horizon can exist for $0 < \beta_m < \sqrt{3}/2$ (third horizon) and $\beta_m > 1$ (first horizon). This behaviour can be seen explicitly in Fig. 2.

Note that, even though we leave only two parameters for characterising the behaviour of the horizon structure, this is very useful for the analytical investigation of how the horizon structure influences the potential form and also the greybody factor. We will show this analysis in the next two sections.

4 Equations of motion of the massless scalar field

In this work, we are interested in a massless uncharged scalar field emitted from the dRGT black string as Hawking radiation. The equation of motion, which describes the motion of the massless uncharged scalar field, is the Klein–Gordon equation,

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right) = 0. \tag{31}$$

By using the solution of the physical metric in Eq. (11), the solutions can be separated in the form

$$\Phi(t, r, \Omega) = T(t)Y(\varphi, z)\frac{\psi(r)}{r}, \tag{32}$$

where $T = e^{\pm i\omega t}$ is the oscillating function and Y satisfies the equation

$$\frac{\partial^2 Y}{\partial\varphi^2} + \frac{1}{\alpha_g^2}\frac{\partial^2 Y}{\partial z^2} = -\ell(\ell + 1)Y. \tag{33}$$

The radial part of the Klein–Gordon equation is

$$\frac{\mathrm{d}^2 \psi(r)}{\mathrm{d}r_*^2} + \left[\omega^2 - V(r)\right]\psi(r) = 0, \tag{34}$$

where r_* is the tortoise coordinate defined by

$$\frac{\mathrm{d}r_*}{\mathrm{d}r} = \frac{1}{f(r)} \tag{35}$$

and $V(r)$ is the potential given by

$$V(r) = f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r}\right]. \tag{36}$$

Surprisingly, the equation of motion for the radial part is of the same form, even though we used the cylindrical coordinates instead of the spherical coordinates. This allows us to perform the investigation for the greybody factor in the same fashion as usually done in spherical coordinates. Moreover, since the form of the radial equation is still in the form of Schrödinger-like equation, one can perform the analysis of the effect of the potential form on the transmission amplitude similar to one in quantum mechanics. It is important to note that the leading contribution to the transmission amplitude or the greybody factor is the mode $\ell = 0$, since the larger the value of ℓ, the higher the value of the potential and the more difficult it is for the wave to transmit. This behaviour is also common in the spherical symmetry case. As a result, we will restrict our attention to the case $\ell = 0$, and then the potential becomes $V = f'f/r$. In order to see the behaviour of the potential in terms of the massive graviton parameters, one can substitute f from Eq. (13), and then reparametrise the parameters in terms of β_m and c_2. As a result, by fixing c_2, and then varying β_m, the behaviour of the potential in both the asymptotic dS and the asymptotic AdS solutions can be illustrated by Fig. 3. From the left panel of this figure (the asymptotic dS case), one can see that the potential becomes lower when the parameter β_m approaches 1. In other words, when the horizons become closer, the potential becomes lower and lower. This gives a hint to us that the greybody factor bound will be higher when the horizons become closer. This analysis is also valid for the asymptotic AdS case. We will consider this analysis in detail in the next section.

5 The rigorous bounds on the greybody factors

There are many methods to calculate the greybody factor, such as the matching technique and the WKB approximation [36–44]. In the present work, we will focus on the method that does not use such an approximation, namely, the rigorous bound on the greybody factor. The advantage of this method is that it provides us with a better way to analyse the greybody
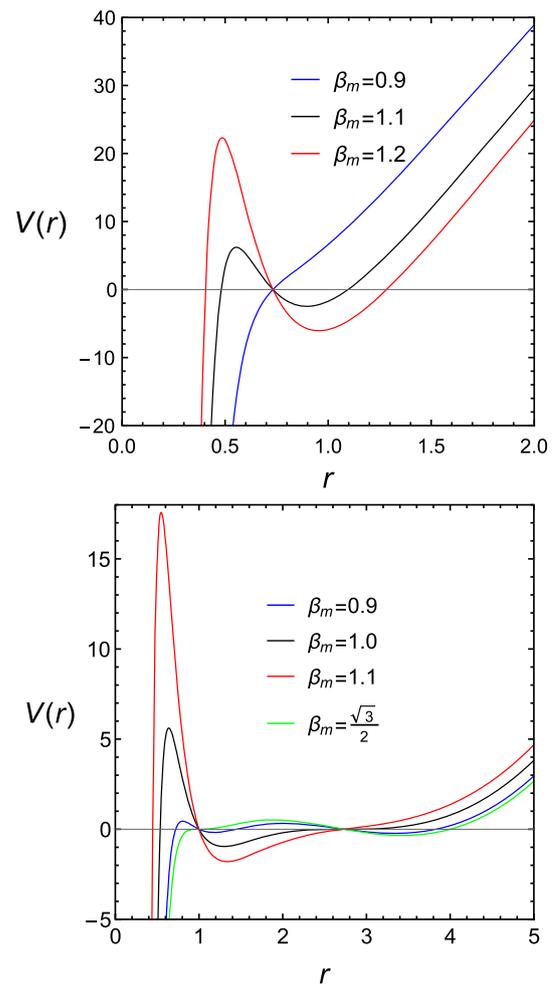
Fig. 3 The left panel shows the potential for the asymptotic dS solution with $\ell = 0$, $c_2 = -1$, $M = 1$. The right panel shows the potential for the asymptotic AdS solution with $\ell = 0$, $c_2 = 1$, $M = 1$

factor qualitatively. Then the influence of the potential form on the greybody factor can be explored. The rigorous bounds on the greybody factors are given by
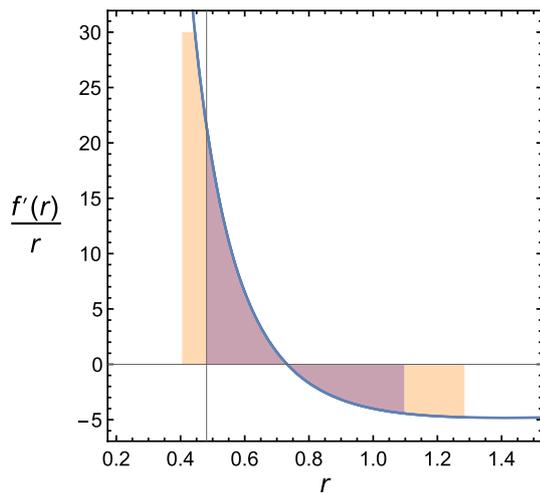
$$T \geq \mathrm{sech}^2\left(\int_{-\infty}^{\infty} \vartheta \, \mathrm{d}r_*\right), \tag{37}$$

where

$$\vartheta = \frac{\sqrt{[h'(r_*)]^2 + \left[\omega^2 - V(r_*) - h^2(r_*)\right]^2}}{2h(r_*)} \tag{38}$$

and $h(r_*)$ is a positive function satisfying $h(-\infty) = h(\infty) = \omega$. See [45] for more details. We select $h = \omega$. Therefore,

$$T \geq \mathrm{sech}^2\left(\frac{1}{2\omega}\int_{-\infty}^{\infty} |V| \, \mathrm{d}r_*\right). \tag{39}$$

Fig. 4 Plot of f'/r using different values of β_m, with $M = 1$ and $c_2 = 1$. The blue filled area corresponds to $\beta_m = 1.1$ and the orange filled area corresponds to $\beta_m = 1.2$

Fig. 5 Plot of T_b using different values of β_m, with $\ell = 0$, $M = 1$ and $c_2 = -1$

From Eq. (36), together with f in Eq. (13), the potential is

$$V(r) = f(r)\left[\frac{\ell(\ell+1)}{r^2} + 2c_2 + \frac{4M}{r^3} - \frac{c_1}{r}\right], \quad (40)$$

where $f(r)$ is given by Eq. (13). From Eq. (35), the rigorous bound on the greybody factor given by Eq. (39) becomes

$$T \geq T_b = \text{sech}^2\left(\frac{1}{2\omega}\int_{r_H}^{R_H}\frac{|V|}{f(r)}dr\right) = \text{sech}^2\left(\frac{A_\ell}{2\omega}\right), \quad (41)$$

where

$$A_\ell = \int_{r_H}^{R_H}\frac{|V|}{f(r)}dr = \int_{r_H}^{R_H}\left|\frac{\ell(\ell+1)}{r^2} + \frac{f'}{r}\right|dr. \quad (42)$$

As is well known, the function sech^2 is maximum at $\text{sech}^2(0)$, so that the function A_ℓ must be close to zero in order to obtain the higher value of the bound T_b. Therefore, one can ignore the contribution from A_ℓ with $\ell \geq 1$. Now, let us consider A_0, which can be written as

$$A_0 = \int_{r_H}^{R_H}\left|\frac{f'}{r}\right|dr. \quad (43)$$

One can see that A_0 is the area filled by the function f'/r. Since the function f'/r does not depend on c_0, it does not depend on β_m. After fixing c_1 and c_2, f'/r is still the same function. Therefore, the filled area is different by the limit of integration as shown in Fig. 4. This can also be seen from Fig. 1, as the value of β_m is close to 1 where two horizons are sunk together. This analysis is also confirmed by using a numerical method as shown in Fig. 5. Moreover, this behaviour is also
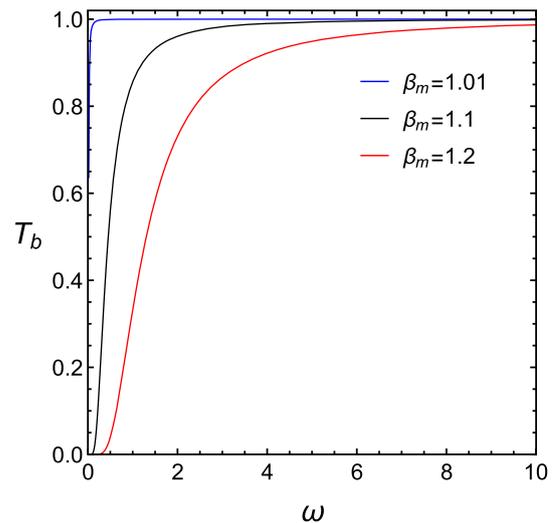
consistent with the shape of the potential as illustrated in the left panel of Fig. 3. From this figure, it can be inferred that if the potential is higher, the value of the transmission amplitude is lower.

In order to find the effect of the parameter c_2, one can fix the parameter as $\beta_m = 1.1$. As a result, the shape of the potential will control the greybody factor bound. This is similar to one in quantum theory, where the higher the potential is, the lower the transmission amplitude and then lower the greybody factor bound becomes. This consistency is shown in Fig. 6. From these figures, one can see that the larger the value of $|c_2|$ is, the higher the value of the potential and then the lower the value of the greybody factor bound becomes.

Now, let us consider the asymptotic AdS solution. As we have discussed, it is possible to obtain the three horizons for this kind of solutions. In this case, one may have to assume the place of the observer. As a result, we can divide our consideration into two cases; the observer being between the first and the second horizons, and the observer being between the second and the third horizons. From Fig. 2, one finds that three horizons exist if $\sqrt{3}/2 < \beta_m < 1$. For $\beta_m = \sqrt{3}/2$, the first and the second horizons are sunk together, and for $\beta_m = 1$, the second and the third horizons are sunk together.

By fixing c_2, one can still use the same analysis as done in the asymptotic dS case, where the greybody factor bound depends crucially on the distance between the horizons. This can be seen explicitly in Fig. 7.

Now, let us fix the parameter β_m. As we have analysed above, the greybody factor bound crucially depends on the maximum value of the potential; the higher the value of the potential, the more difficult it is for the waves to be transmitted and then the lower the bound of the greybody factor
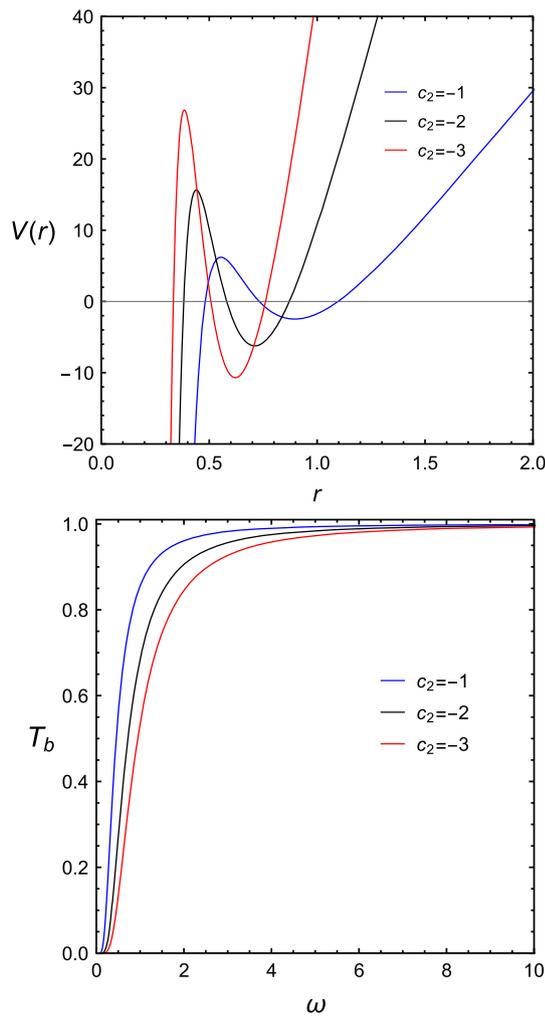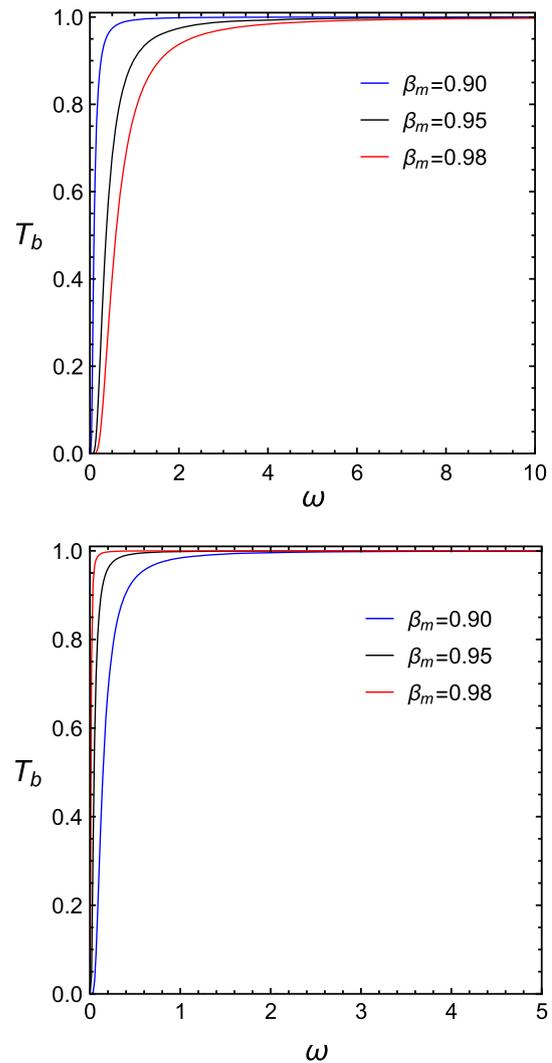
Fig. 6 These plots show the horizon structure, shape of the potential and the greybody factor bound for different values of c_2, with $M = 1$ and $\beta_m = 1.1$

Fig. 7 Plot of T_b using different values of β_m, with $\ell = 0$, $M = 1$ and $c_2 = 1$. The left panel is for the one between the first and the second horizon, and the right panel is for one between the second and the third horizon

becomes. This behaviour is shown explicitly in Fig. 8, where the shape of the potential is in the left panel and the corresponding greybody factor bound is in the right panel.

6 Conclusion

In this paper, we investigated the greybody factor of the black string in dRGT massive gravity theory by using the rigorous bound. In order to properly study the dRGT black string, we first investigated the horizon structures of the dRGT black string. We defined the new model parameter β_m to characterise the existence of the horizons. The results show that, for the asymptotic dS solution, there are two horizons when $\beta_m > 1$, and for the asymptotic AdS solution, there are three horizons when $\sqrt{3}/2 < \beta_m < 1$. By considering a massless uncharged scalar field emitted from the dRGT black

string as Hawking radiation, a Schrödinger-like equation is obtained for the radial part of the solution. This allows us to consider the behaviour of the potential for investigating the greybody factor. It is found that the height of the potential becomes lower when the parameter β_m approaches 1 for the asymptotic dS solution, while β_m approaches 1, $\sqrt{3}/2$ for the asymptotic AdS solution where two horizons are merged. Moreover, rigorous bounds on the greybody factors have also been calculated. It is found that the greybody factor bound can be qualitatively analysed by using a certain form of the potential; the higher the value of the potential, the more difficult it is for the waves to be transmitted and then the lower the bound of the greybody factor. This result is valid for both the asymptotic AdS solution and the asymptotic dS solution, and also it was checked by numerical methods. Since our
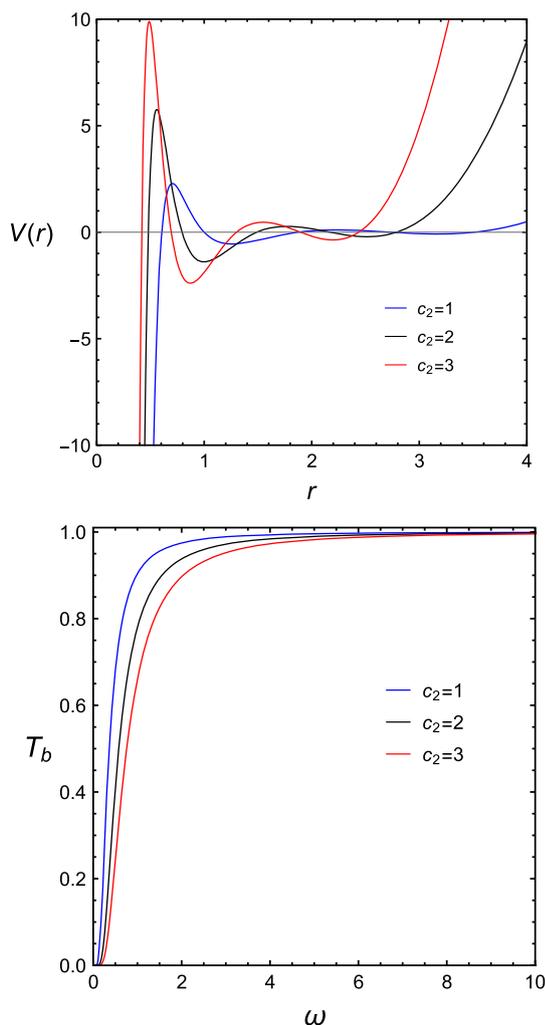
Fig. 8 These plots show the horizon structure, shape of the potential and the greybody factor bound for different values of c_2, with $M = 1$ and $\beta_m = 0.95$

analysis/results are similar to ones in quantum mechanics, it provides us with an easier way to deal with the quantum nature of black holes or black strings, even though a complicated form of spacetime is considered.

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