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Short Note

Propagators, amplitudes and phase accumulation for photon propagation: The Feynman problem

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ABSTRACT

Motivated by Feynman's intuitive and non-technical examination of the strange behavior of light and the underlying physical mystery of "How does a photon make-up its mind where to go?", as Feynman eloquently puts it, we develop the actual technical description of photon propagation, via the Schwinger–Feynman causal propagator, and examine the accumulated phase acquired by a photon upon propagation, as determined from the causal propagator, to provide a probabilistic description of the fate of a photon. Although, due to the probabilistic nature of the problem, it is not known how a photon "makes up its mind" where to go, situations arise where the photon has no option where to go, and the problem becomes deterministic. Illustrations are given in terms of red and blue photons.

The purpose of this note is threefold and addresses the following: (1) The Schwinger–Feynman causal propagator of quantum field theory, in the presence of obstacles, may be used in practical situations in optics in an elegantly and physically clear way. (2) By examining the accumulated phase acquired by a photon upon propagation, as determined from the causal propagator, a probabilistic description of the fate of the photon may be extracted. (3) Although, due to the probabilistic nature of photon propagation, it is not known how a photon "makes up its mind" where to go, as Feynman [1] so eloquently puts it, situations arise where a photon has no option where to go, and the problem becomes deterministic. The motivation of this work came from lectures given by Feynman on this very special topic at McGill University in the late sixties, while I was a graduate student there, and from his later lectures as described below.

In his legendary lectures on the strange theory of light and matter, Feynman [1] spends a great deal of time providing a fascinating but nontechnical treatment of how a photon may reflect-off or go through a layer of glass in terms of amplitudes that acquire phases upon the propagation of a photon, and, in turn, points out the underlying physical mystery of the problem, by raising the question: "How does a photon make-up its mind where to go?". To develop the actual technical language for considering such a problem, we provide a direct Schwinger–Feynman causal propagator description to actually extract *amplitudes together with their acquired phases and actually compute* them in a quantum setting, not just stating them in words, thus providing a quantitative description of the Feynman non-technical intuitive presentation which replaces the standard classical treatment [2]. As the underlying problem being probabilistic, meaning that repeated experiments may, in general, yield different results, Feynman [1] states that how a photon makes up its mind is not known. The exploration of Feynman of this mystery of the strange behavior of light and his statement as how does a photon decide where to go, has even inspired the playwright Peter Parnell to generate a play in recent years [3]. A formulation of light propagation in terms of propagators, in a quantum optics setting, has been also addressed in [4] in the presence of obstacles, which turns out, however, to be too involved. To simplify the admittedly non-trivial problem addressed, and for greater clarity of the presentation and of its development, we consider normal incidence of light on the layer of glass, considered by Feynman, with index of refraction ε,

$$\begin{array}{c} z > 0 \\ -------z = 0 \\ \varepsilon \\ -------z = -D \end{array}$$

with polarization along the x^1 axis. This means, we may consider the photon propagator $D^{11}(zt, z't') \equiv D(zt, z't')$ in one space and

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one time dimensions. The advantage of working with a propagator is that one is not tied down to any wavefunctions [5–9]. For the propagator approach of such related problems for *non*-relativistic particles see, e.g., [10–12].

In the region $z'>0$, the propagator, with the causal arrangement $t'>t$, may be written as

$$D_>(zt, z't') = i\int \frac{dq}{2\,|q|(2\pi)}\, e^{-i|q|(t'-t)}[1\, e^{-iq(z-z')} + A_>(q)\, e^{-iq(z+z')}], \tag{1}$$

where the sign of z', in the last exponential, is reversed to take into consideration of the possible reflection that may occur for $z'>0$, $dq/|q|$ denotes the relativistic measure which does not concern us here, and the amplitude $A_>$ is to be determined. For $z'<-D$,

$$D_<(zt, z't') = i\int \frac{dq}{2\,|q|(2\pi)}\, e^{-i|q|(t'-t)}[e^{-iq(z-z')}A_<(q)]. \tag{2}$$

For $-D<z'<0$, we have for the corresponding part $\underline{D}$ of the propagator, two equations

$$\left[-\left(\frac{\partial}{\partial z'}\right)^2 + \varepsilon\left(\frac{\partial}{\partial t'}\right)^2\right]\underline{D}(zt, z't') = 0,$$

$$\left[-\left(\frac{\partial}{\partial z}\right)^2 + \left(\frac{\partial}{\partial t}\right)^2\right]\underline{D}(zt, z't') = 0, \tag{3}$$

giving rise to two solutions which allow us to write

$$\underline{D}(zt, z't') = i\int \frac{dq}{2\,|q|(2\pi)}\, e^{-i|q|(t'-t)}[e^{-i(qz-Qz')}M_1(q) + e^{-i(qz+Qz')}M_2(q)], \quad Q = \sqrt{\varepsilon}\, q, \tag{4}$$

where the amplitudes $M_1(q)$, $M_2(q)$ are to be determined.

The boundary conditions [2] B.C. may be spelled out here as $D(zt, z't')$ and $\partial D(zt, z't')/\partial z'$ are continuous at $z'=0$ and $z'=-D$. The B.C. give directly

$$1 + A_> = M_1 + M_2, \qquad [1 - A_>] = \sqrt{\varepsilon}\,[M_1 - M_2], \tag{5}$$

$$A_< = e^{-iQD}M_1 + e^{iQD}M_2, \qquad A_< = \sqrt{\varepsilon}\,[e^{-iQD}M_1 - e^{iQD}M_2]. \tag{6}$$

The solutions are given by

$$M_1 = \frac{e^{i(Q-q)D}(\sqrt{\varepsilon}+1)}{[2\sqrt{\varepsilon}\cos QD + i(\varepsilon+1)\sin QD]}, \quad M_2 = \frac{e^{-i(Q+q)D}(\sqrt{\varepsilon}-1)}{[2\sqrt{\varepsilon}\cos QD + i(\varepsilon+1)\sin QD]}, \tag{7}$$

$$A_> = \frac{-i(\varepsilon-1)\sin QD}{[2\sqrt{\varepsilon}\cos QD + i(\varepsilon+1)\sin QD]}, \quad A_< = \frac{2\sqrt{\varepsilon}}{[2\sqrt{\varepsilon}\cos QD + i(\varepsilon+1)\sin QD]}. \tag{8}$$

We note, in particular, that (4) and (7) lead to a non-vanishing probability density of the photon being in the $-D<z'<0$ region before the photon meets its fate, even if it is reflected.

The probabilities of reflection and transmission are, respectively,

$$|A_>|^2 = \frac{(\varepsilon-1)^2\sin^2 QD}{[4\varepsilon + (\varepsilon-1)^2\sin^2 QD]}, \quad |A_<|^2 = \frac{4\,\varepsilon}{[4\varepsilon + (\varepsilon-1)^2\sin^2 QD]}, \tag{9}$$

and we note that the probability of transmission never vanishes, while the probability of reflection may or may not vanish depending on the value of $\sin^2 QD$. More interestingly, we may rewrite the amplitude of reflection as

$$A_> = \frac{(\varepsilon-1)\,|\sin QD|}{\sqrt{[4\varepsilon + (\varepsilon-1)^2\sin^2 QD]}}\, e^{i\vartheta}, \qquad \cos\vartheta = -\frac{(\varepsilon+1)\,|\sin QD|}{\sqrt{[4\varepsilon + (\varepsilon-1)^2\sin^2 QD]}}, \tag{10}$$

where $e^{i\vartheta}$ is the final *phase* factor acquired by the amplitude upon reflection, with the factor multiplying it, representing the contracted magnitude of the amplitude from the initial unit one of emission.

Following Feynman, we consider red and blue light. For the purpose of illustrations, for the permittivity of the glass, we take $\varepsilon = (3+\sqrt{3})/2$. For red and blue light, we may take $\lambda_{\rm red} = 3[3+\sqrt{3}]^{1/2}\,10^{-5}$ cm, $\lambda_{\rm blue} = 3[(3+\sqrt{3})/2]^{1/2}\,10^{-5}$ cm. Let us consider a glass of thickness $D = 3\sqrt{2}/2$ cm, and recall that $Q = \sqrt{\varepsilon}\,q$, and $q = 2\pi/\lambda$.

As a function of QD, $\cos\vartheta$ in (10) is periodic with period π, and the phase angles ϑ and QD are simply related. The above leads to

$$\sqrt{\varepsilon}\, q_{\rm red}\, D = 2\pi(50000), \qquad \sqrt{\varepsilon}\, q_{\rm blue}\, D = 2\pi(70710 + 0.6781). \tag{11}$$

That is, referring to (9), after 50000 full cycles, this red light cannot be reflected, while after 70710 full cycles plus a non-zero angle of 244.1°, the blue light has a non-zero probability of being reflected. The moral of this is the following. Due to the probabilistic nature of the problem, the question "How does a photon make up its mind..." is, in general, not known as pointed out by Feynman [1]. This is unlike the present situation, with this red light, however, where the problem is a *deterministic one, and the photon has no option in making up its mind in which way to go, and is transmitted with a 100% probability*. The quantitative description given here, where *amplitudes and their acquired phases* may be computed directly from the causal Schwinger–Feynman *propagators*, in the presence of obstacles, may be also developed for other situations as well. We hope that the present investigation and its simplicity will inspire other practitioners for causal Schwinger–Feynman propagators descriptions in quantum optics with their inherited quantum mysteries, in general, in the presence of such intervening obstacles. We have particularly in mind to consider different media, as well as the inherited non-linearities encountered in quantum field theory which have no classical counterparts. Such generalizations will be addressed in a future report.

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