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Short Communication

Vacuum-to-vacuum transition probability and radiation in a medium

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HIGHLIGHTS

- Quantum viewpoint of radiation in a medium based on the vacuum-to-vacuum transition probabilities.
- Mathematical method in handling radiation in a medium for arbitrary sources.
- Radiated energy and power for arbitrary current distributions in a medium.
- Explicit power of radiation in a medium in a bounded region.

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ABSTRACT

We recast the vacuum-to-vacuum transition probability for the description of radiation in an isotropic medium of permeability μ, and permittivity ε, in a form which brings us in contact with radiation theory in vacuum. Using the inherited property of such a system, with arbitrary current distributions, of emitting photons via the Poisson distribution, the average number of photons emitted in such a medium is directly obtained from which the power of radiation is readily extracted. As an application, the power of radiation, emitted by a charged particle, in a medium trapped between perfectly conducting neutral parallel plates for arbitrary finite separations is explicitly obtained.

1. Introduction

We consider a homogeneous and isotropic medium of permeability μ and permittivity ε. The Minkowski metric used in this work is defined by $\eta_{\mu\nu} = \mathrm{diag}[-1, 1, 1, 1]$. To describe photons in such a medium, one simply scales $F_{0i}F^{0i} \to \varepsilon F_{0i}F^{0i}$, and $F_{ij}F^{ij} \to F_{ij}F^{ij}/\mu$, in the Lagrangian density, where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. That is, the Lagrangian density becomes ($i, j = 1, 2, 3$)

$$\mathcal{L} = -\frac{1}{4\mu}F_{ij}(x)F^{ij}(x) - \frac{\varepsilon}{2}F_{0i}(x)F^{0i}(x) + J^\mu(x)A_\mu(x). \tag{1}$$

Note that the scaling factors are not ε^2, $1/\mu^2$, respectively, as one may naïvely expect. The reason is that the variations of the action, with respect to the vector potential, involving the quadratic terms $\varepsilon F_{0i}F^{0i}$, $F_{ij}F^{ij}/\mu$, generate the linear terms corresponding to the electric and magnetic field components which are just needed in deriving Maxwell's equations.

We recast the theory in a form which brings us into contact with our earlier treatment (Manoukian, 2015) dealing with radiation in vacuum, from which radiation from an arbitrary current distribution in a medium may be considered in a very general way. We recapitulate the method of study via the vacuum-to-vacuum transition probability in describing radiation. To this end, note that prior to switching on of the current, as a source of photon production, one is dealing with a vacuum state, denoted by $|0_-\rangle$, involving no photons. After switching on of the current, the state of the system may evolve to one involving any number of photons, or it may just stay in the vacuum state, involving no photons, with the latter state now denoted by $|0_+\rangle$. Quantum theory tells us that the vacuum-to-vacuum transition probability satisfies the inequality $|\langle 0_+|0_-\rangle|^2 < 1$, due to conservation of probability, allowing the possibility that the system may evolve to other states as well involving an arbitrary number of photons that may be created by the current source. A very interesting property of this system is that the probability distribution of the photon number N created by the current (Schwinger, 1970) is given by the Poisson distribution (Manoukian, 2011). That is

$$\mathrm{Prob}[N = n] = \frac{(\lambda)^n}{n!}\mathrm{e}^{-\lambda}, \quad n = 0, 1, \ldots, \tag{2}$$

$$\lambda = \langle N \rangle, \tag{3}$$

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where $\lambda = \langle N\rangle$ denotes the average number of photons created by the current source, and

$$\exp[-\langle N\rangle] = |\langle 0_+|0_-\rangle|^2, \tag{4}$$

denotes the probability that no photons are created by the current source, i.e., it represents the vacuum-to-vacuum transition probability $|\langle 0_+|0_-\rangle|^2$ as just stated.

Quantum viewpoint analysis of electromagnetic phenomena and electromagnetic radiation, and of related applications, e.g., Manoukian (1991, 1997, 2013, 2015), Manoukian and Charuchittapan (2000), Manoukian and Viriyasrisuwattana (2006), Feynman (1985), Bialynicki-Birula (1996), Kennedy et al. (1980), Deitsch and Candelas (1979), and Schwinger et al. (1976), turns out to be quite useful in applications and certainly in simplifying, to a large extent, derivations in this field.

The derivation given below is quite general and applies to arbitrary current distributions, and is expected to be of interest in other applications and, in particular, in media involving obstacles and in periodic configuration (Bellucci and Maisheev, 2006) and radiation from varying sources (e.g., Budko, 2009; Gal'tsov et al., 2007; Bessonov, 2006; Manoukian, 1991), in general, as well as for further direct generalizations involving quantum corrections. These and other directions of research mentioned below will be attempted in future work.

In Section 2 a general expression is derived, via the vacuum-to-vacuum transition probability, for the average number of photons emitted from an arbitrary current distributions from which the power of radiation from such currents can be readily extracted. For completeness, and for the convenience of the reader, a direct derivation of the classic Čerenkov power of radiation, in infinite extended media, is derived in Section 3 as a preparation for handling more complicated situations. In Section 4, such radiation is considered in a bounded medium consisting of a slab confined between two parallel conducting neutral plates, separated by a finite distance, and an explicit expression for the power of radiation is derived showing the power of the formalism.

2. Average number of photons emitted

We carry out the following scalings in the action involving the Lagrangian density in (1):

$$x^0 = \sqrt{\mu\varepsilon}\,\underline{x}^0, \quad \mathbf{x} = \underline{\mathbf{x}}, \tag{5}$$

$$\partial_0 = \frac{1}{\sqrt{\mu\varepsilon}}\underline{\partial}_0, \quad \partial_i = \underline{\partial}_i, \tag{6}$$

$$A^0(x) = \frac{1}{\mu^{1/4}\varepsilon^{3/4}}\underline{A}^0(x), \tag{7}$$

$$\mathbf{A}(x) = \frac{\mu^{1/4}}{\varepsilon^{1/4}}\underline{\mathbf{A}}(x), \tag{8}$$

$$J^0(x) = \frac{\varepsilon^{1/4}}{\mu^{1/4}}\underline{J}^0(x), \tag{9}$$

$$\mathbf{J}(x) = \frac{1}{\mu^{3/4}\varepsilon^{1/4}}\underline{\mathbf{J}}(x). \tag{10}$$

The action integral, up to an overall factor, thus takes the form

$$W = \int(\mathrm{d}\underline{x})\Big[-\frac{1}{4}\big(\underline{\partial}_\mu\underline{A}_\nu(x) - \underline{\partial}_\nu\underline{A}_\mu(x)\big)\big(\underline{\partial}^\mu\underline{A}^\nu(x) - \underline{\partial}^\nu\underline{A}_\mu(x)\big) + \underline{J}^\mu(x)\underline{A}_\mu(x)\Big]. \tag{11}$$

Note that the argument of $\underline{A}^\mu(x)$ is x and not $\underline{x}$, also that

$$\underline{\partial}_\mu\underline{J}^\mu(x) = \mu^{3/4}\varepsilon^{1/4}\partial_\mu J^\mu(x). \tag{12}$$

The vacuum-to-vacuum transition amplitude is then simply

$$\langle 0_+|0_-\rangle = \exp\Big[\frac{\mathrm{i}}{2\hbar c^3}\int(\mathrm{d}\underline{x})\,(\mathrm{d}\underline{x}')\underline{J}_\mu(x)D_+^{\mu\nu}(\underline{x},\underline{x}')\underline{J}_\nu(x')\Big], \tag{13}$$

as inferred from theory formulated in vacuum, e.g., Manoukian (2015), where $D^{\mu\nu}(x,x')$ is the photon propagator determined below for the cases considered.

The vacuum-to-vacuum transition probability then follows directly from (13) to be

$$|\langle 0_+|0_-\rangle|^2 = \exp\Big[-\frac{1}{\hbar c^3}\int(\mathrm{d}\underline{x})\,(\mathrm{d}\underline{x}')\underline{J}_\mu(x)\Big(\mathrm{Im}D_+^{\mu\nu}(\underline{x},\underline{x}')\Big)\underline{J}_\nu(x')\Big], \tag{14}$$

where $(\mathrm{d}x) \equiv \mathrm{d}x^0\,\mathrm{d}x^1\,\mathrm{d}x^2\,\mathrm{d}x^3$, from which the average number of photons emitted by the arbitrary current distribution is given by

$$\langle N\rangle = \frac{1}{\hbar c^3}\int(\mathrm{d}\underline{x})\,(\mathrm{d}\underline{x}')\underline{J}_\mu(x)\Big(\mathrm{Im}D_+^{\mu\nu}(\underline{x},\underline{x}')\Big)\underline{J}_\nu(x'). \tag{15}$$

The latter may be more conveniently rewritten as

$$\langle N\rangle = \frac{1}{\mu\varepsilon\hbar c^3}\int(\mathrm{d}x)(\mathrm{d}x')\underline{J}_\mu(x)\Big(\mathrm{Im}\,D_+^{\mu\nu}(\underline{x},\underline{x}')\Big)\underline{J}_\nu(x'). \tag{16}$$

The above expression is valid for any current distribution. We consider a charged particle of charge e in the medium moving, without loss of generality, along the x^1-axis with speed v. The associated current distribution is given by

$$J^i(x) = \mathrm{e}v\delta^{i1}\delta(x^2)\delta(x^3)\delta\Big(x^1 - \frac{v}{c}x^0\Big), \tag{17}$$

$$J^0(x) = \mathrm{e}c\delta(x^2)\delta(x^3)\delta\Big(x^1 - \frac{v}{c}x^0\Big). \tag{18}$$

We work in the celebrated radiation gauge $A^0 = 0$, then the components $D_+^{ij}(x,x')$, $i,j = 1,2,3$, of the photon propagator satisfy, e.g., Lifshitz and Pitaevskii (1984) ($\nu = 0,1,2,3,\ i,j = 1,2,3$) in vacuum

$$[-\partial_\nu\partial^\nu\delta^{ij} + \partial^i\partial^j]D_+^{jk}(x,x') = \delta^{ik}\delta^{(4)}(x,x'). \tag{19}$$

3. Medium of unbounded extension

For an infinite extension, the 4D delta function $\delta^{(4)}(x,x') \equiv \delta^{(4)}(x-x')$ is simply given by

$$\delta^{(4)}(x-x') = \int\frac{(\mathrm{d}Q)}{(2\pi)^4}\mathrm{e}^{\mathrm{i}Q(x-x')} \tag{20}$$

With motion along the x^1-axis, the component of the propagator of interest is $D_+^{11}(\underline{x}-\underline{x}')$ and is given by

$$D_+^{11}(\underline{x}-\underline{x}') = \int\frac{(\mathrm{d}Q)}{(2\pi)^4}\frac{\mathrm{e}^{\mathrm{i}\mathbf{Q}\cdot(\mathbf{x}-\mathbf{x}')}\mathrm{e}^{-\mathrm{i}Q^0(x^0-x'^0)/\sqrt{\mu\varepsilon}}}{\Big[\mathbf{Q}^2 - Q^{0^2} - \mathrm{i}\delta\Big]}, \quad \delta \to 0. \tag{21}$$

From the conservation of the current $\partial_\mu J^\mu(x) = 0 = \underline{\partial}_\mu\underline{J}^\mu(x)$ (see (12)), we obtain for $\langle N\rangle$

$$\langle N\rangle = \frac{\mu^{1/2}}{\varepsilon^{1/2}\hbar c^3}\int(\mathrm{d}x)\big(\mathrm{d}x'\big)\Big(\mathbf{J}(x)\cdot\mathbf{J}\big(x'\big) - \frac{1}{\mu\varepsilon}J^0(x)J^0\big(x'\big)\Big)$$
$$\times\int\frac{\mathrm{d}^3\mathbf{Q}}{(2\pi)^3 2|\mathbf{Q}|}\mathrm{e}^{\mathrm{i}\mathbf{Q}\cdot(\mathbf{x}-\mathbf{x}')}\mathrm{e}^{-\mathrm{i}|\mathbf{Q}|(x^0-x'^0)/\sqrt{\mu\varepsilon}}, \tag{22}$$

using, in the process, the symmetry under the interchange $(x - x') \leftrightarrow (x' - x)$.

Upon inserting the identity

$$1 = \int_0^\infty d\omega\, \delta\left(\omega - \frac{|\mathbf{Q}|c}{\sqrt{\mu\varepsilon}}\right), \tag{23}$$

in the integrand in (22), we get $\langle N\rangle = \int_0^\infty d\omega\, N(\omega)$,with $\mathbf{Q} = |\mathbf{Q}|\mathbf{n}$, $|\mathbf{Q}| = \left(\left(Q^1\right)^2 + |\mathbf{Q}_\parallel|^2\right)^{1/2}$, which from (17), (18)

$$N(\omega) = \frac{e^2 v^2}{\varepsilon\hbar c^2}\left(1 - \frac{c^2}{\mu\varepsilon v^2}\right)\int dx^0\, dx'^0 \frac{dQ^1\left(\pi d|\mathbf{Q}_\parallel|^2\right)}{16\pi^3\omega} \times \delta\left(\omega - \frac{|\mathbf{Q}|c}{\sqrt{\mu\varepsilon}}\right)\exp\left[i\left(\frac{Q^1 v}{c} - \frac{\omega}{c}\right)\left(x^0 - x'^0\right)\right]. \tag{24}$$

To obtain the total energy of radiation $E(\omega)$ of associated angular frequency around a value ω, we simply have to multiply the above expression by $\hbar\omega$. Finally by introducing the variable $\tau = (x^0 - x'^0)/c$ and carrying out the integrals over Q^1, $|\mathbf{Q}_\parallel|^2$, τ, the expression for the celebrated Čerenkov power of radiation emerges

$$P(\omega) = \frac{e^2}{4\pi}\frac{\mu\omega}{c}\frac{v}{c}\left(1 - \frac{c^2}{\mu\varepsilon v^2}\right), \quad v > c/\sqrt{\mu\varepsilon}, \tag{25}$$

and the latter does not vanish only for $v\sqrt{\mu\varepsilon}/c > 1$, as a consequence of the delta function in (24) which gives $|\mathbf{Q}_\parallel|^2 = \mu\varepsilon\omega^2(1 - c^2/\mu\varepsilon v^2)/c^2 > 0$, with $Q^1 = \omega/v$.

The expression in (25) cannot be integrated over for arbitrary large ω. A quantum correction treatment, however, provides a natural cut-off in ω (see, e.g., Manoukian and Charuchittapan, 2000). It is interesting that astronauts during Apollo missions have reported of "seeing" flashes of light even with their eyes closed. An explanation of this was generally attributed to high energy cosmic particles, encountered freely in outer space, that would pass through one's eyelids causing Čerenkov radiation (Čerenkov, 1936; Tamm and Frank, 1937; Jelley, 1958; Belousov, 2006; Lidvansky, 2006; Grichine, 2006) to occur within one's eye itself, see, e.g., Fazio et al. (1970), Pinsky et al. (1974), and McNulty et al. (1976).

4. Medium of bounded extension

It is more practical to consider a bounded extension, such as of radiation emitted, say, within a slab of arbitrary finite width a consisting for simplicity of a region restricted between two parallel perfectly conducting neutral plates. With the plates parallel to the x^1–x^2 plane, situated at $x^3 = \pm a/2$, the 4D dimensional delta function $\delta^{(4)}(x, x')$, for the problem at hand, derived in the appendix, is given by

$$\delta^{(4)}\left(x, x'\right) = \int \frac{d^2\mathbf{Q}_\parallel}{(2\pi)^2} e^{i\mathbf{Q}_\parallel\cdot\left(\mathbf{x}_\parallel - \mathbf{x}'_\parallel\right)} \int \frac{dQ^0}{(2\pi)} e^{-iQ^0\left(x^0 - x'^0\right)} \times \sum_{n=1}^{\infty}\frac{2}{a}\sin\frac{n\pi\left(z - \frac{a}{2}\right)}{a}\sin\frac{n\pi\left(z' - \frac{a}{2}\right)}{a}, \tag{26}$$

breaking translational invariance along the $x^3 \equiv z$-axis, and where, in this case, $\mathbf{x}_\parallel \equiv (x^1, x^2, 0)$, $\mathbf{Q}_\parallel = (Q_1, Q_2, 0)$. The relevant part of the propagator $D_+^{11}(\underline{x}, \underline{x}')$ from (19) reads

$$D_+^{11}(\underline{x}, \underline{x}') = \int \frac{d^2\mathbf{Q}_\parallel}{(2\pi)^2} e^{i\mathbf{Q}_\parallel\cdot(\mathbf{x}_\parallel - \mathbf{x}'_\parallel)} \times \int \frac{dQ^0}{(2\pi)} e^{-iQ^0(x^0 - x'^0)/\sqrt{\mu\varepsilon}} \times \sum_{n=1}^{\infty}\frac{2}{a}\sin\frac{n\pi\left(z - \frac{a}{2}\right)}{a}\sin\frac{n\pi\left(z' - \frac{a}{2}\right)}{a} \times \frac{1}{\left[\mathbf{Q}_\parallel^2 - Q^{0 2} + \frac{n^2\pi^2}{a^2} - i\delta\right]}, \quad \delta \to 0. \tag{27}$$

Again using conservation of the current as in Section 3, making use of the identity

$$1 = \int_0^\infty d\omega\delta\left(\omega - \frac{\sqrt{(Q_1)^2 + (Q_2)^2 + \frac{n^2\pi^2}{a^2}}\,c}{\sqrt{\mu\varepsilon}}\right), \tag{28}$$

and of (17), (18), one readily obtains

$$P(\omega) = \frac{e^2 v^2}{\varepsilon c}\left(1 - \frac{c^2}{\mu\varepsilon v^2}\right)\int_{-\infty}^{\infty} d\left(x^0 - x'^0\right) \times \sum_{n=1}^{\infty}\frac{2}{a}\sin^2\left(\frac{n\pi}{2}\right)\frac{dQ_1\, dQ_2}{8\pi^2}\delta\left(\omega - \frac{\sqrt{(Q_1)^2 + (Q_2)^2 + \frac{n^2\pi^2}{a^2}}\,c}{\sqrt{\mu\varepsilon}}\right) \times \exp\left[i\left(Q_1 - \frac{\omega}{v}\right)\left(x^0 - x'^0\right)/c\right], \tag{29}$$

which upon integrations over $(x^0 - x'^0)$ and over Q_1, one obtains

$$P(\omega) = \frac{e^2 v}{4\pi\varepsilon}\left(1 - \frac{c^2}{\mu\varepsilon v^2}\right)\sum_{n=1}^{\infty}\frac{2}{a}\sin^2\left(\frac{n\pi}{2}\right) \times 2\int_0^\infty d|Q_2|\delta\left(\omega - \frac{\sqrt{\frac{\omega^2}{v^2} + |Q_2|^2 + \frac{n^2\pi^2}{a^2}}\,c}{\sqrt{\mu\varepsilon}}\right). \tag{30}$$

The delta function, as a function of $|Q_2|$, may be rewritten as

$$\delta\left(\omega - \frac{\sqrt{\frac{\omega^2}{v^2} + |Q_2|^2 + \frac{n^2\pi^2}{a^2}}\,c}{\sqrt{\mu\varepsilon}}\right) = \frac{\mu\varepsilon\omega}{c^2|Q_2|}\delta\left(|Q_2| - \sqrt{\frac{\omega^2\mu\varepsilon}{c^2}\left(1 - \frac{c^2}{\mu\varepsilon v^2}\right) - \frac{n^2\pi^2}{a^2}}\right), \tag{31}$$

and

$$\frac{\omega^2\mu\varepsilon}{c^2}\left(1 - \frac{c^2}{\mu\varepsilon v^2}\right) - \frac{n^2\pi^2}{a^2} > 0, \tag{32}$$

which gives

$$P(\omega) = \left(1 - \frac{c^2}{\mu\varepsilon v^2}\right)\frac{\mu e^2}{2\pi}\frac{\omega}{c}\frac{v}{c}\sum_{n=1}^{'}\frac{\frac{2}{a}\sin^2\frac{n\pi}{2}}{\sqrt{\frac{\omega^2\mu\varepsilon}{c^2}\left(1 - \frac{c^2}{\mu\varepsilon v^2}\right) - \frac{n^2\pi^2}{a^2}}}, \tag{33}$$

where $\sum'_{n=1}$ is a sum over all odd positive integers n for which (32) is satisfied. Hence it is a sum over a *finite* number of terms for a given $\omega^2\mu\varepsilon\left(1 - c^2/\mu\varepsilon v^2\right)/c^2 > 0$. For example for $a^2\omega^2\mu\varepsilon\left(1 - c^2/\mu\varepsilon v^2\right)/\pi^2c^2 = 8$, we simply have $n=1$, since for $n=2$ the sine function vanishes. That is, in general, the sum is over all $n = 1, 3, \ldots$, up to the largest odd positive integer for which (32) is satisfied. The singularities that arise in (33), as a function of ω, for the n equal to odd integers, denote the resonant frequencies.

The simplicity of the derivations, and the power of the formalism, given above should be noticed. The results derived in Section 2 are quite general and are valid for arbitrary current distributions and are expected to be applicable in other problems as well. Some of such applications and further generalizations were mentioned at the end of Section 1, and will be attempted in a future report.

The present method of analysis is expected to have also applications in gravitational radiation (Manoukian, 1990; Lambiase, 2001; Gupta et al., 1995), where the polarization modes of gravitation are far more complicated than in electromagnetic ones, especially as arising in various theories of gravitation of higher order derivatives (Capozziello et al., 2009). C̆erenkov-like radiation methods of electrodynamics have been also extended to gluon emission in quantum chromodynamics media (e.g., Kämper and Pavlenko, 2000), where the gluon replaces the photon but with an additional complication that it has also a self-interaction.

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Appendix A. Derivation of Eq. (26)

The vanishing boundary condition due to the perfectly conducting plates, e.g., at $z = a/2$ allows us to expand the one dimensional Dirac delta function $\delta(z, z')$ in a Fourier sine series as follows:

$$\delta(z, z') = \sum_{n=1}^{\infty} a_n(z') \sin \frac{n\pi\left(z - \frac{a}{2}\right)}{a}, \tag{34}$$

where the expansion coefficients $a_n(z')$ obviously depend on z'. [We may equally well carry out a Fourier sine series in $\sin[n\pi(z' - a/2)/a]$ to begin with.] Upon multiplying the above equation by $\sin[m\pi(z - a/2)/a]$ and integrating over z from $-a/2$ to $+a/2$ we obtain

$$\sin \frac{m\pi\left(z' - \frac{a}{2}\right)}{a} = \sum_{n=1}^{\infty} a_n(z') \int_{-a/2}^{+a/2} \mathrm{d}z \times \sin \frac{m\pi\left(z - \frac{a}{2}\right)}{a} \sin \frac{n\pi\left(z - \frac{a}{2}\right)}{a}. \tag{35}$$

Upon introducing the new variable of integration $u = (a/2 - z)/a$, the z-integral in the summand may be rewritten as

$$a \int_0^1 \mathrm{d}u \, \sin(m\pi u)\sin(n\pi u) = \frac{a}{2}\delta_{mn}, \tag{36}$$

from which $a_n(z') = (2/a)\sin\left[n\pi(z' - a/2)/a\right]$. The delta functions corresponding to $\mathbf{x}_{\parallel}$, and along the x^0-direction are given by the well known exponential representations of the Dirac delta function over infinite extensions. The four dimensional Dirac delta function for the problem at hand then becomes as given in Eq. (26).

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