NON-MINIMAL DERIVATIVE COUPLING TO RICCI SCALAR GRAVITY

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Yuttana Jawralee

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ABSTRACT

In this work, we investigate a model of dark energy in which the scalar field kinetic term and a simple NMDC gravity with $\xi R \partial_a \phi \partial^a \phi$ term is considered a flat universe. The model is a trial model by transforming all the field to its logarithm mapping $\phi' = \mu \ln \phi$. Equation of motion, scalar field solution and scalar potential are derived when we assume slow-roll approximation. The field solution and scalar potential are found for power-law, super acceleration and de-Sitter expansions.

CHAPTER I

INTRODUCTION

1.1 Background and motivation

Recently, our universe is not only expanding but also accelerating due to some kind of unknown energy form called *dark energy*, which has been comfirmed by the cosmic acceleration reported in 1998 [1, 2]. This dark energy is usually in the form of either cosmological constant or scalar field [3, 4, 5, 6]. There are many scalar field models proposed to describe the acceleration of the expansion of the universe, for example, quintessence models [7, 8, 9, 10, 11, 12], k-essence models [13, 14].

Modifications of gravity, for instance, $f(R)$ theory [15, 16], the scalar field kinetic term is non-minimally coupled to the curvature [17, 18, 19, 20, 21, 22, 23], and others are as well possible answers of present acceleration. Accomplishing the acceleration needs the effective equation of state of matter species that a scalar field evolving under its potential to give rise to the negative pressure, $p < -\rho c^2/3$.

In the scalar-tensor theories, we can extend the theories for non-minimal coupling (NMC) between scalar fields and Ricci scalar in GR in form of $\sqrt{-g}f(\phi)R$, where $f(\phi)$ is a function of scalar field ϕ and R is the Ricci scalar. The non-minimal coupling (NMC) is motivated by scalar-tensor theories of the Jordan-Brans-Dicke models [24, 25], renormalizing term of quantum field theory in curved space [26] and in multidimensional theories like superstring theory [27, 28, 29, 30, 31].

In the context of inflationary cosmology, first cosmology consideration of non-minimal derivative coupling (NMDC) of scalar field was proposed by Amendola in 1993 [17]. Therein the function of coupling is in form $f(\phi, \partial_a\phi)$. In a derivative coupling to Ricci tensor has been considered to study cosmological on the coupling parameter was proposed to study late time cosmological dynamics.

In this thesis, we consider the simplest NMDC model with only term of $\xi R\partial_a\phi\partial^a\phi$ and free kinetic term. We propose modification to the model to improve the NMDC term in the small field value range. Therefore the NMDC model can be either considered as dark energy model or inflationary model when a field transformation introduced to the model. We will find solutions of the scalar field and scalar potential when the power-law, super-acceleration and de-Sitter expansion were considered .

We use units such that $c = \hbar = 1$, where c is the speed of light and $\hbar$ is reduced Planck's constant. The gravitational constant G is related to the Planck mass $m_{\rm pl} = 1.2211 \times 10^{19}$GeV via $G = 1/m_{\rm pl}^2$ and the reduced Planck mass $M_{\rm pl} = 2.4357 \times 10^{18}$GeV via $8\pi G = 1/M_{\rm pl}^2$, respectively. The signature of the metric is assumed to be $(-,+,+,+)$.

List of natural units

$[\text{mass}] = [\text{energy}] = [\text{M}]$

$[\text{Length}] = [\text{time}] = [\text{L}] = [\text{M}^{-1}]$

$[G] = [\text{L}^2] = [\text{M}^{-2}]$

$[H] = [\text{L}^{-1}] = [\text{M}]$

$[g_{ab}] = [1]$

$[R_{ab}] = [R] = [\Lambda] = [\text{L}^{-2}] = [\text{M}^2]$

$[T_{ab}] = \frac{[\text{M}]}{[\text{L}^3]} = \frac{1}{[\text{L}^4]} = [\text{M}^4]$

$[\phi] = [\partial_a] = [\text{mass}] = [\text{M}]$

$[\partial_a\phi] = [\text{mass}^2] = [\text{M}^2]$

$[\mu] = [\text{mass}] = [\text{M}] = [\text{L}^{-1}]$

$[\xi] = [\text{M}^{-2}] = [\text{L}^2]$

1.2 Objectives

Our objectives are to study general aspects of FLRW cosmology of the NMDC coupling to Ricci curvature R model. Then we study cosmological field equation of this model. After that, we will obtain particular solutions of the scalar field and scalar potential corresponding to power-law, super-acceleration and de-Sitter expansion.

1.3 Frameworks

In this thesis, the introduction and motivation of our works, objectives, and frameworks are shown in Chapter 1. In Chapter 2, we review basic cosmology and introduce the background equations, for example, the Friedmann equations, fluid equation and field equations. Then we review the non-minimal derivative coupling (NMDC) of scalar field and determine the necessarily equations of the model in Chapter 3. Chapter 4 is results and discussions. The last Chapter is the conclusions.

CHAPTER II

STANDARD COSMOLOGY

2.1 Cosmological Principle

The fundamental of modern cosmology is belief that the place that we live in the Universe is not a special location. This is based on cosmological principle, and it follows observations that the Universe on large scales is homogeneous and isotropic. Homogeneity is the states that the Universe looks the same at each point, while isotropy states that the Universe looks the same in all directions.

Note that homogeneity does not imply isotropy and vice versa, for example, a uniform electric field is homogeneous field, at all points are the same, but it is not isotropic at one point because directions of the field can be distinguished from those perpendicular to them. However, the universe looks approximately homogenous and isotropic on the large scales. One strong evidence to support this is the CMB observed in 1995 by the COBE mission, at least to one part in 10^5 [32].

2.2 Hubble's law

In 1920, Edwin Hubble proved that the further galaxies are moving away from the Earth. Hubble realized a relation between the velocity of recession and distance of an object from us

$$\mathbf{v} = H_0 \mathbf{r}, \tag{2.1}$$

where $\mathbf{r}$ is physical distance. This is known as Hubble's law, and the constant H_0 is known as the Hubble's constant at present time t_0. Let consider the velocity of recession is given by

$$\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}. \tag{2.2}$$

and the relationship between physical distance $\mathbf{r}$ and the co-moving distance $\mathbf{x}$ can be written as

$$\mathbf{r} = a(t)\mathbf{x} \tag{2.3}$$

The quantity $a(t)$ is the scale factor. It is a function of time only, while $\mathbf{x}$ is a co-moving coordinate which is fixed by the definition. So

$$\mathbf{v} = \frac{\dot{a}(t)}{a(t)}\mathbf{r} \tag{2.4}$$

The Hubble's parameter defined as $H(t) \equiv \dot{a}(t)/a(t)$, and therefore the Hubble's law can be written as

$$\mathbf{v} = H(t)\mathbf{r}. \tag{2.5}$$

2.3 The Friedmann Robertson Walker Universe

The metric or line-element that describes a 4-dimensional homogeneous and isotropic space-time is called Friedmann-Lemaître-Robertson-Walker (FLRW) space-time and is given by [33, 34]

$$\begin{aligned} \mathrm{d}s^2 &= g_{ab}\mathrm{d}x^a\mathrm{d}x^b, \\ &= -c^2\mathrm{d}t^2 + a^2(t)\mathrm{d}\sigma^2 \end{aligned} \tag{2.6}$$

where g_{ab} is the metric tensor, $a(t)$ is the scale factor with cosmic time t, $\mathrm{d}\sigma^2$ is the time-independent metric of the 3-dimensional space with a constant curvature K.

$$\mathrm{d}\sigma^2 = \frac{\mathrm{d}r^2}{1-Kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2). \tag{2.7}$$

Therefore, the line-element in equation (2.6) now become

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-Kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)\right], \tag{2.8}$$

This is known as the Robertson-Walker metric. Where K is the curvature and takes the values -1, 0, or 1 depending on whether the spatial section has negative, zero or positive curvature respectively. Thus the covariant components g_{ab} of the metric are

$$\begin{aligned} g_{00} &= -c^2, \\ g_{11} &= \frac{a^2(t)}{1-Kr^2}, \\ g_{22} &= a^2(t)r^2, \\ g_{33} &= a^2(t)r^2\sin^2\theta. \end{aligned}$$

The Einstein's field equation has the following form

$$G_{ab} \equiv R_{ab} - \frac{1}{2}g_{ab}R = \frac{8\pi G}{c^4}T_{ab}. \tag{2.9}$$

where G_{ab} is Einstein tensor, R and R_{ab} are Ricci scalar and the Ricci tensor of the metric g_{ab}. G is the gravitational constant and T_{ab} is the matter stress-energy tensor.

From g_{ab} we can obtain the connection or the Chritoffel symbol as

$$\Gamma^e_{ab} = \frac{1}{2}g^{ec}(\partial_b g_{ca} + \partial_a g_{cb} - \partial_c g_{ab}) \tag{2.10}$$

and we have the only non-zero coefficients are

$$\begin{aligned} \Gamma^0_{11} &= \frac{a\dot{a}}{c(1-Kr^2)}, \quad \Gamma^0_{22} = \frac{a\dot{a}r^2}{c}, \quad \Gamma^0_{33} = \frac{a\dot{a}r^2\sin^2\theta}{c}, \\ \Gamma^1_{01} &= \frac{\dot{a}}{ca}, \quad \Gamma^1_{11} = \frac{Kr}{1-Kr^2}, \quad \Gamma^1_{22} = -r(1-Kr^2), \\ \Gamma^1_{33} &= -r(1-Kr^2)\sin^2\theta, \\ \Gamma^2_{02} &= \frac{\dot{a}}{ca}, \quad \Gamma^2_{12} = \frac{1}{r}, \quad \Gamma^2_{33} = \sin\theta\cos\theta, \\ \Gamma^3_{03} &= \frac{\dot{a}}{ca}, \quad \Gamma^3_{13} = \frac{1}{r}, \quad \Gamma^3_{23} = \cot\theta. \end{aligned}$$

Next we need to find the components of the Ricci tensor which is defined by

$$R^d{}_{abc} = \partial_c\Gamma^d_{ab} - \partial_b\Gamma^d_{ac} + \Gamma^e_{ab}\Gamma^d_{ec} - \Gamma^e_{ac}\Gamma^d_{eb} \tag{2.11}$$

when the upper index is the same as the last index of the Reimann tensor $R^d{}_{abc}$ we will obtain the Ricci tensor, $R^c{}_{abc} = R_{ab}$, as

$$R_{ab} = \partial_c \Gamma^c_{ab} - \partial_b \Gamma^c_{ac} + \Gamma^e_{ab}\Gamma^c_{ec} - \Gamma^e_{ac}\Gamma^c_{eb} \tag{2.12}$$

We find that the off-diagonal components of the Ricci tensor are zero and the diagonal components are given by

$$\begin{aligned} R_{00} &= -\frac{3}{c^2}\frac{\ddot{a}}{a}, \\ R_{11} &= \frac{a\ddot{a} + 2\dot{a}^2 + 2Kc^2}{(1-Kr^2)c^2}, \\ R_{22} &= \frac{(a\ddot{a} + 2\dot{a}^2 + 2Kc^2)r^2}{c^2}, \\ R_{33} &= \frac{(a\ddot{a} + 2\dot{a}^2 + 2Kc^2)r^2\sin^2\theta}{c^2}. \end{aligned}$$

and the Ricci tensor is then

$$R = R^a_a = g^{ab}R_{ab} = 6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2}\right]. \tag{2.13}$$

Let us now consider equation (2.9),

$$R_{ab} - \frac{1}{2}g_{ab}R \;=\; \frac{8\pi G}{c^4}T_{ab}. \tag{2.14}$$

It is convenient to express the field equations in the alternative form

$$R_{ab} \;=\; \frac{8\pi G}{c^4}\left(T_{ab} - \frac{1}{2}g_{ab}T\right). \tag{2.15}$$

In the FRW spacetime the energy-momentum tensor of the perfect fluid takes the form [35]

$$T^{ab} = \left(\rho + \frac{P}{c^2}\right)u^a u^b + Pg^{ab} \tag{2.16}$$

where u^a is 4-velocity, ρ and P are mass density and pressure respectively. When we contracted the energy-momentum tensor by g_{ab} we will obtain the trace of the energy-momentum tensor

$$\begin{aligned} g_{ab}T^{ab} &= g_{ab}\left(\rho + \frac{P}{c^2}\right)u^a u^b + Pg_{ab}g^{ab} \\ T &= \left(\rho + \frac{P}{c^2}\right)(-c^2) + 4P \end{aligned} \tag{2.17}$$

where $u^a u_a = u^a g_{ab} u^b = -c^2$, and therefore

$$T = -\rho c^2 + 3P. \tag{2.18}$$

From equation (2.15) we can write the energy-momentum tensor as

$$T_{ab} = \left(\rho + \frac{P}{c^2}\right) u_a u_b + P g_{ab} \tag{2.19}$$

and we find that T_{00}, T_{11}, T_{22}, and T_{33} components are

$$\begin{aligned} T_{00} &= \rho c^2, \\ T_{11} &= (\frac{a^2}{1 - Kr^2})P, \\ T_{22} &= a^2 r^2 P, \\ T_{33} &= (a^2 r^2 \sin^2\theta)P. \end{aligned}$$

we substitute the 00-component of the energy-momentum tensor into equation (2.14), and given by

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{2.20}$$

This equation is known as the *acceleration equation*. Similarly, the ii-components are

$$\begin{aligned} R_{ii} &= \frac{8\pi G}{c^4}\left(T_{ii} - \frac{1}{2} g_{ii} T\right) \\ \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + \frac{2Kc^2}{a^2} &= -\frac{4\pi G}{c^2}\left(P - \rho c^2\right) \end{aligned} \tag{2.21}$$

where $i = 1, 2, 3$, we see that the components 11, 22, and 33 are the same because the homogeneity and isotropy of the FLRW metric. Then we substitute equation (2.19) into equation (2.20), and we will obtain

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} \tag{2.22}$$

where $H = \dot{a}/a$. We finally arrive at the *Friedmann equation*.

Taking the time derivative of the Friedmann equation and substitute $\ddot{a}$ from the acceleration equation. We will obtain

$$-\left(\rho+3\frac{P}{c^2}\right)a\dot{a}=\dot{\rho}a^2+2\rho a\dot{a} \tag{2.23}$$

and dividing through by a^2, hence

$$\dot{\rho}+3\frac{\dot{a}}{a}\left(\rho+\frac{P}{c^2}\right)=0$$

or

$$\dot{\rho}+3H\left(\rho+\frac{P}{c^2}\right) = 0. \tag{2.24}$$

This is called *continuity equation*.

Another useful quantity is the *density parameter*, which is ratio of density and critical density, the quantity is defined as

$$\Omega=\frac{8\pi G}{3H^2}\rho=\frac{\rho}{\rho_c}, \tag{2.25}$$

where the *critical density* is defined as density that is just enough for flat geometry.

$$\rho_c=\frac{3H^2}{8\pi G}. \tag{2.26}$$

Hence, the Friedmann equation (2.22) can be written as

$$\Omega-1=\frac{K}{H^2a^2}. \tag{2.27}$$

The sign of K is therefore determined if Ω is greater than, equal to, or less than, one. We have

$$\begin{aligned}
\rho < \rho_c &\leftrightarrow \Omega < 1 \leftrightarrow K < 0 \leftrightarrow \text{open}\\
\rho = \rho_c &\leftrightarrow \Omega = 1 \leftrightarrow K = 0 \leftrightarrow \text{flat}\\
\rho > \rho_c &\leftrightarrow \Omega > 1 \leftrightarrow K > 0 \leftrightarrow \text{closed}
\end{aligned}$$

The density parameter tells us which one of the three FRW geometries describes our universe. Recent measurement of the cosmic microwave background found consistent with flat geometry, i.e. or Ω is very close to unity.

2.4 Power-Law expansion

In this section we introduce the power-law expansion. The power-law expansion were also studied in context of scalar field cosmology [36] and phantom scalar field cosmology [37]. We use the power parameters p and q for two forms of power-law to separated between the canonical and phantom power-law, $a \propto t^p$ and $a \propto (t_s - t)^q$ respectively.

The power-law expansion used in the models under the assumptions that our flat FLRW universe is filled with dust matter and scalar fields, and dominated by dark energy.

The power-law is defined as

$$\frac{a(t)}{a_0(t_0)} = \left(\frac{t}{t_0}\right)^p, \tag{2.28}$$

where $a_0(t_0)$ is the value of the scale factor at present time t_0 and p is a number which described the acceleration of the universe, where $p > 1$.

The flat universe dominated by the dark energy and the Friedmann equation gives $1 < p < \infty$. We will consider the constant value of p in the range $0 < p < \infty$ and in the short range of redshift $z \lesssim 0.45$ to present, $z = 0$. Then we can write the power-law cosmology of the cosmic speed as

$$\begin{aligned} \dot{a} &= a_0 p\left(\frac{t^{p-1}}{t_0^p}\right), \\ &= a_0 p\left(\frac{t}{t_0}\right)^p \frac{1}{t}, \\ &= a\frac{p}{t}, \end{aligned} \tag{2.29}$$

and the cosmic acceleration

$$\begin{aligned} \ddot{a} &= a_0 p(p-1)\left(\frac{t^{p-2}}{t_0^p}\right), \\ &= a_0 p(p-1)\left(\frac{t}{t_0}\right)^p \frac{1}{t^2}, \\ &= a\frac{p(p-1)}{t^2}. \end{aligned} \tag{2.30}$$

Hence, the Hubble parameter and its time derivative in the power-law expansion reads

$$\begin{aligned} H &= \frac{\dot{a}}{a}, \\ &= \frac{p}{t}, \end{aligned} \tag{2.31}$$

$$\dot{H} = -\frac{p}{t^2}. \tag{2.32}$$

From equation (2.29), p is calculated at the present H_0, t_0 as $p = H_0 t_0$. Therefore,we can write the dust matter density in the power-law as

$$\rho_{\mathrm{m}} = \rho_{\mathrm{m},0}\left(\frac{t_0}{t}\right)^{3p} = \rho_{\mathrm{m},0}\left(\frac{a_0}{a}\right)^3. \tag{2.33}$$

where $\rho_{\mathrm{m},0}$ is the dust matter density at present time t_0.

2.5 Super-acceleration expansion

In the case of super-acceleration expansion, the power-law is defined slightly different from previous and the scale factor becomes

$$\frac{a(t)}{a_0(t_0)} = \left(\frac{t_{\mathrm{s}} - t}{t_{\mathrm{s}} - t_0}\right)^q, \tag{2.34}$$

where t_{s} is called the big-rip time which is defined as [38]

$$t_{\mathrm{s}} \equiv t_0 + \frac{|q|}{H(t_0)}, \tag{2.35}$$

In the same as previous, then we can written a cosmic speed and the cosmic acceleration as

$$\begin{aligned} \dot{a} &= -a_0 q \frac{(t_{\mathrm{s}} - t)^{q-1}}{(t_{\mathrm{s}} - t_0)^q}, \\ &= -q\frac{a}{(t_{\mathrm{s}} - t)}, \end{aligned} \tag{2.36}$$

and

$$\begin{aligned}\ddot{a} &= a_0 q(q-1)\frac{(t_{\rm s}-t)^{q-2}}{(t_{\rm s}-t_0)^q}, \\ &= a\frac{q(q-1)}{(t_{\rm s}-t)^2}. \end{aligned} \tag{2.37}$$

The Hubble parameter and its time derivative in this case are

$$H = \frac{q}{t_{\rm s}-t}, \tag{2.38}$$

$$\dot{H} = -\frac{q}{(t_{\rm s}-t)^2}. \tag{2.39}$$

At present, $q = H_0(t_0 - t_{\rm s})$. The dust matter density in the phantom power-law is

$$\begin{aligned}\rho_{\rm m} &= \rho_{\rm m,0}\left(\frac{a_0}{a}\right)^3, \\ &= \rho_{\rm m,0}\left(\frac{a_0}{a_0[(t_{\rm s}-t)/(t_{\rm s}-t_0)]^q}\right)^3, \\ &= \rho_{\rm m,0}\left(\frac{t_{\rm s}-t_0}{t_{\rm s}-t}\right)^{3q}. \end{aligned} \tag{2.40}$$

where $\rho_{\rm m,0}$ is the dust matter density at present time t_0.

2.6 Scalar field model

In this section we will discuss about scalar field, $\phi = \phi(x, t)$. Now the scalar field is a model of dark energy with the time dependence equation of state. It is assumed to be spatial homogeneous or invariance under transformation like $\phi(x') = \phi(x)$. Therefore, scalar field can be written as $\phi = \phi(t)$, we roughly discuss about homogeneous scalar field, for example, quintessence field.

Quintessence field

Cosmological model of dark energy with a canonical scalar field ϕ is called quintessence. The action for quintessence is given by [3, 39]

$$S_{\rm q} = \int {\rm d}^4{\rm x}\sqrt{-g}\left[\frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi - V(\phi)\right], \tag{2.41}$$

where ϕ is the quintessence field with potential $V(\phi)$, and g is determinant of g_{ab}. Variation of the action (2.41) with respect to ϕ gives

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi) = 0, \tag{2.42}$$

where $\mathrm{d}V(\phi)/\mathrm{d}\phi$ stands for the derivative of potential with respect to scalar field and over dot represents a derivative with respect to time.

The energy momentum tensor of scalar field can be derive by varying the action (2.41) respect to the metric g^{ab} , then the energy momentum tensor is taken form

$$T_{ab} = \partial_a\phi\partial_b\phi - g_{ab}\left[\frac{1}{2}g^{cd}\partial_c\phi\partial_d\phi + V(\phi)\right]. \tag{2.43}$$

Energy density and pressure for the scalar field are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{2.44}$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{2.45}$$

The Friedmann equation (2.22) and the acceleration equation (2.20)

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right], \tag{2.46}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left[\dot{\phi}^2 - V(\phi)\right]. \tag{2.47}$$

Therefore we will see that accelerating expansion of the universe takes place when $\dot{\phi}^2 < V(\phi)$, that is $\ddot{a} > 0$. The equation of state for the field ϕ is given by

$$w_\phi \equiv \frac{P_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \tag{2.48}$$

For the case of $\dot{\phi}^2 \ll V(\phi)$ and $\ddot{\phi} \simeq 0$, Eq. (2.42) and (2.46) can be approximated as

$$3H\dot{\phi} \simeq -\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi), \tag{2.49}$$

and

$$3H^2 \simeq 8\pi G V(\phi). \tag{2.50}$$

Hence the EoS parameter Eq. (2.48) gives the approximation

$$w_\phi \simeq -1 + \frac{2}{3}\epsilon, \tag{2.51}$$

where $\epsilon \equiv [(\mathrm{d}V/\mathrm{d}\phi)/V]^2/(16\pi G)$ is known as slow-roll parameter.

Let us consider the fluid equation of the field is taken the form

$$\dot{\rho}_\phi + 3H\rho_\phi(1 + w_\phi) = 0, \tag{2.52}$$

which can be written in an integrated form as

$$\rho = \rho_0 \exp\left[-\int 3(1 + w_\phi)\left(\frac{\dot{a}}{a}\right)\mathrm{dt}\right], \tag{2.53}$$

or

$$\rho = \rho_0 \exp\left[-\int 3(1 + w_\phi)\left(\frac{da}{a}\right)\right]. \tag{2.54}$$

where ρ_0 is an integration constant.

Consider the Friedmann and the acceleration equation, Eqs. (2.46). We can express the scalar potential $V(\phi)$ and the field ϕ it follows

$$V = \frac{3H^2}{8\pi G}\left[1 + \frac{\dot{H}}{3H^2}\right], \tag{2.55}$$

$$\phi = \int \mathrm{dt}\left[-\frac{2\dot{H}}{8\pi G}\right]^{\frac{1}{2}}. \tag{2.56}$$

2.7 Slow-roll approximation

In cosmological inflation, the simplest way for inflation to happen is to approximate that $\dot{\phi}^2 \ll V(\phi)$, so that the Friedmann equation is dominated by the potential term, i.e.

$$H^2 \simeq \frac{8\pi G}{3}V(\phi). \tag{2.57}$$

If we also assume that the second order time derivative of the field value can be negligible, i.e.

$$\ddot{\phi} \ll 3H\dot{\phi}, \tag{2.58}$$

Then we have the "slow-roll approximation". The Klein-Gordon equation under slow-roll approximation becomes

$$\dot{\phi} \simeq -\frac{1}{3H}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi). \tag{2.59}$$

The $3H\dot{\phi}$ term is the friction term given by the expansion rate of the universe. This means that even when the potential is steep inflation could happen, satisfying the condition $\dot{\phi}^2 < V(\phi)$, due to the friction created from the rapid expansion. Another case is that inflation happens when the field moves slowly due to the almost flat or slowly varying potential, i.e. $\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)$ is very small.

The approximation is governed by the slow-roll parameters. When the scalar field potential term dominates the Friedmann equation, the slow-roll parameters are defined as

$$-\frac{\dot{H}}{H^2} \simeq \frac{1}{16\pi G}\left(\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}\phi}\right)^2 \equiv \varepsilon(\phi), \tag{2.60}$$

$$\frac{1}{3H^2}\frac{\mathrm{d}^2V}{\mathrm{d}\phi^2} \simeq \frac{1}{8\pi GV}\frac{\mathrm{d}^2V}{\mathrm{d}\phi^2} \equiv \eta(\phi). \tag{2.61}$$

These parameters, in the slow-roll approximation, satisfy

$$\varepsilon(\phi) \ll 1, \tag{2.62}$$

$$\eta(\phi) \ll 1. \tag{2.63}$$

These conditions are called the "slow-roll conditions". The statements in the slow-roll conditions are equivalent to slowly moving field and slowly-varying potential. Inflation comes to an end when the condition $\dot{\phi}^2 < V(\phi)$ is violated and hence $\dot{\phi}^2$ becomes greater than $V(\phi)$.

CHAPTER III

NON-MINIMAL DERIVATIVE COUPLING

We have known that our universe is under the accelerating expansion phase due to the dark energy [1, 2, 3, 4, 5, 6]. In present, there are many scalar field models proposed to describe accelerating expansion of the universe, for example, quintessence [7, 8, 9, 10, 11, 12] and k-essence models [13, 14]. We have seen that there are many models represent various modifications of scalar-tensor theories [24, 25]. One of the scalar-tensor theories is the non-minimal couplings (NMC) between the curvature and the scalar fields [17, 22]. The NMC is motivated by scalar-tensor theories of the Jordan-Brans-Dicke models [24, 25].

The non-minimal derivative coupling (NMDC) is the extended model of the NMC. The coupling function $f(\phi)$ is not only the function of the scalar field but it is also the function of the derivative of ϕ as well; $f = f(\phi, \partial_a\phi)$. The simplest form of the NMDC is the coupling between Ricci scalar and the derivative of scalar eld i.e. $R\partial_a\phi\partial^a\phi$. The NMDC term can phenomenologically be dominant either at late time (for the quadratic or Higgs potential) or at inflation (for runaway potential). Hence the model can be either considered as dark energy model or inflationary model of which the scalar field derivative is both non-minimally self-coupling and coupling to Ricci scalar.

3.1 Literature review of the NMDC models

In this section, we give a brief review of the recent NMDC gravity models.

3.1.1 Amendola's Model

In 1993 [17], Amendola studied the scalar-tensor theory with the Lagrangian linear in the Ricci scalar R, quadratic in ϕ, and containing terms as follows

$$R\,\nabla_a\phi\,\nabla^a\phi,\quad R_{ab}\,\nabla^a\phi\,\nabla^b\phi,\quad R\,\phi\nabla^2\phi,$$

$$R_{ab}\,\phi\,\nabla^a\nabla^b\phi,\quad \nabla_a R\,\nabla^a\phi,\quad \nabla^2 R\phi.$$

Amendola showed that the non-minimal derivative couplings are an interesting source of new cosmological dynamics. He investigated a model with the only derivative coupling term $R_{ab}\,\partial^a\phi\,\partial^b\phi$ and presented some analytical inflationary solutions.

3.1.2 Capozziello, Lambiase and Schmidt's Result

In 2000 [18], Capozziello, Lambiase and Schmidt studied theories of gravity where non-minimal derivative couplings of the form $R\,\partial_a\phi\,\partial^a\phi$ and $R^{ab}\,\partial_a\phi\,\partial_b\phi$ are presented in the Lagrangian. They showed that the de Sitter space-time is an attractor solution. When considering only $R\,\partial_a\phi\,\partial^a\phi$ with free Ricci scalar, free kinetic term, potential and matter terms, the equation of state parameter close to -1. Assuming slow-roll condition and power law expansion, the scalar field potential is found [21].

3.1.3 Granda's Two Coupling constant Model

In 2010 [19], Granda studied on scalar field with kinetic term coupled to itself and the curvature. The model consists of the Einstein Hilbert term, a kinetic term of scalar field, a potential term and two couplings term, ξ, η, in form of $-(1/2)\xi R\phi^{-2}\partial_a\phi\,\partial^a\phi$ and $-(1/2)\eta R_{ab}\phi^{-2}\partial^a\phi\,\partial^b\phi$. This model gives late time accelerated expansion even with no potential. In the case of scalar field dominated, the scalar field and potential are found and an expression are given by a power-law expansion. This implies that the NMDC gives an important role in the explanation of the dark energy or cosmological constant problem.

3.1.4 Granda's One Coupling constant Model

In 2010 [20], Granda studied on scalar field with kinetic term coupled to itself and the curvature. From the model we can find solution for Friedmann equations with the particular restriction on the scalar field $\dot{\phi} = \text{constant} = \dot{\phi}_0$. This model gives late time accelerated expansion even without potential and at limit $t \to \infty$ the system reproduces an effective cosmological constant, $w \to -1$. At last the scalar field potential is found by power-law expansion.

3.2 Background Equations

In this work, we consider one coupling of Granda's model. We assumed the universe is flatted and filled with a perfect fluid and scalar field ϕ with the non-minimal derivative coupling (NMDC) to the scalar curvature R. We considered the action [20] and transformed all the field value to its logarithm, $\phi' \to \phi' = \mu \ln \phi$. The mass dimension of the field prior to the transformation is with the constant μ. Let start with the action with re-scaling field

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\Bigg[\frac{1}{16\pi G}R - \frac{1}{2}\left(\frac{\mu^2}{\phi^2}\right)\partial_a\phi\partial^a\phi - \frac{1}{2}\xi R\left(\frac{\mu^2}{\phi^2}\right)\partial_a\phi\partial^a\phi \\ &\quad -V(\phi')\Bigg] + S_m, \end{aligned} \tag{3.1}$$

where S_m is the dark matter action which describes a fluid with barotropic equation of state, $g = \det(g_{ab})$, G is the gravitational constant, R is the scalar curvature, ξ is the coupling parameter, and $V(\phi')$ is the potential of scalar field. In the cosmological context, by using the Friedmann-Robertson-Walker metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2 \tag{3.2}$$

where $\mathrm{d}\mathbf{x}^2$ is the metric in three space, $a(t)$ is the scale factor. From the (00) and (ii) components of the Einstein equations, the Friedmann equations and the acceleration equation respectively, are given by

$$3H^2 = 8\pi G(\rho_\phi + \rho_{\mathrm{m}}), \tag{3.3}$$

$$2\dot{H} + 3H^2 = -8\pi G(P_\phi + P_{\mathrm{m}}). \tag{3.4}$$

where P_{m}, P_ϕ are the pressure of the matter and scalar field, ρ_{m} and ρ_ϕ are respectively the energy density of matter and scalar field. We consider dust matter here, hence $P_{\rm m} = 0$.

Let us derive the Einstein equations by varying for each term of the action (3.1), $\phi' \to \phi' = \mu \ln \phi$, consider on the first term of the brace we obtain

$$\begin{aligned}
\frac{1}{16\pi G}\delta(\sqrt{-g}R) &= \frac{1}{16\pi G}\bigg[(\delta\sqrt{-g})R + \sqrt{-g}(\delta R)\bigg], \\
&= \frac{1}{16\pi G}\bigg[(-\frac{1}{2}\sqrt{-g}g_{ab}\delta g^{ab})R \\
&\quad +\sqrt{-g}(R_{ab} + g_{ab}\nabla_a\nabla^a - \nabla_a\nabla_b)\delta g^{ab}\bigg], \\
&= \frac{1}{16\pi G}\bigg[\sqrt{-g}\bigg(R_{ab} - \frac{1}{2}g_{ab}R\bigg)\delta g^{ab} \\
&\quad +\sqrt{-g}g_{ab}\delta g^{ab}\nabla_a\nabla^a - \sqrt{-g}\delta g^{ab}\nabla_a\nabla_b\bigg], \\
&= \frac{1}{16\pi G}\sqrt{-g}\bigg(R_{ab} - \frac{1}{2}g_{ab}R\bigg)\delta g^{ab}.
\end{aligned} \tag{3.5}$$

The second term is

$$\begin{aligned}
\delta\bigg[-\frac{1}{2}\sqrt{-g}(\partial_a\mu\ln\phi)(\partial^a\mu\ln\phi)\bigg] &= \delta\bigg[-\frac{1}{2}\sqrt{-g}(\frac{\mu}{\phi}\partial_a\phi)(\frac{\mu}{\phi}\partial^a\phi)\bigg] \\
&= -\frac{1}{2}\bigg[(\delta\sqrt{-g})g^{ab}\frac{\mu^2}{\phi^2}(\partial_a\phi\partial_b\phi) \\
&\quad +\sqrt{-g}(\delta g^{ab})\frac{\mu^2}{\phi^2}(\partial_a\phi\partial_b\phi)\bigg], \\
&= -\frac{1}{2}\bigg[(-\frac{1}{2}\sqrt{-g}g_{cd}\delta g^{cd})g^{ab}\frac{\mu^2}{\phi^2}(\partial_a\phi\partial_b\phi) \\
&\quad +\sqrt{-g}(\delta g^{ab})\frac{\mu^2}{\phi^2}(\partial_a\phi\partial_b\phi)\bigg], \\
&= \frac{1}{2}\sqrt{-g}\bigg[\frac{1}{2}g_{ab}g^{ab}\frac{\mu^2}{\phi^2}(\partial_a\phi\partial_b\phi) \\
&\quad -\frac{\mu^2}{\phi^2}(\partial_a\phi\partial_b\phi)\bigg]\delta g^{ab}, \\
&= \frac{1}{2}\sqrt{-g}\bigg[\frac{1}{2}\frac{\mu^2}{\phi^2}g_{ab}(\partial_a\phi\partial^a\phi) \\
&\quad -\frac{\mu^2}{\phi^2}(\partial_a\phi\partial_b\phi)\bigg]\delta g^{ab}, \\
&= \frac{1}{2}\sqrt{-g}\bigg[\frac{1}{2}g_{ab}(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla^a\phi) \\
&\quad -(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi)\bigg]\delta g^{ab}.
\end{aligned} \tag{3.6}$$

The third term is

$$
\begin{aligned}
\delta\Big[-\frac{1}{2}\xi R\sqrt{-g}\frac{\mu^2}{\phi^2}g^{ab}\partial_a\phi\partial_b\phi\Big] &= -\frac{1}{2}\Big[\xi(\delta R)\sqrt{-g}\frac{\mu^2}{\phi^2}g^{ab}(\partial_a\phi\partial_b\phi) \\
&\quad +\xi R(\delta\sqrt{-g})\frac{\mu^2}{\phi^2}g^{ab}(\partial_a\phi\partial_b\phi) \\
&\quad +\xi R\sqrt{-g}\frac{\mu^2}{\phi^2}(\delta g^{ab})(\partial_a\phi\partial_b\phi)\Big], \\
&= -\frac{1}{2}\xi\Big[R_{ab}\delta g^{ab}\sqrt{-g}\frac{\mu^2}{\phi^2}g^{ab}(\partial_a\phi\partial_b\phi) \\
&\quad -g_{ab}\nabla_a\nabla^a\delta g^{ab}\sqrt{-g}\frac{\mu^2}{\phi^2}g^{ab}(\partial_a\phi\partial_b\phi) \\
&\quad +\nabla_a\nabla_b\delta g^{ab}\sqrt{-g}\frac{\mu^2}{\phi^2}g^{ab}(\partial_a\phi\partial_b\phi) \\
&\quad -\frac{1}{2}R\sqrt{-g}g_{ab}\delta g^{ab}\frac{\mu^2}{\phi^2}g^{ab}(\partial_a\phi\partial_b\phi) \\
&\quad +R\sqrt{-g}\frac{\mu^2}{\phi^2}\delta g^{ab}(\partial_a\phi\partial_b\phi)\Big], \\
&= -\frac{1}{2}\xi\Big[R_{ab}\delta g^{ab}\sqrt{-g}\frac{\mu^2}{\phi^2}g^{ab}(\nabla_a\phi\nabla_b\phi) \\
&\quad -\sqrt{-g}g_{ab}g^{ab}\delta g^{ab}\nabla_a\nabla^a(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi) \\
&\quad +\sqrt{-g}g^{ab}\delta g^{ab}\nabla_a\nabla_b(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi) \\
&\quad -\frac{1}{2}R\sqrt{-g}g_{ab}\delta g^{ab}\frac{\mu^2}{\phi^2}g^{ab}(\nabla_a\phi\nabla_b\phi) \\
&\quad +R\sqrt{-g}\delta g^{ab}\frac{\mu^2}{\phi^2}(\nabla_a\phi\nabla_b\phi)\Big], \\
&= -\frac{1}{2}\xi\sqrt{-g}\Big[(R_{ab}-\frac{1}{2}g_{ab}R)\frac{\mu^2}{\phi^2}(\nabla_a\phi\nabla^a\phi) \\
&\quad -g_{ab}\nabla_a\nabla^a(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla^a\phi)+R\frac{\mu^2}{\phi^2}(\nabla_a\phi\nabla_b\phi) \\
&\quad +\nabla_a\nabla_b(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla^a\phi)\Big]\delta g^{ab}. \qquad (3.7)
\end{aligned}
$$

and the last term we get

$$
\begin{aligned}
\delta(-\sqrt{-g}V(\phi')) &= -[(\delta\sqrt{-g})V(\phi')+\sqrt{-g}(\delta V(\phi'))], \\
&= -[(-\frac{1}{2}\sqrt{-g}g_{ab}\delta g^{ab})V(\phi')], \\
&= \frac{1}{2}\sqrt{-g}g_{ab}\delta g^{ab}V(\phi'). \qquad (3.8)
\end{aligned}
$$

Now, by putting together for each term into the action

$$
\begin{aligned}
\delta S &= \delta \int \mathrm{d^4x}\sqrt{-g}\Bigg[\frac{1}{16\pi G}R - \frac{1}{2}\bigg(\frac{\mu^2}{\phi^2}\bigg)\partial_a\phi\partial^a\phi - \frac{1}{2}\xi R\bigg(\frac{\mu^2}{\phi^2}\bigg)\partial_a\phi\partial^a\phi - V(\phi')\Bigg] + \delta S_m, \\
0 &= \frac{1}{2}\int \mathrm{d^4x}\sqrt{-g}\Bigg[\frac{1}{8\pi G}\bigg(R_{ab} - \frac{1}{2}g_{ab}R\bigg) + \frac{1}{2}g_{ab}(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla^a\phi) - (\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi) \\
&\quad -\xi\bigg((R_{ab} - \frac{1}{2}g_{ab}R)(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla^a\phi) - g_{ab}\nabla_a\nabla^a(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla^a\phi) \\
&\quad +R(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi) + \nabla_a\nabla_b(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla^a\phi)\bigg) + g_{ab}V(\phi')\Bigg]\delta g^{ab}. \qquad (3.9)
\end{aligned}
$$

Therefore, by the action principle, it can be written as

$$
\begin{aligned}
0 &= \frac{1}{8\pi G}\bigg(R_{ab} - \frac{1}{2}g_{ab}R\bigg) + \frac{1}{2}g_{ab}(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla^a\phi) - \frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi \\
&\quad -\xi\bigg((R_{ab} - \frac{1}{2}g_{ab}R)(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla^a\phi) - g_{ab}\nabla_a\nabla^a(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla^a\phi) \\
&\quad +R(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi) + \nabla_a\nabla_b(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla^a\phi)\bigg) + g_{ab}V(\phi'),
\end{aligned}
$$

We can change index and rearrange as

$$
\begin{aligned}
\frac{1}{8\pi G}\bigg(R_{ab} - \frac{1}{2}g_{ab}R\bigg) &= \frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi - \frac{1}{2}\frac{\mu^2}{\phi^2}g_{ab}\nabla_d\phi\nabla^d\phi - g_{ab}V(\phi') \\
&\quad +\xi\bigg(R_{ab} - \frac{1}{2}g_{ab}R\bigg)\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi + \xi R\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi \\
&\quad -g_{ab}\xi\nabla_d\nabla^d(\frac{\mu^2}{\phi^2}\nabla_c\phi\nabla^c\phi) + \xi\nabla_a\nabla_b(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi).
\end{aligned}
\qquad (3.10)
$$

From Einstein's field equation, we get

$$
\frac{1}{8\pi G}\bigg(R_{ab} - \frac{1}{2}g_{ab}R\bigg) = T_{ab}. \qquad (3.11)
$$

Finally, after varying the action (3.1) with respect to the metric and from Eq. (3.10), the effective energy-momentum tensor for the scalar field is given by

$$
\begin{aligned}
T_{ab} &= \frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\bigg(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi\bigg) - g_{ab}V(\phi') \\
&\quad +\xi\Bigg[\bigg(R_{ab} - \frac{1}{2}g_{ab}R\bigg)\bigg(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi\bigg) + R\bigg(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi\bigg) \\
&\quad -g_{ab}\nabla_d\nabla^d\bigg(\frac{\mu^2}{\phi^2}\nabla_c\phi\nabla^c\phi\bigg) + \nabla_a\nabla_b\bigg(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi\bigg)\Bigg]. \qquad (3.12)
\end{aligned}
$$

Assuming the spatially-flat Friedmann Robertson Walker (FRW) metric, we can write the components of tensor T_{ab}, as follows, (See detail in appendix C)

$$\rho_\phi = T_{00} = \frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') + 3\xi\left[(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 2H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} - 2H\mu^2\frac{\dot{\phi}^3}{\phi^3}\right] \quad (3.13)$$

and

$$P_\phi = \frac{T_{11}}{a^2} = \frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 - V(\phi') + \xi\left[(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 4H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 4H\mu^2\frac{\dot{\phi}^3}{\phi^3}\right.$$
$$\left. +2\mu^2\left(\frac{\ddot{\phi}^2}{\phi^2} + \frac{\dot{\phi}}{\phi^2}\dddot{\phi} - 5\frac{\dot{\phi}^2}{\phi^3}\ddot{\phi} + 3\frac{\dot{\phi}^4}{\phi^4}\right)\right]. \quad (3.14)$$

Let us consider the action (3.1) varying this action with respect to ϕ', and we get

$$\frac{\partial \mathcal{L}_{\phi'}}{\partial \phi'} = -\frac{\partial V(\phi')}{\partial \phi'}. \quad (3.15)$$

and

$$\begin{aligned}
\frac{\partial \mathcal{L}_{\phi'}}{\partial(\nabla_a\phi')} &= \frac{\partial}{\partial(\nabla_a\phi')}\left[-\frac{1}{2}\nabla_b\phi'\nabla^b\phi' - \frac{1}{2}\xi R\nabla_b\phi'\nabla^b\phi'\right],\\
&= -\frac{1}{2}\left[\frac{\partial}{\partial(\nabla_a\phi')}\left(g^{bc}\nabla_b\phi'\nabla_c\phi' + \xi R g^{bc}\nabla_b\phi'\nabla_c\phi'\right)\right],\\
&= -\frac{1}{2}g^{bc}\left[(\nabla_c\phi')\frac{\partial(\nabla_b\phi')}{\partial(\nabla_a\phi')} + (\nabla_b\phi')\frac{\partial(\nabla_c\phi')}{\partial(\nabla_a\phi)} + (\nabla_c\phi')\frac{\partial(\xi R\nabla_b\phi')}{\partial(\nabla_a\phi')}\right.\\
&\quad \left.+\xi R(\nabla_b\phi')\frac{\partial(\nabla_c\phi')}{\partial(\nabla_a\phi')}\right],\\
&= -\frac{1}{2}g^{bc}\left[(\nabla_c\phi')\delta^b_a + (\nabla_b\phi')\delta^c_a + (\nabla_c\phi')\xi R\delta^b_a + \xi R(\nabla_b\phi')\delta^c_a\right],\\
&= -\frac{1}{2}\left[g^{ac}(\nabla_c\phi') + g^{ab}(\nabla_b\phi') + g^{ac}\xi R(\nabla_c\phi') + g^{ab}\xi R(\nabla_b\phi')\right],\\
&= -\frac{1}{2}\left[2g^{ac}(\nabla_c\phi') + 2g^{ac}\xi R(\nabla_c\phi')\right]\\
&= -\left[g^{ac}(\nabla_c\phi') + g^{ac}\xi R(\nabla_c\phi')\right],\\
-\nabla_a\left[\frac{\partial \mathcal{L}_{\phi'}}{\partial(\nabla_a\phi')}\right] &= \nabla_a\left[g^{ac}(\nabla_c\phi') + g^{ac}\xi R(\nabla_c\phi')\right],\\
&= g^{ac}\nabla_a(\nabla_c\phi') + g^{ac}\xi(\nabla_a R)(\nabla_c\phi'),\\
&\quad +g^{ac}\xi R\nabla_a(\nabla_c\phi')\\
&= \nabla_a(\nabla^a\phi') + \xi(\nabla_a R)(\nabla^a\phi') + \xi R\nabla_a(\nabla^a\phi'). \qquad (3.16)
\end{aligned}$$

Let us consider

$$\begin{aligned}
\nabla_a(\nabla^a\phi') &= \nabla_a(\partial^a\phi'),\\
&= \partial_a(\partial^a\phi') + \Gamma^a_{ca}(\partial^c\phi'),\\
&= \partial_0(\partial^0\phi') + (\Gamma^1_{01} + \Gamma^2_{02} + \Gamma^3_{03})(\partial^0\phi'),\\
&= \partial_0(-\dot{\phi}') + \left(\frac{\dot{a}}{a} + \frac{\dot{a}}{a} + \frac{\dot{a}}{a}\right)(-\dot{\phi}'),\\
&= -(\ddot{\phi}' + 3H\dot{\phi}'). \qquad (3.17)
\end{aligned}$$

where the field is function of time only, and

$$\begin{aligned}
\nabla_a R &= \partial_0(6\dot{H} + 12H^2),\\
&= 6\ddot{H} + 24H\dot{H}. \qquad (3.18)
\end{aligned}$$

where $R = g^{ab}R_{ab} = 6\dot{H} + 12H^2$,

We put Eq. (3.17) and Eq. (3.18) into Eq. (3.16) and then we will obtain

$$\begin{aligned}
\nabla_a\left[\frac{\partial\mathcal{L}_{\phi'}}{\partial(\nabla_a\phi')}\right] &= \ddot{\phi}' + 3H\dot{\phi}' + \xi(6\ddot{H} + 24H\dot{H})\dot{\phi}' + \xi(6\dot{H} + 12H^2)\ddot{\phi}'\\
&\quad +3\xi H(6\dot{H} + 12H^2)\dot{\phi}',\\
&= \ddot{\phi}' + 3H\dot{\phi}' + 6\xi\ddot{H}\dot{\phi}' + 24\xi\dot{H}H\dot{\phi}' + 6\xi\dot{H}\ddot{\phi}' + 12\xi H^2\ddot{\phi}'\\
&\quad +18\xi\dot{H}H\dot{\phi}' + 36\xi H^3\dot{\phi}',\\
&= \ddot{\phi}' + 3H\dot{\phi}' + 6\xi\ddot{H}\dot{\phi}' + 6\xi H(7\dot{H} + 6H^2)\dot{\phi}'\\
&\quad +6\xi(\dot{H} + 2H^2)\ddot{\phi}'. \qquad (3.19)
\end{aligned}$$

From the Euler-Lagrange equations

$$\nabla_a\left[\frac{\partial\mathcal{L}_{\phi'}}{\partial(\nabla_a\phi')}\right] - \frac{\partial\mathcal{L}_{\phi'}}{\partial\phi'} = 0 \qquad (3.20)$$

we replace Eq. (3.15) and Eq. (3.19) into Eq. (3.20). Therefore we can write the equation of motion (EoM) of the system for the field ϕ' as

$$\begin{aligned}
&\ddot{\phi}' + 3H\dot{\phi}' + 6\xi\ddot{H}\dot{\phi}' + 6\xi H(7\dot{H} + 6H^2)\dot{\phi}'\\
&\quad +6\xi(\dot{H} + 2H^2)\ddot{\phi}' + \frac{\mathrm{d}}{\mathrm{d}\phi'}V(\phi') = 0. \qquad (3.21)
\end{aligned}$$

By transforming, $\phi' = \mu \ln \phi$, we obtain

$$\begin{aligned}\dot{\phi}' &= \frac{\mathrm{d}}{\mathrm{dt}}(\ln \phi) \\ &= \mu \frac{\dot{\phi}}{\phi},\end{aligned}$$

and

$$\begin{aligned}\ddot{\phi}' &= \frac{\mathrm{d}}{\mathrm{dt}}\left(\mu \frac{\dot{\phi}}{\phi}\right) \\ &= \mu\left(\frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2}\right).\end{aligned}$$

Finally, we substitute $\ddot{\phi}'$ and $\dot{\phi}'$ into Eq. (3.21) and then we can write the equation of motion for the field ϕ as

$$\begin{aligned}\ddot{\phi} + 3H\dot{\phi} - \frac{\dot{\phi}^2}{\phi} - \frac{\dot{\phi}^2}{\phi}\xi(6\dot{H} + 12H^2) + 6\xi\ddot{H}\dot{\phi} + 6\xi H(7\dot{H} + 6H^2)\dot{\phi} \\ +\xi(6\dot{H} + 12H^2)\ddot{\phi} + \left(\frac{\phi^2}{\mu^2}\right)\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi) = 0. \qquad (3.22)\end{aligned}$$

where $\mathrm{d}V(\phi')/\mathrm{d}\phi' \equiv (\phi/\mu)\mathrm{d}V(\phi)/\mathrm{d}\phi$, is derivative of potential with respect to scalar field ϕ, and over dot represents a derivative with respect to time t.

3.3 Non-Minimal Derivative Coupling model

This section we will study about the field ϕ, the potential of the scalar field $V(\phi)$, and the equation of state (E.o.S) w_ϕ, respectively.

3.3.1 Shape of the field ϕ

Let us look for the shape of the field, ϕ, which compatible with late time or early time solution. Under slow-roll assumption of $0 < |\dot{\phi}| \ll 1$ and $|\dddot{\phi}| \ll |\ddot{\phi}| \ll |\dot{\phi}|$, we neglect the terms with $\dot{\phi}^3, \ddot{\phi}^2, \dot{\phi}\dddot{\phi}, \dot{\phi}^2\ddot{\phi}$ and $\dot{\phi}^4$ hence the pressure and density now become

$$\rho_\phi \simeq \frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') + 3\xi\left[(2\dot{H} + 3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 2H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi}\right], \qquad (3.23)$$

and

$$P_\phi \simeq \frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 - V(\phi') + \xi\left[(2\dot{H} + 3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 4H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi}\right]. \qquad (3.24)$$

Therefore, we start from Eq. (3.4) with the pressure and energy density

$$\dot{H} \simeq -\frac{8\pi G}{2}\left[\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 4\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 2\xi H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + \rho_{\rm m}\right], \quad (3.25)$$

by using linear-equation approximation of the NMDC field equation

$$\ddot{\phi} \simeq -3H\dot{\phi} - \left(\frac{\phi^2}{\mu^2}\right)\frac{\rm d}{{\rm d}\phi}V(\phi). \quad (3.26)$$

we substitute the equation of motion, Eq. (3.26), into Eq. (3.25), therefore

$$\begin{aligned}\dot{H} &\simeq -\frac{8\pi G}{2}\left[\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 4\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 2\xi H\frac{\mu^2}{\phi^2}\dot{\phi}\left(3H\dot{\phi} + \frac{\phi^2}{\mu^2}\frac{\rm d}{{\rm d}\phi}V(\phi)\right) + \rho_{\rm m}\right], \\ &\simeq -\frac{8\pi G}{2}\left[\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 8\xi\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 6\xi H^2\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 2\xi H\dot{\phi}\frac{\rm d}{{\rm d}\phi}V(\phi) + \rho_m\right], \\ &\simeq -\frac{8\pi G}{2}\left[\left(1+6\xi H^2+8\xi\dot{H}\right)\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 2\xi H\dot{\phi}\frac{\rm d}{{\rm d}\phi}V(\phi)\right] - \frac{8\pi G}{2}\rho_{\rm m}.\end{aligned} \quad (3.27)$$

From the Friedmann equation, Eq.(3.3), we take time derivative of its

$$2H\dot{H} = \frac{8\pi G}{3}\left(\dot{\rho}_\phi + \dot{\rho}_{\rm m}\right), \quad (3.28)$$

Let consider time derivative of the energy density, ρ_ϕ, from equation (3.23) we will obtain

$$\begin{aligned}\dot{\rho}_\phi &= \frac{\rm d}{\rm dt}\left[\frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') + 3\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 6\xi H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} - 6\xi H\frac{\mu^2}{\phi^3}\dot{\phi}^3\right], \\ &= \mu^2\frac{\dot{\phi}}{\phi^2}\ddot{\phi} - \mu^2\frac{\dot{\phi}^3}{\phi^3} + \dot{\phi}\frac{\rm d}{{\rm d}\phi}V(\phi) + 12\xi\dot{H}\frac{\dot{\phi}}{\phi^2}\ddot{\phi} - 12\mu^2\xi\dot{H}\frac{\dot{\phi}^3}{\phi^3} \\ &\quad +6\mu^2\xi\ddot{H}\frac{\dot{\phi}^2}{\phi^2} + 18\mu^2\xi H^2\frac{\dot{\phi}}{\phi^2}\ddot{\phi} - 18\mu^2\xi H^2\frac{\dot{\phi}^3}{\phi^3} + 18\mu^2\xi H\dot{H}\frac{\dot{\phi}^2}{\phi^2} \\ &\quad +6\mu^2\xi\dot{H}\frac{\dot{\phi}}{\phi^2}\ddot{\phi} + 6\mu^2\xi H\frac{\ddot{\phi}^2}{\phi^2} - 12\mu^2\xi H\frac{\dot{\phi}^2}{\phi^3}\ddot{\phi} + 6\mu^2\xi H\frac{\dot{\phi}}{\phi^2}\dddot{\phi} \\ &\quad -18\mu^2\xi H\frac{\dot{\phi}^2}{\phi^3}\ddot{\phi} + 6\mu^2\xi\dot{H}\frac{\dot{\phi}^3}{\phi^3} + 18\mu^2\xi H\frac{\dot{\phi}^4}{\phi^4},\end{aligned} \quad (3.29)$$

By neglecting terms of order higher than the second one, the equation (3.29) reduces to

$$\begin{aligned}\dot{\rho}_\phi &\simeq \frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + \dot{\phi}\frac{\rm d}{{\rm d}\phi}V(\phi) + 18\xi\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 6\xi\ddot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2 \\ &\quad +18\xi H^2\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 18\xi H\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2.\end{aligned} \quad (3.30)$$

Similarly, we can use approximation from Eq. (3.26) into Eq. (3.30) to obtain

$$\begin{aligned}
\dot{\rho}_\phi &\simeq \frac{\mu^2}{\phi^2}\dot{\phi}\bigg(-3H\dot{\phi}-\frac{\phi^2}{\mu^2}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)\bigg)+\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi) \\
&\quad +18\xi\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}\bigg(-3H\dot{\phi}-\frac{\phi^2}{\mu^2}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)\bigg) \\
&\quad +6\xi\ddot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2+18\xi H^2\frac{\mu^2}{\phi^2}\dot{\phi}\bigg(-3H\dot{\phi}-\frac{\phi^2}{\mu^2}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)\bigg) \\
&\quad +18\xi H\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2, \\
&\simeq -3H\frac{\mu^2}{\phi^2}\dot{\phi}^2-(18\xi\dot{H})(3H)\frac{\mu^2}{\phi^2}\dot{\phi}^2-18\xi\dot{H}\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)+6\xi\ddot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2 \\
&\quad -(18\xi H^2)(3H)\frac{\mu^2}{\phi^2}\dot{\phi}^2-18\xi H^2\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)+18\xi\dot{H}H\frac{\mu^2}{\phi^2}\dot{\phi}^2, \\
&\simeq 3\bigg[-H\frac{\mu^2}{\phi^2}\dot{\phi}^2-18\xi H\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2+2\xi\ddot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2-18\xi H^3\frac{\mu^2}{\phi^2}\dot{\phi}^2 \\
&\quad +6\xi H\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2-6\xi\dot{H}\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)-6\xi H^2\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)\bigg], \\
&\simeq 3\bigg[-H\frac{\mu^2}{\phi^2}\dot{\phi}^2-12\xi H\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2+2\xi\ddot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2-18\xi H^3\frac{\mu^2}{\phi^2}\dot{\phi}^2 \\
&\quad -6\xi\dot{H}\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)-6\xi H^2\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)\bigg]. \qquad (3.31)
\end{aligned}$$

Therefore, substituting Eq. (3.31) into Eq. (3.28), we get

$$\begin{aligned}
2H\dot{H} &= \frac{8\pi G}{3}\bigg(\dot{\rho}_\phi+\dot{\rho}_{\mathrm{m}}\bigg), \\
&\simeq 8\pi G\bigg(-H\frac{\mu^2}{\phi^2}\dot{\phi}^2-12\xi H\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2+2\xi\ddot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2-18\xi H^3\frac{\mu^2}{\phi^2}\dot{\phi}^2\bigg) \\
&\quad -6(8\pi G)\xi(\dot{H}+H^2)\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)+\frac{8\pi G}{3}\dot{\rho}_{\mathrm{m}}, \qquad (3.32)
\end{aligned}$$

Finally, the equation (3.32) can be written as

$$\begin{aligned}
H\dot{H} &\simeq \frac{8\pi G}{2}\bigg(-H-12\xi H\dot{H}+2\xi\ddot{H}-18\xi H^3\bigg)\frac{\mu^2}{\phi^2}\dot{\phi}^2 \\
&\quad -8\pi G\xi(3\dot{H}+3H^2)\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)+\frac{8\pi G}{6}\dot{\rho}_{\mathrm{m}}. \qquad (3.33)
\end{aligned}$$

we can rearrange the Eq. (3.33), reads

$$-\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)\simeq\frac{H\dot{H}-\frac{8\pi G}{2}\bigg(-H-12\xi H\dot{H}+2\xi\ddot{H}-18\xi H^3\bigg)\frac{\mu^2}{\phi^2}\dot{\phi}^2-\frac{8\pi G}{6}\dot{\rho}_{\mathrm{m}}}{8\pi G\xi(3\dot{H}+3H^2)\dot{\phi}}, \qquad (3.34)$$

By replacing $\mathrm{d}V(\phi)/\mathrm{d}\phi$ from Eq. (3.34) into Eq. (3.27), and then

$$\begin{aligned}
\dot{H} &\simeq -\frac{8\pi G}{2}\left(1+8\xi\dot{H}+6\xi H^2\right)\frac{\mu^2}{\phi^2}\dot{\phi}^2-\frac{8\pi G}{2}\rho_{\mathrm{m}} \\
&\quad -8\pi G\xi H\dot{\phi}\frac{\left(H\dot{H}-\frac{8\pi G}{2}(-H-12\xi H\dot{H}+2\xi\ddot{H}-18\xi H^3)\frac{\mu^2}{\phi^2}\dot{\phi}^2-\frac{8\pi G}{6}\dot{\rho}_{\mathrm{m}}\right)}{8\pi G\xi(3\dot{H}+3H^2)\dot{\phi}}, \\
&\simeq -\frac{8\pi G}{2}\left(1+8\xi\dot{H}+6\xi H^2\right)\frac{\mu^2}{\phi^2}\dot{\phi}^2-\frac{H^2\dot{H}}{(3\dot{H}+3H^2)}+\frac{8\pi G}{6}\frac{H\dot{\rho}_{\mathrm{m}}}{(3\dot{H}+3H^2)} \\
&\quad +\frac{8\pi G(-H^2-12\xi H^2\dot{H}+2\xi H\ddot{H}-18\xi H^4)\frac{\mu^2}{\phi^2}\dot{\phi}^2}{2(3\dot{H}+3H^2)}-\frac{8\pi G}{2}\rho_{\mathrm{m}}, \\
2\dot{H} &\simeq -8\pi G\left(1+8\xi\dot{H}+6\xi H^2\right)\frac{\mu^2}{\phi^2}\dot{\phi}^2-\frac{2H^2\dot{H}}{(3\dot{H}+3H^2)}+\frac{8\pi G}{3}\frac{H\dot{\rho}_{\mathrm{m}}}{(3\dot{H}+3H^2)} \\
&\quad +\frac{8\pi G(-H^2-12\xi H^2\dot{H}+2\xi H\ddot{H}-18\xi H^4)\frac{\mu^2}{\phi^2}\dot{\phi}^2}{(3\dot{H}+3H^2)}-8\pi G\rho_{\mathrm{m}},
\end{aligned}$$

Multiply above equation by $(3\dot{H}+3H^2)$

$$\begin{aligned}
2\dot{H}(3\dot{H}+3H^2) &\simeq -8\pi G(3\dot{H}+3H^2)\left(1+8\xi\dot{H}+6\xi H^2\right)\frac{\mu^2}{\phi^2}\dot{\phi}^2 \\
&\quad +8\pi G(-H^2-12\xi H^2\dot{H}+2\xi H\ddot{H}-18\xi H^4)\frac{\mu^2}{\phi^2}\dot{\phi}^2 \\
&\quad +\frac{8\pi GH\dot{\rho}_{\mathrm{m}}}{3}-8\pi G(3\dot{H}+3H^2)\rho_{\mathrm{m}}-2H^2\dot{H},
\end{aligned}$$

Therefore, by using the continuity equation of matter, $\dot{\rho}_{\mathrm{m}}=-3H\rho_{\mathrm{m}}$, then the kinetic term of scalar field in term of the Hubble parameter is

$$\frac{\mu^2}{\phi^2}\dot{\phi}^2 \simeq \frac{-6\dot{H}^2-8H^2\dot{H}-8\pi GH^2\rho_{\mathrm{m}}-8\pi G(3\dot{H}+3H^2)\rho_{\mathrm{m}}}{8\pi G\left(3\dot{H}+4H^2+\xi(36H^4+54H^2\dot{H}+24\dot{H}^2-2H\ddot{H})\right)}. \tag{3.35}$$

In other word, we can write down

$$\frac{\mu}{\phi}\frac{\mathrm{d}\phi}{\mathrm{dt}} \simeq \left[\frac{-6\dot{H}^2-8H^2\dot{H}-8\pi GH^2\rho_{\mathrm{m}}-8\pi G(3\dot{H}+3H^2)\rho_{\mathrm{m}}}{8\pi G(3\dot{H}+4H^2+\xi(36H^4+54H^2\dot{H}+24\dot{H}^2-2H\ddot{H}))}\right]^{\frac{1}{2}}, \tag{3.36}$$

which can be integrated with respect to time for finding the shape of the field as following

$$\int \mathrm{d}(\ln\phi) \simeq \frac{1}{\mu}\int \mathrm{dt}\left[\frac{-6\dot{H}^2-8H^2\dot{H}-8\pi GH^2\rho_{\mathrm{m}}-8\pi G(3\dot{H}+3H^2)\rho_{\mathrm{m}}}{8\pi G(3\dot{H}+4H^2+\xi(36H^4+54H^2\dot{H}+24\dot{H}^2-2H\ddot{H}))}\right]^{\frac{1}{2}}, \tag{3.37}$$

Therefore, we obtain the following expression for ϕ as function of time is

$$\phi(t) \simeq \exp\left\{\frac{1}{\mu}\int\left[\frac{-6\dot{H}^2-8H^2\dot{H}-8\pi G\rho_{\rm m}\left(3\dot{H}+4H^2\right)}{8\pi G\left[3\dot{H}+4H^2+\xi\left(36H^4+54H^2\dot{H}+24\dot{H}^2-2H\ddot{H}\right)\right]}\right]^{1/2}\right\}. \tag{3.38}$$

This is an exact solution of the field in an integrated form for the Non-Minimal Derivative Coupling model.

3.3.2 The scalar potential $V(\phi)$

Let us start from the Friedmann equation, Eq. (3.3), with Eq. (3.23) and substitute $\ddot{\phi}$, Eq. (3.26), into the Friedmann equation

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\left[\rho_\phi+\rho_{\rm m}\right], \\ &\simeq \frac{8\pi G}{3}\left[\frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2+V(\phi')+3\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2+6\xi H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi}+\rho_{\rm m}\right], \\ &\simeq \frac{8\pi G}{3}\left[\frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2+V(\phi')+3\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2\right. \\ &\quad \left.+6\xi H\frac{\mu^2}{\phi^2}\dot{\phi}\left(-3H\dot{\phi}-\frac{\phi^2}{\mu^2}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)\right)+\rho_{\rm m}\right], \\ &\simeq \frac{8\pi G}{6}\frac{\mu^2}{\phi^2}\dot{\phi}^2+\frac{8\pi G}{3}V(\phi')+8\pi G\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2-6(8\pi G)\xi H^2\frac{\mu^2}{\phi^2}\dot{\phi}^2 \\ &\quad -2(8\pi G)\xi H\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)+\frac{8\pi G}{3}\rho_{\rm m}, \\ &\simeq \frac{8\pi G}{6}\left[\frac{\mu^2}{\phi^2}\dot{\phi}^2+2V(\phi')+6\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2-36\xi H^2\frac{\mu^2}{\phi^2}\dot{\phi}^2\right. \\ &\quad \left.-12\xi H\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)\right]+\frac{8\pi G}{3}\rho_{\rm m}, \end{aligned}$$

Finally, the Friedmann equation is

$$H^2 \simeq \frac{8\pi G}{6}\left[\left(1+12\xi\dot{H}-18\xi H^2\right)\frac{\mu^2}{\phi^2}\dot{\phi}^2+2V(\phi')-12\xi H\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)\right]+\frac{8\pi G}{3}\rho_{\rm m}. \tag{3.39}$$

Similarly as before, we can rearrange Eq. (3.27) as

$$\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi) \simeq \frac{\dot{H}+\frac{8\pi G}{2}\left(1+6\xi H^2+8\xi\dot{H}\right)\frac{\mu^2}{\phi^2}\dot{\phi}^2+\frac{8\pi G}{2}\rho_{\rm m}}{8\pi G\xi H\dot{\phi}}. \tag{3.40}$$

Substituting Eqs (3.35) and (3.40) into Eq. (3.39) the Friedmann equation can be written as

$$\begin{aligned}
H^2 &\simeq \frac{8\pi G}{6}\Bigg\{ \left(1+12\xi\dot{H}-18\xi H^2\right) \\
&\times\left(\frac{-6\dot{H}^2-8H^2\dot{H}-8\pi G\rho_{\rm m}(3\dot{H}+4H^2)}{8\pi G[3\dot{H}+4H^2+\xi(36H^4+54H^2\dot{H}+24\dot{H}^2-2H\ddot{H})]}\right)-12\xi H\dot{\phi} \\
&\times\frac{\dot{H}+\frac{8\pi G}{2}\left(1+6\xi H^2+8\xi\dot{H}\right)\left(\frac{-6\dot{H}^2-8H^2\dot{H}-8\pi G\rho_{\rm m}(3\dot{H}+4H^2)}{8\pi G[3\dot{H}+4H^2+\xi(36H^4+54H^2\dot{H}+24\dot{H}^2-2H\ddot{H})]}\right)+\frac{8\pi G}{2}\rho_{\rm m}}{8\pi G\xi H\dot{\phi}} \\
&+2V(\phi')\Bigg\}+\frac{8\pi G}{3}\rho_{\rm m}, \\
&\simeq \frac{8\pi G}{6}\Bigg\{ \left(1+12\xi\dot{H}-18\xi H^2\right) \\
&\times\left(\frac{-6\dot{H}^2-8H^2\dot{H}-8\pi G\rho_{\rm m}(3\dot{H}+4H^2)}{8\pi G[3\dot{H}+4H^2+\xi(36H^4+54H^2\dot{H}+24\dot{H}^2-2H\ddot{H})]}\right) \\
&-6\left(1+8\xi\dot{H}+6\xi H^2\right) \\
&\times\left(\frac{-6\dot{H}^2-8H^2\dot{H}-8\pi G\rho_{\rm m}(3\dot{H}+4H^2)}{8\pi G\left[3\dot{H}+4H^2+\xi(36H^4+54H^2\dot{H}+24\dot{H}^2-2H\ddot{H})\right]}\right) \\
&-6\rho_{\rm m}-\frac{12\dot{H}}{8\pi G}+2V(\phi')\Bigg\}+\frac{8\pi G}{3}\rho_{\rm m}, \\
6H^2 &\simeq 8\pi G\Bigg[\left(-5-54\xi H^2-36\xi\dot{H}\right) \\
&\times\left(\frac{-6\dot{H}^2-8H^2\dot{H}-8\pi G\rho_{\rm m}(3\dot{H}+4H^2)}{8\pi G[3\dot{H}+4H^2+\xi(36H^4+54H^2\dot{H}+24\dot{H}^2-2H\ddot{H})]}\right) \\
&-4\rho_{\rm m}-\frac{12\dot{H}}{8\pi G}+2V(\phi')\Bigg].
\end{aligned}$$

Now we can write down the potential of the scalar field in terms of the Hubble parameter, $H, \dot{H}, \ddot{H}$, and $\rho_{\rm m}$ as

$$\begin{aligned}
V(a,H,\dot{H},\ddot{H}) &\simeq \frac{6\dot{H}}{8\pi G}+\frac{3H^2}{8\pi G}+4\rho_{\rm m}+\frac{1}{2}\left(5+54\xi H^2+36\xi\dot{H}\right) \\
&\times\left[\frac{-6\dot{H}^2-8H^2\dot{H}-8\pi G\rho_{\rm m}\left(3\dot{H}+4H^2\right)}{8\pi G\left[3\dot{H}+4H^2+\xi\left(36H^4+54H^2\dot{H}+24\dot{H}^2-2H\ddot{H}\right)\right]}\right].
\end{aligned} \tag{3.41}$$

3.3.3 The equation of state of scalar field w_ϕ

This subsection we study the equation of state of the model. Consider equations (3.23) and (3.30)

$$\rho_\phi \simeq \frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') + 3\xi\Big[(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 2H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi}\Big], \qquad (3.42)$$

$$\dot{\rho}_\phi \simeq \frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + \dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi) + 18\xi\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 6\xi\ddot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 18\xi H^2\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 18\xi H\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2, \qquad (3.43)$$

Consider the standard continuity equation of the scalar field

$$\dot{\rho}_\phi + 3H\rho_\phi(1+w_\phi) = 0. \qquad (3.44)$$

Substituting Eq. (3.42) into the continuity equation (3.44) gives

$$\dot{\rho}_\phi + 3H\Big[\frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') + 3\xi\Big((2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 2H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi}\Big)\Big](1+w_\phi) = 0,$$

and we obtain

$$\dot{\rho}_\phi \simeq -3H\Big[\frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') + 3\xi\Big((2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 2H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi}\Big)\Big](1+w_\phi). \qquad (3.45)$$

compare equations between (3.43) and (3.45), we obtain

$$\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + \dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi) + 18\xi\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 6\xi\ddot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 18\xi H^2\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 18\xi H\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2 \simeq -3H\Big[\frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') + 3\xi\Big((2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 2H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi}\Big)\Big](1+w_\phi),$$

or we can write as

$$1+w_\phi \simeq \frac{\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + \dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi) + 18\xi\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 6\xi\ddot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 18\xi H^2\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 18\xi H\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2}{-3H\Big[\frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') + 3\xi\Big((2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 2H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi}\Big)\Big]},$$

Therefore, the equation of state parameter (E.o.S) of the model is

$$w_\phi \simeq \frac{\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + \dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi) + 18\xi\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 6\xi\ddot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 18\xi H^2\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 18\xi H\dot{H}\frac{\mu^2}{\phi^2}\dot{\phi}^2}{-3H\Big[\frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') + 3\xi\Big((2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 2H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi}\Big)\Big]} - 1. \qquad (3.46)$$

Substituting $\ddot{\phi}$ from Eq. (3.26) into Eq. (3.46), the equation of state parameter (E.o.S) of the model now becomes

$$w_\phi \simeq \frac{\Big[-3H - 54\xi H^3 - 36\xi \dot{H}H + 6\xi\ddot{H}\Big]\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 18\xi(\dot{H}+H^2)\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)}{\Big[-\frac{3}{2}H + 27\xi H^3 - 18\xi\dot{H}H\Big]\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 3HV(\phi') + 18\xi H^2\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)} - 1. \tag{3.47}$$

In addition, we can find the equation of state in the general kinematical form by using the Friedmann equation, Eq. (3.3).

$$\begin{aligned} H^2 &\simeq \frac{8\pi G}{3}\left[\frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') + 3\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 6\xi H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + \rho_{\mathrm{m}}\right], \\ 3H^2 &\simeq \frac{8\pi G}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 8\pi G V(\phi') + 3(8\pi G)\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 \\ &\quad + 6(8\pi G)\xi H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 8\pi G\rho_{\mathrm{m}}, \end{aligned}$$

Therefore, the potential is

$$V(\phi') \simeq \frac{3H^2}{8\pi G} - \frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 3\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 6\xi H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} - \rho_{\mathrm{m}}. \tag{3.48}$$

we substitute Eq. (3.48) into the energy density, ρ_ϕ. Therefore, the equation (3.23) now becomes

$$\begin{aligned} \rho_\phi &\simeq \frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') + 3\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 6\xi H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi}, \\ &\simeq \frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + \frac{3H^2}{8\pi G} - \frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 3\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 6\xi H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} \\ &\quad -\rho_{\mathrm{m}} + 3\xi(2\dot{H}+3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 6\xi H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi}, \\ \rho_\phi + \rho_{\mathrm{m}} &\simeq \frac{3H^2}{8\pi G}. \end{aligned} \tag{3.49}$$

Then we put Eq. (3.49) into Eq. (3.4)

$$\begin{aligned} \dot{H} &\simeq -\frac{8\pi G}{2}\left(\frac{3H^2}{8\pi G} + P_\phi + P_{\mathrm{m}}\right), \\ -\frac{2\dot{H}}{8\pi G} &\simeq \frac{3H^2}{8\pi G} + P_\phi + P_{\mathrm{m}}, \end{aligned}$$

Hence, the pressure is

$$P_\phi \simeq -\frac{3H^2}{8\pi G} - \frac{2\dot{H}}{8\pi G} - P_{\rm m}. \tag{3.50}$$

From the relation of the equation of state, $w_\phi = P_\phi/\rho_\phi$, and in the absence of matter, $w_{\rm m} = 0$. We obtain the equation of state as

$$\begin{aligned} w_\phi &\simeq \frac{P_\phi}{\rho_\phi}, \\ &\simeq \frac{-\frac{3H^2}{8\pi G} - \frac{2\dot{H}}{8\pi G}}{\frac{3H^2}{8\pi G} - \rho_{\rm m}}, \\ w_\phi(H, \dot{H}, \rho_{\rm m}) &\simeq -\left[\frac{3H^2 + 2\dot{H}}{3H^2 - 8\pi G\rho_{\rm m}}\right]. \end{aligned} \tag{3.51}$$

From all information above, we can write the effective equation of state parameter, $w_{\rm eff}$, with dust-matter content as

$$\begin{aligned} w_{\rm eff} &\simeq \frac{w_\phi \rho_\phi}{\rho_\phi + \rho_{\rm m}}, \\ &\simeq \frac{w_\phi\left[\frac{3H^2}{8\pi G} - \rho_{\rm m}\right]}{\frac{3H^2}{8\pi G}}, \\ &\simeq w_\phi\left[1 - \frac{8\pi G\rho_{\rm m}}{3H^2}\right], \end{aligned} \tag{3.52}$$

or simplify to

$$\begin{aligned} w_{\rm eff} &\simeq -\left[\frac{3H^2 + 2\dot{H}}{3H^2 - 8\pi G\rho_{\rm m}}\right]\left[1 - \frac{8\pi G\rho_{\rm m}}{3H^2}\right], \\ &\simeq -\left[\frac{3H^2 + 2\dot{H}}{3H^2 - 8\pi G\rho_{\rm m}}\right]\left[\frac{3H^2 - 8\pi G\rho_{\rm m}}{3H^2}\right], \\ &\simeq -\frac{3H^2 + 2\dot{H}}{3H^2}, \\ &\simeq -1 - \frac{2\dot{H}}{3H^2}. \end{aligned} \tag{3.53}$$

In the case of power-law expansion, we can write the equation of state as

$$\begin{aligned} w_{\rm eff} &\simeq -1 - \frac{2(-p/t^2)}{3(p^2/t^2)}, \\ &\simeq -1 + \frac{2}{3p}. \end{aligned} \tag{3.54}$$

and the accelerated expansion occurs for $p > 1$ which contributes to $w_{\rm eff} < -1/3$.

3.4 Non-Minimal Derivative Coupling and Cosmology

3.4.1 NMDC with power-law expansion

In this subsection, we present the NMDC cosmology under power-law expansion, $a \propto t^p$. We use the information of the power-law expansion from sec. (2.4) and the continuity equation of matter, $\dot{\rho_{\rm m}} = -3H\rho_{\rm m}$. Since we consider on small red-shifts, the radiation sector is ignored. Therefore, the kinetic term of the model from Eq. (3.35) can be written as

$$\begin{aligned}
\frac{\mu^2}{\phi^2}\dot{\phi}^2 &\simeq \frac{-6\dot{H}^2 - 8H^2\dot{H} - 8\pi G\rho_{\rm m}H^2 - 8\pi G\rho_{\rm m}(3\dot{H}+3H^2)}{8\pi G\left(3\dot{H}+4H^2+\xi(36H^4+54H^2\dot{H}+24\dot{H}^2-2H\ddot{H})\right)}, \\
&\simeq \frac{2(\frac{-3p^2+4p^3}{t^4}) - 8\pi G\rho_{\rm m}(\frac{p^2}{t^2}) - 8\pi G\rho_{\rm m}(\frac{3p^2-3p}{t^2})}{8\pi G\left[(\frac{4p^2-3p}{t^2}) + \xi(\frac{36p^4-54p^3+20p^2}{t^4})\right]}, \\
&\simeq \frac{2(\frac{-3p^2+4p^3}{t^4}) - 8\pi G\rho_{\rm m}(\frac{4p^2-3p}{t^2})}{8\pi G\left[(\frac{4p^2-3p}{t^2}) + \xi(\frac{36p^4-54p^3+20p^2}{t^4})\right]}, \\
&\simeq \frac{2p(4p-3) - 8\pi G\rho_{\rm m}t^2(4p-3)}{8\pi G\left[(4p-3)t^2 + p\xi(36p^2-54p+20)\right]}. \qquad (3.55)
\end{aligned}$$

and then

$$\begin{aligned}
\frac{\mu}{\phi}\dot{\phi} &\simeq \frac{1}{\sqrt{8\pi G}}\left[\frac{2p(4p-3) - 8\pi G\rho_{\rm m}t^2(4p-3)}{(4p-3)t^2 + p\xi(36p^2-54p+20)}\right]^{\frac{1}{2}}, \\
&\simeq \frac{1}{\sqrt{8\pi G}}\left[\frac{2p - 8\pi G\rho_{\rm m}t^2}{t^2 + \frac{p\xi(36p^2-54p+20)}{(4p-3)}}\right]^{\frac{1}{2}}. \qquad (3.56)
\end{aligned}$$

Let $\alpha = \sqrt{p\xi(36p^2-54p+20)/(4p-3)}$,

In the case of scalar field dominated or the matter term can be neglected, we get

$$\mu\int d(\ln\phi) \simeq \frac{\sqrt{2p}}{\sqrt{8\pi G}}\int dt\frac{1}{\sqrt{t^2+\alpha^2}}, \qquad (3.57)$$

Integration of above equation, Eq. (3.57), which gives the scalar field as a function of time

$$\ln\left(\frac{\phi}{\phi_0}\right) \simeq \frac{\sqrt{2p}}{\mu\sqrt{8\pi G}}\sinh^{-1}\left(\frac{t}{\alpha}\right), \qquad (3.58)$$

Finally, we obtain the field as

$$\phi(t) \simeq \phi_0 \exp\left[\left(\frac{\sqrt{2p}}{\mu\sqrt{8\pi G}}\right)\sinh^{-1}\left(\frac{t}{\alpha}\right)\right]. \tag{3.59}$$

where ϕ_0 is an integration constant.

Inverting of Eq. (3.58) and define $\chi = \mu\sqrt{8\pi G}/\sqrt{2p}$ we can obtain the time parameter as a function of ϕ as

$$t \simeq \alpha \sinh\left[\chi \ln\left(\frac{\phi}{\phi_0}\right)\right] \tag{3.60}$$

Consider the scalar potential of the model, Eq. (3.41). We use the power-law expansion, $a \propto t^p$, and write the scalar potential as the function of time. Hence, the scalar potential is given by

$$\begin{aligned} V(t) &\simeq \frac{-6p}{8\pi G t^2} + \frac{3p^2}{8\pi G t^2} - \frac{\left[\frac{-4p^3}{t^4} + \frac{3p^2}{t^4}\right]\left[5 + 36\xi(-\frac{p}{t^2}) + 54\xi(\frac{p^2}{t^2})\right]}{8\pi G\left[\frac{4p^2-3p}{t^2} + \xi\left(\frac{36p^4-54p^3+20p^2}{t^4}\right)\right]}, \\ &\simeq \frac{3p(p-2)}{8\pi G t^2} - \frac{3p^2}{8\pi G t^2} - \frac{\left[\frac{4p^3-3p^2}{t^4}\right]\left[\frac{5t^2+54\xi p^2-36\xi p}{t^2}\right]}{8\pi G\left[\frac{4p^2-3p}{t^2} + \xi\left(\frac{36p^4-54p^3+20p^2}{t^4}\right)\right]}, \\ &\simeq \frac{3p(p-2)}{8\pi G t^2} + \frac{\left[\frac{(4p^3-3p^2)(5t^2+54\xi p^2-36\xi p)}{t^6}\right]}{8\pi G\left[\frac{p(4p-3)t^2+\xi p^2(36p^2-54p+20)}{t^4}\right]}, \\ &\simeq \frac{3p(p-2)}{8\pi G t^2} + \frac{p^2(4p-3)\left[5t^2 + 54\xi p^2 - 36\xi p\right]}{8\pi G\left[p(4p-3)t^4 + \xi p^2 t^2(36p^2 - 54p + 20)\right]}. \end{aligned} \tag{3.61}$$

The corresponding the scalar potential in term of ϕ, which is obtained after substituting t from Eq. (3.60) into Eq. (3.61), is given by

$$\begin{aligned} V(\phi) \simeq\ & \frac{3p(p-2)}{8\pi G\alpha^2 \sinh^2\left[\chi \ln(\frac{\phi}{\phi_0})\right]} + \\ & \frac{p^2(4p-3)\left\{5\alpha^2\sinh^2\left[\chi\ln(\frac{\phi}{\phi_0})\right] + 54\xi p^2 - 36\xi p\right\}}{8\pi G\left\{p(4p-3)\alpha^4\sinh^4\left[\chi\ln(\frac{\phi}{\phi_0})\right] + p^2\xi\alpha^2\sinh^2\left[\chi\ln(\frac{\phi}{\phi_0})\right](36p^2-54p+20)\right\}}. \end{aligned} \tag{3.62}$$

Next we want to derive the equation of state (E.o.S) of the model by using the power-law expansion. The Hubble parameter is $H(t) = \dot{a}/a = p/t$ with $\dot{H} = -p/t^2$ and $\ddot{H} = -2p/t^3$. Hence the E.o.S parameter, Eq. (3.47), is

$$
\begin{aligned}
w_\phi &\simeq \frac{\Big[-3H - 54\xi H^3 - 36\xi \dot{H}H + 6\xi\ddot{H}\Big]\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 18\xi\left(\dot{H}+H^2\right)\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)}{\Big[-\frac{3}{2}H + 27\xi H^3 - 18\xi\dot{H}H\Big]\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 3HV(\phi') + 18\xi H^2\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)} - 1,\\
&\simeq \frac{\Big[18\xi\left(\frac{p^3}{t^3}\right) - 12\xi\left(\frac{p^2}{t^3}\right) - 4\xi\left(\frac{p}{t^3}\right) + \left(\frac{p}{t}\right)\Big]\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 6\xi\left(\frac{p^2}{t^2} - \frac{p}{t^2}\right)\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)}{\Big[-9\xi\left(\frac{p^3}{t^3}\right) - 6\xi\left(\frac{p^2}{t^3}\right) + \frac{1}{2}\left(\frac{p}{t}\right)\Big]\frac{\mu^2}{\phi^2}\dot{\phi}^2 + \left(\frac{p}{t}\right)V(\phi') - 6\xi\left(\frac{p^2}{t^2}\right)\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)} - 1,\\
&\simeq \frac{\Big[18\xi\left(\frac{p^2}{t^2}\right) - 12\xi\left(\frac{p}{t^2}\right) - 4\xi\left(\frac{1}{t^2}\right) + 1\Big]\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 6\xi\left(\frac{p}{t} - \frac{1}{t}\right)\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)}{\Big[-9\xi\left(\frac{p^2}{t^2}\right) - 6\xi\left(\frac{p}{t^2}\right) + \frac{1}{2}\Big]\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') - 6\xi\left(\frac{p}{t}\right)\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)} - 1,
\end{aligned}
\tag{3.63}
$$

Substituting the scalar field kinetic term, $\mu^2(\dot{\phi}^2/\phi^2)$, from Eq. (3.55) to above equation. For the case of potential is not equal to zero, we get

$$
\begin{aligned}
w_\phi &\simeq \frac{\Big[\frac{18\xi p^2 - 12\xi p - 4\xi + t^2}{t^2}\Big]\Big[\frac{2p(4p-3)}{8\pi G[(4p-3)t^2 + p\xi(36p^2-54p+20)]}\Big] + 6\xi\frac{(p-1)}{t}\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)}{\Big[\frac{-9\xi p^2 - 6\xi p + t^2/2}{t^2}\Big]\Big[\frac{2p(4p-3)}{8\pi G[(4p-3)t^2 + p\xi(36p^2-54p+20)]}\Big] + V(\phi') - 6\xi\left(\frac{p}{t}\right)\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)} - 1,\\
&\simeq \frac{\frac{(18\xi p^2 - 12\xi p - 4\xi + t^2)[2p(4p-3)]}{8\pi G[(4p-3)t^4 + p\xi t^2(36p^2-54p+20)]} + 6\xi\frac{(p-1)}{t}\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)}{\frac{(-9\xi p^2 - 6\xi p + t^2/2)[2p(4p-3)]}{8\pi G[(4p-3)t^4 + p\xi t^2(36p^2-54p+20)]} + V(\phi') - 6\xi\left(\frac{p}{t}\right)\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)} - 1,
\end{aligned}
\tag{3.64}
$$

In the case of with out the potential, $V(\phi') = 0$, the consequence of its derivative is $\mathrm{d}V(\phi)/\mathrm{d}\phi = 0$. Therefore, the EoS parameter reduces to

$$
\begin{aligned}
w_\phi &\simeq -1 + \frac{\Big[\frac{18\xi p^2 - 12\xi p - 4\xi + t^2}{t^2}\Big]\Big[\frac{2p(4p-3)}{8\pi G[(4p-3)t^2 + p\xi(36p^2-54p+20)]}\Big]}{\Big[\frac{-9\xi p^2 - 6\xi p + t^2/2}{t^2}\Big]\Big[\frac{2p(4p-3)}{8\pi G[(4p-3)t^2 + p\xi(36p^2-54p+20)]}\Big]},\\
w_\phi &\simeq -1 - \Bigg[\frac{18\xi p^2 - 12\xi p - 4\xi + t^2}{9\xi p^2 + 6\xi p - t^2/2}\Bigg].
\end{aligned}
\tag{3.65}
$$

3.4.2 NMDC with de-Sitter expansion

Let us now consider the kinetic term of the scalar field in term of the de-Sitter expansion, $a \propto \exp(H_0 t)$. The Hubble parameter is

$$\begin{aligned} H &= \frac{a_0 H_0 \exp(H_0 t)}{a_0 \exp(H_0 t)}, \\ H &= H_0. \end{aligned} \tag{3.66}$$

where H_0 is the Hubble constant at the present time, and we use $\rho_{\rm m} = \rho_{\rm m,0} a^{-3}$, then we get

$$\rho_{\rm m} = \rho_{\rm m,0} \exp(-3H_0 t) \tag{3.67}$$

Therefore the kinetic term of the model, Eq. (3.35), is

$$\begin{aligned} \frac{\mu^2}{\phi^2}\dot{\phi}^2 &\simeq \frac{-6\dot{H}^2 - 8H^2\dot{H} - 8\pi G\rho_{\rm m}\left(3\dot{H} + 4H^2\right)}{8\pi G\left(3\dot{H} + 4H^2 + \xi\left(36H^4 + 54H^2\dot{H} + 24\dot{H}^2 - 2H\ddot{H}\right)\right)}, \\ &\simeq \frac{-32\pi G\rho_{\rm m}H^2}{8\pi G\left(4H^2 + 36\xi H^4\right)}, \\ &\simeq -\frac{\rho_{\rm m}}{(1 + 9\xi H^2)}. \end{aligned} \tag{3.68}$$

Substituting Eq. (3.67) into Eq. (3.68) and integrating these equation with respect to time is given by

$$\begin{aligned} -\int \mathrm{d}(\ln\phi) &\simeq \frac{1}{\mu}\int \mathrm{dt}\left[\frac{\rho_{\rm m,0}\exp(-3H_0 t)}{(1 + 9\xi H_0^2)}\right]^{\frac{1}{2}} \\ \ln\left(\frac{\phi}{\phi_0}\right) &\simeq \frac{2}{3}\left[\frac{\rho_{\rm m,0}\exp(-3H_0 t)}{\mu^2 H_0^2(1 + 9\xi H_0^2)}\right]^{\frac{1}{2}}. \end{aligned} \tag{3.69}$$

Finally, we obtain the field

$$\phi(t) \simeq \phi_0 \exp\left\{\frac{2}{3}\left[\frac{\rho_{\rm m,0}\exp(-3H_0 t)}{\mu^2 H_0^2(1 + 9\xi H_0^2)}\right]^{\frac{1}{2}}\right\} \tag{3.70}$$

Inverting Eq. (3.69), we can write the time parameter as function of field ϕ as

$$t \simeq -\frac{1}{3H_0}\ln\left\{\left[\frac{\mu^2 H_0^2(1 + 9\xi H_0^2)}{\rho_{\rm m,0}}\right]\left[\frac{3}{2}\ln\left(\frac{\phi}{\phi_0}\right)\right]^2\right\}. \tag{3.71}$$

Next we will find solutions giving rise to accelerated expansion and look for the scalar potential, Eq. (3.41). The scalar potential of the model is given by

$$\begin{aligned} V(t) &\simeq \frac{3H_0^2}{8\pi G} + 4\rho_{\mathrm{m},0}\exp\left(-3H_0 t\right) + \frac{1}{2}\frac{(5+54\xi H_0^2)\,(-4H_0^2)}{(4H_0^2+36\xi H_0^4)}\rho_{\mathrm{m},0}\exp\left(-3H_0 t\right), \\ &\simeq \frac{3H_0^2}{8\pi G} - \left[\frac{5+54\xi H_0^2}{2(1+9\xi H_0^2)} - 4\right]\rho_{\mathrm{m},0}\exp\left(-3H_0 t\right). \end{aligned} \tag{3.72}$$

To find the scalar potential in term of ϕ, which is obtained after substituting t from Eq. (3.71) into Eq. (3.72)

$$\begin{aligned} V(\phi) &\simeq \frac{3H_0^2}{8\pi G} - \left[\frac{5+54\xi H_0^2}{2+18\xi H_0^2} - 4\right]\rho_{\mathrm{m},0}\exp\left\{\ln\left[\frac{\mu^2(H_0^2+9\xi H_0^4)}{\rho_{\mathrm{m},0}}\right]\left[\frac{3}{2}\ln\left(\frac{\phi}{\phi_0}\right)\right]^2\right\}, \\ &\simeq \frac{3H_0^2}{8\pi G} - \mu^2 H_0^2(1+9\xi H_0^2)\left[\frac{5+54\xi H_0^2}{2+18\xi H_0^2} - 4\right]\left[\frac{3}{2}\ln\left(\frac{\phi}{\phi_0}\right)\right]^2, \\ &\simeq \frac{3H_0^2}{8\pi G} + \frac{27}{8}\mu^2 H_0^2(1+6\xi H_0^2)\left[\ln\left(\frac{\phi}{\phi_0}\right)\right]^2. \end{aligned} \tag{3.73}$$

Now we will derive the equation of state of the model with function of the de-Sitter expansion. Let us consider Eq. (3.47)

$$w_\phi \simeq \frac{\left[-3H - 54\xi H^3 - 36\xi \dot{H}H + 6\xi\ddot{H}\right]\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 18\xi(\dot{H}+H^2)\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)}{\left[-\frac{3}{2}H + 27\xi H^3 - 18\xi\dot{H}H\right]\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 3HV(\phi') + 18\xi H^2\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)} - 1,$$

By using Eq. (3.68), and $H = \dot{a}/a = H_0$, in the case of non zero potential and zero potential, respectively. we obtain

$$w_\phi \simeq \frac{\left[-3H_0 - 54\xi H_0^3\right]\left[\frac{-\rho_{\mathrm{m},0}\exp(-3H_0 t)}{(1+9\xi H_0^2)}\right] - 18\xi H_0^2\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)}{\left[-\frac{3}{2}H_0 + 27\xi H_0^3\right]\left[\frac{-\rho_{\mathrm{m},0}\exp(-3H_0 t)}{(1+9\xi H_0^2)}\right] - 3H_0 V(\phi') + 18\xi H_0^2\dot{\phi}\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi)} - 1, \tag{3.74}$$

and

$$\begin{aligned} w_\phi &\simeq -1 - \left[\frac{-3H_0 - 54\xi H_0^3}{\frac{3}{2}H_0 - 27\xi H_0^3}\right], \\ &\simeq -1 - \left[\frac{-1 - 18\xi H_0^2}{\frac{1}{2} - 9\xi H_0^2}\right]. \end{aligned} \tag{3.75}$$

3.4.3 NMDC with super-acceleration expansion

In this subsection, let us consider a model with function of the super-acceleration expansion. The Hubble in this case is

$$H = \frac{\dot{a}}{a} = -\frac{q}{t_s - t}, \tag{3.76}$$

time derivative of Hubble parameter is

$$\dot{H} = -\frac{q}{(t_s - t)^2}, \tag{3.77}$$

and

$$\ddot{H} = -\frac{2q}{(t_s - t)^3}. \tag{3.78}$$

where $q < 0$ and the dust matter density, $\rho_{\rm m} = \rho_{\rm m,0}\,(a_0^3/a^3)$ is then

$$\rho_{\rm m} = \rho_{\rm m,0}\left(\frac{t_{\rm s} - t_0}{t_{\rm s} - t}\right)^{3q} \tag{3.79}$$

Therefore, the kinetic term of the model, Eq. (3.35), can be written as

$$\begin{aligned}
\frac{\mu^2}{\phi^2}\dot{\phi}^2 &\simeq \frac{-6\dot{H}^2 - 8H^2\dot{H} - 8\pi G\rho_{\rm m}H^2 - 8\pi G\rho_{\rm m}(3\dot{H} + 3H^2)}{8\pi G\left(3\dot{H} + 4H^2 + \xi(36H^4 + 54H^2\dot{H} + 24\dot{H}^2 - 2H\ddot{H})\right)}, \\
&\simeq \frac{2\left(\frac{-3q^2+4q^3}{(t_s-t)^4}\right) - 8\pi G\rho_{\rm m}\left(\frac{q^2}{(t_s-t)^2}\right) - 8\pi G\rho_{\rm m}(\frac{3q^2-3q}{(t_s-t)^2})}{8\pi G\left[\left(\frac{4q^2-3q}{(t_s-t)^2}\right) + \xi\left(\frac{36q^4-54q^3+20q^2}{(t_s-t)^4}\right)\right]}, \\
&\simeq \frac{2\left(\frac{-3q^2+4q^3}{(t_s-t)^4}\right) - 8\pi G\rho_{\rm m}\left(\frac{4q^2-3q}{(t_s-t)^2}\right)}{8\pi G\left[\left(\frac{4q^2-3q}{(t_s-t)^2}\right) + \xi\left(\frac{36q^4-54q^3+20q^2}{(t_s-t)^4}\right)\right]}, \\
&\simeq \frac{2q(4q-3) - 8\pi G\rho_{\rm m}(t_s-t)^2(4q-3)}{8\pi G\left[(4q-3)(t_s-t)^2 + q\xi(36q^2-54q+20)\right]}.
\end{aligned} \tag{3.80}$$

and

$$\begin{aligned}
\frac{\mu}{\phi}\dot{\phi} &\simeq \frac{1}{\sqrt{8\pi G}}\left[\frac{2q(4q-3) - 8\pi G\rho_{\rm m}(t_s-t)^2(4q-3)}{(4q-3)(t_s-t)^2 + q\xi(36q^2-54q+20)}\right]^{\frac{1}{2}}, \\
&\simeq \frac{1}{\sqrt{8\pi G}}\left[\frac{2q - 8\pi G\rho_{\rm m}(t_s-t)^2}{(t_s-t)^2 + \frac{q\xi(36q^2-54q+20)}{(4q-3)}}\right]^{\frac{1}{2}}.
\end{aligned} \tag{3.81}$$

Let $\beta^2 = (36q^2 - 54q + 20)/(4q - 3)$ and $\sqrt{q} = i\sqrt{|q|}$, in this case we assume no the matter contribution , $\rho_{\rm m} = 0$, so that

$$\mu \int d(\ln \phi) \;\simeq\; -\frac{i\sqrt{2|q|}}{\sqrt{8\pi G}} \int d(\mathrm{t_s} - \mathrm{t}) \frac{1}{\sqrt{(t_s - t)^2 + q\xi\beta^2}}, \tag{3.82}$$

Integration of these equation which gives the scalar field as function of time is

$$\ln\left(\frac{\phi}{\phi_0}\right) \;\simeq\; -\frac{i\sqrt{2|q|}}{\mu\sqrt{8\pi G}} \sinh^{-1}\left(\frac{t_s - t}{i\sqrt{|q|\xi}\beta}\right) \tag{3.83}$$

Finally, the field is found

$$\phi(t) \;\simeq\; \phi_0 \exp\left[-\left(\frac{\sqrt{2|q|}}{\mu\sqrt{8\pi G}}\right) \sin^{-1}\left(\frac{t_s - t}{\sqrt{|q|\xi}\beta}\right)\right]. \tag{3.84}$$

Similarly, we define $\chi = \mu\sqrt{8\pi G}/\sqrt{2|q|}$ and we can write the time parameter as function of field as

$$t_s - t \;\simeq\; \beta \sinh\left[-\chi \ln\left(\frac{\phi}{\phi_0}\right)\right]. \tag{3.85}$$

or

$$t \;\simeq\; t_s - \beta \sinh\left[-\chi \ln\left(\frac{\phi}{\phi_0}\right)\right]. \tag{3.86}$$

As the scalar potential of the model, Eq. (3.61) we have the scalar potential as function of time

$$V(t) \;\simeq\; \frac{3q\,(q-2)}{8\pi G\,(t_s - t)^2} + \frac{q^2(4q-3)\Big[5(t_s - t)^2 + 54\xi q^2 - 36\xi q\Big]}{8\pi G\Big[q(4q-3)(t_s - t)^4 + \xi q^2(t_s - t)^2(36q^2 - 54q + 20)\Big]}, \tag{3.87}$$

Substituting Eq. (3.85) into Eq. (3.87) and we have used relations, $\sinh(-ix) = -i\sin(x)$ and $\sin^{-1}(-x) = i\sinh^{-1}(ix)$. Hence we obtain the scalar potential is

$$V(\phi) \;\simeq\; \frac{3(q-2)}{8\pi G\xi\beta^2 \sin^2\left[\chi \ln(\frac{\phi}{\phi_0})\right]} + \frac{-5\beta^2 \sin^2\left[\chi \ln(\frac{\phi}{\phi_0})\right] + 54q - 36}{-8\pi G\xi\beta^4 \sin^2\left[\chi \ln(\frac{\phi}{\phi_0})\right] \cos^2\left[\chi \ln(\frac{\phi}{\phi_0})\right]}. \tag{3.88}$$

Similar method to the power-law expansion, we want to derive the equation of state (E.o.S) from the case of super-acceleration expansion. By using Eqs. (3.76), (3.77), and (3.78) into Eq. (3.47). Therefore the E.o.S parameter for the simple case, without potential, is given by

$$
\begin{aligned}
w_\phi &\simeq -1 + \frac{\left[\frac{18\xi q^2 - 12\xi q - 4\xi + (t_s - t)^2}{(t_s - t)^2}\right]\left[\frac{2q(4q-3)}{8\pi G[(4q-3)(t_s-t)^2 + q\xi(36q^2 - 54q + 20)]}\right]}{\left[\frac{-9\xi q^2 - 6\xi q + (t_s - t)^2/2}{t^2}\right]\left[\frac{2q(4q-3)}{8\pi G[(4p-3)(t_s-t)^2 + q\xi(36q^2 - 54q + 20)]}\right]}, \\
w_\phi &\simeq -1 - \left[\frac{18\xi q^2 - 12\xi q - 4\xi + (t_s - t)^2}{9\xi q^2 + 6\xi q - (t_s - t)^2/2}\right]. \qquad (3.89)
\end{aligned}
$$

CHAPTER IV

RESULTS AND DISCUSSIONS

In the previous chapter, we have presented the derived essential solutions, e.g. the scalar field and the scalar potential, from our investigation in the NMDC model. In the present chapter we show the results and discussions.

The power-law function has been widely considered in astrophysical observation, see detail [36, 40]. Previously, Granda has studied simplest $\xi R\partial_a\phi\partial^a\phi$ model and has found scalar potential by assuming power-law expansion [20]. In our study, we have considered a result of a field transformation introduced to the model. After that we have found the scalar potential when power-law, super-acceleration, and de-Sitter expansion were considered.

4.1 Results

As can be seen from the previous chapter, if we know the exact form of scale factor $a = a(t)$ we can derive the field solution and the scalar potential. We show cosmological solutions where the ingredients are dark energy and dark matter could be viewed as the presence of the NMDC field with dust matter fluid.

In this model, we assume the known expansions form are power-law, de-Sitter and phantom power-law or super- acceleration. Hence the scalar field and the scalar potential are found.

4.1.1 Power-law expansion

We consider an expansion function, $a \propto t^p$ where $p > 0$. To find the exact scalar field solution of this case, we need to disregard small contribution of $\rho_{\rm m}$ in order to obtain exact solution,

$$\phi(t) \simeq \phi_0 \exp\left[\left(\frac{\sqrt{2p}}{\mu\sqrt{8\pi G}}\right)\sinh^{-1}\left(\frac{t}{\alpha}\right)\right], \qquad (4.1)$$

where $\alpha = \sqrt{p\xi\left(36p^2 - 54p + 20\right)/\left(4p - 3\right)}$ and $\chi = \mu\sqrt{8\pi G}/\sqrt{2p}$. Then the scalar potential is

$$V(\phi) \simeq \frac{3p(p-2)}{8\pi G\alpha^2 \sinh^2\left[\chi \ln(\frac{\phi}{\phi_0})\right]} + \frac{p^2(4p-3)\left\{5\alpha^2 \sinh^2\left[\chi \ln(\frac{\phi}{\phi_0})\right] + 54\xi p^2 - 36\xi p\right\}}{8\pi G\left\{p(4p-3)\alpha^4 \sinh^4\left[\chi \ln(\frac{\phi}{\phi_0})\right] + p^2\xi\alpha^2 \sinh^2\left[\chi \ln(\frac{\phi}{\phi_0})\right](36p^2 - 54p + 20)\right\}}. \tag{4.2}$$

4.1.2 de-Sitter expansion

Assuming the de-Sitter expansion, $a \propto e^{H_0 t}$ and we keep the dust matter contribution in the solution here. The scalar solution is

$$\phi(t) \simeq \phi_0 \exp\left\{\frac{2}{3}\left[\frac{\rho_{\mathrm{m},0}\exp\left(-3H_0 t\right)}{\mu^2 H_0^2(1 + 9\xi H_0^2)}\right]^{\frac{1}{2}}\right\}, \tag{4.3}$$

with the potential

$$V(\phi) \simeq \frac{3H_0^2}{8\pi G} + \frac{9}{4}\mu^2 H_0^2(3 + 18\xi H_0^2)\left[\ln\left(\frac{\phi}{\phi_0}\right)\right]^2. \tag{4.4}$$

4.1.3 Super-acceleration expansion

The super-acceleration expansion is assumed, $a \propto (t_s - t)^q$ where $q < 0$ and t_s is the future singularity, $t_s > t$. The solution is

$$\phi(t) \simeq \phi_0 \exp\left[-\left(\frac{\sqrt{2|q|}}{\mu\sqrt{8\pi G}}\right)\sin^{-1}\left(\frac{t_s - t}{\sqrt{|q|\xi\beta}}\right)\right]. \tag{4.5}$$

where $\beta^2 = (36q^2 - 54q + 20)/(4q - 3)$ with $\chi = \mu\sqrt{8\pi G}/\sqrt{2|q|}$. Therefore we obtain the potential

$$V(\phi) \simeq \frac{3(q-2)}{8\pi G\xi\beta^2 \sin^2\left[\chi \ln(\frac{\phi}{\phi_0})\right]} + \frac{-5\beta^2 \sin^2\left[\chi \ln(\frac{\phi}{\phi_0})\right] + 54q - 36}{-8\pi G\xi\beta^4 \sin^2\left[\chi \ln(\frac{\phi}{\phi_0})\right]\cos^2\left[\chi \ln(\frac{\phi}{\phi_0})\right]}. \tag{4.6}$$

4.2 Discussions

According to this results, our results give slightly different from the result given in [20]. The complicated structure of the FLRW equations in this model is not easy to analysis of their solutions and in general we cannot find an explicit of the potential. Nevertheless, when power-law, super-acceleration, and de-Sitter expansion are assumed with under slow-roll assumption of $0 < |\dot{\phi}| \ll 1$ and $|\dddot{\phi}| \ll |\ddot{\phi}| \ll |\dot{\phi}|$, an explicit of potential can be derived in (4.2), (4.4) and (4.6), respectively.

However, all cases of this studies have only considered in the case of without the matter contribution, $\rho_{\rm m} = 0$, except for a de-Sitter case. Including a matter term might completely changes the evolution of the scale factor, possibly giving rise a past deceleration followed by the present acceleration.

In the future work, we will use employ the skills learn from this work to approach NMDC palatini model and find scalar field exact solution and scalar field potential. The stability analysis of the model will be investigated.

CHAPTER V

CONCLUSIONS

In this thesis, we study general aspect of FLRW cosmology of the NMDC model for explanation the accelerated expansion of the universe.

In our work, we are interested in and starting with the action of Granda's model [20]. We propose a transformation $\phi' = \mu \ln \phi$ which enhance domination of the NMDC terms. We assumed a flat FLRW universe filled with scalar field and pressureless matter. Our scalar fields are in the non-minimal derivative coupling to Ricci scalar with the coupling constant ξ.

After constructing the scenario, we derived field equations and equation of motion for this model under slow-roll approximation. Then we find cosmological solutions of the scalar field and scalar potential as function of $a, H, \dot{H}$ and $\ddot{H}$ when considering power-law, super-acceleration, and de-Sitter expansion.

In conclusion, the field solution, $\phi(t)$ and the potential, $V(\phi)$ can be found for an explicit forms by using three types of expansion functions: power-law, de-Sitter and super-acceleration expansions [41].

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APPENDIX

APPENDIX A VARIATIONAL APPROACH

We start from the Einstein-Hilbert action

$$S_{EH} = \int \mathrm{d}^4x\sqrt{-g}R \tag{A.1}$$

Writing the Ricci scalar as $R = g^{ab}R_{ab}$, the first-order variation in the Einstein-Hilbert action can be written as

$$\begin{aligned} \delta S_{EH} &= \int \mathrm{d}^4x(\delta\sqrt{-g})g^{ab}R_{ab} + \int \mathrm{d}^4x\sqrt{-g}(\delta g^{ab})R_{ab} + \int \mathrm{d}^4x\sqrt{-g}g^{ab}(\delta R_{ab}), \\ &= \delta S_1 + \delta S_2 + \delta S_3. \end{aligned} \tag{A.2}$$

the first term: $\delta\sqrt{-g}$

the first term of equation (A.2) is

$$\delta S_1 = \int \mathrm{d}^4x(\delta\sqrt{-g})g^{ab}R_{ab} \tag{A.3}$$

Let consider

$$\delta\sqrt{-g} = -\frac{1}{2}\frac{1}{\sqrt{-g}}\delta g \tag{A.4}$$

From the relation,

$$\begin{aligned} \ln g &= \mathrm{Tr}\Big[\ln(g_{ab})\Big], \\ \frac{\delta g}{g} &= \mathrm{Tr}\Big[\frac{1}{g^{ab}}\delta g_{ab}\Big], \\ &= g^{ab}\delta g_{ab}, \\ \delta g &= g g^{ab}\delta g_{ab}. \end{aligned} \tag{A.5}$$

then equation (A.4) becomes

$$\delta\sqrt{-g} = -\frac{1}{2}\frac{1}{\sqrt{-g}}g g^{ab}\delta g_{ab} \tag{A.6}$$

and from the relation

$$g_{ab}g^{bc} = \delta^c_a = 0 \tag{A.7}$$

take the variation to equation (A.7),

$$\begin{aligned}
(\delta g_{ab})g^{bc} + g_{ab}(\delta g^{bc}) &= 0 \\
(\delta g_{ab})g^{bc} &= -g_{ab}(\delta g^{bc}) \\
g_{cd}(\delta g_{ab})g^{bc} &= -g_{cd}g_{ab}\delta g^{bc} \\
\delta^b_d \delta g_{ab} &= -g_{cd}g_{ab}\delta g^{bc} \\
\delta g_{ad} &= -g_{cd}g_{ab}\delta g^{bc} \qquad \text{(A.8)}
\end{aligned}$$

substitute into equation (A.6) and then

$$\begin{aligned}
\delta\sqrt{-g} &= -\frac{1}{2}\frac{1}{\sqrt{-g}}gg^{ab}(-g_{cb}g_{ad}\delta g^{dc}) \\
\delta\sqrt{-g} &= \frac{1}{2}\frac{1}{\sqrt{-g}}g\delta^a_c g_{ad}\delta g^{dc} \\
\delta\sqrt{-g} &= -\frac{1}{2}\sqrt{-g}g_{cd}\delta g^{dc}
\end{aligned}$$

changing the indices above equation $c \to a$ and $d \to b$ and we can write down

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{ab}\delta g^{ab} \qquad \text{(A.9)}$$

Finally, we will obtain the equation for the equation (A.3) as

$$\begin{aligned}
\delta S_1 &= \int \mathrm{d}^4x \Big[\Big(-\frac{1}{2}\sqrt{-g}g_{ab}\delta g^{ab}\Big)g^{cd}R_{cd}\Big] \\
&= \int \mathrm{d}^4x \sqrt{-g}\Big[\Big(-\frac{1}{2}g_{ab}R\Big)\delta g^{ab}\Big]. \qquad \text{(A.10)}
\end{aligned}$$

the second term: δg^{ab}

the second term of equation (A.2) is

$$\delta S_2 = \int \mathrm{d}^4x\sqrt{-g}R_{ab}\delta g^{ab} \qquad \text{(A.11)}$$

the third term: δR_{ab}

the third term of equation (A.2) is

$$\delta S_3 = \int \mathrm{d}^4x\sqrt{-g}g^{ab}(\delta R_{ab}) \qquad \text{(A.12)}$$

consider

$$R_{ab} = R^c_{abc} = \partial_b\Gamma^c_{ac} - \partial_c\Gamma^c_{ab} + \Gamma^e_{ac}\Gamma^c_{eb} - \Gamma^e_{ab}\Gamma^c_{ec} \quad \text{(A.13)}$$

therefore,

$$\begin{aligned}
\delta R_{ab} &= \partial_b(\delta\Gamma^c_{ac}) - \partial_c(\delta\Gamma^c_{ab}) + \Gamma^e_{ac}(\delta\Gamma^c_{eb}) + (\delta\Gamma^e_{ac})\Gamma^c_{eb} \\
&\quad -\Gamma^e_{ab}(\delta\Gamma^c_{ec}) - (\delta\Gamma^e_{ab})\Gamma^c_{ec} \\
&= \{\partial_b(\delta\Gamma^c_{ac}) - \Gamma^e_{ab}(\delta\Gamma^c_{ec}) + \Gamma^c_{eb}(\delta\Gamma^e_{ac}) - [\Gamma^e_{cb}(\delta\Gamma^c_{ea})]\} \\
&\quad -\{\partial_c(\delta\Gamma^c_{ab}) + \Gamma^c_{ec}(\delta\Gamma^e_{ab}) - \Gamma^e_{ac}(\delta\Gamma^c_{eb}) - [\Gamma^e_{cb}(\delta\Gamma^c_{ea})]\} \\
\delta R_{ab} &= \nabla_b(\delta\Gamma^c_{ac}) - \nabla_c(\delta\Gamma^c_{ab}) \quad \text{(A.14)}
\end{aligned}$$

and then

$$g^{ab}\delta R_{ab} = \nabla_b(g^{ab}\delta\Gamma^c_{ac}) - \nabla_c(g^{ab}\delta\Gamma^c_{ab}), \quad \text{(A.15)}$$

from the relation

$$\begin{aligned}
\delta\Gamma^c_{ab} &= \frac{1}{2}g^{cd}(\nabla_a\delta g_{db} + \nabla_b\delta g_{da} - \nabla_d\delta g_{ab}), \quad \text{(A.16)} \\
\delta\Gamma^c_{ac} &= \frac{1}{2}g^{cd}(\nabla_a\delta g_{dc} + \nabla_c\delta g_{da} - \nabla_d\delta g_{ac}). \quad \text{(A.17)}
\end{aligned}$$

we substitute above equations into (A.15)

$$\begin{aligned}
g^{ab}\delta R_{ab} &= \frac{1}{2}\nabla_b(g^{ab}g^{cd}\nabla_a\delta g_{cd}) - \frac{1}{2}\nabla_c[g^{ab}g^{cd}(2\nabla_a\delta g_{bd} - \nabla_d\delta g_{ab})] \\
&= \nabla_c\nabla^c(g^{ab}\delta g_{ab}) - \nabla^a\nabla^b(\delta g_{ab}) \\
&= [\nabla_a\nabla_b - g^{ab}\nabla_a\nabla^a]\delta g^{ab} \quad \text{(A.18)}
\end{aligned}$$

Therefore, the equation (A.12) become

$$\delta S_3 = \int d^4x\sqrt{-g}[\nabla_a\nabla_b - g^{ab}\nabla_a\nabla^a]\delta g^{ab} \quad \text{(A.19)}$$

we will obtain

$$\begin{aligned}
\delta S &= \delta S_1 + \delta S_2 + \delta S_3 \\
&= \int d^4x\sqrt{-g}\left[R_{ab} - \frac{1}{2}g_{ab}R + [\nabla_a\nabla_b - g^{ab}\nabla_a\nabla^a]\right]\delta g^{ab} \quad \text{(A.20)}
\end{aligned}$$

Finally, from the action principle, $\delta S = 0$, we can write down

$$R_{ab} - \frac{1}{2}g_{ab}R = 0 \equiv G_{ab}. \tag{A.21}$$

this is called the Einstein's field equation in vacuum.

Let us consider matter term

$$\begin{aligned} S &= S_{EH} + S_m \\ S &= \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{16\pi G}\mathcal{L}_{EH} + \mathcal{L}_m\right) \end{aligned} \tag{A.22}$$

and vary this action

$$\begin{aligned} \delta S &= \int \mathrm{d}^4x\left[\sqrt{-g}\frac{1}{16\pi G}G_{ab}\delta g^{ab} + \delta\mathcal{L}_m\right] \\ 0 &= \int \mathrm{d}^4x\frac{\sqrt{-g}}{16\pi G}\left[G_{ab} + \frac{16\pi G}{\sqrt{-g}}\frac{\delta\mathcal{L}_m}{\delta g^{ab}}\right]\delta g^{ab}. \end{aligned} \tag{A.23}$$

From definition of the energy-momentum tensor of matter field

$$T^{(m)}_{ab} = \frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_m}{\delta g^{ab}}, \tag{A.24}$$

and therefore (A.23) read

$$\delta S = \int \mathrm{d}^4x\frac{\sqrt{-g}}{16\pi G}\left[G_{ab} + 8\pi G T^{(m)}_{ab}\right]\delta g^{ab} \tag{A.25}$$

From above equation, we now can write the Einstein's field equation in matter field as follow

$$\begin{aligned} G_{ab} + 8\pi G T^{(m)}_{ab} &= 0 \\ G_{ab} &= 8\pi G T^{(m)}_{ab} \end{aligned}$$

or in full form is

$$R_{ab} - \frac{1}{2}g_{ab}R = -8\pi G T^{(m)}_{ab}. \tag{A.26}$$

APPENDIX B FIELD THEORY

The simplest example of field theory is a single real scalar field $\phi(x^a)$ defined on the space-time. We start from the Lagrangian

$$\mathcal{L} = \frac{1}{2}g^{ab}\nabla_a\phi\nabla_b\phi - V(\phi), \tag{B.1}$$

and the action is given by

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}g^{ab}\nabla_a\phi\nabla_b\phi - V(\phi)\right], \tag{B.2}$$

Varying this action, (B.2), with respect to ϕ, we use the convenient from the Euler-Lagrange equations

$$\frac{\partial\mathcal{L}}{\partial\phi} = -\frac{dV}{d\phi},$$

and

$$\frac{\partial\mathcal{L}}{\partial(\nabla_a\phi)} = \frac{\partial}{\partial(\nabla_a\phi)}\left[\frac{1}{2}g^{cd}\nabla_c\phi\nabla_d\phi\right],$$

where in the second equation we have relabelled the dummy indices in order to make the differentiation more transparent. Evaluating this derivative gives

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial(\nabla_a\phi)} &= \frac{1}{2}g^{cd}\left[\delta^a_c\nabla_d\phi + \nabla_c\phi\delta^a_d\right],\\ &= g^{ab}\nabla_b\phi,\end{aligned} \tag{B.3}$$

and therefore the EL equations become

$$-\frac{dV}{d\phi} - \nabla_a(g^{ab}\nabla_b\phi) = 0. \tag{B.4}$$

we find that the dynamical field equation satisfied by ϕ is

$$\nabla^2\phi + \frac{dV}{d\phi} = 0, \tag{B.5}$$

where $\nabla^2 \equiv \nabla_a\nabla^a = g^{ab}\nabla_a\nabla_b$ is covariant d' Alembertian operator.

APPENDIX C THE ENERGY DENSITY AND PRESSURE

After varying the action (3.1) with respect to metric, we will get

$$\begin{aligned} T_{ab} &= \frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\left(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi\right) - g_{ab}V(\mu\ln\phi) \\ &+\xi\left[\left(R_{ab} - \frac{1}{2}g_{ab}R\right)\left(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi\right) + R\left(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi\right)\right. \\ &\left. -g_{ab}\nabla_d\nabla^d\left(\frac{\mu^2}{\phi^2}\nabla_c\phi\nabla^c\phi\right) + \nabla_a\nabla_b\left(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi\right)\right]. \end{aligned} \tag{C.1}$$

then we can extract the energy density and pressure from above equation. Let us start with

Finding the energy density, $T_{00} = \rho_\phi$

the 1^{st} **term :**

$$\begin{aligned} \frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi &= \frac{\mu^2}{\phi^2}\partial_0\phi\partial_0\phi, \\ &= \frac{\mu^2}{\phi^2}\dot{\phi}^2. \end{aligned} \tag{C.2}$$

where we have used the scalar field as the function of time only.

the 2^{nd} **term:**

$$\begin{aligned} -\frac{1}{2}g_{ab}\left(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi\right) &= -\frac{1}{2}g_{ab}\left(\frac{\mu^2}{\phi^2}g^{cd}\partial_d\phi\partial_c\phi\right) \\ &= -\frac{1}{2}g_{00}\left(\frac{\mu^2}{\phi^2}g^{00}\partial_0\phi\partial_0\phi\right) \\ &= -\frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2. \end{aligned} \tag{C.3}$$

the 3^{rd} **term:**

$$\begin{aligned} -g_{ab}V(\phi') &= g_{00}V(\phi'), \\ &= V(\phi'). \end{aligned} \tag{C.4}$$

the 4^{th} **term:**

$$\begin{aligned}
\xi\left(R_{ab}-\frac{1}{2}g_{ab}R\right)\left(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi\right) &= \xi R_{00}g^{00}\frac{\mu^2}{\phi^2}\partial_0\phi\partial_0\phi-\frac{1}{2}g_{00}Rg^{00}\frac{\mu^2}{\phi^2}\partial_0\phi\partial_0\phi,\\
&= 3\xi(\dot{H}+H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2-\frac{1}{2}\xi(6\dot{H}+12H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2,\\
&= -3\xi H^2\frac{\mu^2}{\phi^2}\dot{\phi}^2. \qquad \text{(C.5)}
\end{aligned}$$

where $R = R_{ab}g^{ab} = 6\dot{H} + 12H^2$.

the 5^{th} **term:**

$$\begin{aligned}
\xi R\left(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi\right) &= \xi R_{ab}g^{ab}\left(\frac{\mu^2}{\phi^2}\partial_0\phi\partial_0\phi\right),\\
&= \xi(6\dot{H}+12H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2. \qquad \text{(C.6)}
\end{aligned}$$

the 6^{th} **term:**

$$\begin{aligned}
-\xi g_{ab}\nabla_d\nabla^d\left(\frac{\mu^2}{\phi^2}\nabla_c\phi\nabla^c\phi\right) &= -\xi g_{ab}\nabla_d\nabla^d\left(\frac{\mu^2}{\phi^2}g^{cd}\partial_c\phi\partial_d\phi\right),\\
&= -\xi g_{00}\nabla_d\nabla^d\left(\frac{\mu^2}{\phi^2}g^{00}\partial_0\phi\partial_0\phi\right),\\
&= -\xi\nabla_d\nabla^d\left(\frac{\mu^2}{\phi^2}\dot{\phi}^2\right),\\
&= -\xi\mu^2\left[\partial_d\partial^0\left(\frac{\dot{\phi}^2}{\phi^2}\right)+\Gamma^d_{cd}\partial^0\left(\frac{\dot{\phi}^2}{\phi^2}\right)\right],\\
&= -\xi\mu^2\left\{\partial_0\left[-2\left(\frac{\dot{\phi}\ddot{\phi}}{\phi^2}-\frac{\dot{\phi}^3}{\phi^3}\right)\right]-6H\left(\frac{\dot{\phi}\ddot{\phi}}{\phi^2}-\frac{\dot{\phi}^3}{\phi^3}\right)\right\},\\
&= 2\xi\mu^2\left[\frac{\dot{\phi}\dddot{\phi}}{\phi^2}+\frac{\ddot{\phi}^2}{\phi^2}-2\frac{\dot{\phi}^2\ddot{\phi}}{\phi^3}-3\frac{\dot{\phi}^2\ddot{\phi}}{\phi^3}\right.\\
&\left.+3\frac{\dot{\phi}^4}{\phi^4}+3H\frac{\dot{\phi}}{\phi^2}\ddot{\phi}-3H\frac{\dot{\phi}^3}{\phi^3}\right]. \qquad \text{(C.7)}
\end{aligned}$$

the 7^{th} term:

$$\begin{aligned}
\xi\nabla_a\nabla_b\left(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi\right) &= \xi\nabla_a\nabla_b\left(\frac{\mu^2}{\phi^2}g^{cd}\partial_d\phi\partial_c\phi\right), \\
&= -\xi\nabla_a\nabla_b\left(\frac{\mu^2}{\phi^2}\dot{\phi}^2\right), \\
&= -\xi\mu^2\left[\partial_a\partial_0\left(\frac{\dot{\phi}^2}{\phi^2}\right) - \Gamma^c_{ab}\partial_0\left(\frac{\dot{\phi}^2}{\phi^2}\right)\right], \\
&= -\xi\mu^2\left\{\partial_0\left[2\left(\frac{\dot{\phi}\ddot{\phi}}{\phi^2} - \frac{\dot{\phi}^3}{\phi^3}\right)\right]\right\}, \\
&= -2\xi\mu^2\left[\frac{\dot{\phi}\dddot{\phi}}{\phi^2} + \frac{\ddot{\phi}^2}{\phi^2} - 2\frac{\dot{\phi}^2\ddot{\phi}}{\phi^3} - 3\frac{\dot{\phi}^2\ddot{\phi}}{\phi^3} + 3\frac{\dot{\phi}^4}{\phi^4}\right]. \quad (C.8)
\end{aligned}$$

where $\Gamma^0_{00} = 0$. So that the energy density, ρ_ϕ, is the combining all terms. It can be written as

$$\rho_\phi = \frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 + V(\phi') + 3\xi\left[(2\dot{H} + 3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 + 2H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} - 2H\mu^2\frac{\dot{\phi}^3}{\phi^3}\right]. \quad (C.9)$$

Finding the pressure, $T_{11} = P_\phi$

the 1^{st} term:

$$\begin{aligned}
\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi &= \frac{\mu^2}{\phi^2}\partial_1\phi\partial_1\phi, \\
&= 0, \quad (C.10)
\end{aligned}$$

the 2^{nd} term:

$$\begin{aligned}
-\frac{1}{2}g_{ab}\left(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi\right) &= -\frac{1}{2}g_{ab}\left(\frac{\mu^2}{\phi^2}g^{cd}\partial_d\phi\partial^c\phi\right) \\
&= -\frac{1}{2}g_{11}\left(\frac{\mu^2}{\phi^2}g^{00}\partial_0\phi\partial_0\phi\right) \\
&= \frac{1}{2}a^2\frac{\mu^2}{\phi^2}\dot{\phi}^2, \quad (C.11)
\end{aligned}$$

the 3^{rd} term:

$$\begin{aligned}
-g_{ab}V(\phi') &= g_{11}V(\phi'), \\
&= -a^2V(\phi'), \quad (C.12)
\end{aligned}$$

the 4^{th} term:

$$\begin{aligned}
\xi\left(R_{ab}-\frac{1}{2}g_{ab}R\right)\left(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi\right) &= \xi R_{11}g^{00}\frac{\mu^2}{\phi^2}\partial_0\phi\partial_0\phi-\frac{1}{2}g_{11}Rg^{00}\frac{\mu^2}{\phi^2}\partial_0\phi\partial_0\phi, \\
&= -\xi(a\dot{a}+2\dot{a}^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2+\frac{1}{2}\xi a^2(6\dot{H}+12H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2, \\
&= -\xi(a\dot{a}+2\dot{a}^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2+\xi a^2(3\dot{H}+6H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2,
\end{aligned} \tag{C.13}$$

where $R = R_{ab}g^{ab} = 6\dot{H}+12H^2$.

the 5^{th} term:

$$\begin{aligned}
\xi R\left(\frac{\mu^2}{\phi^2}\nabla_a\phi\nabla_b\phi\right) &= \xi R_{ab}g^{ab}\left(\frac{\mu^2}{\phi^2}\partial_1\phi\partial_1\phi\right), \\
&= 0,
\end{aligned} \tag{C.14}$$

the 6^{th} term:

$$\begin{aligned}
-\xi g_{ab}\nabla_d\nabla^d\left(\frac{\mu^2}{\phi^2}\nabla_c\phi\nabla^c\phi\right) &= -\xi g_{ab}\nabla_d\nabla^d\left(\frac{\mu^2}{\phi^2}g^{cd}\partial_c\phi\partial_d\phi\right), \\
&= -\xi g_{11}\nabla_d\nabla^d\left(\frac{\mu^2}{\phi^2}g^{00}\partial_0\phi\partial_0\phi\right), \\
&= \xi a^2\mu^2\nabla_d\nabla^d\left(\frac{\mu^2}{\phi^2}\dot{\phi}^2\right), \\
&= \xi a^2\mu^2\left[\partial_d\partial^0\left(\frac{\dot{\phi}^2}{\phi^2}\right)+\Gamma^d_{cd}\partial^0\left(\frac{\dot{\phi}^2}{\phi^2}\right)\right], \\
&= \xi a^2\mu^2\left\{\partial_0\left[-2\left(\frac{\dot{\phi}\ddot{\phi}}{\phi^2}-\frac{\dot{\phi}^3}{\phi^3}\right)\right]-6H\left(\frac{\dot{\phi}\ddot{\phi}}{\phi^2}-\frac{\dot{\phi}^3}{\phi^3}\right)\right\}, \\
&= -2\xi a^2\mu^2\left[\frac{\dot{\phi}\dddot{\phi}}{\phi^2}+\frac{\ddot{\phi}^2}{\phi^2}-2\frac{\dot{\phi}^2\ddot{\phi}}{\phi^3}-3\frac{\dot{\phi}^2\ddot{\phi}}{\phi^3}+3\frac{\dot{\phi}^4}{\phi^4}\right. \\
&\left.\quad +3H\frac{\dot{\phi}}{\phi^2}\ddot{\phi}-3H\frac{\dot{\phi}^3}{\phi^3}\right],
\end{aligned} \tag{C.15}$$

the 7^{th} term:

$$\begin{aligned}
\xi\nabla_a\nabla_b\left(\frac{\mu^2}{\phi^2}\nabla_d\phi\nabla^d\phi\right) &= \xi\nabla_a\nabla_b\left(\frac{\mu^2}{\phi^2}g^{00}\partial_0\phi\partial_0\phi\right), \\
&= -\xi\mu^2\nabla_a\nabla_b\left(\frac{\dot{\phi}^2}{\phi^2}\right), \\
&= -\xi\mu^2\left[\partial_1\partial_1\left(\frac{\dot{\phi}^2}{\phi^2}\right) - \Gamma^0_{11}\partial_0\left(\frac{\dot{\phi}^2}{\phi^2}\right)\right], \\
&= \xi\mu^2 a\dot{a}\left[2\left(\frac{\dot{\phi}\ddot{\phi}}{\phi^2} - \frac{\dot{\phi}^3}{\phi^3}\right)\right], \qquad \text{(C.16)}
\end{aligned}$$

where $\Gamma^0_{11} = a\dot{a}$. So that the energy density, P_ϕ, is the collecting all terms. It can be written as

$$\begin{aligned}
P_\phi = \frac{1}{2}\frac{\mu^2}{\phi^2}\dot{\phi}^2 - V(\phi') + \xi\Bigg[(2\dot{H} + 3H^2)\frac{\mu^2}{\phi^2}\dot{\phi}^2 - 4H\frac{\mu^2}{\phi^2}\dot{\phi}\ddot{\phi} + 4H\mu^2\frac{\dot{\phi}^3}{\phi^3} \\
+2\mu^2\left(\frac{\ddot{\phi}^2}{\phi^2} + \frac{\dot{\phi}}{\phi^2}\dddot{\phi} - 5\frac{\dot{\phi}^2}{\phi^3}\ddot{\phi} + 3\frac{\dot{\phi}^4}{\phi^4}\right)\Bigg]. \qquad \text{(C.17)}
\end{aligned}$$

BIOGRAPHY

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