Arbitrariness of potentials in interacting quintessence models

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Though we have entered an era of high precision cosmology the form of the quintessence potentials still remain arbitrary. In this work we explore the interacting quintessence models considering a parametrization of the quintessence potentials and tried to constrain the form of potentials using the recent cosmological observations. The particular parametrization which we have used includes many popular quintessence potentials. By constraining the parameters in the parametrization one can find which class of potentials are more favorable. Our findings reconfirm the arbitrariness of the quintessence potentials even for the interacting dark energy models as the recent cosmological observations are not able to put any constraint on the parameters in the parametrization. As a result, it is shown that the current observations are able to put an upper bound to the interaction parameter for both of the interactions we consider, although it is not possible to constrain the form of the potentials.

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I. INTRODUCTION

Dark energy is probably the biggest unsolved mystery in modern cosmology. Many cosmological observations have confirmed its existence [1–6] but the exact nature of it is still unknown. The lambda cold dark matter model (ΛCDM) [7–12] is the most popular and observationally consistent model of dark energy in which the cosmological constant is considered as the candidate of the dark energy.

Although ΛCDM is successful and consistent in explaining the accelerated expansion of the universe, it still has to overcome some challenges coming from both observations and theoretical interpretation of the cosmological constant. Recently a discordance at the level of 2.3σ in the (ΛCDM) model is reported between the Planck 2015 CMB data and KiDS(Kilo Degree Survey) data [6,13–16]. There are other observational challenges of the ΛCDM model, one of which comes from the 3σ tension between the direct measurement Hubble constant from Hubble Space Telescope data (HST) and the value of Hubble constant obtained from the CMB data considering the ΛCDM model [17]. In field theory, the cosmological constant can be interpreted as constant vacuum energy. The interpretation of the cosmological constant as vacuum energy also gives rise to a theoretical challenge to the ΛCDM model which is known as the famous cosmological constant problem. The problem comes from the discrepancy between the theoretical prediction and the observed value of the cosmological constant which is of the order of 10^{54}.

Consideration of dynamical dark energy models is one way to alleviate the cosmological constant problem [18]. Another way is the modification of the theory of gravity, which can also explain the accelerated expansion without considering any exotic matter component, but recently these models are weakened by the estimation of the speed of gravitational wave as the speed of the light from the observation of the two binary systems of neutron stars colliding [19–23]. For recent reviews on the issue of dark energy and the modification of gravity theory to explain the late-time cosmic acceleration, see, for instance, [24–30].

Quintessence dark energy models are the most popular amongst the dynamical dark energy models [28,31,32]. In these models of dark energy the accelerated expansion is driven by a minimally coupled scalar field and an associated quintessence potential. The form of the quintessence potential is arbitrary as there is no identification of a particular form of the quintessence potentials either from the cosmological observations or from basic physics. This allowed researchers to consider a wide range of potentials and all of them in certain parameter range satisfy the observations [33–39]. Since the true nature of the dark matter (DM) and dark energy (DE) is unknown, interaction between them cannot be ruled out *a priori* and it has been

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already shown that consideration of an interaction between them can help us to alleviate the cosmic coincident problem. Like the form of the quintessence potentials, the form of the interaction is also don't have any general consensus. In the literature a different form of the interaction has been considered from a variety of motivations [40].

In this work, we have tried to constrain the form of the potentials in the interacting quintessence models. In doing so we have used a parametrization of the quintessence potentials proposed by Roy *et al.* [41] to study the noninteracting quintessence models along with two different kinds of interactions between the dark sectors. Consideration of an interaction between the dark sectors not only helps us to alleviate the coincidence problem but it also has significant effects on the cosmological observations. For example, it can affect the Hubble parameter, sequence of radiation, dark matter and dark energy dominated eras, cosmic microwave background (CMB), matter power spectrum, the growth of structure, nonlinear perturbations etc. [42–47]. For a comprehensive review, one can look at [40]. Recently it has been also shown that the interaction between the dark sectors can be the possible explanation of the 21 cm deep absorption at the cosmic dawn [48]. The particular parametrization of our consideration has three arbitrary parameters, named as dynamical parameters or α parameters. By tuning these α parameters one can switch between different quintessence potentials. Thus a constraint on the α parameters can help us to find which class of potentials are more favorable. We have implemented this parametrization in the Boltzmann code CLASS and MCMC code MONTEPYTHON to check if there is any preference on the values of the α parameters. A similar approach for the noninteracting quintessence models has done in [41]. There it has been shown that the form of the potentials remains arbitrary. As it was already mentioned the interaction between the dark sectors can influence the cosmological dynamics both in background and perturbation level it is worthy to check if there is any effect of interaction on the choice of the form of potentials. In this work, we study it for the background level and leave the study of linear perturbations as the future scope of work. We have considered two well-known forms of interactions from the existing literature.

The field equations are written as a set of autonomous equations through a suitable variable transformation. Later on, these autonomous equations are transferred in to polar form [41,49–53]. The particular importance of the polar form of the system is the direct relation of the cosmological variables to the dynamical variables. We have considered two different types of interactions between the dark sectors and used the cosmological observations to constrain the parameters in the model.

Following is the summary of the paper. In Sec. II, we have discussed about the mathematical background of the system and the form of the potentials. Section III deals with the interactions and the corresponding system of equations. In Sec. IV, we have shown how to estimate a set of initial conditions of the system by matching matter dominated and radiation dominated approximate solutions. Section V presents a full Bayesian analysis of the model using diverse cosmological observations to constrain different parameters in the model. In Sec. VI we give a summary and conclusion of the results we obtained.

II. MATHEMATICAL BACKGROUND

Let us consider the universe to be filled with radiation, dark matter, and the dark energy. The dark energy is in the form of quintessence scalar field. We also consider that there is an interaction between the dark matter and the dark energy. The components of the universe are barotropic in nature, hence they obey the relation $p_j = w_j \rho_j$, for radiation $w_r = 1/3$ and for dark matter $w_m = 0$. In such a universe which is spatially flat, homogeneous and isotropic the Einstein field equations are written as

$$H^2 = \frac{\kappa^2}{3}\left(\sum_j \rho_j + \rho_\phi\right), \tag{1a}$$

$$\dot{H} = -\frac{\kappa^2}{2}\left[\sum_j (\rho_j + p_j) + (\rho_\phi + p_\phi)\right], \tag{1b}$$

where $\kappa^2 = 8\pi G$ and a is the scale factor of the Universe, and $H \equiv \dot{a}/a$ is the Hubble parameter. A dot means derivative with respect to cosmic time. The continuity equations of the radiation, matter and the scalar field can be respectively written as

$$\dot{\rho_r} + 3H\rho_r(1 + w_r) = 0, \tag{2a}$$

$$\dot{\rho_m} + 3H\rho_m(1 + w_m) = -Q, \tag{2b}$$

$$\dot{\rho_\phi} + 3H(\rho_\phi + p_\phi) = +Q. \tag{2c}$$

The wave equation of the scalar field is written as

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = \frac{Q}{\dot{\phi}}, \tag{3}$$

where V is the scalar field potential related to the quintessence field and Q is the interaction term between the dark energy and dark matter. A positive coupling Q indicates an exchange of energy from dark matter to dark energy and a negative Q indicates an exchange of energy from dark energy to dark matter.

We introduce the following sets of dimensionless variables to write the systems of equations as an set of autonomous equations,

$$x \equiv \frac{\kappa\dot{\phi}}{\sqrt{6}H}, \qquad y \equiv \frac{\kappa V^{1/2}}{\sqrt{3}H}, \tag{4a}$$

TABLE I. A classification of different potentials depending on the choice of α parameters given in [41].

No	Structure of y_2/y	Form of the potentials $V(\phi)$
Ia	$\alpha_0 = 0, \alpha_1 = 0, \alpha_2 \neq -\frac{1}{2}$	$(A + B\phi)^{\frac{2}{(2\alpha_2+1)}}$
Ib	$\alpha_0 = 0, \alpha_1 = 0, \alpha_2 = -\frac{1}{2}$	$A^2 e^{2B\phi}$
IIa	$\alpha_0 \neq 0, \alpha_1 = 0, \alpha_2 \neq -\frac{1}{2}$	$A^2 \cos\left[\sqrt{\alpha_0\kappa^2(1+2\alpha_2)}(\phi - B)/2\sqrt{3}\right]^{\frac{2}{1+2\alpha_2}}$
IIb	$\alpha_0 \neq 0, \alpha_1 = 0, \alpha_2 = -\frac{1}{2}$	$A^2 \exp(-\kappa^2\alpha_0\phi^2/12)\exp(2B\phi)$
IIIa	$\alpha_0 = 0, \alpha_1 \neq 0, \alpha_2 \neq -\frac{1}{2}$	$[A\exp(\alpha_1\kappa\phi/\sqrt{6}) + B]^{\frac{2}{1+2\alpha_2}}$
IIIb	$\alpha_0 = 0, \alpha_1 \neq 0, \alpha_2 = -\frac{1}{2}$	$A^2 \exp[2B\exp(\kappa\alpha_1\phi/\sqrt{6})]$
IVa	$\alpha_0 \neq 0, \alpha_1 \neq 0, \alpha_2 \neq -\frac{1}{2}$	$A^2 \exp\left(\frac{\kappa\alpha_1\phi}{\sqrt{6}(1+2\alpha_2)}\right)\left\{\cos\left[\left(-\frac{\kappa^2\alpha_1^2}{24} + \frac{\kappa^2\alpha_0}{12}(1+2\alpha_2)\right)^{\frac{1}{2}}(\phi - B)\right]\right\}^{\frac{2}{1+2\alpha_2}}$
IVb	$\alpha_0 \neq 0, \alpha_1 \neq 0, \alpha_2 = -\frac{1}{2}$	$A^2 \exp\left[\frac{\kappa\alpha_0\phi}{\sqrt{6}\alpha_1} + 2B\exp(\frac{\kappa\alpha_1\phi}{\sqrt{6}})\right]$

$$y_1 \equiv -2\sqrt{2}\frac{\partial_\phi V^{1/2}}{H}, \qquad y_2 \equiv -4\sqrt{3}\frac{\partial_\phi^2 V^{1/2}}{\kappa H}. \tag{4b}$$

This particular transformation was first used in [50] and later it is used in [41]. Using these sets of new variables the system of equations which governs the dynamics of the scalar field reduces to the following set of autonomous equations,

$$x' = -\frac{3}{2}(1 - w_{\mathrm{tot}})x + \frac{1}{2}yy_1 + q, \tag{5a}$$

$$y' = \frac{3}{2}(1 + w_{\mathrm{tot}})y - \frac{1}{2}xy_1, \tag{5b}$$

$$y_1' = \frac{3}{2}(1 + w_{\mathrm{tot}})y_1 + xy_2, \tag{5c}$$

where $q = \frac{\kappa Q}{\sqrt{6}H^2\dot{\phi}}$ and a "prime" represents differentiation with respect to the e-foldings $N = \ln(a)$ and the present value of the scale factor is scaled to be unity. The w_{tot} is the total equation of state of the system which is defined as $w_{\mathrm{tot}} \equiv \frac{p_{\mathrm{tot}}}{\rho_{\mathrm{tot}}} = \frac{1}{3}\Omega_r + x^2 - y^2$.

A. Polar form

Further we introduced a set of polar transformation to write Eqs. (5) in a polar form. Following are the polar transformations, $x = \Omega_\phi^{1/2}\sin(\theta/2)$ and $y = \Omega_\phi^{1/2}\cos(\theta/2)$, where $\Omega_\phi = \kappa^2\rho_\phi/3H^2$ and the θ is angular degree of freedom. With these transformations the equations in (5) reduce to

$$\theta' = -3\sin\theta + y_1 + q\Omega_\phi^{1/2}\cos(\theta/2), \tag{6a}$$

$$y_1' = \frac{3}{2}(1 + w_{\mathrm{tot}})y_1 + \Omega_\phi^{1/2}\sin(\theta/2)y_2, \tag{6b}$$

$$\Omega_\phi' = 3(w_{\mathrm{tot}} - w_\phi)\Omega_\phi + q\Omega_\phi^{1/2}\sin(\theta/2). \tag{6c}$$

The advantage of writing down the equations (5) in polar form is the direct connection between the cosmological variables and the dynamical variables whereas in the previous transformations the x and y does not carry any direct physical meaning. The Ω_ϕ is itself the scalar field energy density and the θ can be directly related to the equation of state of the scalar field as $w_\phi = -\cos\theta$. Apart from this θ can also give us information about the ratio of K.E and potential energy of the scalar field as $\tan^2\theta = \frac{\frac{1}{2}\dot{\phi}}{V(\phi)} = \frac{x^2}{y^2}$.

B. Form of the potential

One can see from the equations in Eq. (6) that the system of equations is not close until one has the information about the functional form of the potential variable y_2 related to the potential $V(\phi)$ and the interaction variable q related to the interaction term Q. Unfortunately both of $V(\phi)$ and Q are arbitrary as there is no preferential functional from of these two from cosmological observations. Though there is no general agreement some hints about the form of potentials from fundamental physics can be found in [54–56]. The functional form of the potential function y_2 can be considered in two different ways. One can consider a form of the potential $V(\phi)$ and using the definition of y_2 corresponding y_2 can be calculated. In another way one can consider a particular form of the y_2 and get back a form of potential by integrating it back. In Roy *et al.* [41] both of these two ways have been used to find a parametrization of the potential function y_2 which includes large class of potentials exist in the literature. They have first considered some particular potentials and found out their corresponding y_2 (Please see Table I, in [41]). All the potentials which they have considered in this work, the corresponding y_2 of them follows a particular functional form $\frac{y_2}{y} = \alpha_0 + \alpha_1(\frac{y_1}{y}) + (\frac{y_1}{y})^2$, where the α parameters α_0, α_1, and α_2 are the arbitrary parameters. The α parameters are called dynamical parameters as they are the ones which influence the field dynamics. Now using the other way one can get back different functional form of potentials by integrating back for different combinations of α parameters. For example in Table I, we have included the same classifications of the potential form the [41]. The A and B are the constants coming from the integration. They are called passive parameters as they do not influence the behavior of the field dynamics except the initial conditions. It is shown in [41] that the passive parameters may be subjected to constraints if we fixed either the initial conditions or the final conditions. It is also worth mentioning here that though this particular form of y_2 includes quite a good number of quintessence potentials, the list is not comprehensive. For example, the potentials of the form $V(\phi) = \sum_i^n \Lambda_i \exp(\lambda_i \phi)$ [57,58] and $V(\phi) = V_0 \tanh^2(\frac{\lambda_1 \phi}{m_p}) \cosh(\frac{\lambda_2 \phi}{m_p})$ [59] cannot be included in this form. An advantage of consideration of this form of $\frac{y_2}{y} = \alpha_0 + \alpha_1 (\frac{y_1}{y}) + (\frac{y_1}{y})^2$ is that it can be easily implemented in the CLASS code to constrain the α parameters using MONTEPYTHON.

III. INTERACTION

We have considered two different interaction terms for the analysis. Likewise the quintessence potentials and the form of the interaction term are also arbitrary and each one of them can explain accelerated expansion for certain parameter range. For a list of different types of interactions in quintessence models we refer to [60]. The reason behind our choice of interaction terms is the mathematical simplicity of the field equations particularly to be able to write the interaction term q as a function of Ω_ϕ and θ so that we can close the systems of equations in Eq. (6).

A. $Q = \beta H \dot{\phi}^2$

We consider the interaction to be of the form $Q = \beta H \dot{\phi}^2$. This particular form of interaction was first introduced in [61] to study the asymptotic behavior of the warm inflation scenario with viscous pressure. Dynamical systems analysis of interacting phantom models considering a general form of this coupling can be found here [62]. After considering this particular form of the coupling the equations in (6) reduces to

$$\theta' = -3 \sin\theta + y_1 + \frac{\beta}{2} \Omega_\phi \sin\theta, \tag{7a}$$

$$y_1' = \frac{3}{2}(1 + w_{\rm tot}) y_1 + \Omega_\phi^{1/2} \sin(\theta/2) y_2, \tag{7b}$$

$$\Omega_\phi' = 3(w_{\rm tot} - w_\phi)\Omega_\phi + \frac{\beta}{2}\Omega_\phi (1 + w_\phi). \tag{7c}$$

B. $Q = \beta \kappa^2 (\rho_m + \rho_r) \dot{\phi}^2 / H$

With the consideration of the above functional form of the interaction the system of equations in Eq. (6) reduces to the following sets of equations,

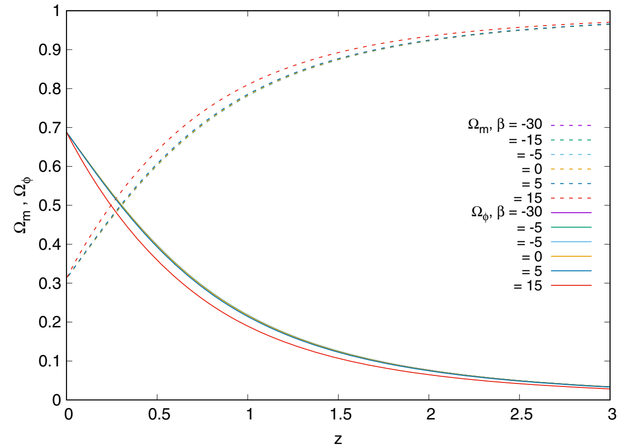

FIG. 1. Plot of Ω_m and Ω_ϕ for different values of β parameters for the interaction A. All the α parameters are set to unity ($\alpha_0 = \alpha_1 = \alpha_2 = 1$) and the present value of the EOS of scalar field is chosen to be $w_\phi = -0.95$.

$$\theta' = -3 \sin\theta + y_1 + \frac{3\beta}{2} \Omega_\phi (1 - \Omega_\phi) \sin\theta, \tag{8a}$$

$$y_1' = \frac{3}{2}(1 + w_{\rm tot}) y_1 + \Omega_\phi^{1/2} \sin(\theta/2) y_2, \tag{8b}$$

$$\Omega_\phi' = 3(w_{\rm tot} - w_\phi)\Omega_\phi + \frac{3\beta}{2}\Omega_\phi (1 - \Omega_\phi)(1 + w_\phi). \tag{8c}$$

Consideration of this interaction has been motivated from the type of interaction $Q = \beta \rho_m \dot{\phi}^2 / H$ used in [62] in the context of interacting phantom models. In [62] the contribution from the radiation has been neglected but here we do not neglect the radiation so a ρ_r term is present in the interaction.

We have modified the publicly available CLASS [63] code to incorporate these two sets of equations in Eq. (7) and

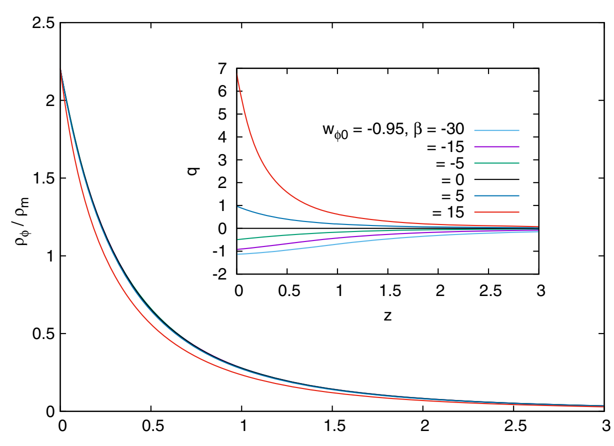

FIG. 2. Plot of ratio of ρ_ϕ and ρ_m and in the inset of the plot is for the interaction variable q for different values of β parameters for the interaction A. All the α parameters are set to unity ($\alpha_0 = \alpha_1 = \alpha_2 = 1$) and the present value of the EOS of scalar field is chosen to be $w_\phi = -0.95$.

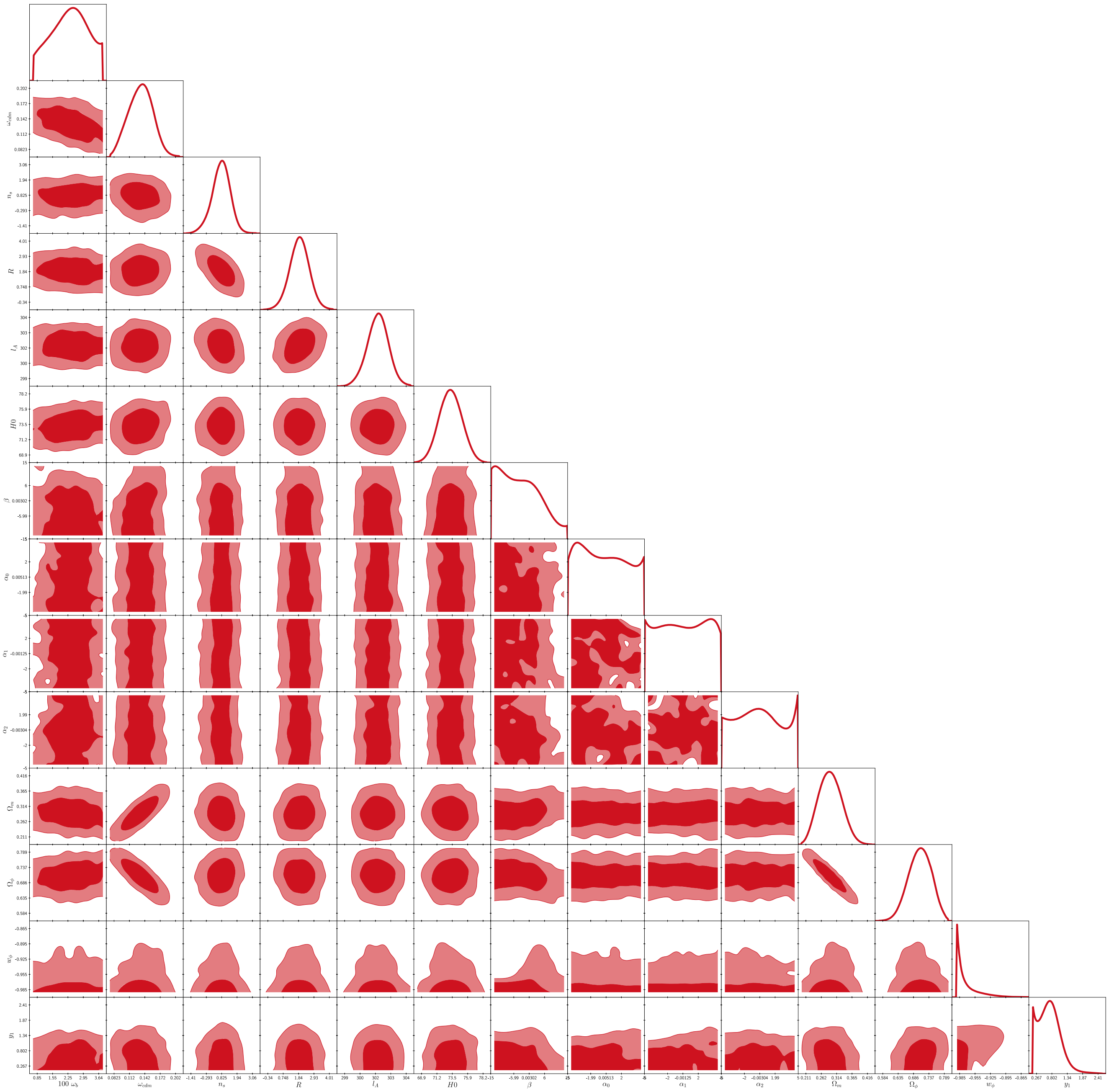

FIG. 3. Plot of Posterior (1D and 2D) distributions of the constrained cosmological parameters for interaction A. The datasets used are BAO + JLA + RSD + H(z) and a PLANCK15 prior is imposed on ω_b and ω_{cdm}.

Eq. (8) separately. In the Sec. V more details about the modification of the CLASS code are discussed. For the numerical analysis with the CLASS code to be done one has to supply a viable initial condition to the CLASS code. Rather than choosing the initial condition arbitrarily in the next section, we follow the method of estimating the initial condition from [41].

IV. INITIAL CONDITIONS

In this section we will try to estimate the initial conditions of the universe based on the approximate solution of the matter and radiation dominated era and match them at the radiation-matter equality. From the recent cosmological observations the present value of the dark energy equation of state $w_\phi \simeq -1$ and this implies that $\theta < 1$. We also assume that when the universe entered in the matter dominated phase from the radiation dominated era till then the dark energy density was very subdominant $\Omega_\phi \ll 1$ and $\theta \ll 1$. This means the EOS of the quintessence field started to evolve from $w_\phi \simeq -1$ and at the late time it will differ from the cosmological constant value. A choice of these initial assumptions $\Omega_\phi \ll 1$ and $\theta \ll 1$

TABLE II. Best fit value of the cosmological parameter for the Interaction A (95% C.L.).

Cosmological parameters	JLA + BAO + RSD + $H(z)$
$100\omega_b$	$2.34^{+1.02}_{-0.913}$
ω_{cdm}	$0.134^{+0.0243}_{-0.0248}$
n_s	$0.777^{+0.723}_{-0.633}$
R	$1.9^{+0.728}_{-0.744}$
l_A	$302^{+1.02}_{-0.916}$
$H_0\ (kmS^{-1}/Mpc)$	$73.3^{+1.81}_{-1.8}$
β	≤ 6.5
Ω_m	$0.292^{+0.0366}_{-0.0409}$
Ω_ϕ	$0.707^{+0.0409}_{0.0366}$
w_ϕ	$-0.983^{+0.0018}_{-0.0168}$

makes our analysis hereafter to be valid for thawing type potentials. Thawing type potentials are those for which the EOS of the quintessence field starts with $w_\phi \simeq -1$ which is a value for the EOS of cosmological constant and at late times it differs from the cosmological constant value. This formalism can also be applied to other cases like freezing, tracker etc. by considering proper initial conditions.

By considering these two facts $\theta \ll 1$ and $\Omega_\phi \ll 1$ the equations in (7) and (8) reduce to the following form,

$$\theta' = -3\theta + y_1, \tag{9a}$$

$$y_1' = \frac{3}{2}(1 + w_{\rm tot})y_1, \tag{9b}$$

$$\Omega_\phi' = 3(w_{\rm tot} - w_\phi)\Omega_\phi. \tag{9c}$$

Considering the fact that $\theta \ll 1$, $\sin(\theta) \simeq \sin(\theta/2) \ll 1$, and $\Omega_\phi \ll 1$ we have neglected the last term from the

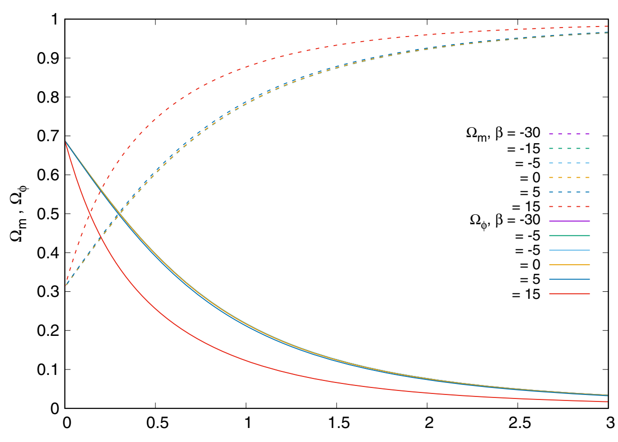

FIG. 4. Plot of Ω_m and Ω_ϕ for different values of β parameters for the interaction B. All the α parameters are set to unity ($\alpha_0 = \alpha_1 = \alpha_2 = 1$) and the present value of the EOS of scalar field is chosen to be $w_\phi = -0.95$.

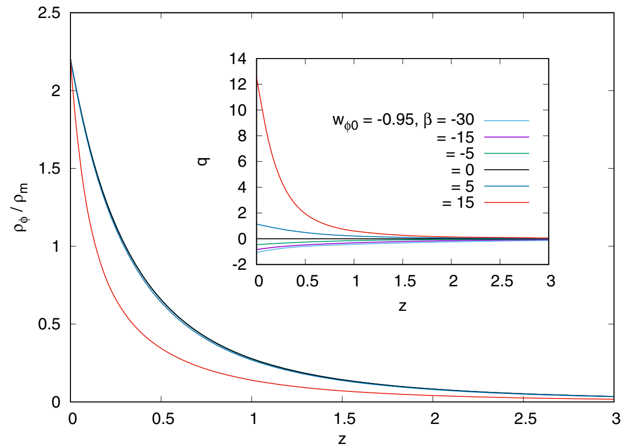

FIG. 5. Plot of ratio of ρ_ϕ and ρ_m and in the inset the plot is for the interaction variable q for different values of β parameters for the interaction B. All the α parameters are set to unity ($\alpha_0 = \alpha_1 = \alpha_2 = 1$) and the present value of the EOS of the scalar field is chosen to be $w_\phi = -0.95$.

equations in (7) and (8). The approximate form of the equations in (9) is valid for both forms of the interaction. Now we shall try to find out solution for radiation and matter dominated era separately and later match them at the radiation matter equality which will allow us to make a good guess about the initial condition of the universe that can evolve to give us a present day accelerating universe. The e-folding is different for radiation and matter dominated era. For radiation dominated era $N_r = \ln(a/a_i)$ and for matter dominated era $N_m = \ln(a/a_{\rm eq})$ where a_i is the initial value of the scale factor, whereas $a_{\rm eq}$ is the value of the scale factor at the radiation-matter equality.

A. Radiation dominated era

In a radiation dominated universe the total EOS is $w_{\rm tot} = 1/3$ and the equations in (9) reduce to

$$\theta' = -3\theta + y_1, \qquad y_1' = 2y_1, \qquad \Omega_\phi' = 4\Omega_\phi. \tag{10}$$

Considering the growing solutions in the radiation dominated era the approximate solutions are given by,

$$\theta_r = \theta_i(a/a_i)^2, \qquad y_{1r} = y_{1i}(a/a_i)^2,$$
$$\Omega_{\phi r} = \Omega_{\phi i}(a/a_i)^4, \tag{11}$$

The subindex "r" represents the solutions during the radiation dominated era and "i" represents initial value of the cosmological parameters. In addition to these solutions we found another relation between y_1 and θ as $y_1 = 5\theta$.

B. Matter dominated era

Once the universe is matter dominated the total EOS is $w_{\rm tot} \simeq 0$. The equations in (9) reduce to

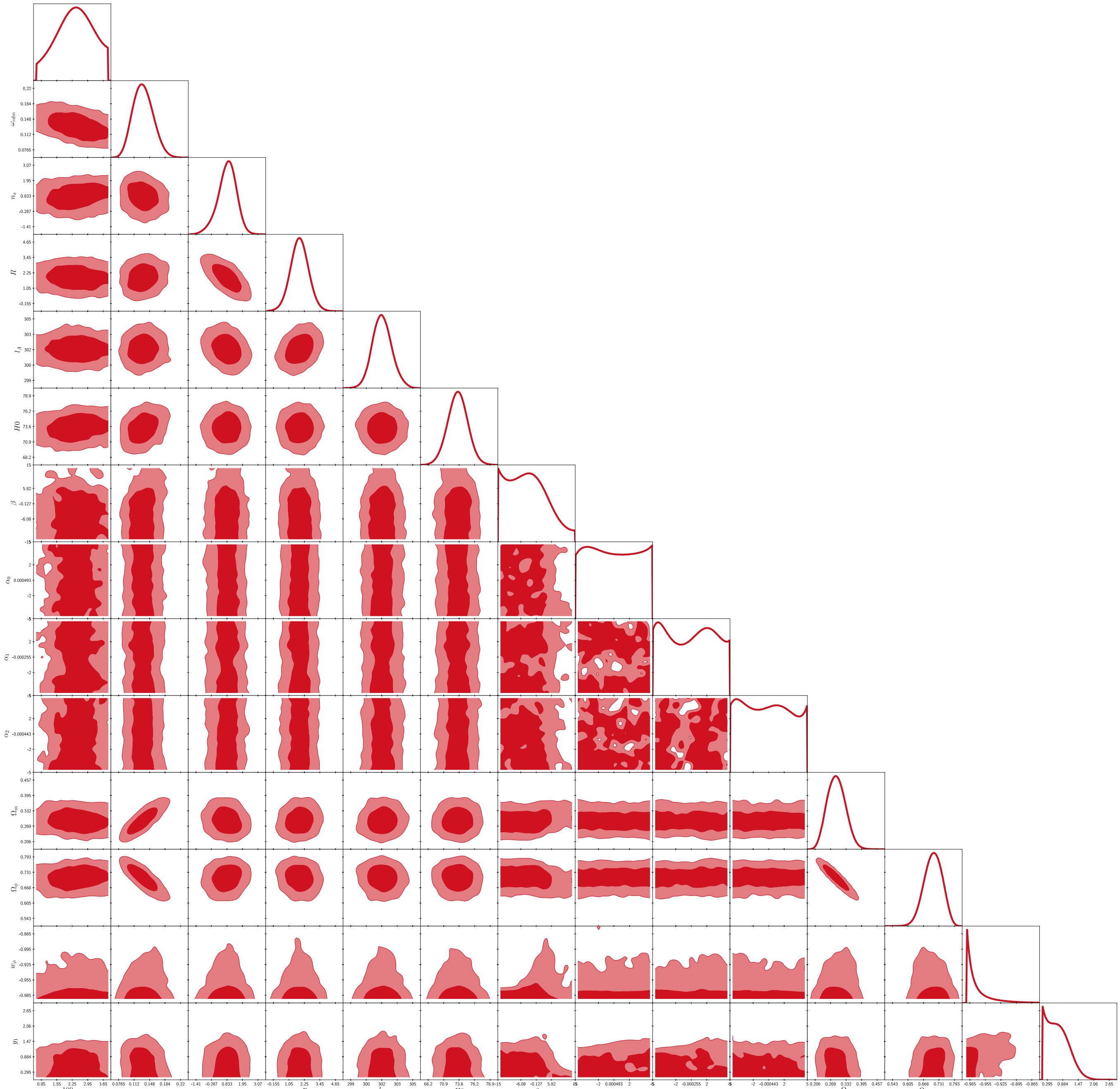

FIG. 6. Plot of Posterior (1D and 2D) distributions of the constrained cosmological parameters for interaction B. The datasets used are BAO + JLA + RSD + H(z) and a PLANCK15 prior is imposed on ω_b and ω_{cdm}.

$$\theta' = -3\theta + y_1, \qquad y_1' = \frac{3}{2} y_1, \qquad \Omega_\phi' = 3\Omega_\phi. \tag{12}$$

The solution of these equations is given by,

$$\theta_m = \left(\theta_{\rm eq} - \frac{2}{9} y_{1\rm eq}\right)(a/a_{\rm eq})^{-3} + \frac{2}{9} y_{1\rm eq}(a/a_{\rm eq})^{3/2},$$
$$y_{1m} = y_{1\rm eq}(a/a_{\rm eq})^{3/2}, \qquad \Omega_{\phi m} = \Omega_{\phi\rm eq}(a/a_{\rm eq})^3. \tag{13}$$

A subindex "m" represents solutions at the matter dominated era and "eq" represents value of the cosmological parameters at the radiation matter equality. We do not neglect the decaying solutions in contrast with the radiation dominated solution as it will make the matching of the solutions at the radiation-matter equality simpler. Once we perform the matching of the Eqs. (11) and (13) at the radiation matter equality $a_{\rm eq} = \Omega_{r0}/\Omega_{m0}$ it allowed us to find a solution at the matter domination which has the information about the initial sate of the universe. From the matching of the solutions we found $\theta_{\rm eq} = \theta_i(a_{\rm eq}/a_i)^2$, $y_{1\rm eq} = 5\theta_{\rm eq} = y_{1i}(a_{\rm eq}/a_i)^2$, and $\Omega_{\phi\rm eq} = \Omega_{\phi i}(a_{\rm eq}/a_i)^4$, and substituting it in (13) obtain

$$\theta_m = \frac{10}{9}\left(\frac{a_{\rm eq}}{a_i}\right)^2 \theta_i \left[\left(\frac{a}{a_{\rm eq}}\right)^{3/2} - \frac{1}{10}\left(\frac{a}{a_{\rm eq}}\right)^{-3}\right], \quad (14a)$$

$$y_{1m} = \frac{a_{\rm eq}^{1/2}}{a_i^2} y_{1i} a^{3/2}, \quad (14b)$$

$$\Omega_{\phi m} = \frac{a_{\rm eq}}{a_i^4}\Omega_{\phi i} a^3. \quad (14c)$$

Hence by considering the present value of the scale factor to be unity or $a = 1$ in (14) we estimate the initial condition for the dynamical variables as

$$\theta_i \simeq \frac{9}{10} a_i^2 \frac{\Omega_{m0}^{1/2}}{\Omega_{r0}^{1/2}} \theta_0, \quad (15a)$$

$$\Omega_{\phi i} \simeq a_i^4 \frac{\Omega_{m0}}{\Omega_{r0}} \Omega_{\phi 0}. \quad (15b)$$

The initial values of the dynamical variables θ and Ω_ϕ can be estimated from the present values of them and one can estimate y_{1i} from the relation $y_{1i} = 5\theta_i$.

V. NUMERICAL INVESTIGATION

In this section we discuss the general method we adopt to constrain the dynamical variables and the cosmological parameters using cosmological observations.

A. Datasets

For the numerical investigation of the system we have used an modified version of the Boltzmann code CLASS [63] and the MCMC code MONTEPYTHON [64]. Modification to the CLASS code is done separately for two different interactions. In order to maintain the interface with MONTEPYTHON so that it can sample all extra dynamical parameters which we are having in our analysis, the necessary modifications are also done to the CLASS code.

At the beginning of any numerical run it is necessary to fine tune the initial values of the dynamical variables. To fine tune the initial conditions we write $y_{1i} = 5\theta_i$, $\theta_i = P\times$ Eq. (15a) and $\Omega_{\phi i} = Q\times$ Eq. (15b). The values of P and Q are adjusted by the shooting method which is already implemented in the CLASS code for the scalar field. Generally the P, $Q = \mathcal{O}(1)$ is enough to find a successful initial condition which can give us $\Omega_{\phi 0}$ and $w_{\phi 0}$ with a very high precision.

We will be sampling all the α parameters α_0, α_1, α_2 in the general form of the potential together with other Ω_ϕ, w_ϕ, y_1 and the interaction parameter β. Sampling of the α parameters will allow us to sample the general form of the potential and sampling of the interaction parameter β will carry information about the energy transfer between dark matter and dark energy.

TABLE III. Best fit value of the cosmological parameter for the Interaction B (95% C.L.).

Cosmological parameters	JLA + BAO + RSD + $H(z)$
$100\omega_b$	$2.39^{+0.956}_{-0.86}$
ω_{cdm}	$0.134^{+0.0221}_{-0.0249}$
n_s	$0.81^{+0.753}_{-0.659}$
R	$1.87^{+0.888}_{-0.936}$
l_A	$302^{+0.888}_{-0.936}$
H_0 (kmS^{-1}/Mpc)	$73.3^{+1.77}_{-1.83}$
β	≤ 5.9
Ω_m	$0.293^{+0.0339}_{-0.0396}$
Ω_ϕ	$0.707^{+0.0396}_{0.0339}$
w_ϕ	$-0.983^{+0.000236}_{-0.0172}$

We have used following datasets: (i) the SDSS-II/SNLS3 JLA supernova data [65] and (ii) BAO measurements (baryonic acoustic oscillations) and Redshift Space Distortion data, BOSS DR 12: LOWZ & CMASS [66] and (iii) Hubble Space Telescope (HST) data [67]. The background quantities are sensitive to these datasets.

We have imposed the Planck compressed likelihood on CMB shift parameter (R), angular scale of sound horizon at the last scattering surface (l_A), baryon density (ω_b) and the scalar spectral index (n_s) using the mean value with the standard deviation on the parameters ($R = 1.7488 \pm 0.0074$, $l_A = 301.76 \pm 0.14$, $\omega_b = 0.02228 \pm 0.00023$, $n_s = 0.9660 \pm 0.0061$) and the correlation between the parameters given in the Table 4 of [68] for the smooth dark energy models. This likelihood has been widely used to reduce the full Planck likelihood information to few parameters.

A flat prior has been imposed on the α parameters and the β parameter. Following is the prior which we have considered for the data analysis $-5 < \alpha_0 < 5$, $-5 < \alpha_1 < 5$, $-5 < \alpha_2 < 5$, and $-15 < \beta < 15$. All these parameters are sampled by the MCMC code MONTE PYTHON. The set of derived parameters are Ω_m, Ω_ϕ, w_ϕ and y_1. In what follows we discuss about the result we obtain from the numerical analysis.

B. $Q = \beta H\dot{\phi}^2$

In the Fig. 1 we have plotted the matter density parameter Ω_m and the scalar field density parameter Ω_ϕ with respect to the redshift (z) for different values of β parameter. It is interesting to note from Fig. 1 that the matter and dark energy equality redshift decreases with increase of β. From this observation in the plot we expect to have a constraint on the allowed higher value of the parameter β while we use the observations to constrain the cosmological parameters. Figure 2 shows the ratio of ρ_ϕ and ρ_m, in the inset, we have plotted the evolution of the

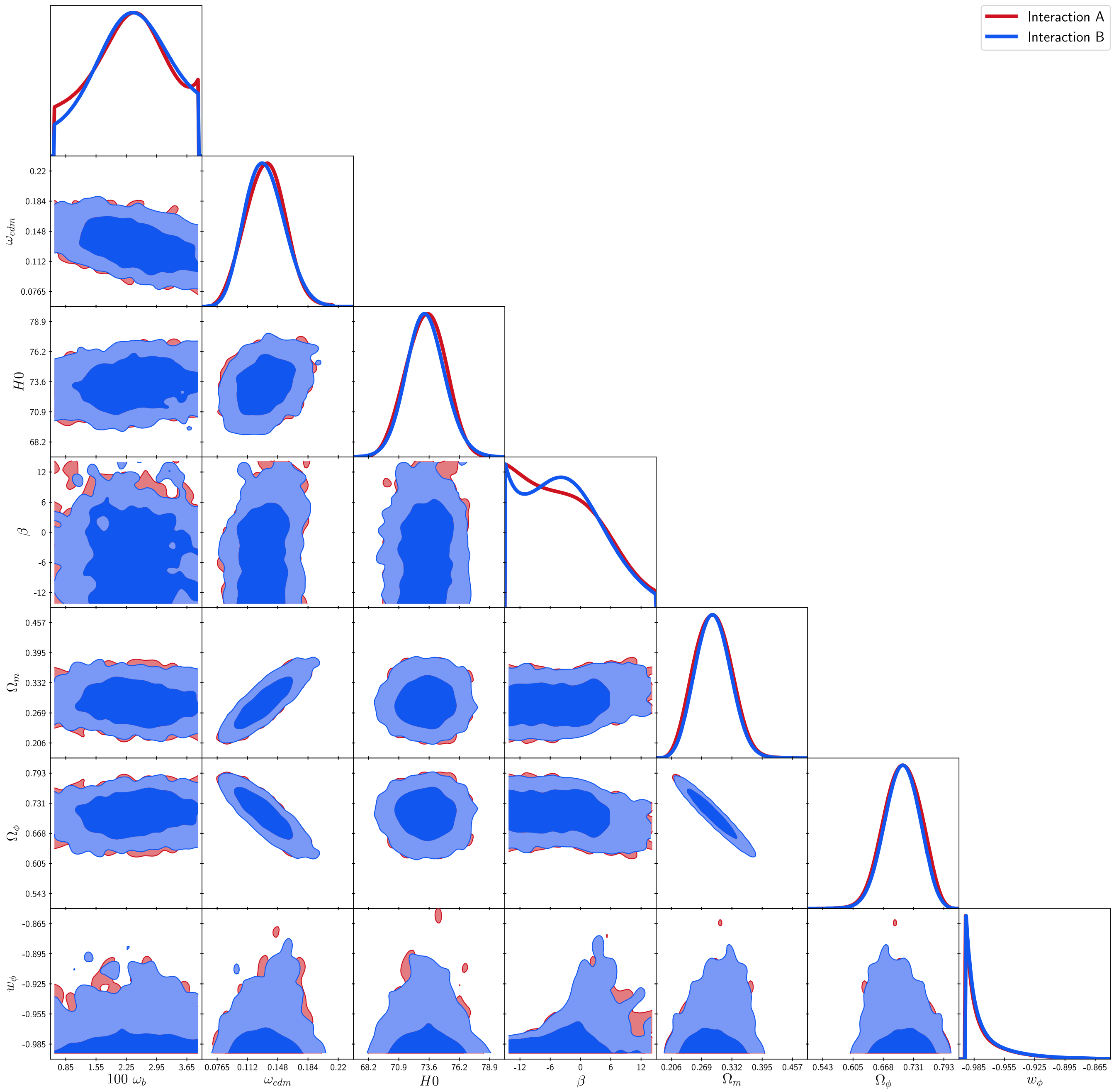

FIG. 7. Comparison of posterior of different cosmological parameters from the two classes of the interaction. The datasets used are BAO + JLA + H(z) + RSD and a PLANCK15 prior is imposed on ω_b and ω_{cdm}.

interaction parameter q for different values of the β parameter. In the remote past, the interaction term q was close to zero for any value of β but it has started to evolve recently. This nature of the interaction parameter suggests to us that if there is a transfer of energy from the dark matter to dark energy it has started very recently and this could be the reason why the universe at present is dominated by the dark energy. From Fig. 2 one can also notice that the consideration of a larger negative value of β does not have a significant change in the cosmological dynamics from this behavior of the system, and we expect no lower bound on the β parameter.

The constraint on the cosmological parameters for the interaction A is shown in Fig. 3 (95% C.L.). We get back the same result of the [41] on the constraint on α parameters. It is found that the cosmological parameters like, ω_{cdm}, H_0, Ω_ϕ, Ω_m are very well constrained whereas the constraint on the parameters ω_b, n_s, R, l_A are significantly weaker than [68]. The α parameters are unconstrained for these data sets. Table II is the best fit values of the cosmological parameter for the interaction A. The interaction parameter β is having a maximum cutoff value $\beta \leq 6.5$.

C. $Q = \beta\kappa^2(\rho_m + \rho_r)\dot{\phi}^2/H$

The plot of the Ω_m and Ω_ϕ is shown in Fig. 4 for the interaction B with different values of the β parameter. Like the interaction A similar nature of the plot is observed. The dark matter and dark energy equality redshift decreases with increase in β and hence a similar constraint on the maximum allowed value of the β parameter is expected from the cosmological data analysis. In Fig. 5 the ratio of ρ_ϕ and ρ_m has been plotted. Like type A interaction the dynamics does not change significantly for negative β so a lower limit on the β from the data analysis is not expected. An interaction between the dark matter and the dark energy has started recently and a transfer of energy from the dark matter to dark energy happens due to this.

Figure 6 shows the constraint on the cosmological parameter for the interaction II (95% C.L.). Similar to the Interaction B the α parameters are not constrained so the choice of potential remains arbitrary. This plot also indicates an upper bound on the interaction parameter $\beta \leq 5.9$, hence a transfer of energy from dark matter to dark energy cannot be arbitrary large. The best fit values are given in Table III.

A plot of the comparison of selected cosmological parameters can be found in Fig. 7. It can be seen that for the two interactions of our consideration the change in the background dynamics is insignificant.

VI. CONCLUSION

In a very recent work [41], it has been shown that the present cosmological observations are not enough to constrain any particular form of the quintessence potentials. The arbitrary nature of the quintessence potentials remains unresolved even though we have entered an era of high precision cosmology. To search the favorable form of quintessence potentials in [41] a general form of the quintessence potentials has been proposed. This general parametrization consists of three arbitrary parameters which are called as α parameters or the dynamical parameters as these parameters effect the cosmological dynamics. Different combinations of these α parameters corresponds to different potentials. The original idea was to constrain these α parameters and which will allow some one to find which class of the potentials are more favorable. But the present observations has failed to do so as it cannot put any constraint on the alpha parameters.

In this work, we have extended the above mentioned study of quintessence scalar field using the general parametrization to the interacting quintessence models. The interaction between dark matter and dark energy can have a significant effect on the cosmological observations. We have tried to check if an interaction between the dark sectors can break the degeneracy of quintessence potentials. Two different forms of the interaction are considered as an example since the functional form of the interaction is also arbitrary. While choosing the form of the interaction, particular importance was given to those functional form of the interactions which make the system equations simpler. As for our knowledge, we expect the result we obtained from this exercise will remain qualitatively the same for any other interactions at least at the background level.

Our results reconfirm the findings in [41] as our analysis also fails to constrain the α parameters. It is interesting to note that we have found a constraint on the upper bound on the β parameter which tells us that the transfer of energy from DM to DE cannot be arbitrarily large. This result is quite expected as in the Fig. 2 and Fig. 5 one can notice that the matter and dark energy equality redshift decrease with an increase in β for both the cases of the interactions. The matte and dark energy equality cannot be arbitrarily small so an upper bound on the β is natural to find. The morale of this exercise is that it not possible to break the degeneracy in the quintessence potentials even we consider an interaction between the dark sectors. This degeneracy cannot be broken as the recent cosmological observations can only constrain the present value of the equation of state parameter (EOS). For a given set of initial values there will be always a set of α parameters and β parameters which will satisfy the cosmological observations. Unless there is cosmological data on the evolution of the EOS of the dark energy it will not be possible to constrain the form of the quintessence potentials.

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