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Spin correlations in elastic e$^+$e$^-$ scattering in QED

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Abstract. Spin correlations are carefully investigated in elastic e^+e^- scattering in QED, for initially *polarized* as well as *unpolarized* particles, with emphasis placed on energy or speed of the underlying particles involved in the process. An explicit expression is derived for the corresponding transition probabilities in closed form to the leading order. These expressions differ from those obtained from simply combining the spins of the relevant particles, which are of kinematic nature. It is remarkable that these explicit results obtained from quantum field theory show a clear violation of Bell's inequality at *all* energies, in support of quantum theory in the relativistic regime. We hope that our explicit expression will lead to experiments, of the type described in the bulk of this paper, that monitor speed.

PACS. 12.20.Ds Specific calculations – 12.20.Fv Experimental tests – 12.20.-m Quantum electrodynamics

1 Introduction

In recent years, we have been interested in studying joint polarization correlations of fundamental processes in quantum electrodynamics (QED) and in the electro-weak theory [1–4], for initially *polarized* and *unpolarized* particles. Our main conclusion, based on explicit computations in quantum field theory, is that the mere fact that particles emerging from a process have non-zero speeds upon reaching the detectors, implies, in general, that their spin polarization correlation probabilities *depend on speed* [1–4] and *may also depend on the underlying couplings* [4]. The explicit expressions of polarization correlations follow from these *dynamical* computations, and are non-speculative, involving no arbitrary input assumptions, and are seen to depend on speed, and possibly on the couplings as well. These are unlike those from the rather naïve method of simply combining the spins of the particles in question, which are of kinematical nature. Such a method is, in general, not applicable to relativistic particles, and the regularly used formal arguments (based on combining spins only) completely fail. In the limit of low energies, our earlier expressions [1–4] for the polarization correlations were shown to be reduced to the naïve ones just mentioned by simply combining spins. In our previous investigation [4], in which a process for the creation of a $\mu^+\mu^-$ pair (from e^+e^- scattering, for example) was considered under the electro-weak theory, it was noted that, due to the threshold needed to create such a pair, the zero-energy limit may not be taken, and that the study of polarization correlations by simply combining spins (without recourse to quantum field theory) has no meaning. The focus of this paper is the derivation of the *explicit* polarization correlation probabilities in elastic e^+e^- scattering in QED, for initially *polarized* as well as *unpolarized* particles, with emphasis put on the *energy* available in the process so that a detailed study can be carried out in the relativistic regime as well. The reasons for our present investigation are twofold. Firstly, several experiments on $e^+e^- \to e^+e^-$ have been carried out over the years [5–9], and it is expected that our explicit new expression for the polarization correlations, depending on speeds, may lead to new experiments on polarization correlations that monitor the speed of the underlying particles. Secondly, such a study may be relevant to experiments in the light of Bell's theorem (monitoring speed), as discussed below.

The relevant quantity of interest here in testing Bell's inequality [10,11] is, in a standard notation,

$$S = \frac{p_{12}(a_1,a_2)}{p_{12}(\infty,\infty)} - \frac{p_{12}(a_1,a_2')}{p_{12}(\infty,\infty)} + \frac{p_{12}(a_1',a_2)}{p_{12}(\infty,\infty)} + \frac{p_{12}(a_1',a_2')}{p_{12}(\infty,\infty)} - \frac{p_{12}(a_1',\infty)}{p_{12}(\infty,\infty)} - \frac{p_{12}(\infty,a_2)}{p_{12}(\infty,\infty)} \quad (1)$$

as is *computed from* QED. Here a_1, a_2 (a_1', a_2') specify directions along which the polarizations of two particles are measured, with $p_{12}(a_1,a_2)/p_{12}(\infty,\infty)$ denoting the joint probability, and $p_{12}(a_1,\infty)/p_{12}(\infty,\infty)$ and $p_{12}(\infty,a_2)/p_{12}(\infty,\infty)$ denoting the probabilities when the polarization of only one of the particles is measured ($p_{12}(\infty,\infty)$ is normalization factor). The corresponding probabilities, as computed from QED, will be denoted by $P[\chi_1,\chi_2]$, $P[\chi_1,-]$, and $P[-,\chi_2]$, with χ_1 and χ_2 denoting

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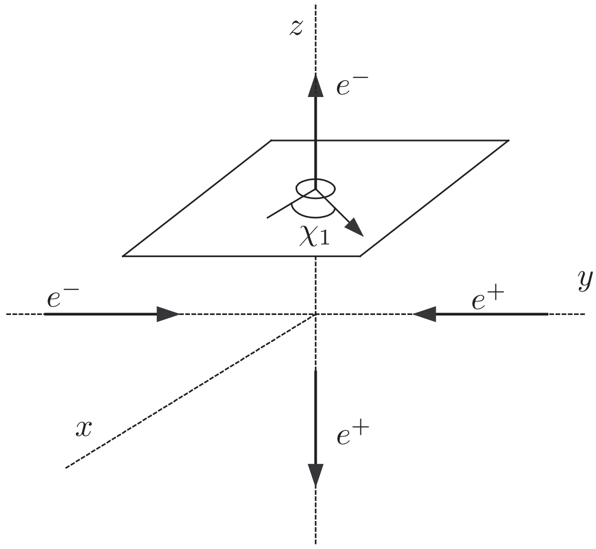

Fig. 1. The figure depicts e^+e^- scattering, with the electron and positron initially moving the y-axis, while the emerging electron and positron moving along the z-axis. The angle χ_1, measured relative to the x-axis, denotes the orientation of spin of the emerging electron.

angles specifying directions along which spin measurements are carried out with respect to certain axes spelled out in the bulk of the paper. To show that the QED process is in violation with Bell's inequality of local hidden variables (LHV), it is sufficient to find one set of angles χ_1, χ_2, χ_1', and χ_2', such that S, as computed in QED, has a value outside of the interval $[-1, 0]$. In this work, it is implicitly assumed that the polarization parameters in the particle states are directly observable and may be used for Bell-type measurements (as discussed). We show a clear violation of Bell's inequality for *all* speeds, in support of quantum theory in the relativistic regime, i.e. of quantum field theory.

2 Spin correlations; initially polarized particles

We consider the process $e^+e^- \to e^+e^-$ in the center of mass frame (see Fig. 1), with an electron-positron pair initially polarized with one spin up (along z-axis) and one spin down. With $\boldsymbol{p}_1 = \gamma m\beta(0,1,0) = -\boldsymbol{p}_2$ denoting the momenta of initial electron and positron, respectively, and $\gamma = 1/\sqrt{1-\beta^2}$, we consider the momenta of the emerging electron and positron as

$$\boldsymbol{k}_1 = \gamma m\beta(0,0,1) = -\boldsymbol{k}_2. \tag{2}$$

For four-spinors of the initial electron and positron, respectively, we have

$$u(p_1) \sim \begin{pmatrix} \uparrow \\ i\rho\downarrow \end{pmatrix} \quad \text{and} \quad \overline{v}(p_2) \sim \begin{pmatrix} i\rho\uparrow^\dagger - \downarrow^\dagger \end{pmatrix} \tag{3}$$

$$\rho = \frac{\gamma\beta}{\gamma+1} = \frac{\beta}{1+\sqrt{1-\beta^2}} \tag{4}$$

where $\uparrow \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is a spin up, $\downarrow \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is a spin down, $\uparrow^\dagger \equiv \begin{pmatrix} 1 & 0 \end{pmatrix}$ is a transpose matrix of spin up, and $\downarrow^\dagger \equiv \begin{pmatrix} 0 & 1 \end{pmatrix}$ is a transpose matrix of spin down. For four-spinors of the emerging electron and positron, respectively, we have

$$\overline{u}(k_1) \sim \begin{pmatrix} \zeta_1^\dagger & \rho\sigma_3\zeta_1^\dagger \end{pmatrix} \quad \text{and} \quad v(k_2) \sim \begin{pmatrix} \rho\sigma_3\zeta_2 \\ \zeta_2 \end{pmatrix} \tag{5}$$

$$\rho = \frac{\gamma\beta}{\gamma+1} = \frac{\beta}{1+\sqrt{1-\beta^2}} \tag{6}$$

where the two-spinors ζ_1, ζ_2 will be specified later.

The well-known expression for the amplitude of the process is [12,13]

$$\mathcal{M} \propto \overline{v}(p_2)\gamma^\mu u(p_1)\overline{u}(k_1)\gamma_\mu v(k_2)\frac{1}{(p_1+p_2)^2} - \overline{u}(k_1)\gamma^\mu u(p_1)\overline{v}(p_2)\gamma_\mu v(k_2)\frac{1}{(p_1-k_1)^2}. \tag{7}$$

Given the amplitude of the process above, we compute the conditional joint probability of spin measurements of e^+, e^- along directions specified by the angles χ_1 and χ_2, as shown in Figure 1. We then have two-spinors as follows:

$$\zeta_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\chi_1/2} \\ e^{i\chi_1/2} \end{pmatrix} \text{ and } \zeta_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\chi_2/2} \\ e^{i\chi_2/2} \end{pmatrix}. \tag{8}$$

Here we have considered only the single state (cf. [2,4]). Using the four-spinors of initial and emerging e^+, e^-, and the two-spinors in term of the angles χ_1 and χ_2 to calculate the invariant amplitude of the process in Figure 1, gives

$$\mathcal{M} \propto \left[A(\beta)\cos\left(\frac{\chi_1+\chi_2}{2}\right) + B(\beta)\sin\left(\frac{\chi_1-\chi_2}{2}\right)\right] + i\left[C(\beta)\sin\left(\frac{\chi_1+\chi_2}{2}\right) + D(\beta)\cos\left(\frac{\chi_1-\chi_2}{2}\right)\right] \tag{9}$$

where

$$\begin{aligned} A(\beta) &= 1 - \rho^2(1-\rho) + 2\beta^2(1-\rho^2)^2 \\ B(\beta) &= \rho(1+\rho) + 8\beta^2\rho^2 \\ C(\beta) &= 1 + \rho^2(1-\rho) + 2\beta(1-\rho^4) \\ D(\beta) &= \rho(1+\rho). \end{aligned}$$

Using the notation $F[\chi_1, \chi_2]$ for the absolute value square of the right-hand side of (9), the conditional joint probability distribution of spin measurements along the directions specified by angles χ_1 and χ_2 is given by

$$P[\chi_1, \chi_2] = \frac{F[\chi_1, \chi_2]}{N(\beta)}. \tag{10}$$

The normalization factor $N(\beta)$ is obtained by summing over all the polarizations of the emerging particles. This is equivalent to the summing of $F[\chi_1, \chi_2]$ over the pairs of angles

$$(\chi_1, \chi_2), \quad (\chi_1, \chi_2), \quad (\chi_1 + \pi, \chi_2), \quad (\chi_1, \chi_2 + \pi) \tag{11}$$

which leads to

$$\begin{aligned} N(\beta) &= F[\chi_1, \chi_2] + F[\chi_1 + \pi, \chi_2] \\ &\quad + F[\chi_1, \chi_2 + \pi] + P[\chi_1 + \pi, \chi_2 + \pi] \\ &= 2[A^2(\beta) + B^2(\beta) + C^2(\beta) + D^2(\beta)] \end{aligned} \tag{12}$$

giving

$$\begin{aligned} P[\chi_1, \chi_2] &= \frac{\left[A(\beta)\cos\left(\frac{\chi_1+\chi_2}{2}\right) + B(\beta)\sin\left(\frac{\chi_1-\chi_2}{2}\right)\right]^2}{2[A^2(\beta) + B^2(\beta) + C^2(\beta) + D^2(\beta)]} \\ &\quad + \frac{\left[C(\beta)\sin\left(\frac{\chi_1+\chi_2}{2}\right) + D(\beta)\cos\left(\frac{\chi_1-\chi_2}{2}\right)\right]^2}{2[A^2(\beta) + B^2(\beta) + C^2(\beta) + D^2(\beta)]}. \end{aligned} \tag{13}$$

If only one of the spins is measured, say, corresponding to χ_1, the probability can be written as

$$\begin{aligned} P[\chi_1, -] &= P[\chi_1, \chi_2] + P[\chi_1, \chi_2 + \pi] \\ &= \frac{1}{2} + \frac{2[A(\beta)B(\beta) + C(\beta)D(\beta)]\sin\chi_1}{2[A^2(\beta) + B^2(\beta) + C^2(\beta) + D^2(\beta)]}. \end{aligned} \tag{14}$$

Similarly, if only one of the spins is measured corresponding to χ_2, the probability can be written as

$$\begin{aligned} P[-, \chi_2] &= P[\chi_1, \chi_2] + P[\chi_1 + \pi, \chi_2] \\ &= \frac{1}{2} + \frac{2[C(\beta)D(\beta) - A(\beta)B(\beta)]\sin\chi_2}{2[A^2(\beta) + B^2(\beta) + C^2(\beta) + D^2(\beta)]}. \end{aligned} \tag{15}$$

For all $0 \leq \beta \leq 1$, angles χ_1, χ_2, χ_1' and χ_2' are readily found to lead to a violation of Bell's inequality of LHV theories. For example, for $\beta = 0.9$, $\chi_1 = 0°$, $\chi_2 = 45°$, $\chi_1' = 69°$, and $\chi_2' = 200°$, $S = -1.311$, violating the inequality from below.

3 Spin correlations; initially unpolarized particles

For the process $e^+e^- \to e^+e^-$, in the center of mass (c.m.), with initially unpolarized spins, with momenta $\boldsymbol{p}_1 = \gamma m\beta(0, 1, 0) = -\boldsymbol{p}_2$, we take for the final electron and positron

$$\boldsymbol{k}_1 = \gamma m\beta(1, 0, 0) = -\boldsymbol{k}_2 \tag{16}$$

and for the four-spinors

$$u(k_1) = \left(\frac{k^0 + m}{2m}\right)^{1/2} \begin{pmatrix} \xi_1 \\ \frac{\boldsymbol{k}_1\cdot\boldsymbol{\sigma}}{k^0+m}\xi_1 \end{pmatrix}, \ \xi_1 = \begin{pmatrix} -i\cos\chi_1/2 \\ \sin\chi_1/2 \end{pmatrix} \tag{17}$$

$$v(k_2) = \left(\frac{k^0 + m}{2m}\right)^{1/2} \begin{pmatrix} \frac{\boldsymbol{k}_2\cdot\boldsymbol{\sigma}}{k^0+m}\xi_2 \\ \xi_2 \end{pmatrix}, \ \xi_2 = \begin{pmatrix} -i\cos\chi_2/2 \\ \sin\chi_2/2 \end{pmatrix}. \tag{18}$$

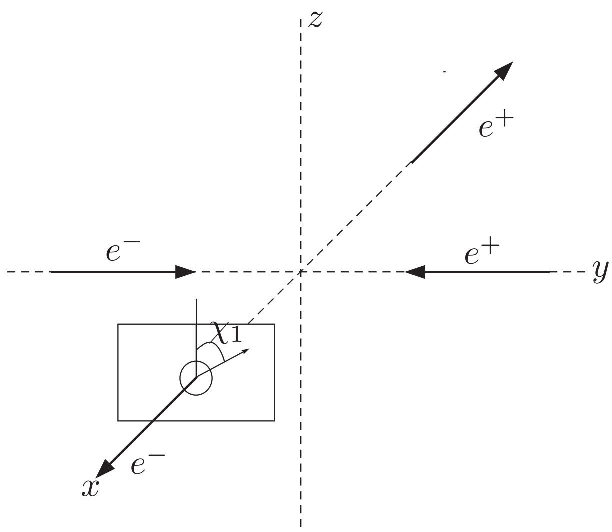

Fig. 2. The figure depicts e^+e^- scattering, with e^+, e^- moving along the y-axis, and the emerging electron and positron moving along the x-axis. The angle χ_1, measured relative to the z-axis, denotes the orientation of spin of the emerging electron.

The absolute value square of the right-hand side of (9), with initially unpolarized electrons and positron, leads to

$$\begin{aligned} \text{Prob} \propto & \\ & \frac{\mathrm{Tr}\left[\gamma^\sigma(\gamma p_2+m)\gamma^\mu(-\gamma p_1 + m)\right]\overline{u}(k_1)\gamma_\mu v(k_2)\overline{v}(k_2)\gamma_\sigma u(k_1)}{(p_1+p_2)^4} \\ & - \frac{\mathrm{Tr}[(\gamma p_2+m)\gamma^\mu(-\gamma p_1+m)\gamma^\sigma]\,\overline{v}(k_2)\gamma_\sigma u(k_1)\overline{u}(k_1)\gamma_\mu v(k_2)}{(p_1+p_2)^2(p_1-k_1)^2} \\ & - \frac{\mathrm{Tr}[\gamma^\mu(-\gamma p_1+m)\gamma^\sigma(\gamma p_2+m)]\,\overline{u}(k_1)\gamma_\mu v(k_2)\overline{v}(k_2)\gamma_\sigma u(k_1)}{(p_1+p_2)^2(p_1-k_1)^2} \\ & + \frac{\overline{u}(k_1)\gamma^\mu(-\gamma p_1+m)\gamma^\sigma u(k_1)\overline{v}(k_2)\gamma_\sigma(\gamma p_2+m)\gamma_\mu v(k_2)}{(p_1-k_1)^4} \end{aligned} \tag{19}$$

which, after simplification and collecting terms, reduces to

$$\begin{aligned} \text{Prob} \ &\propto [2\beta^4(1 + 2\beta^2) - 3(1 + \beta^2)]\sin^2\left(\frac{\chi_1 - \chi_2}{2}\right) \\ &\quad + (1 + \beta^2 + 2\beta^4)\cos^2\left(\frac{\chi_1 + \chi_2}{2}\right) + 5(1 - \beta^2) \\ &\equiv F[\chi_1, \chi_2] \end{aligned} \tag{20}$$

where we have used the expressions for the spinors in (17) and (18).

Given that the process has occurred, the conditional probability that the spins of the emerging electron and positron make angles χ_1 and χ_2, respectively, with the z-axis is obtained directly from (20) as

$$P[\chi_1, \chi_2] = \frac{F[\chi_1, \chi_2]}{C}. \tag{21}$$

The normalization constant C is obtained by summing over the polarizations of the emerging electrons. This is equivalent to the summing of $F[\chi_1, \chi_2]$ over the pairs of angles in (11) for any arbitrarily chosen fixed χ_1 and χ_2, corresponding to the orthonormal spinors

$$\begin{pmatrix} -i\cos\chi_j/2 \\ \sin\chi_j/2 \end{pmatrix}, \quad \begin{pmatrix} -i\cos(\chi_j+\pi)/2 \\ \sin(\chi_j+\pi)/2 \end{pmatrix} = \begin{pmatrix} i\sin\chi_j/2 \\ \cos\chi_j/2 \end{pmatrix} \tag{22}$$

providing a complete set, for each $j = 1, 2$, in reference to (17) and (18). This is,

$$\begin{aligned} C &= F[\chi_1, \chi_2] + F[\chi_1+\pi, \chi_2] \\ &\quad + F[\chi_1, \chi_2+\pi] + F[\chi_1+\pi, \chi_2+\pi] \\ &= 8[2 - 3\beta^2 + \beta^4 + \beta^6] \end{aligned} \tag{23}$$

which as expected is independent of χ_1 and χ_2. This gives

$$\begin{aligned} P[\chi_1, \chi_2] &= \frac{[2\beta^4(1+2\beta^2) - 3(1+\beta^2)]\sin^2\left(\frac{\chi_1-\chi_2}{2}\right)}{8[2-3\beta^2+\beta^4+\beta^6]} \\ &\quad + \frac{[1+\beta^2+2\beta^4]\cos^2\left(\frac{\chi_1+\chi_2}{2}\right) + 5(1-\beta^2)}{8[2-3\beta^2+\beta^4+\beta^6]}. \end{aligned} \tag{24}$$

By summing over

$$\chi_2, \quad \chi_2 + \pi \tag{25}$$

for any arbitrarily fixed χ_2, we obtain

$$P[\chi_1, -] = \frac{1}{2} \tag{26}$$

and similarly,

$$P[-, \chi_2] = \frac{1}{2} \tag{27}$$

for the probabilities when only one of the photons polarizations is measured.

A clear violation of Bell's inequality of LHV theories was obtained for all $0 \leq \beta \leq 1$. For example, for $\beta = 0.8$, with $\chi_1 = 0°$, $\chi_2 = 45°$, $\chi_1' = 210°$, and $\chi_2' = 15°$, $S = -1.167$, violating the inequality from below.

4 Conclusion

A critical study of polarization correlation probabilities in elastic e^+e^- scattering was carried out for initially *polarized* as well as *unpolarized* particles, emphasizing their dependence on speed, and an explicit expression was obtained for them in QED. The necessity of such a study within the realm of quantum field theory cannot be overemphasized, as estimates of such correlations from simply combining spins (as is often done), have no meaning, as they do not involve dynamical considerations. The relevant dynamics is, of course, dictated directly from quantum field theory. The explicit expression for the polarization correlation obtained is interesting in its own right, but may also lead to experiments that investigate such correlations by monitoring speed, not only for initially polarized particles, but also for unpolarized ones. Our results may also be relevant in the realm of Bell's inequality with emphasis put on relativistic aspects of quant! um theory, that is, of quantum field theory. Our expressions have shown clear violations of Bell's inequality of LHV theories, in support of quantum theory in the relativistic regime. In recent years, several experiments have been already performed (cf. [14–18]) on particles' polarization correlations. It is expected that the novel properties recorded here by explicit calculations following directly from field theory (which is based on the principle of relativity and quantum theory) will lead to new experiments on polarization correlations monitoring speed in the light of Bell's Theorem. We hope that these computations, within the general setting of quantum field theory, will also be useful in areas of physics such as quantum teleportation and quantum information.

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