DARK ENERGY AND ACCELERATING UNIVERSE

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ABSTRACT

Our universe today is under the accelerating expansion phase. Most physicists and cosmologists believe that this behavior is due to an unknown form of energy known as dark energy. There are many models proposed to explain this behavior of the universe. In our work, we consider the two different cosmological models in the scenario of canonical and phantom power-law cosmology.

First, the tachyonic scalar field-driven late universe with dust matter content is considered. The cosmic expansion is modeled with power-law and phantom power-law expansion at late time, i.e. $z \lesssim 0.45$. WMAP7 and its combination with other data are used to constraint the model. The forms of potential and the field solution are different for quintessence and tachyonic cases. Power-law cosmology model (driven by either quintessence or tachyonic field) predicts an equation of state parameter that does not match the observed value and hence the power-law model is excluded for both quintessence and tachyonic field. In the opposite, the phantom power-law model predicts agreeing valued of equation of state parameter with the observational data for both quintessence and tachyonic cases, i.e. $w_{\phi,0} = -1.49^{+11.64}_{-4.08}$ (WMAP7+BAO+H_0) and $w_{\phi,0} = -1.51^{+3.89}_{-6.72}$ (WMAP7). The phantom-power law exponent β must be less than about -6, so that the $-2 < w_{\phi,0} < -1$. The phantom power-law tachyonic potential is reconstructed. We found that dimensionless potential slope variable Γ at present is about 1.5. The tachyonic potential reduces to $V = V_0\phi^{-2}$ in the limit $\Omega_{\mathrm{m},0} \to 0$.

In addition, we give a brief review of the non-minimal derivative coupling (NMDC) scalar field theory of where there is non-minimal coupling between scalar field derivative term to the Einstein tensor. We estimate that the expansion is of power-law type or super-acceleration type in a very recent range of redshifts. The Lagrangian includes NMDC term, free kinetic term, constant

potential, $V = \Lambda/(8\pi G)$ and barotropic matter term. With inflation-suggested value of the coupling constant $\kappa \approx 10^{-74}$ sec^2, we use the combined WMAP9 (WMAP9+eCMB+BAO+ H_0) dataset, PLANCK+WP dataset, and PLANCK TT, TE, EE+lowP+Lensing+ext datasets to find value of cosmological constant in the model. Modeling the expansion with power-law gives negative cosmological constants while the phantom power-law (super-acceleration) expansion gives positive cosmological constant with large error bar. The value obtained is of the same order as of the ΛCDM model since at late time NMDC effect is tiny due to small curvature.

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CHAPTER I

INTRODUCTION

1.1 Background and Motivation

Recently, we knew that the universe is under an accelerating expansion and composed of three main ingredients, 5% of ordinary matter, 27% of unknown matter called dark matter and the unknown energy occupied most part of our universe with 68% and it is called *dark energy* from the evidences, for example, the supernova type Ia (SNIa) [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], the large-scale structure surveys [11, 12], the cosmic microwave background (CMB) [13, 14, 15, 16] and the X-ray luminosity from galaxy clusters [15, 17, 18]. The accelerating expansion of the universe is responsible by dark energy [19, 20, 21] which typically is in form of either cosmological constant or scalar field [19, 20, 21, 22]. For a word "scalar field", $\phi = \phi(x,t)$, it is assumed to be spatial homogeneous or invariance under transformation like $\phi(x') = \phi(x)$. Therefore scalar field can be written as $\phi(x,t) = \phi(t)$. Scalar field is responsible for symmetry breaking mechanisms and super-fast expansion in inflationary scenario, resolving horizon and flatness problems as well as explaining the origin of structures [23, 24, 25, 26, 27].

There are many models proposed to explain the accelerating expansion of the universe, for example, the quintessence [28, 29, 30, 31], k-essence and classes of k-essence type models [32, 33]. Modifications of gravity, for instance, braneworlds, $f(R)$ theory [34, 35], the non-minimal kinetic scalar field term coupling to curvature [36, 37, 38, 39, 40, 41, 42, 43] are the models proposed to explain the accelerating expansion of the universe as well. The interesting explanation is that the acceleration is an effect of a scalar field evolving under its potential to give rise the negative pressure, $p < -\rho c^2/3$. If dark energy is the scalar field, the field could have non-canonical kinetic part like tachyon, for example, which is classified in a type of k-essence models. The tachyon field is a negative mass-square mode of an unstable non-BPS D3-brane in string theory [44, 45, 46] or it is a massive scalar field on anti-D3 brane [47]. It was found that the potential of the tachyonic field must less steep than the inverse field square, ϕ^{-2}, in order to account for the late acceleration [48, 49, 50, 51, 52, 53].

Most models we can notice that they are represented the various modification of the scalar-tensor theories. In the scalar-tensor theories, we can extend the theories to allowing for non-minimal coupling (NMC) between scalar fields to Einstein's tensor or to Ricci scalar in GR in form of $\sqrt{-g}f(\phi)R$ which is motivated by scalar-tensor theories of the Jordan-Brans-Dicke models [54, 55], renormalizing term of quantum field in curved space [56] or supersymmetries, superstring and induced gravity theories [57, 58, 59, 60, 61]. It was applied to extended inflations with first-order phase transition and other inflationary models [62, 63, 64, 65, 66, 67, 68]. In context of quintessence field for present acceleration, non-minimal coupling to curvature is also investigated [69, 70, 71, 72]. First cosmological consideration of the non-minimal curvature coupling to the kinetic term of scalar field was proposed by Amendola in 1993 [36] of which the coupling function is in form of $f(\phi, \phi_{,\mu}, \phi_{,\mu\nu}, \ldots)$. The derivative coupling is required in scalar quantum electrodynamics to satisfy U(1) invariance of the theory and in models of which the gravitational constant is function of the mass density of the gravitational source. It is called non-minimal derivative coupling-NMDC model. NMDC terms are found as lower energy limits of higher dimensional theories which make quantum gravity possible to be investigated perturbatively. Moreover, they are found in Weyl anomaly in $\mathcal{N} = 4$ conformal supergravity [73, 74]. With simplest NMDC term, $R\phi_{,\mu}\phi^{,\mu}$, class of inflationary attractors is enlarged from the previous NMC model of [68] and the NMDC renders non-scale invariant spectrum without requirement of multiple scalar fields. Moreover it is possible to realize double inflation without adding more fields to the theory [36]. However conformal transformation can not change the NMDC theory to the standard field equation in Einstein frame. The conformal (metric) re-scaling transformation needs to be generalized to Legendre transformation in order to recover the Einstein frame equations [36, 75].

The power-law cosmology was also studied in context of scalar field cosmology [76, 77], phantom scalar field cosmology [78]. There is also slightly different form of the power-law function which α can be evolved with time, $\alpha = \alpha(t)$, so that it can parameterize cosmological observables [79]. For the power-law to be valid throughout the cosmic evolution, it is not possible with constant exponent. For example, at big bang primordial nucleosynthesis (BBN), α is allowed to have maximum value at approximately 0.55 in order to be capable of light element abundances [80, 81]. The value is about 1/2 at highly-radiation dominated era, about 2/3 at highly-dust dominated era and greater than one at present. Low value of α results in much younger cosmic age and does not give acceleration. On the other hand a power-law with exponent $\alpha \geq 1$ is needed to solve age problem in the CDM model [82] without flatness and horizon problems. In general, the power-law has the scale factor scaling as $a \propto t^{\alpha}$ with $0 \leq \alpha \leq \infty$, it is called the canonical power-law and is corresponding to the acceleration if $\alpha > 1$. In addition we also consider the phantom power-law cosmology where the scale factor scaling as $a \propto (t_{\rm s} - t)^{\beta}$. The power-law has proved to be a very good phenomenological description of the cosmic evolution, for example, it easily to tune in a short period of red-shift and can describe radiation epoch, dark matter epoch, and dark energy epoch according to value of the exponent [82, 83]. Previously linear-coasting cosmology, $\alpha \approx 1$ was analyzed [84, 85, 86, 87] with motivation from SU(2) instanton cosmology [88], higher order (Weyl) gravity [89], or from scalar-tensor theories [90]. However the universe expanding with $\alpha = 1$ [91] was not able to agree with observational constraint from Type Ia supernovae, Hubble rate data from cosmic chronometers and BAO [92] which indicates that $H'(z) = const$ and $q(z) = 0$ are not favored by the observations. We consider power-law cosmology with a brief period of recent cosmic era when dark energy began to dominate, i.e. from $z \lesssim 0.45$ to present. We are using results from WMAP7 [13] and WMAP7+BAO+H_0 combined datasets [14] in the tachyonic power-law model and using the results observed by combined WMAP9+eCMB+BAO+H_0 [93] and PLANCK satellite [94, 95] datasets in the

NMDC power-law model.

1.2 Objectives

We investigate the possible models that can be a source of accelerating expansion of our universe. In the tachyon model, we consider a short period of recent cosmic era when the dark energy began to dominate, i.e. $z \lesssim 0.45$. There are various of models we investigated but, in general, we consider only two models in the scenario of flat FLRW universe filled with dust and scalar field with the canonical and phantom power-law cosmology. We aim

1. to test whether the power-law cosmology is still valid in the scenario of tachyonic scalar field by looking at the value of the equation of state (EoS) parameter predicted by the power-law tachyonic cosmology and that of varying dark energy equation of state direct-observational result.
2. To constrain the EoS parameter and the cosmological parameters in the tachyonic power-law and phantom power-law cosmology by using data from WMAP7 [13] and WMAP7+BAO+H_0 combined datasets [14].
3. To give a brief review on the non-minimal derivative coupling (NMDC) model from various behaviours since it has been proposed by Amendola in 1993 [36].
4. To evaluate value of the cosmological constant by using the recent cosmological parameters observed by WMAP9 (combined WMAP9+eCMB+BAO+H_0) dataset [93], PLANCK+WP dataset [94], and PLANCK including polarization and other external parameters (TT, TE, EE+lowP+Lensing+ext.) [95].

1.3 Frameworks

In this dissertation, the introduction and motivation of our works, objectives, and frameworks are shown in chapter 1. In chapter 2, we considered dark energy in form of tachyonic scalar field in power-law cosmology of which the scale factor scaled as $a \propto t^{\alpha}$ with $0 \leq \alpha \leq \infty$, corresponding to acceleration if $\alpha > 1$. In addition we also consider phantom power-law cosmology with $a \propto (t_{\mathrm{s}} - t)^{\beta}$ and we require $\beta < 0$ for the acceleration phase. In cosmic history, there were epoch when radiation or dust was dominant component in the universe for which the scale factor evolves as power-law $a \propto t^{1/2}$ and $a \propto t^{2/3}$. The universe with mixed combination of different cosmic ingredients can be modeled using power-law expansion with some approximately constant α during a brief period of cosmic time. These are such as non-minimally coupled scalar-tensor theory in which the scalar field couples to the curvature contributing to energy density that cancels out the vacuum energy [96, 97, 98, 99] and simple inflationary model in which the power-law cosmology can avoid flatness and horizon problems and can give simple spectrum [100]. The power-law has proved to be a very good phenomenological description of the cosmic evolution, since it can describe radiation epoch, dark matter epoch, and dark energy epoch according to value of the exponent [82, 83]. In our universe, there are tachyonic scalar field evolving under potential $V(\phi)$ and dust barotropic fluid (cold dark matter and baryonic matter) as two major ingredients. In this chapter, we consider the tachyonic model with the canonical power-law and when the field is phantom power-law expansion [101, 102], where $a \propto (t_{\rm s} - t)^{\beta}$, $\beta < 0$ from $z \lesssim 0.45$ till present.

We give a brief review on NMDC gravity models in the first part of chapter 3. We are interesting in the model in which the Einstein tensor couples to the kinetic scalar field term with a free kinetic term with and without potential term. In general, we assumed that our universe is filled with dust matter and scalar field in the flat FLRW universe. The coupling constant, κ, of the scalar field and Einstein's tensor still remain and constantly values up until present since the end of the inflationary epoch, $t_f \simeq 10^{-35}$ sec, $H_\kappa \simeq 6{\times}10^{36}$sec^{-1} and $\kappa \simeq 10^{-75}$sec^2. And assuming power-law expansion as well as phantom case in addition the non-phantom case, we estimate the theoretical value of the cosmological constant, Λ, with recent cosmological parameters observed by WMAP9 (combined WMAP9+eCMB+BAO+H_0) dataset [93] and PLANCK satellite datasets [94, 95].

Chapter 4, we show the derived cosmological parameters from WMAP7 and WMAP7+BAO+H_0 combined datasets and from the present values observed by WMAP9 (combined WMAP9+eCMB+BAO+H_0) and PLANCK satellite datasets for both canonical and phantom power-law scenarios. We also show the results and parametric plots from both tachyonic with power-law cosmology and NMDC with power-law cosmology. We also present the parametric plots of cosmological constant versus the power-law exponents i.e. $\alpha(\beta)$ in canonical (phantom) power-law. We make discussions of both, tachyonic (phantom) power-law and NMDC with power-law cosmology, models in this chapter as well.

Finally, chapter 5 is the last chapter, we will summarize all models we explored and make the conclusions of each model with the possibility of our future researches and outlooks.

CHAPTER II

THE TACHYONIC POWER-LAW COSMOLOGY

The tachyon field is a negative mass-square mode of an unstable non-BPS D3-brane in string theory [44, 45, 46] or a massive scalar field on anti-D3 brane [47]. It was found that the tachyonic field potential must not be too steep, i.e. less steep than $V(\phi) \propto \phi^{-2}$ in order to account for the late acceleration [48, 49, 50, 51, 52, 53]. In this work, we considered dark energy in form of tachyonic scalar field in power-law cosmology and then reduced the form of background equations to the present time, $t = t_0$. For those reduced form of equations, we will use the observed data from WMAP7 and its combined dataset to constrain the EoS parameter. All results are shown in Chapter 4.

In the first section, we will introduce the basic model of tachyon and how to find the energy density and pressure of tachyon via its action. The background equations i.e. Friedmann equation can be found by varying the full form action [103, 104] of the tachyon and those procedures are shown in this section. In the second section, we will introduce the power-law cosmology of which the scale factor scaled as $a \propto t^{\alpha}$ with $0 \leq \alpha \leq \infty$, corresponding to acceleration if $\alpha > 1$ and in addition we also consider phantom power-law cosmology with $a \propto (t_{\rm s} - t)^{\beta}$. In the next two sections, we combine the tachyonic model to the power-law cosmology and find the background equations i.e. the EoS parameter and its effective EoS parameter in the canonical form, Section (2.3), and phantom form, Section (2.4). In the last section, we will reduce the form of the background equations by setting the cosmic time to present, $t = t_0$, and we will use these reduced background equations to constrain the values of the EoS parameter in Chapter 4.

2.1 Tachyonic Cosmology

We consider standard FLRW universe containing dust matters (cold dark matter and baryonic matter) with tachyonic field. The Lagrangian density of tachyon can be written as

$$\mathcal{L}_{\rm tachyon} = -V(\phi)\sqrt{1 + \varepsilon \partial_{\mu}\phi\partial^{\mu}\phi}, \tag{2.1}$$

where $V(\phi)$ is tachyon potential, ε is a constant with values ± 1 indicating the case when kinetic term of the tachyon is phantom when $\varepsilon = -1$ and ϕ is a tachyonic scalar field. Therefore the action of this model is

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\mathcal{L}, \\ &= \int d^4x\sqrt{-g}\left(-V(\phi)\sqrt{1 + \varepsilon \partial_{\mu}\phi\partial^{\mu}\phi}\right). \end{aligned} \tag{2.2}$$

To derive the tachyon field energy density and pressure, we need to vary the action, Eq.(2.2), with respect to the metric tensor $g^{\mu\nu}$. But we dropped subscript g and use only δ instead of δ_g which is the same meaning, *variation with respect to metric tensor* $g^{\mu\nu}$. Therefore,

$$\begin{aligned}\delta S &= \delta\left[\int d^4x\sqrt{-g}\left(-V(\phi)\sqrt{1+\varepsilon\partial_\mu\phi\partial^\mu\phi}\right)\right],\\ &= \int d^4x\Big[\left(\delta\sqrt{-g}\right)\left(-V(\phi)\sqrt{1+\varepsilon\partial_\mu\phi\partial^\mu\phi}\right)+\sqrt{-g}\left(-\delta V(\phi)\right)\sqrt{1+\varepsilon\partial_\mu\phi\partial^\mu\phi}\\ &\quad+\sqrt{-g}\left(-V(\phi)\delta\sqrt{1+\varepsilon\partial_\mu\phi\partial^\mu\phi}\right)\Big]. \end{aligned} \tag{2.3}$$

Let consider a first term of Eq.(2.3),

$$\delta\sqrt{-g} = -\frac{1}{2\sqrt{-g}}\delta g. \tag{2.4}$$

From the relation,

$$\ln(\det M) = \mathrm{Tr}(\ln M), \tag{2.5}$$

and take a variation to Eq.(2.5),

$$\frac{1}{\det M}\delta(\det M) = \mathrm{Tr}(M^{-1}\delta M). \tag{2.6}$$

If we set $M = g_{\mu\nu}$ and $\det M = g$, we obtained

$$\begin{aligned}\frac{1}{g}\delta g &= g^{-1}_{\mu\nu}\delta g_{\mu\nu},\\ \delta g &= g g^{\mu\nu}\delta g_{\mu\nu}.\end{aligned} \tag{2.7}$$

Then Eq.(2.4) becomes

$$\begin{aligned}\delta\sqrt{-g} &= -\frac{1}{2\sqrt{-g}}g g^{\mu\nu}\delta g_{\mu\nu},\\ &= -\frac{1}{2}\frac{(-1)(-g)}{\sqrt{-g}}g^{\mu\nu}\delta g_{\mu\nu},\\ &= \frac{\sqrt{-g}}{2}g^{\mu\nu}\delta g_{\mu\nu}.\end{aligned} \tag{2.8}$$

From the relation

$$g_{\mu\rho}g^{\rho\nu} = \delta_\mu^{\ \nu}. \tag{2.9}$$

Take the variation to above equation,

$$\begin{aligned}(\delta g_{\mu\rho})g^{\rho\nu} + g_{\mu\rho}(\delta g^{\rho\nu}) &= 0,\\ (\delta g_{\mu\rho})g^{\rho\nu} = -g_{\mu\rho}\delta g^{\rho\nu},\end{aligned}$$

multiply both sides by $g_{\nu\sigma}$,

$$\begin{aligned}(\delta g_{\mu\rho})g^{\rho\nu}g_{\nu\sigma} &= -g_{\nu\sigma}g_{\mu\rho}\delta g^{\rho\nu},\\ (\delta g_{\mu\rho})\delta^\rho_{\ \sigma} &= -g_{\nu\sigma}g_{\mu\rho}\delta g^{\rho\nu},\\ \delta g_{\mu\sigma} &= -g_{\nu\sigma}g_{\mu\rho}\delta g^{\rho\nu}.\end{aligned} \tag{2.10}$$

Then Eq.(2.8) becomes

$$\begin{aligned}
\delta\sqrt{-g} &= \frac{\sqrt{-g}}{2}g^{\mu\nu}\left(-g_{\mu\rho}g_{\nu\sigma}\delta g^{\rho\sigma}\right), \\
&= -\frac{\sqrt{-g}}{2}g_{\rho\sigma}\delta g^{\rho\sigma}, \\
&= -\frac{\sqrt{-g}}{2}g_{\mu\nu}\delta g^{\mu\nu}. \qquad (2.11)
\end{aligned}$$

Above equation is coming from changing the indices from $\rho \to \mu$ and $\sigma \to \nu$. The varying of a second term of Eq.(2.3) is zero because the potential is a function of scalar field only, hence $\delta V(\phi) = 0$. Let consider the varying of the third term of Eq.(2.3),

$$\begin{aligned}
\delta\sqrt{1+\varepsilon\partial_\mu\phi\partial^\mu\phi} &= \frac{\delta(1+\varepsilon\partial_\mu\phi\partial^\mu\phi)}{2\sqrt{1+\varepsilon\partial_\mu\phi\partial^\mu\phi}}, \\
&= \frac{\varepsilon\delta(\partial_\mu\phi)\partial^\mu\phi+\varepsilon\partial_\mu\phi(\delta\partial^\mu\phi)}{2\sqrt{1+\varepsilon\partial_\mu\phi\partial^\mu\phi}}, \\
&= \frac{\varepsilon\partial_\mu\phi\delta(g^{\mu\nu}\partial_\nu\phi)}{2\sqrt{1+\varepsilon\partial_\mu\phi\partial^\mu\phi}}, \\
&= \frac{\varepsilon\partial_\mu\phi\partial_\nu\phi}{2\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}}\delta g^{\mu\nu}. \qquad (2.12)
\end{aligned}$$

The last form of above equation we have to change the index of the denominator from μ to σ because the index μ of the denominator is different from the numerator index. Then substituting Eqs.(2.11) and (2.12) into Eq.(2.3), we obtain

$$\begin{aligned}
\delta S &= \int d^4x\left[\left(-\frac{\sqrt{-g}}{2}g_{\mu\nu}\delta g^{\mu\nu}\right)\left(-V(\phi)\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}\right)+0\right. \\
&\quad \left.-\sqrt{-g}V(\phi)\frac{\varepsilon\partial_\mu\phi\partial_\nu\phi}{2\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}}\delta g^{\mu\nu}\right], \\
&= \int d^4x\frac{\sqrt{-g}}{2}\left[g_{\mu\nu}V(\phi)\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}-\frac{V(\phi)\varepsilon\partial_\mu\phi\partial_\nu\phi}{\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}}\right]\delta g^{\mu\nu}, \\
&= -\int d^4x\frac{\sqrt{-g}}{2}\left[\frac{V(\phi)\varepsilon\partial_\mu\phi\partial_\nu\phi}{\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}}-g_{\mu\nu}V(\phi)\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}\right]\delta g^{\mu\nu}. \qquad (2.13)
\end{aligned}$$

Then the energy-momentum tensor, $T^{(\phi)}_{\mu\nu}$, of tachyon field is

$$T^{(\phi)}_{\mu\nu} = \frac{V(\phi)\varepsilon\partial_\mu\phi\partial_\nu\phi}{2\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}} - g_{\mu\nu}V(\phi)\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}. \qquad (2.14)$$

Then the energy density of tachyon field, the 00-component of $T^{(\phi)}_{\mu\nu}$, is[1]

$$\rho_\phi c^2 = T^{(\phi)}_{00} = \frac{V(\phi)\varepsilon\partial_0\phi\partial_0\phi}{\sqrt{1+\varepsilon g^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi}} - g_{00}V(\phi)\sqrt{1+\varepsilon g^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi},$$

[1]You will see the prove of this equation later, Eq.(2.42).

$$
\begin{aligned}
\rho_\phi c^2 &= \frac{V(\phi)\varepsilon\dot{\phi}^2}{\sqrt{1+g^{00}\varepsilon\partial_0\phi\partial_0\phi+g^{ii}\varepsilon\partial_i\phi\partial_i\phi}} \\
&\quad -(-1)V(\phi)\sqrt{1+g^{00}\varepsilon\partial_0\phi\partial_0\phi+g^{ii}\varepsilon\partial_i\phi\partial_i\phi}, \\
&= \frac{V(\phi)\varepsilon\dot{\phi}^2}{\sqrt{1+(-1)\varepsilon\dot{\phi}^2}} + V(\phi)\sqrt{1+(-1)\varepsilon\dot{\phi}^2}, \\
&= \frac{V(\phi)\varepsilon\dot{\phi}^2}{\sqrt{1-\varepsilon\dot{\phi}^2}} + V(\phi)\sqrt{1-\varepsilon\dot{\phi}^2}, \\
&= \frac{V(\phi)\left(\varepsilon\dot{\phi}^2+1-\varepsilon\dot{\phi}^2\right)}{\sqrt{1-\varepsilon\dot{\phi}^2}}, \\
\rho_\phi c^2 &= \frac{V(\phi)}{\sqrt{1-\varepsilon\dot{\phi}^2}}. \qquad (2.15)
\end{aligned}
$$

The pressure of tachyon field, the ii-component of $T^{(\phi)}_{\mu\nu}$, is

$$
\begin{aligned}
p_\phi = T^\phi_{ii} &= \frac{V(\phi)\varepsilon\partial_i\phi\partial_i\phi}{\sqrt{1+\varepsilon g^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi}} - g_{ii}V(\phi)\sqrt{1+\varepsilon g^{\sigma\rho}\partial_\sigma\phi\partial_\rho\phi}, \\
&= -(1)V(\phi)\sqrt{1+g^{00}\varepsilon\partial_0\phi\partial_0\phi+g^{ii}\varepsilon\partial_i\phi\partial_i\phi}, \\
&= -V(\phi)\sqrt{1+(-1)\varepsilon\dot{\phi}^2}, \\
p_\phi &= -V(\phi)\sqrt{1-\varepsilon\dot{\phi}^2}. \qquad (2.16)
\end{aligned}
$$

From Eq.(2.15), take time derivative,

$$
\begin{aligned}
\dot{\rho}_\phi c^2 &= \frac{\sqrt{1-\varepsilon\dot{\phi}^2}\left(\frac{dV(\phi)}{d\phi}\right)\left(\frac{d\phi}{dt}\right) - \frac{V(\phi)(-2\varepsilon\dot{\phi}\ddot{\phi})}{2\sqrt{1-\varepsilon\dot{\phi}^2}}}{\left(\sqrt{1-\varepsilon\dot{\phi}^2}\right)^2} \\
&= \frac{\sqrt{1-\varepsilon\dot{\phi}^2}V'\dot{\phi}}{1-\varepsilon\dot{\phi}^2} + \frac{V(\phi)\varepsilon\dot{\phi}\ddot{\phi}}{\sqrt{1-\varepsilon\dot{\phi}^2}\left(1-\varepsilon\dot{\phi}^2\right)} \\
&= \frac{V'\dot{\phi}}{\sqrt{1-\varepsilon\dot{\phi}^2}} + \frac{\varepsilon\dot{\phi}\ddot{\phi}V(\phi)}{(1-\varepsilon\dot{\phi}^2)^{3/2}}, \qquad (2.17)
\end{aligned}
$$

where V' is defined as a derivative of tachyonic potential with respect to scalar field, $dV/d\phi$.

To obtain the background equation i.e. Friedmann equation we need to vary the full form action [103, 104] of the tachyon field with matter in the universe,

$$
S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - V(\phi)\sqrt{1+\varepsilon\partial_\mu\phi\partial^\mu\phi}\right] + S_{\mathrm{m}}, \qquad (2.18)
$$

where R is Ricci scalar. Vary the above action with respect to metric tensor $g^{\mu\nu}$,

$$\begin{aligned}\delta S &= \int d^4x \Bigg\{ \delta\sqrt{-g}\left[\frac{R}{16\pi G} - V(\phi)\sqrt{1+\varepsilon\partial_\mu\phi\partial^\mu\phi}\right] \\ &+ \sqrt{-g}\Big[\frac{\delta R}{16\pi G} + \delta\left(-V(\phi)\sqrt{1+\varepsilon\partial_\mu\phi\partial^\mu\phi}\right)\Big]\Bigg\} + \delta S_{\rm m}. \end{aligned} \tag{2.19}$$

We obtain the variation $\delta\sqrt{-g} = -\frac{\sqrt{-g}}{2} g_{\mu\nu}\delta g^{\mu\nu}$ from Eq.(2.11). Then we will consider the variation of the Ricci scalar,

$$\delta R = \delta\left(g^{\mu\nu}R_{\mu\nu}\right) = \left(\delta g^{\mu\nu}\right)R_{\mu\nu} + g^{\mu\nu}\left(\delta R_{\mu\nu}\right), \tag{2.20}$$

and from

$$\begin{aligned} R_{\mu\nu} &= R^{\rho}_{\mu\nu\rho}, \\ &= \partial_\nu\Gamma^{\rho}_{\mu\rho} - \partial_\rho\Gamma^{\rho}_{\mu\nu} + \Gamma^{\sigma}_{\mu\rho}\Gamma^{\rho}_{\sigma\nu} - \Gamma^{\sigma}_{\mu\nu}\Gamma^{\rho}_{\sigma\rho}, \end{aligned} \tag{2.21}$$

where $\Gamma^{\sigma}_{\mu\nu}$ is called the *Christoffel symbols* and it is a tensor-like object that used to study the geometry of the Riemannian metric. Next, we vary the Ricci tensor with respect to metric tensor,

$$\begin{aligned} \delta R_{\mu\nu} &= \partial_\nu\delta\Gamma^{\rho}_{\mu\rho} - \partial_\rho\delta\Gamma^{\rho}_{\mu\nu} + \left(\delta\Gamma^{\sigma}_{\mu\rho}\right)\Gamma^{\rho}_{\sigma\nu} + \Gamma^{\sigma}_{\mu\rho}\left(\delta\Gamma^{\rho}_{\sigma\nu}\right) - \left(\delta\Gamma^{\sigma}_{\mu\nu}\right)\Gamma^{\rho}_{\sigma\rho} - \Gamma^{\sigma}_{\mu\nu}\left(\delta\Gamma^{\rho}_{\sigma\rho}\right), \\ &= \left\{\partial_\nu\delta\Gamma^{\rho}_{\mu\rho} + \Gamma^{\rho}_{\sigma\nu}\left(\delta\Gamma^{\sigma}_{\mu\rho}\right) - \Gamma^{\rho}_{\sigma\rho}\left(\delta\Gamma^{\sigma}_{\mu\nu}\right) - \left[\Gamma^{\beta}_{\nu\rho}\left(\delta\Gamma^{\rho}_{\mu\beta}\right)\right]\right\} \\ &\quad - \left\{\partial_\rho\delta\Gamma^{\rho}_{\mu\nu} - \Gamma^{\sigma}_{\mu\rho}\left(\delta\Gamma^{\rho}_{\sigma\nu}\right) + \Gamma^{\sigma}_{\mu\nu}\left(\delta\Gamma^{\rho}_{\sigma\rho}\right) - \left[\Gamma^{\beta}_{\nu\rho}\left(\delta\Gamma^{\rho}_{\mu\beta}\right)\right]\right\}, \\ &= \nabla_\nu\left(\delta\Gamma^{\rho}_{\mu\rho}\right) - \nabla_\rho\left(\delta\Gamma^{\rho}_{\mu\nu}\right). \end{aligned} \tag{2.22}$$

From above equation, we multiply the tensor metric, $g^{\mu\nu}$, on both sides of above equation,

$$\begin{aligned} g^{\mu\nu}\delta R_{\mu\nu} &= g^{\mu\nu}\nabla_\nu\left(\delta\Gamma^{\rho}_{\mu\rho}\right) - g^{\mu\nu}\nabla_\rho\left(\delta\Gamma^{\rho}_{\mu\nu}\right), \\ &= \nabla_\nu\left(g^{\mu\nu}\delta\Gamma^{\rho}_{\mu\rho}\right) - \left(\delta\Gamma^{\rho}_{\mu\rho}\right)\nabla_\nu g^{\mu\nu} - \nabla\left(g^{\mu\nu}\delta\Gamma^{\rho}_{\mu\nu}\right) + \left(\delta\Gamma^{\rho}_{\mu\nu}\right)\nabla_\rho g^{\mu\nu}, \\ &= \nabla_\sigma\left[\delta^{\sigma}_{\nu}g^{\mu\nu}\delta\Gamma^{\rho}_{\mu\rho} - \delta^{\sigma}_{\rho}g^{\mu\nu}\delta\Gamma^{\rho}_{\mu\nu}\right], \\ &= \nabla_\sigma\left[g^{\mu\sigma}\delta\Gamma^{\rho}_{\mu\rho} - g^{\mu\nu}\delta\Gamma^{\sigma}_{\mu\nu}\right]. \end{aligned} \tag{2.23}$$

Next we consider the variation of matter action,

$$\begin{aligned} \delta S_{\rm m} &= \int d^4x\,\delta\left(\sqrt{-g}L_{\rm m}\right), \\ &= \int d^4x\left[\left(\delta\sqrt{-g}\right)L_{\rm m} + \sqrt{-g}\left(\delta L_{\rm m}\right)\right]. \end{aligned} \tag{2.24}$$

Using Eq.(2.11), we obtain

$$\begin{aligned} \delta S_{\rm m} &= \int d^4x\left[-\frac{\sqrt{-g}}{2}g_{\mu\nu}\delta g^{\mu\nu}L_{\rm m} + \sqrt{-g}L_{\rm m}\right], \\ &= -\frac{1}{2}\int d^4x\sqrt{-g}\,\delta g^{\mu\nu}\left[g_{\mu\nu}L_{\rm m} - 2\frac{\delta L_{\rm m}}{\delta g^{\mu\nu}}\right]. \end{aligned} \tag{2.25}$$

By the definition of energy-momentum tensor of matter,

$$\begin{aligned}
T^{(\mathrm{m})}_{\mu\nu} &= -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_{\mathrm{m}}}{\delta g^{\mu\nu}}, \\
&= -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_{\mathrm{m}}\right)}{\delta g^{\mu\nu}}, \\
&= -\frac{2}{\sqrt{-g}}\left[\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}}L_{\mathrm{m}} + \sqrt{-g}\frac{\delta L_{\mathrm{m}}}{\delta g^{\mu\nu}}\right], \\
&= -\frac{2}{\sqrt{-g}}\left[-\frac{\sqrt{-g}}{2}g_{\mu\nu}L_{\mathrm{m}} + \sqrt{-g}\frac{\delta L_{\mathrm{m}}}{\delta g^{\mu\nu}}\right], \\
&= g_{\mu\nu}L_{\mathrm{m}} - 2\frac{\delta L_{\mathrm{m}}}{\delta g^{\mu\nu}}, \qquad (2.26)
\end{aligned}$$

where $\mathcal{L}_{\mathrm{m}} = \sqrt{-g}L_{\mathrm{m}}$ is the Lagrangian density of matter. Then Eq.(2.25) becomes

$$\delta S_{\mathrm{m}} = -\frac{1}{2}\int d^4x\sqrt{-g}\delta g^{\mu\nu}T^{(\mathrm{m})}_{\mu\nu}. \qquad (2.27)$$

Using Eq.(2.13), Eq.(2.23) into Eq.(2.20), and Eq.(2.27), then the variation of action Eq.(2.19) becomes

$$\begin{aligned}
\delta S &= \int d^4x\sqrt{-g}\Bigg\{-\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}\left[\frac{c^4R}{16\pi G} - V(\phi)\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}\right] \\
&\quad + \frac{c^4}{16\pi G}\Big[\left(\delta g^{\mu\nu}\right)R_{\mu\nu} + \nabla_\sigma\left(g^{\mu\sigma}\delta\Gamma^{\rho}_{\mu\rho} - g^{\mu\nu}\delta\Gamma^{\sigma}_{\mu\nu}\right)\Big] - \frac{V(\phi)\varepsilon\partial_\mu\phi\partial_\nu\phi}{2\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}}\delta g^{\mu\nu}\Bigg\} \\
&\quad - \frac{1}{2}\int d^4x\sqrt{-g}\delta g^{\mu\nu}T^{(\mathrm{m})}_{\mu\nu}. \qquad (2.28)
\end{aligned}$$

The integral of total derivative is zero when $\nabla_\sigma g^{\mu\nu} = 0$, are assumed [105]. Then Eq.(2.28) becomes

$$\begin{aligned}
\delta S &= \int d^4x\sqrt{-g}\Bigg\{-\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}\left[\frac{c^4R}{16\pi G} - V(\phi)\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}\right] \\
&\quad + \frac{c^4R_{\mu\nu}}{16\pi G}\delta g^{\mu\nu} - \frac{V(\phi)\varepsilon\partial_\mu\phi\partial_\nu\phi}{2\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}}\delta g^{\mu\nu} - \frac{1}{2}T^{(\mathrm{m})}_{\mu\nu}\delta g^{\mu\nu}\Bigg\}, \\
&= \int d^4x\sqrt{-g}\delta g^{\mu\nu}\Bigg\{-\frac{1}{2}g_{\mu\nu}\frac{c^4R}{16\pi G} + \frac{1}{2}g_{\mu\nu}V(\phi)\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi} \\
&\quad + \frac{c^4R_{\mu\nu}}{16\pi G} - \frac{V(\phi)\varepsilon\partial_\mu\phi\partial_\nu\phi}{2\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}} - \frac{1}{2}T^{(\mathrm{m})}_{\mu\nu}\Bigg\}, \\
&= \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\delta g^{\mu\nu}\Bigg[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{8\pi G}{c^4}\Big(\frac{V(\phi)\varepsilon\partial_\mu\phi\partial_\nu\phi}{\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}} \\
&\quad - g_{\mu\nu}V(\phi)\sqrt{1+\varepsilon\partial_\sigma\phi\partial^\sigma\phi}\Big) - \frac{8\pi G}{c^4}T^{(\mathrm{m})}_{\mu\nu}\Bigg],
\end{aligned}$$

$$\delta S = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\delta g^{\mu\nu}\left[\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)-\frac{8\pi G}{c^4}\left(T^{(\phi)}_{\mu\nu}+T^{(\mathrm{m})}_{\mu\nu}\right)\right] \quad (2.29)$$

where $T^{(\phi)}_{\mu\nu}$ is the energy-momentum tensor of tachyon scalar field coming from Eq.(2.14). Therefore from Eq.(2.29) above, the Einstein's field equation is

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R = G_{\mu\nu} = \frac{8\pi G}{c^4}\left(T^{(\phi)}_{\mu\nu}+T^{(\mathrm{m})}_{\mu\nu}\right). \quad (2.30)$$

Take trace to above equation, we obtain

$$\begin{aligned} R-\frac{1}{2}(4)R &= \frac{8\pi G}{c^4}\left(T^{(\phi)}+T^{(\mathrm{m})}\right), \\ R &= -\frac{8\pi G}{c^4}\left(T^{(\phi)}+T^{(\mathrm{m})}\right). \end{aligned} \quad (2.31)$$

Then Eq.(2.29) becomes

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}\left[-\frac{8\pi G}{c^4}\left(T^{(\phi)}+T^{(\mathrm{m})}\right)\right] = \frac{8\pi G}{c^4}\left(T^{(\phi)}_{\mu\nu}+T^{(\mathrm{m})}_{\mu\nu}\right),$$

$$R_{\mu\nu} = \frac{8\pi G}{c^4}\Big[T^{(\phi)}_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T^{(\phi)}+T^{(\mathrm{m})}_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T^{(\mathrm{m})}\Big]. \quad (2.32)$$

We have the energy-momentum tensor of the perfect fluid (no viscosity, no shear stresses, and no heat conduction) in thermodynamics equilibrium in a simple form as

$$T^{\mu\nu} = \left(\rho+\frac{P}{c^2}\right)u^\mu u^\nu + Pg^{\mu\nu}, \quad (2.33)$$

where ρ is the mass-energy density of fluid, P is the pressure, u^μ is the fluid's 4-velocity[2] Trace of Eq.(2.33) is

$$\begin{aligned} g_{\mu\nu}T^{\mu\nu} &= \left(\rho+\frac{P}{c^2}\right)g_{\mu\nu}u^\mu u^\nu + Pg_{\mu\nu}g^{\mu\nu}, \\ T &= \left(\rho+\frac{P}{c^2}\right)\left(g_{00}u^0u^0+g_{ii}u^iu^j\right)+P\delta^\mu_\mu, \\ &= \left(\rho+\frac{P}{c^2}\right)(-1)(c)^2+4P, \\ &= -\rho c^2+3P. \end{aligned} \quad (2.34)$$

Next we need to find the components of Ricci tensor, $R_{\mu\nu}$, from Eq.(2.21),

$$R_{\mu\nu} = \partial_\nu\Gamma^\rho_{\mu\rho}-\partial_\rho\Gamma^\rho_{\mu\nu}+\Gamma^\sigma_{\mu\rho}\Gamma^\rho_{\sigma\nu}-\Gamma^\sigma_{\mu\nu}\Gamma^\rho_{\sigma\rho},$$

where

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}\left(\partial_\mu g_{\sigma\nu}+\partial_\nu g_{\mu\sigma}-\partial_\sigma g_{\mu\nu}\right). \quad (2.35)$$

---

[2]In the inertial frame of reference the 4-velocity is $u^\mu = (1,0,0,0)$.

In the flat FLRW universe, $k=0$ but we keep it in equation for completeness, we have the line element

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \tag{2.36}$$

where $a(t)$ is scale factor and this line element give us the metric tensor

$$g_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & a^2\left(1-kr^2\right) & 0 & 0 \\ 0 & 0 & a^2r^2 & 0 \\ 0 & 0 & 0 & a^2r^2\sin^2\theta \end{pmatrix} \tag{2.37}$$

By using non-zero connection terms[3], we find the 00-component of Ricci tensor

$$\begin{aligned} R_{00} &= \partial_\sigma\Gamma^\sigma_{00} - \partial_0\Gamma^\sigma_{0\sigma} + \Gamma^\rho_{00}\Gamma^\sigma_{\rho\sigma} - \Gamma^\rho_{0\sigma}\Gamma^\sigma_{\rho 0}, \\ &= -\partial_0\Gamma^\sigma_{0\sigma} - \Gamma^\rho_{0\sigma}\Gamma^\sigma_{\rho 0}, \\ &= -\left(\partial_0\Gamma^0_{00} + \partial_0\Gamma^1_{01} + \partial_0\Gamma^2_{02} + \partial_0\Gamma^3_{03}\right) \\ &\quad -\Big[\Gamma^1_{01}\Gamma^1_{10} + \Gamma^1_{02}\Gamma^2_{10} + \Gamma^1_{03}\Gamma^3_{03} \\ &\quad + \Gamma^2_{01}\Gamma^1_{20} + \Gamma^2_{02}\Gamma^2_{20} + \Gamma^2_{03}\Gamma^3_{20} \\ &\quad + \Gamma^3_{01}\Gamma^1_{30} + \Gamma^3_{02}\Gamma^2_{30} + \Gamma^3_{03}\Gamma^3_{30}\Big], \\ &= -\left[\partial_0\left(\frac{\dot{a}}{ca}\right) + \partial_0\left(\frac{\dot{a}}{ca}\right) + \partial_0\left(\frac{\dot{a}}{ca}\right)\right] \\ &\quad -\left[\left(\frac{\dot{a}}{ca}\right)^2 + \left(\frac{\dot{a}}{ca}\right)^2 + \left(\frac{\dot{a}}{ca}\right)^2\right], \\ &= -\frac{3}{c^2}\left[\frac{a\ddot{a} - a^2}{a^2}\right] - \frac{3}{c^2}\left(\frac{\dot{a}^2}{a^2}\right), \\ &= -\frac{3}{c^2}\frac{\ddot{a}}{a}, \end{aligned} \tag{2.38}$$

where $\partial_0 \equiv \partial/\partial(ct)$ and with the same procedures we obtained the 11-component

$$R_{11} = \frac{a\ddot{a} + 2\dot{a}^2 + 2kc^2}{c^2(1-kr^2)}, \tag{2.39}$$

---

[3]The non-zero connection terms are [106]

$\Gamma^0_{11} = \frac{a\dot{a}}{c(1-kr^2)}$ $\Gamma^1_{11} = \frac{kr}{1-kr^2}$ $\Gamma^2_{12} = \frac{1}{r} = \Gamma^3_{13}$

$\Gamma^0_{22} = \frac{a\dot{a}r^2}{c}$ $\Gamma^1_{22} = -r(1-kr^2)$ $\Gamma^2_{33} = -\sin\theta\cos\theta$

$\Gamma^0_{33} = \frac{1}{c}(a\dot{a}r^2\sin^2\theta)$ $\Gamma^1_{33} = -r(1-kr^2)\sin^2\theta$ $\Gamma^3_{23} = \cot\theta$

$\Gamma^1_{01} = \frac{\dot{a}}{ca} = \Gamma^2_{02} = \Gamma^3_{03}$

the 22-component

$$R_{22} = \frac{r^2}{c^2}\left(a\ddot{a} + 2\dot{a}^2 + 2kc^2\right), \tag{2.40}$$

and the 33-component

$$R_{33} = \frac{r^2\sin^2\theta}{c^2}\left(a\ddot{a} + 2\dot{a}^2 + 2kc^2\right). \tag{2.41}$$

From Eq.(2.33), we find the 00-component of the energy-momentum tensor

$$\begin{aligned} T_{00} &= \left(\rho + \frac{P}{c^2}\right) g_{0\sigma} g_{0\rho} u^\sigma u^\rho + g_{00}P, \\ &= \left(\rho + \frac{P}{c^2}\right) g_{00} g_{00} u^0 u^0 + (-1)P, \\ &= \left(\rho + \frac{P}{c^2}\right) (-1)^2 c^2 - P, \\ &= \rho c^2. \end{aligned} \tag{2.42}$$

That means $T_{00}^{(\phi)} = \rho_\phi c^2$ [4] and $T_{00}^{(\mathrm{m})} = \rho_{\mathrm{m}} c^2$. With the same procedures where $u^\mu = (c, 0, 0, 0)$, we obtain the 11-component

$$T_{11} = \left(\frac{a^2}{1 - kr^2}\right) P, \tag{2.43}$$

the 22-component

$$T_{22} = \left(a^2 r^2\right) P, \tag{2.44}$$

and the 33-component

$$T_{33} = \left(a^2 r^2 \sin^2\theta\right) P. \tag{2.45}$$

Now we obtain all components both on the right hand and the left hand sides of Eq.(2.32), therefore the 00-component is

$$\begin{aligned} R_{00} &= \frac{8\pi G}{c^4}\left[T_{00}^{(\phi)} - \frac{1}{2}g_{00}T^{(\phi)} + T_{00}^{(\mathrm{m})} - \frac{1}{2}g_{00}T^{(\mathrm{m})}\right], \\ -\frac{3}{c^2}\ddot{a}a &= \frac{8\pi G}{c^4}\left[\rho_\phi c^2 - \frac{1}{2}(-1)(-\rho_\phi c^2 + 3P_\phi) + \rho_{\mathrm{m}} c^2 - \frac{1}{2}(-1)(-\rho_{\mathrm{m}} c^2 + 3P_{\mathrm{m}})\right], \\ -3\frac{\ddot{a}}{a} &= \frac{8\pi G}{c^2}\left[\rho_\phi c^2 - \frac{1}{2}\rho_\phi c^2 + \frac{3}{2}P_\phi + \rho_{\mathrm{m}} c^2 - \frac{1}{2}\rho_{\mathrm{m}} c^2\right], \\ \frac{\ddot{a}}{a} &= -\frac{8\pi G}{3c^2}\left[\frac{1}{2}\rho_\phi c^2 + \frac{3}{2}P_\phi + \frac{1}{2}\rho_{\mathrm{m}} c^2\right], \\ &= -\frac{4\pi G}{3c^2}\left(\rho_\phi c^2 + 3P_\phi + \rho_{\mathrm{m}} c^2\right). \end{aligned} \tag{2.46}$$

[4]The Eq.(2.15) was proved.

The 11-component is

$$
\begin{aligned}
R_{11} &= \frac{8\pi G}{c^4}\left[T^{(\phi)}_{11} - \frac{1}{2}g_{11}T^{(\phi)} + T^{(\mathrm{m})}_{11} - \frac{1}{2}g_{11}T^{(\mathrm{m})}\right], \\
\frac{a\ddot{a} + 2\dot{a}^2 + 2kc^2}{c^2(1-kr^2)} &= \frac{8\pi G}{c^4}\Big[\frac{a^2}{1-kr^2}P_\phi - \frac{1}{2}\left(\frac{a^2}{1-kr^2}\right)\left(-\rho_\phi c^2 + 3P_\phi\right) \\
&\quad + \frac{a^2}{1-kr^2}P_\mathrm{m} - \frac{1}{2}\left(\frac{a^2}{1-kr^2}\right)\left(-\rho_\mathrm{m} c^2 + 3P_\mathrm{m}\right)\Big], \\
a\ddot{a} + 2\dot{a}^2 + 2kc^2 &= \frac{8\pi G}{c^2}\left[a^2P_\phi - \frac{1}{2}a^2\left(-\rho_\phi c^2 + 3P_\phi\right) - \frac{1}{2}a^2(-\rho_\mathrm{m}c^2)\right], \\
&= \frac{8\pi G}{c^2}a^2\left[P_\phi + \frac{1}{2}\rho_\phi c^2 - \frac{3}{2}P_\phi + \frac{1}{2}\rho_\mathrm{m}c^2\right], \\
\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{2kc^2}{a^2} &= \frac{4\pi G}{c^2}\left[\rho_\phi c^2 - P_\phi + \rho_\mathrm{m}c^2\right]. \qquad (2.47)
\end{aligned}
$$

This equation is the 11-component of the Ricci tensor and is the same to the other 22- and 33-components. Then substituting the 00-component, Eq.(2.46), into 11-component, Eq.(2.47),

$$
\begin{aligned}
\frac{2\dot{a}^2}{a^2} + \frac{2kc^2}{a^2} &= \frac{4\pi G}{c^2}\left[\rho_\phi c^2 - P_\phi + \rho_\mathrm{m}c^2\right] - \frac{\ddot{a}}{a}, \\
&= \frac{4\pi G}{c^2}\left(\rho_\phi - P_\phi + \rho_\mathrm{m}c^2\right) + \frac{4\pi G}{3c^2}\left(\rho_\phi c^2 + 3P_\phi + \rho_\mathrm{m}c^2\right), \\
&= \frac{4\pi G}{3c^2}\Big(3\rho_\phi c^2 - 3P_\phi + 3\rho_\mathrm{m}c^2 + \rho_\phi c^2 + 3P_\phi + \rho_\mathrm{m}c^2\Big), \\
&= \frac{4\pi G}{3c^2}\left(4\rho_\phi c^2 + 4\rho_\mathrm{m}c^2\right), \\
\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} &= \frac{8\pi G}{3c^2}\left(\rho_\phi c^2 + \rho_\mathrm{m}c^2\right), \\
H^2 &= \frac{8\pi G}{3c^2}\left(\rho_\phi c^2 + \rho_\mathrm{m}c^2\right) - \frac{kc^2}{a^2}, \qquad (2.48)
\end{aligned}
$$

where $H = \dot{a}/a$ is the Hubble parameter and this equation is called the Friedmann equation. Taking time derivative on both sides,

$$
\begin{aligned}
\frac{dH^2}{dt} &= \frac{8\pi G}{3c^2}\left(\dot{\rho}_\phi c^2 + \dot{\rho}_\mathrm{m}c^2\right) - kc^2\frac{da^{-2}}{dt}, \\
2H\dot{H} &= \frac{8\pi G}{3c^2}\left(\dot{\rho}_\phi c^2 + \dot{\rho}_\mathrm{m}c^2\right) - kc^2(-2)a^{-3}\dot{a}, \\
&= \frac{8\pi G}{3c^2}\left(\dot{\rho}_\phi c^2 + \dot{\rho}_\mathrm{m}c^2\right) + 2H\frac{kc^2}{a^2},
\end{aligned}
$$

and the acceleration equation,

$$
\dot{H} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = \frac{4\pi G}{3Hc^2}\left(\dot{\rho}_\phi c^2 + \dot{\rho}_\mathrm{m}c^2\right) + \frac{kc^2}{a^2}. \qquad (2.49)
$$

Consider the fluid equations

$$
\dot{\rho} + 3H(\rho + P) = 0. \qquad (2.50)
$$

Therefore, the fluid equation of matter ($P_{\rm m} = 0$) and tachyon field,

$$\dot{\rho}_{\rm m} = -3H\rho_{\rm m}, \tag{2.51}$$

$$\dot{\rho}_{\phi} = -3H\left(\rho_{\phi} + p_{\phi}\right). \tag{2.52}$$

Substituting Eqs.(2.15), (2.16), and (2.17) into Eq.(2.52), we obtain

$$\begin{aligned}
\frac{V'\dot{\phi}}{\sqrt{1-\varepsilon\dot{\phi}^2}} + \frac{\varepsilon\dot{\phi}\ddot{\phi}V}{(1-\varepsilon\dot{\phi}^2)^{3/2}} + 3H\left[\frac{V(\phi)}{\sqrt{1-\varepsilon\dot{\phi}^2}} - V(\phi)\sqrt{1-\varepsilon\dot{\phi}^2}\right] &= 0, \\
\frac{V'\dot{\phi}}{\sqrt{1-\varepsilon\dot{\phi}^2}} + \frac{\varepsilon\dot{\phi}\ddot{\phi}V}{(1-\varepsilon\dot{\phi}^2)^{3/2}} + 3H\frac{V}{\sqrt{1-\varepsilon\dot{\phi}^2}}\left[1 - 1 + \varepsilon\dot{\phi}^2\right] &= 0, \\
\frac{V'\dot{\phi}}{\sqrt{1-\varepsilon\dot{\phi}^2}} + \frac{\varepsilon\dot{\phi}\ddot{\phi}V}{(1-\varepsilon\dot{\phi}^2)^{3/2}} + 3H\frac{\varepsilon\dot{\phi}^2 V}{\sqrt{1-\varepsilon\dot{\phi}^2}} &= 0, \quad (2.53)
\end{aligned}$$

where $V = V(\phi)$. From above equation, multiply with $\frac{\sqrt{1-\varepsilon\dot{\phi}^2}}{\dot{\phi}V}$. Then we obtain the equation of motion (EoM) or Klien-Gordon equation of the tachyon field reads

$$\frac{\varepsilon\ddot{\phi}}{1-\varepsilon\dot{\phi}^2} + 3H\varepsilon\dot{\phi} + \frac{V'}{V} = 0. \tag{2.54}$$

Next substituting Eqs.(2.17), (2.51), and (2.53) into Eq.(2.49), we obtain

$$\begin{aligned}
\dot{H} &= \frac{4\pi G}{3Hc^2}\left[-\frac{3H\varepsilon\dot{\phi}^2 V}{\sqrt{1-\varepsilon\dot{\phi}^2}} - 3H\rho_{\rm m}c^2\right] + \frac{kc^2}{a^2}, \\
&= -\frac{4\pi G}{c^2}\left[\frac{\varepsilon\dot{\phi}^2 V}{\sqrt{1-\varepsilon\dot{\phi}^2}} + \rho_{\rm m}c^2\right] + \frac{kc^2}{a^2}. \quad (2.55)
\end{aligned}$$

Consider the Friedmann equation, Eq.(2.48),

$$\begin{aligned}
H^2 &= \frac{8\pi G}{3c^2}\left[\frac{V}{\sqrt{1-\varepsilon\dot{\phi}^2}} + \rho_{\rm m}c^2\right] - \frac{kc^2}{a^2}, \\
H^2 + \frac{kc^2}{a^2} &= \frac{8\pi G}{3c^2}\left[\frac{V}{\sqrt{1-\varepsilon\dot{\phi}^2}} + \rho_{\rm m}c^2\right], \\
\frac{3c^2}{8\pi G}\left[H^2 + \frac{kc^2}{a^2}\right] &= \frac{V}{\sqrt{1-\varepsilon\dot{\phi}^2}} + \rho_{\rm m}c^2,
\end{aligned}$$

therefore

$$\frac{V}{\sqrt{1-\varepsilon\dot{\phi}^2}} = \frac{3c^2}{8\pi G}\left[H^2 + \frac{kc^2}{a^2}\right] - \rho_{\rm m}c^2. \tag{2.56}$$

Substituting Eq.(2.56) into Eq.(2.55),

$$\dot{H} = -\frac{4\pi G}{c^2}\left[\frac{3\varepsilon\dot{\phi}^2 c^2}{8\pi G}\left(H^2 + \frac{kc^2}{a^2}\right) + \left(1 - \varepsilon\dot{\phi}^2\right)\rho_{\rm m}c^2\right] + \frac{kc^2}{a^2}, \tag{2.57}$$

which can be rearrange,

$$\begin{aligned}
\dot{H} - \frac{kc^2}{a^2} &= -\frac{4\pi G}{c^2}\left[\frac{3\varepsilon\dot{\phi}^2 c^2}{8\pi G}\left(H^2 + \frac{kc^2}{a^2}\right) + \left(1 - \varepsilon\dot{\phi}^2\right)\rho_{\rm m}c^2\right], \\
-\frac{c^2}{4\pi G}\left[\dot{H} - \frac{kc^2}{a^2}\right] &= \varepsilon\dot{\phi}^2\left[\frac{3c^2}{8\pi G}\left(H^2 + \frac{kc^2}{a^2}\right) - \rho_{\rm m}c^2\right] + \rho_{\rm m}c^2, \\
\varepsilon\dot{\phi}^2\left[\frac{3c^2}{8\pi G}\left(H^2 + \frac{kc^2}{a^2}\right) - \rho_{\rm m}c^2\right] &= -\frac{c^2}{4\pi G}\left[\dot{H} - \frac{kc^2}{a^2}\right] - \rho_{\rm m}c^2,
\end{aligned}$$

or the kinetic term can be written as

$$\begin{aligned}
\varepsilon\dot{\phi}^2 &= \frac{-\frac{c^2}{4\pi G}\left[\dot{H} - \frac{kc^2}{a^2}\right] - \rho_{\rm m}c^2}{\frac{3c^2}{8\pi G}\left[H^2 + \frac{kc^2}{a^2}\right] - \rho_{\rm m}c^2}, \\
&= -\frac{2}{3}\frac{\dot{H} - \frac{kc^2}{a^2} + 4\pi G\rho_{\rm m}}{H^2 + \frac{kc^2}{a^2} - \frac{8\pi G}{3}\rho_{\rm m}}, \\
\varepsilon\dot{\phi}^2 &= -\left[\frac{2\dot{H} - (2kc^2/a^2) + 8\pi G\rho_{\rm m}}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_{\rm m}}\right].
\end{aligned} \tag{2.58}$$

Therefore

$$\begin{aligned}
1 - \varepsilon\dot{\phi}^2 &= 1 + \frac{2\dot{H} - (2kc^2/a^2) + 8\pi G\rho_{\rm m}}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_{\rm m}}, \\
&= \frac{3H^2 + (3kc^2/a^2) - 8\pi G\rho_{\rm m} + 2\dot{H} - (2kc^2/a^2) + 8\pi G\rho_{\rm m}}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_{\rm m}}, \\
1 - \varepsilon\dot{\phi}^2 &= \frac{3H^2 + 2\dot{H} + (kc^2/a^2)}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_{\rm m}},
\end{aligned} \tag{2.59}$$

and hence

$$\sqrt{1 - \varepsilon\dot{\phi}^2} = \sqrt{\frac{3H^2 + 2\dot{H} + (kc^2/a^2)}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_{\rm m}}}. \tag{2.60}$$

We use the above expression in Eq.(2.56), as a result we get the potential only if H and a and $\dot{\phi}$ and $\rho_{\rm m}$ are given

$$\begin{aligned}
V &= \left[\frac{3c^2}{8\pi G}\left(H^2 + \frac{kc^2}{a^2}\right) - \rho_{\rm m}c^2\right]\sqrt{1 - \varepsilon\dot{\phi}^2}, \\
&= \left[\frac{3c^2}{8\pi G}\left(H^2 + \frac{kc^2}{a^2}\right) - \rho_{\rm m}c^2\right]\sqrt{\frac{3H^2 + 2\dot{H} + (kc^2/a^2)}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_{\rm m}}}.
\end{aligned} \tag{2.61}$$

Then the tachyonic equation of state (EoS) parameter, w_ϕ, is

$$
\begin{aligned}
w_\phi &= \frac{p_\phi}{\rho_\phi c^2}, \\
&= \frac{-V\sqrt{1-\varepsilon\dot{\phi}^2}}{V/\sqrt{1-\varepsilon\dot{\phi}^2}}, \\
&= -\left(1-\varepsilon\dot{\phi}^2\right), \\
w_\phi(H,\dot{H},\rho_{\rm m}) &= -\left[\frac{3H^2+2\dot{H}+(kc^2/a^2)}{3H^2+(3kc^2/a^2)-8\pi G\rho_{\rm m}}\right].
\end{aligned} \tag{2.62}
$$

We see that for both quintessence and tachyonic cases, w_ϕ does not depend on the scalar field model but depends on the form of expansion function. This is also true for $w_{\rm eff,0}$. The equation of state is also independent of the sign of ε which indicates negative kinetic energy. This can be weighted with the dust-matter content to give effective equation of state, $w_{\rm eff}$, with all information above,

$$
\begin{aligned}
w_{\rm eff} &= \frac{\rho_\phi c^2 w_\phi + \rho_{\rm m} c^2 w_{\rm m}}{\rho_\phi c^2 + \rho_{\rm m} c^2}, \\
&= \frac{\rho_\phi c^2 w_\phi}{\rho_\phi c^2 + \rho_{\rm m} c^2}, \\
&= \frac{w_\phi}{1+\rho_{\rm m}c^2/(\rho_\phi c^2)}, \\
&= \frac{w_\phi}{1+\rho_{\rm m}c^2(\sqrt{1-\varepsilon\dot{\phi}^2}/V)}.
\end{aligned} \tag{2.63}
$$

Let consider a second term of a denominator by using Eqs.(2.60) and (2.61),

$$
\begin{aligned}
\frac{\sqrt{1-\varepsilon\dot{\phi}^2}}{V} &= \frac{\sqrt{\frac{3H^2+2\dot{H}+(kc^2/a^2)}{3H^2+(3kc^2/a^2)-8\pi G\rho_{\rm m}}}}{\left[\frac{3c^2}{8\pi G}\left(H^2+\frac{kc^2}{a^2}\right)-\rho_{\rm m}c^2\right]\sqrt{\frac{3H^2+2\dot{H}+(kc^2/a^2)}{3H^2+(3kc^2/a^2)-8\pi G\rho_{\rm m}}}}, \\
&= \left[\frac{3c^2}{8\pi G}\left(H^2+\frac{kc^2}{a^2}\right)-\rho_{\rm m}c^2\right]^{-1}.
\end{aligned} \tag{2.64}
$$

Then

$$
\rho_{\rm m}c^2\frac{\sqrt{1-\varepsilon\dot{\phi}^2}}{V} = \rho_{\rm m}c^2\left[\frac{3c^2}{8\pi G}\left(H^2+\frac{kc^2}{a^2}\right)-\rho_{\rm m}c^2\right]^{-1}, \tag{2.65}
$$

and hence

$$
\begin{aligned}
1+\rho_{\rm m}c^2\frac{\sqrt{1-\varepsilon\dot{\phi}^2}}{V} &= 1+\frac{\rho_{\rm m}c^2}{\frac{3c^2}{8\pi G}\left(H^2+\frac{kc^2}{a^2}\right)-\rho_{\rm m}c^2}, \\
&= \frac{\frac{3c^2}{8\pi G}\left(H^2+\frac{kc^2}{a^2}\right)-\rho_{\rm m}c^2+\rho_{\rm m}c^2}{\frac{3c^2}{8\pi G}\left(H^2+\frac{kc^2}{a^2}\right)-\rho_{\rm m}c^2},
\end{aligned}
$$

$$1+\rho_{\rm m}c^2\frac{\sqrt{1-\varepsilon\dot{\phi}^2}}{V} = \frac{\frac{3c^2}{8\pi G}\left(H^2+\frac{kc^2}{a^2}\right)}{\frac{3c^2}{8\pi G}\left(H^2+\frac{kc^2}{a^2}\right)-\rho_{\rm m}c^2},$$
$$= \frac{H^2+kc^2/a^2}{H^2+(kc^2/a^2)-(8\pi G/3)\rho_{\rm m}}. \quad (2.66)$$

By using Eqs.(2.62) and (2.66), $w_{\rm eff}$ can be written as

$$w_{\rm eff} = -\left(\frac{3H^2+2\dot{H}+(kc^2/a^2)}{3H^2+(3kc^2/a^2)-8\pi G\rho_{\rm m}}\right)\Bigg/\left[\frac{H^2+kc^2/a^2}{H^2+(kc^2/a^2)-(8\pi G/3)\rho_{\rm m}}\right],$$
$$= -\left(\frac{3H^2+2\dot{H}+(kc^2/a^2)}{3H^2+(3kc^2/a^2)-8\pi G\rho_{\rm m}}\right)\left[\frac{H^2+(kc^2/a^2)-(8\pi G/3)\rho_{\rm m}}{H^2+kc^2/a^2}\right],$$
$$= -\left(\frac{3H^2+2\dot{H}+(kc^2/a^2)}{3H^2+(3kc^2/a^2)-8\pi G\rho_{\rm m}}\right)\left[1-\frac{(8\pi G/3)\rho_{\rm m}}{H^2+kc^2/a^2}\right],$$
$$w_{\rm eff} = w_\phi\left[1-\frac{(8\pi G/3)\rho_{\rm m}}{H^2+kc^2/a^2}\right]. \quad (2.67)$$

This equation we can be written in the form of a dimensionless parameter, density parameter Ω, where $\rho_{\rm c}=\frac{3H^2}{8\pi G}$ then

$$w_{\rm eff} = w_\phi\left[1-\frac{\Omega_{\rm m}H^2}{H^2(\Omega_\phi+\Omega_{\rm m})}\right],$$
$$= w_\phi\left[1-\frac{\Omega_{\rm m}}{\Omega_\phi+\Omega_{\rm m}}\right],$$
$$= w_\phi\left[\frac{\Omega_\phi}{\Omega_\phi+\Omega_{\rm m}}\right],$$
$$= w_\phi\Omega_\phi, \quad (2.68)$$

with a constraint equation $\Omega_\phi+\Omega_{\rm m}=1$. We also simplify, from Eq.(2.67), as

$$w_{\rm eff} = -\left[\frac{3H^2+2\dot{H}+(kc^2/a^2)}{3H^2+(3kc^2/a^2)-8\pi G\rho_{\rm m}}\right]\left[1-\frac{(8\pi G/3)\rho_{\rm m}}{H^2+kc^2/a^2}\right],$$
$$= -\left[\frac{3H^2+2\dot{H}+(kc^2/a^2)}{3H^2+(3kc^2/a^2)-8\pi G\rho_{\rm m}}\right]\left[\frac{H^2+kc^2/a^2-(8\pi G/3)\rho_{\rm m}}{H^2+kc^2/a^2}\right],$$
$$= -\left[\frac{3H^2+2\dot{H}+(kc^2/a^2)}{3(H^2+kc^2/a^2-(8\pi G/3)\rho_{\rm m})}\right]\left[\frac{H^2+kc^2/a^2-(8\pi G/3)\rho_{\rm m}}{H^2+kc^2/a^2}\right],$$
$$= -\left[\frac{3H^2+2\dot{H}+(kc^2/a^2)}{3H^2+3kc^2/a^2}\right]. \quad (2.69)$$

In this equation we can rewrite in a form of a dimensionless deceleration parameter q by using a definition

$$\frac{\dot{H}}{H^2}\equiv-(1+q). \quad (2.70)$$

Therefore Eq.(2.69) becomes

$$
\begin{aligned}
w_{\text{eff}} &= -\left[\frac{1+\frac{2\dot{H}}{3H^2}+\frac{kc^2}{3H^2a^2}}{1+\frac{kc^2}{H^2a^2}}\right], \\
&= -\left[\frac{1-\frac{2}{3}(1+q)+\frac{kc^2}{3H^2a^2}}{1+\frac{kc^2}{H^2a^2}}\right], \\
&= -\left[\frac{\frac{1}{3}-q+\frac{kc^2}{3H^2a^2}}{1+\frac{kc^2}{H^2a^2}}\right], \\
w_{\text{eff}} &= \frac{q-\frac{1}{3}-\frac{kc^2}{3H^2a^2}}{1+\frac{kc^2}{H^2a^2}}. \qquad (2.71)
\end{aligned}
$$

When we set the curvature $k = 0$ in the case of flat FLRW universe. Our parameters can be reduced to more simpler forms. The kinetic term, Eq.(2.58), reduced to

$$
\varepsilon\dot{\phi}^2 = -\left[\frac{2\dot{H}+8\pi G\rho_{\text{m}}}{3H^2-8\pi G\rho_{\text{m}}}\right], \qquad (2.72)
$$

the tachyonic potential,

$$
V = \left[\frac{3c^2}{8\pi G}H^2-\rho_{\text{m}}c^2\right]\sqrt{\frac{3H^2+2\dot{H}}{3H^2-8\pi G\rho_{\text{m}}}}, \qquad (2.73)
$$

the EoS parameter,

$$
w_\phi = -\left[\frac{3H^2+2\dot{H}}{3H^2-8\pi G\rho_{\text{m}}}\right], \qquad (2.74)
$$

and the effective EoS parameter,

$$
\begin{aligned}
w_{\text{eff}} &= w_\phi\left[1-\frac{8\pi G\rho_{\text{m}}}{3H^2}\right], \\
&= w_\phi\left[1-\Omega_{\text{m}}\right], \\
&= w_\phi\Omega_\phi, \qquad (2.75)
\end{aligned}
$$

which is the same as in Eq.(2.68) and it imply that the effective of EoS is independent of the curvature value. We also simplify in the form of deceleration parameter, from Eq.(2.71), as

$$
w_{\text{eff}} = q-\frac{1}{3}. \qquad (2.76)
$$

We found that Eqs. (2.62) and (2.67) are the same for both quintessence scalar field [77] and tachyonic field cases, albeit the $\dot{\phi}$ and $V(\phi)$ are expressed differently in both cases.

2.2 Power-Law Cosmology

In this section we introduce the power-law cosmology and its phantom scenario. Two forms of power-law are different by the definition of the scale factor and we use the power parameters α and β, to avoiding the confusion, to separated between the canonical and phantom power-law, $a \propto t^{\alpha}$ and $a \propto (t_{\rm s} - t)^{\beta}$ respectively. We first introduce the canonical power-law in a first subsection and then the phantom power-law in the second subsection.

2.2.1 Canonical Power-Law Cosmology

The power-law used in the models are under the assumptions that our flat FLRW universe is filled with dust matter and scalar fields, and dominated by dark energy. The power-law is defined as

$$a = a_0 \left(\frac{t}{t_0}\right)^{\alpha}, \tag{2.77}$$

where a_0 is a scale factor at a present time t_0 and α is a constant which described the acceleration phase of the universe when $\alpha > 1$. In the flat FLRW universe dominated by the dark energy and the flat Friedmann equation gives $1 < \alpha < \infty$. Here we consider the constant value of α in the range $0 < \alpha < \infty$ and we will consider the power-law cosmology scenario in a short range of redshift $z \lesssim 0.45$ to present, $z = 0$.

In the power-law cosmology the cosmic speed is

$$\begin{aligned} \dot{a} &= a_0 \alpha (t^{\alpha-1}/t_0^{\alpha}), \\ &= a_0 \alpha \left(\frac{t}{t_0}\right)^{\alpha} \frac{1}{t}, \\ &= \frac{\alpha a}{t}, \end{aligned} \tag{2.78}$$

and the cosmic acceleration

$$\begin{aligned} \ddot{a} &= a_0 \alpha (\alpha - 1)(t^{\alpha-2}/t_0^{\alpha}), \\ &= a_0 \alpha (\alpha - 1) \left(\frac{t}{t_0}\right)^{\alpha} \frac{1}{t^2}, \\ &= \frac{\alpha(\alpha - 1) a}{t^2}. \end{aligned} \tag{2.79}$$

Then the Hubble parameter and its time derivative in the power-law cosmology are

$$\begin{aligned} H &= \frac{\dot{a}}{a}, \\ &= \frac{(\alpha a)/t}{a}, \\ &= \frac{\alpha}{t}. \end{aligned} \tag{2.80}$$

with

$$\dot{H} = -\alpha t^{-2} = -\frac{\alpha}{t^2}. \tag{2.81}$$

The deceleration in this scenario is

$$q \equiv -\frac{a\ddot{a}}{\dot{a}^2},$$

$$
\begin{aligned}
q &= -\left[a\frac{\alpha(\alpha-1)a}{t^2}\right] \Bigg/ \left[\frac{\alpha^2 a^2}{t^2}\right], \\
&= -\left(1-1/\alpha\right), \\
&= \frac{1}{\alpha} - 1,
\end{aligned} \tag{2.82}
$$

that is

$$
\alpha = \frac{1}{q+1}. \tag{2.83}
$$

As $\alpha \geq 0$ is required in power-law cosmology, hence $q \geq -1$ and $H_0 \geq 0$. In general, the testings of power-law cosmology indicating that the value of α are performed by observing $H(z)$ data of SNIa or high-redshift objects such as distant globular clusters [107, 108, 109]. So, to convert the scale factor into redshift z we use the relation

$$
\begin{aligned}
1+z &= \frac{a_0}{a}, \\
&= \frac{a_0}{a_0(t/t_0)^\alpha}, \\
&= \left(\frac{t_0}{t}\right)^\alpha.
\end{aligned} \tag{2.84}
$$

From above equation $t = t_0/(1+z)^{1/\alpha}$ and the Hubble parameter can be written as

$$
\begin{aligned}
H(z) &= \frac{\alpha}{t_0/(1+z)^{1/\alpha}}, \\
&= \frac{\alpha}{t_0}(1+z)^{1/\alpha}.
\end{aligned} \tag{2.85}
$$

In our study α is calculated at the present H_0, t_0 as $\alpha = H_0 t_0$. The dust matter density in the power-law can be written as

$$
\rho_{\rm m} = \rho_{\rm m,0}\left(\frac{t_0}{t}\right)^{3\alpha}, \tag{2.86}
$$

where $\rho_{\rm m,0}$ is the dust matter density at present time t_0.

2.2.2 Phantom Power-Law Cosmology

In the case of phantom power-law, the scale factor is defined different from previous as

$$
a = a_0\left(\frac{t_{\rm s}-t}{t_{\rm s}-t_0}\right)^\beta, \tag{2.87}
$$

where $t_{\rm s}$ is the future singularity time called *the big-rip time* which is defined as [110]

$$
t_{\rm s} \equiv t_0 + \frac{|\beta|}{H(t_0)}, \tag{2.88}
$$

and β is a constant (we use β here to avoid a confusing with α.) Then a cosmic speed,

$$\dot{a} = -a_0\beta\frac{(t_\mathrm{s}-t)^{\beta-1}}{(t_\mathrm{s}-t_0)^{\beta}} = -\beta\frac{a}{(t_\mathrm{s}-t)}, \tag{2.89}$$

and the cosmic acceleration,

$$\ddot{a} = a_0\beta(\beta-1)\frac{(t_\mathrm{s}-t)^{\beta-2}}{(t_\mathrm{s}-t_0)^{\beta}} = \frac{\beta(\beta-1)a}{(t_\mathrm{s}-t)^2}. \tag{2.90}$$

For both $\dot{a}$ and $\ddot{a}$ to be greater than zero, i.e. both expanding and accelerating universe in the phantom power-law cosmology is required the condition $\beta < 0$. The Hubble parameter in this case is

$$H = \frac{\dot{a}}{a} = -\frac{\beta}{t_\mathrm{s}-t}, \tag{2.91}$$

and the time derivative of Hubble parameter,

$$\dot{H} = -(-1)\beta(t_\mathrm{s}-t)^{-2}(-1) = -\frac{\beta}{(t_\mathrm{s}-t)^2}. \tag{2.92}$$

At present, $\beta = H_0(t_0 - t_\mathrm{s})$. The deceleration parameter is

$$\begin{aligned} q &= -\frac{a\ddot{a}}{\dot{a}^2}, \\ &= -a\left[\frac{\beta(\beta-1)a}{(t_\mathrm{s}-t)^2}\right] \Bigg/ \left[\beta^2\frac{a^2}{(t_\mathrm{s}-t)^2}\right], \\ &= -\left[1-\frac{1}{\beta}\right], \\ &= \frac{1}{\beta} - 1. \end{aligned} \tag{2.93}$$

The dust matter density in the phantom power-law is

$$\begin{aligned} \rho_\mathrm{m} &= \rho_\mathrm{m,0}\left(\frac{a_0}{a}\right)^3, \\ &= \rho_\mathrm{m,0}\left(\frac{a_0}{a_0\left[(t_\mathrm{s}-t)/(t_\mathrm{s}-t_0)\right]^{\beta}}\right)^3, \\ &= \rho_\mathrm{m,0}\left(\frac{t_\mathrm{s}-t_0}{t_\mathrm{s}-t}\right)^{3\beta}. \end{aligned} \tag{2.94}$$

To convert to redshift we can use

$$\begin{aligned} 1+z &= \frac{a_0}{a}, \\ &= \frac{a_0}{a_0\left((t_\mathrm{s}-t)/(t_\mathrm{s}-t_0)\right)^{\beta}}, \\ &= \left(\frac{t_\mathrm{s}-t_0}{t_\mathrm{s}-t}\right)^{\beta}, \end{aligned} \tag{2.95}$$

and from above equation we can be written

$$t_{\rm s} - t = \frac{t_{\rm s} - t_0}{(1+z)^{1/\beta}}. \tag{2.96}$$

Then the Hubble parameter

$$\begin{aligned} H(z) &= -\frac{\beta}{(t_{\rm s} - t_0)/(1+z)^{1/\beta}}, \\ &= -\frac{\beta(1+z)^{1/\beta}}{t_{\rm s} - t_0}. \end{aligned} \tag{2.97}$$

At present, $t = t_0$, the big-rip time $t_{\rm s}$ can be calculated from

$$t_{\rm s} \approx t_0 - \frac{2}{3(1+w_{\rm DE})} \frac{1}{H_0\sqrt{1-\Omega_{\rm m,0}}} \tag{2.98}$$

Here, $w_{\rm DE}$ must be less than -1 and to derive the above expression the flat geometry and constant dark energy equation of state is assumed [101, 102].

2.3 Tachyonic Power-Law

In this section, our universe is filled with dust and tachyon scalar field in the flat FLRW universe, $k = 0$. The tachyonic scalar field is acted as the dark energy dominated at late time. We combine the tachyonic scalar field with the power-law cosmology, $\varepsilon = +1$. By using details from Subsection (2.2.1) in Section (2.2), the kinetic term, Eq.(2.58), (here we keep k in the equation for completeness and will set to be zero later) can be written as

$$\begin{aligned} \dot{\phi}^2 &= -\frac{2\dot{H} - (2kc^2/a^2) + 8\pi G\rho_{\rm m}}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_{\rm m}}, \\ &= -\frac{2(-\alpha/t^2) - (2kc^2/a_0^2)t_0^{2\alpha}t^{-2\alpha} + 8\pi G\rho_{\rm m,0}t_0^{3\alpha}t^{-3\alpha}}{3(\alpha/t)^2 + (3kc^2/a_0^2)t_0^{2\alpha}t^{-2\alpha} - 8\pi G\rho_{\rm m,0}t_0^{3\alpha}t^{-3\alpha}}, \\ &= \frac{2\alpha + (2kc^2/a_0^2)t_0^{2\alpha}t^{2-2\alpha} - 8\pi G\rho_{\rm m,0}t_0^{3\alpha}t^{2-3\alpha}}{3\alpha^2 + (3kc^2/a_0^2)t_0^{2\alpha}t^{2-2\alpha} - 8\pi G\rho_{\rm m,0}t_0^{3\alpha}t^{2-3\alpha}}. \end{aligned} \tag{2.99}$$

From above equation,

$$\dot{\phi} = \sqrt{\frac{2\alpha + (2kc^2/a_0^2)t_0^{2\alpha}t^{2-2\alpha} - 8\pi G\rho_{\rm m,0}t_0^{3\alpha}t^{2-3\alpha}}{3\alpha^2 + (3kc^2/a_0^2)t_0^{2\alpha}t^{2-2\alpha} - 8\pi G\rho_{\rm m,0}t_0^{3\alpha}t^{2-3\alpha}}}. \tag{2.100}$$

We can integrate with respect to time to finding the scalar field as a function of time, $\phi(t)$.

$$\phi(t) = \int dt \sqrt{\frac{2\alpha + (2kc^2/a_0^2)t_0^{2\alpha}t^{2-2\alpha} - 8\pi G\rho_{\rm m,0}t_0^{3\alpha}t^{2-3\alpha}}{3\alpha^2 + (3kc^2/a_0^2)t_0^{2\alpha}t^{2-2\alpha} - 8\pi G\rho_{\rm m,0}t_0^{3\alpha}t^{2-3\alpha}}}, \tag{2.101}$$

but it is not easy to solve from above equation. Therefore we left it here and will find its solution later. Hence the potential of tachyonic power-law, Eq.(2.61), can be written as

$$\begin{aligned} V &= \left[\frac{3c^2}{8\pi G}\left(\frac{\alpha^2}{t^2}+\frac{kc^2t_0^{2\alpha}}{a_0^2t^{2\alpha}}\right)-\frac{\rho_{\mathrm{m},0}c^2t_0^{3\alpha}}{t^{3\alpha}}\right] \\ &\quad\times\sqrt{\frac{3\alpha^2-2\alpha+(kc^2/a_0^2)t_0^{2\alpha}t^{2-2\alpha}}{3\alpha^2+(3kc^2/a_0^2)t_0^{2\alpha}t^{2-2\alpha}-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{2-3\alpha}}} \end{aligned} \qquad (2.102)$$

Therefore the EoS, Eq.(2.62), and the effective EoS, Eq.(2.69), parameters are

$$\begin{aligned} w_\phi &= -\left[\frac{3(\alpha/t)^2+2(-\alpha/t^2)+(kc^2/a_0^2)t_0^{2\alpha}t^{-2\alpha}}{3(\alpha/t)^2+(3kc^2/a_0^2)t_0^{2\alpha}t^{-2\alpha}-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{-3\alpha}}\right], \\ &= -\left[\frac{3\alpha^2/t^2-2\alpha/t^2+(kc^2/a_0^2)t_0^{2\alpha}t^{-2\alpha}}{3\alpha^2/t^2+(3kc^2/a_0^2)t_0^{2\alpha}t^{-2\alpha}-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{-3\alpha}}\right], \\ &= -\left[\frac{3\alpha^2-2\alpha+(kc^2/a_0^2)t_0^{2\alpha}t^{2-2\alpha}}{3\alpha^2+(3kc^2/a_0^2)t_0^{2\alpha}t^{2-2\alpha}-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{2-3\alpha}}\right], \end{aligned} \qquad (2.103)$$

and

$$\begin{aligned} w_{\mathrm{eff}} &= -\left[\frac{3(\alpha/t)^2+2(-\alpha/t^2)+(kc^2/a_0^2)t_0^{2\alpha}t^{-2\alpha}}{3(\alpha/t)^2+(3kc^2/a_0^2)t_0^{2\alpha}t^{-2\alpha}}\right], \\ &= -\left[\frac{3\alpha^2/t^2-2\alpha/t^2+(kc^2/a_0^2)t_0^{2\alpha}t^{-2\alpha}}{3\alpha^2/t^2+(3kc^2/a_0^2)t_0^{2\alpha}t^{-2\alpha}}\right], \\ &= -\left[\frac{3\alpha^2-2\alpha+(kc^2/a_0^2)t_0^{2\alpha}t^{2-2\alpha}}{3\alpha^2+(3kc^2/a_0^2)t_0^{2\alpha}t^{2-2\alpha}}\right]. \end{aligned} \qquad (2.104)$$

For the flat FLRW universe, now we apply the value of curvature $k = 0$ into above equations. Then the kinetic term, Eq.(2.99), reduced to

$$\dot{\phi}^2 = \frac{2\alpha-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{2-3\alpha}}{3\alpha^2-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{2-3\alpha}}, \qquad (2.105)$$

and tachyonic scalar field

$$\phi(t) = \int dt\sqrt{\frac{2\alpha-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{2-3\alpha}}{3\alpha^2-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{2-3\alpha}}}. \qquad (2.106)$$

Hence the potential of tachyonic power-law, Eq.(2.102), reduced to

$$\begin{aligned} V &= \left[\frac{3c^2}{8\pi G}\frac{\alpha^2}{t^2}-\frac{\rho_{\mathrm{m},0}c^2t_0^{3\alpha}}{t^{3\alpha}}\right]\sqrt{\frac{3\alpha^2-2\alpha}{3\alpha^2-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{2-3\alpha}}}, \\ &= \left[\frac{3c^2\alpha^2t^{3\alpha}-8\pi Gt^2\rho_{\mathrm{m},0}c^2t_0^{3\alpha}}{8\pi Gt^{2+3\alpha}}\right]\sqrt{\frac{3\alpha^2-2\alpha}{3\alpha^2-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{2-3\alpha}}}, \\ &= \left[\frac{c^2t^{3\alpha}(3\alpha^2-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{2-3\alpha})}{8\pi Gt^{2+3\alpha}}\right]\sqrt{\frac{3\alpha^2-2\alpha}{3\alpha^2-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{2-3\alpha}}}, \\ V &= \frac{c^2}{8\pi Gt^2}\sqrt{(3\alpha^2-2\alpha)(3\alpha^2-8\pi G\rho_{\mathrm{m},0}t_0^{3\alpha}t^{2-3\alpha})} \end{aligned} \qquad (2.107)$$

The EoS, Eq.(2.103) and the effective EoS, Eq.(2.67), reduced to

$$w_\phi = -\left[\frac{3\alpha^2 - 2\alpha}{3\alpha^2 - 8\pi G\rho_{\mathrm{m},0} t_0^{3\alpha} t^{2-3\alpha}}\right], \tag{2.108}$$

and

$$\begin{aligned} w_{\mathrm{eff}} &= -\left[\frac{3\alpha^2 - 2\alpha}{3\alpha^2 - 8\pi G\rho_{\mathrm{m},0} t_0^{3\alpha} t^{2-3\alpha}}\right]\left[1 - \frac{(8\pi G/3)\rho_{\mathrm{m},0} t_0^{3\alpha} t^{-3\alpha}}{(\alpha/t)^2}\right], \\ &= -\left[\frac{3\alpha^2 - 2\alpha}{3\alpha^2 - 8\pi G\rho_{\mathrm{m},0} t_0^{3\alpha} t^{2-3\alpha}}\right]\left[1 - \frac{8\pi G\rho_{\mathrm{m},0} t_0^{3\alpha} t^{2-3\alpha}}{3\alpha^2}\right], \end{aligned}$$

$$\begin{aligned} w_{\mathrm{eff}} &= -\left[\frac{3\alpha^2 - 2\alpha}{3\alpha^2 - 8\pi G\rho_{\mathrm{m},0} t_0^{3\alpha} t^{2-3\alpha}}\right]\left[\frac{3\alpha^2 - 8\pi G\rho_{\mathrm{m},0} t_0^{3\alpha} t^{2-3\alpha}}{3\alpha^2}\right], \\ &= -\frac{3\alpha^2 - 2\alpha}{3\alpha^2}, \\ &= -1 + \frac{2}{3\alpha}. \end{aligned} \tag{2.109}$$

This equation is independent of any field but depends only on the values of the exponent of the power-law. In another words, it is regardless of the type of field.

2.4 Tachyonic Phantom Power-Law

In this section, we do the same procedures but using the scale factor in a form of phantom power-law, $\varepsilon = -1$ in this case. Substituting the scale factor and Hubble parameter from Subsection (2.2.2) into the kinetic term, the tachyonic potential, the EoS parameter w_ϕ, and the effective EoS parameter w_{eff}. Therefore the kinetic term, from Eq.(2.58), becomes

$$\begin{aligned} -\dot\phi^2 &= -\left[\frac{2\dot H - (2kc^2/a^2) + 8\pi G\rho_{\mathrm{m}}}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_{\mathrm{m}}}\right], \\ \dot\phi^2 &= \frac{2\left(-\beta/(t_{\mathrm{s}}-t)^2\right) - \frac{2kc^2}{\left[a_0^2((t_{\mathrm{s}}-t)/(t_{\mathrm{s}}-t_0))^{2\beta}\right]} + 8\pi G\rho_{\mathrm{m},0}\left((t_{\mathrm{s}}-t_0)/(t_{\mathrm{s}}-t)\right)^{3\beta}}{3\left(-\beta/(t_{\mathrm{s}}-t)\right)^2 + \frac{3kc^2}{\left[a_0^2((t_{\mathrm{s}}-t)/(t_{\mathrm{s}}-t_0))^{2\beta}\right]} - 8\pi G\rho_{\mathrm{m},0}\left((t_{\mathrm{s}}-t_0)/(t_{\mathrm{s}}-t)\right)^{3\beta}}, \end{aligned}$$

$$\dot\phi^2 = \frac{-2\beta - \frac{2kc^2}{a_0^2}(t_{\mathrm{s}}-t_0)^{2\beta}(t_{\mathrm{s}}-t)^{2-2\beta} + 8\pi G\rho_{\mathrm{m},0}(t_{\mathrm{s}}-t_0)^{3\beta}(t_{\mathrm{s}}-t)^{2-3\beta}}{3\beta^2 + \frac{3kc^2}{a_0^2}(t_{\mathrm{s}}-t_0)^{2\beta}(t_{\mathrm{s}}-t)^{2-2\beta} - 8\pi G\rho_{\mathrm{m},0}(t_{\mathrm{s}}-t_0)^{3\beta}(t_{\mathrm{s}}-t)^{2-3\beta}}. \tag{2.110}$$

When we set the curvature, $k = 0$, in the flat FLRW universe, above equation can be reduced to

$$\dot\phi^2 = \frac{-2\beta + 8\pi G\rho_{\mathrm{m},0}(t_{\mathrm{s}}-t_0)^{3\beta}(t_{\mathrm{s}}-t)^{2-3\beta}}{3\beta^2 - 8\pi G\rho_{\mathrm{m},0}(t_{\mathrm{s}}-t_0)^{3\beta}(t_{\mathrm{s}}-t)^{2-3\beta}} \tag{2.111}$$

The tachyonic potential, Eq.(2.61), becomes

$$V = \left[\frac{3c^2}{8\pi G}\left(H^2 + \frac{kc^2}{a^2}\right) - \rho_{\mathrm{m}} c^2\right]\sqrt{\frac{3H^2 + 2\dot H + (kc^2/a^2)}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_{\mathrm{m}}}},$$

$$
\begin{aligned}
V &= \left[\frac{3c^2}{8\pi G}\left(\left(\frac{-\beta}{t_\mathrm{s}-t}\right)^2+\frac{kc^2}{a_0^2(t_\mathrm{s}-t)^{2\beta}(t_\mathrm{s}-t_0)^{-2\beta}}\right)-\rho_{\mathrm{m},0}c^2\left(\frac{t_\mathrm{s}-t_0}{t_\mathrm{s}-t}\right)^{3\beta}\right] \\
&\quad\times\sqrt{\frac{3(-\beta/(t_\mathrm{s}-t))^2+2(-\beta/(t_\mathrm{s}-t)^2)+(kc^2/[a_0^2(t_\mathrm{s}-t)^{2\beta}(t_\mathrm{s}-t_0)^{-2\beta}])}{3(-\beta/(t_\mathrm{s}-t))^2+(3kc^2/[a_0^2(t_\mathrm{s}-t)^{2\beta}(t_\mathrm{s}-t_0)^{-2\beta}])-8\pi G\rho_{\mathrm{m},0}\left(\frac{t_\mathrm{s}-t_0}{t_\mathrm{s}-t}\right)^{3\beta}}}, \\
&= \left[\frac{3c^2}{8\pi G}\left(\frac{\beta^2}{(t_\mathrm{s}-t)^2}+\frac{kc^2}{a_0^2}\frac{(t_\mathrm{s}-t_0)^{2\beta}}{(t_\mathrm{s}-t)^{2\beta}}\right)-\rho_{\mathrm{m},0}c^2\frac{(t_\mathrm{s}-t_0)^{3\beta}}{(t_\mathrm{s}-t)^{3\beta}}\right] \\
&\quad\times\sqrt{\frac{3\beta^2-2\beta+\frac{kc^2}{a_0^2}(t_\mathrm{s}-t)^{2-2\beta}(t_\mathrm{s}-t_0)^{2\beta}}{3\beta^2+\frac{3kc^2}{a_0^2}(t_\mathrm{s}-t)^{2-2\beta}(t_\mathrm{s}-t_0)^{2\beta}-8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s}-t_0)^{3\beta}(t_\mathrm{s}-t)^{2-3\beta}}}. \qquad (2.112)
\end{aligned}
$$

It can be reduced, when $k=0$, to

$$
\begin{aligned}
V &= \left[\frac{3c^2}{8\pi G}\frac{\beta^2}{(t_\mathrm{s}-t)^2}-\rho_{\mathrm{m},0}c^2\frac{(t_\mathrm{s}-t_0)^{3\beta}}{(t_\mathrm{s}-t)^{3\beta}}\right] \\
&\quad\times\sqrt{\frac{3\beta^2-2\beta}{3\beta^2-8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s}-t_0)^{3\beta}(t_\mathrm{s}-t)^{2-3\beta}}}, \\
&= \frac{c^2}{(t_\mathrm{s}-t)^2}\left[\frac{3\beta^2}{8\pi G}-\rho_{\mathrm{m},0}(t_\mathrm{s}-t_0)^{3\beta}(t_\mathrm{s}-t)^{2-3\beta}\right] \\
&\quad\times\sqrt{\frac{3\beta^2-2\beta}{3\beta^2-8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s}-t_0)^{3\beta}(t_\mathrm{s}-t)^{2-3\beta}}}, \\
&= \frac{c^2}{(t_\mathrm{s}-t)^2}\left[\frac{3\beta^2-8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s}-t_0)^{3\beta}(t_\mathrm{s}-t)^{2-3\beta}}{8\pi G}\right] \\
&\quad\times\sqrt{\frac{3\beta^2-2\beta}{3\beta^2-8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s}-t_0)^{3\beta}(t_\mathrm{s}-t)^{2-3\beta}}}, \\
V &= \frac{c^2\sqrt{\beta(3\beta-2)(3\beta^2-8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s}-t_0)^{3\beta}(t_\mathrm{s}-t)^{2-3\beta})}}{8\pi G(t_\mathrm{s}-t)^2}. \qquad (2.113)
\end{aligned}
$$

The equation of state parameter, Eq.(2.62), is

$$
\begin{aligned}
w_\phi &= -\frac{3H^2+2\dot{H}+(kc^2/a^2)}{3H^2+(3kc^2/a^2)-8\pi G\rho_\mathrm{m}}, \\
&= -\frac{3(-\beta/(t_\mathrm{s}-t))^2+2(-\beta/(t_\mathrm{s}-t)^2)+kc^2/[a_0^2((t_\mathrm{s}-t)/(t_\mathrm{s}-t_0))^{2\beta}]}{3(-\beta/(t_\mathrm{s}-t))^2+3kc^2/[a_0^2((t_\mathrm{s}-t)/(t_\mathrm{s}-t_0))^{2\beta}]-8\pi G\rho_{\mathrm{m},0}((t_\mathrm{s}-t_0)/(t_\mathrm{s}-t))^{3\beta}}, \\
&= -\frac{3\beta^2-2\beta+\frac{kc^2}{a_0^2}(t_\mathrm{s}-t_0)^{2\beta}(t_\mathrm{s}-t)^{2-2\beta}}{3\beta^2+\frac{3kc^2}{a_0^2}(t_\mathrm{s}-t_0)^{2\beta}(t_\mathrm{s}-t)^{2-2\beta}-8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s}-t_0)^{3\beta}(t_\mathrm{s}-t)^{2-3\beta}}. \qquad (2.114)
\end{aligned}
$$

It can be reduced, for $k=0$, to

$$
w_\phi = -\frac{3\beta^2-2\beta}{3\beta^2-8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s}-t_0)^{3\beta}(t_\mathrm{s}-t)^{2-3\beta}}. \qquad (2.115)
$$

Finally, the effective EoS parameter, Eq.(2.69), becomes

$$\begin{aligned} w_{\rm eff} &= -\left[\frac{3H^2+2\dot{H}+(kc^2/a^2)}{3H^2+3kc^2/a^2}\right], \\ &= -\left[\frac{3(-\beta/(t_{\rm s}-t))^2+2(-\beta/(t_{\rm s}-t)^2)+kc^2/[a_0^2((t_{\rm s}-t)/(t_{\rm s}-t_0))^{2\beta}]}{3(-\beta/(txt))^2+3kc^2/[a_0^2((t_{\rm s}-t)/(t_{\rm s}-t_0))^{2\beta}]}\right], \\ &= -\left[\frac{3\beta^2-2\beta+\frac{kc^2}{a_0^2}(t_{\rm s}-t_0)^{2\beta}(t_{\rm s}-t)^{2-2\beta}}{3\beta^2+3\frac{kc^2}{a_0^2}(t_{\rm s}-t_0)^{2\beta}(t_{\rm s}-t)^{2-2\beta}}\right], \qquad (2.116) \end{aligned}$$

and it can be reduced, for $k=0$, to

$$\begin{aligned} w_{\rm eff} &= -\left[\frac{3\beta^2-2\beta}{3\beta^2}\right], \\ &= -\left[1-\frac{2}{3\beta}\right], \\ &= -1+\frac{2}{3\beta}, \qquad (2.117) \end{aligned}$$

this equation is the same as Eq.(2.109) but the exponent is β instead of α.

2.5 Cosmological Background Equations at Present

In the case of derive the cosmological parameters, we use the WMAP7 and combined WMAP7 datasets. The background equations as in Section (2.3) and Section (2.4) we will set the cosmic time t to present time t_0 within the flat FLRW universe, $k=0$. Therefore those equations are reducing to more simpler form as follow:

2.5.1 Tachyonic Power-Law Cosmology

The kinetic term of tachyon, from Eq.(2.105), is

$$\dot{\phi}^2 = \frac{2\alpha-8\pi G\rho_{\rm m,0}t_0^{3\alpha}t_0^{2-3\alpha}}{3\alpha^2-8\pi G\rho_{\rm m,0}t_0^{3\alpha}t_0^{2-3\alpha}},$$

$$\dot{\phi}^2 = \frac{2\alpha-8\pi G\rho_{\rm m,0}t_0^2}{3\alpha^2-8\pi G\rho_{\rm m,0}t_0^2}. \qquad (2.118)$$

Therefore we can integrate the above equation to obtain the solution of the scalar field. But that solution is not a general solution, it just a case that we replace the cosmic time $t=t_0$. To obtain the general solution, we have to find the solution of Eq.(2.106). The potential of tachyon field is

$$\begin{aligned} V &= \frac{c^2}{8\pi G t_0^2}\sqrt{(3\alpha^2-2\alpha)(3\alpha^2-8\pi G\rho_{\rm m,0}t_0^{3\alpha}t_0^{2-3\alpha})}, \\ &= \frac{c^2}{8\pi G t_0^2}\sqrt{(3\alpha^2-2\alpha)(3\alpha^2-8\pi G\rho_{\rm m,0}t_0^2)}. \qquad (2.119) \end{aligned}$$

The equation of state parameter of tachyon field becomes

$$w_\phi = -\left[\frac{3\alpha^2 - 2\alpha}{3\alpha^2 - 8\pi G\rho_{\mathrm{m},0} t_0^{3\alpha} t_0^{2-3\alpha}}\right],$$
$$= -\left[\frac{3\alpha^2 - 2\alpha}{3\alpha^2 - 8\pi G\rho_{\mathrm{m},0} t_0^2}\right], \quad (2.120)$$

while the effective equation of state parameter still the same as in Eq.(2.109) because it is independent of the cosmic time but depends only on α. Eq.(2.120) gives us the values of the EoS parameter when we apply the observational data.

2.5.2 Tachyonic Phantom Power-Law Cosmology

In the phantom power-law, the kinetic term is

$$\dot{\phi}^2 = \frac{-2\beta + 8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s} - t_0)^{3\beta}(t_\mathrm{s} - t_0)^{2-3\beta}}{3\beta^2 - 8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s} - t_0)^{3\beta}(t_\mathrm{s} - t_0)^{2-3\beta}},$$
$$= \frac{-2\beta + 8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s} - t_0)^2}{3\beta^2 - 8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s} - t_0)^2}, \quad (2.121)$$

and we can find the specific solution of the scalar field. In order to find the general solution we have to integrate Eq.(2.111). The potential of tachyon is

$$V = \frac{c^2}{8\pi G(t_\mathrm{s} - t_0)^2}\sqrt{\beta(3\beta - 2)(3\beta^2 - 8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s} - t_0)^{3\beta}(t_\mathrm{s} - t_0)^{2-3\beta})},$$
$$= \frac{c^2}{8\pi G(t_\mathrm{s} - t_0)^2}\sqrt{\beta(3\beta - 2)(3\beta^2 - 8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s} - t_0)^2)}. \quad (2.122)$$

Finally, the equation of state parameter becomes

$$w_\phi = -\frac{3\beta^2 - 2\beta}{3\beta^2 - 8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s} - t_0)^{3\beta}(t_\mathrm{s} - t_0)^{2-3\beta}},$$
$$= -\frac{3\beta^2 - 2\beta}{3\beta^2 - 8\pi G\rho_{\mathrm{m},0}(t_\mathrm{s} - t_0)^2}, \quad (2.123)$$

and the effective equation of state still the same as in Eq.(2.117) because it is independent of cosmic time but depends only on β.

CHAPTER III

NON-MINIMAL DERIVATIVE COUPLING

The non-minimal derivative coupling (NMDC) model where curvature coupling to the derivative of scalar field was proposed by Amendola in 1993 [36]. It was developed from the non-minimal coupling (NMC) between scalar field to Ricci scalar in GR in form of $\sqrt{-g}f(\phi)R$ where $f(\phi)$ is a function of scalar field ϕ. In the NMDC, the coupling function is in form of the derivative of scalar field, $f = f(\phi, \phi_{,\mu}, \phi_{,\mu\nu}, ...)$. The simplest form of the NMDC is the coupling between Ricci scalar and the derivative of scalar field i.e. $R\phi_{,\mu}\phi^{,\mu}$.

In this chapter, we will start with a brief review of the various form of the NMDC gravity models in the first section. In Section (3.2), we will give the background equation of the NMDC model where the coupling constant is κ. Those background equations we will use to constrain the present cosmological constant, Λ. We will combine the NMDC model with the power-law cosmology in both of canonical and phantom scenarios in Section (3.3) and Section (3.4) respectively. To estimate the present value of the cosmological constant, we require to proposing the constant potential $V \equiv \Lambda/(8\pi G)$ and using the observed data from the combined WMAP9 (WMAP9+eCMB+BAO+H_0), PLANCK+WP, and PLANCK including polarization and other external parameters (*TT,TE,EE*+lowP+Lensing+ext.) dataset. All the results are shown in Chapter 4.

3.1 Review of the Non-Minimal Derivative Coupling Theory

In this section, we give a brief review of the recently and more interesting NMDC gravity models within each subsection.

3.1.1 Capozziello, Lambiase and Schmidt's Result

Capozziello, Lambiase and Schmidt [38] found that the possible coupling Lagrangian terms are only $R\phi_{,\mu}\phi^{,\mu}$ and $R^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ terms in the Lagrangian without losing its generality. There is a free canonical kinetic term without either scalar field potential $V(\phi)$ or Λ and there is no self-interaction in the Lagrangian of those two new terms. In the case of there is the effective cosmological constant, the general solution without potential is giving de-Sitter expansion [37]. The conditions for which de-Sitter expansion is a late time attractor are given in [38]. In the case of considering $R\phi_{,\mu}\phi^{,\mu}$ term with free Ricci scalar, free kinetic term, free potential and free matter terms, the equation of state goes to -1 at late time for a zero potential and goes to $-1 + 2/3p$ in the case of power-law expansion with the acceleration expansion for $p > 1$ [111]. Another case is when we consider the $R^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ term with a free Ricci scalar, a free kinetic scalar term and a free potential, the field equation contains third-order derivatives of scalar field, $\nabla^{\gamma}\nabla^{\mu}\nabla^{\nu}\phi$, and the scalar field equation contains third-order derivative of metric $g_{\mu\nu}$. This model is severely constrained for weakly coupling and display an instabilities with strong negative coupling and absence of potential and unsuitable for present acceleration [39].

3.1.2 Granda's Two Coupling Constant Model

Granda, in 2010 [112], was proposed the another NMDC model. The model contains a free kinetic term, a free potential term and two coupling terms that re-scaled by ϕ^{-2} with two different coupling parameters ξ and η in form of $-(1/2)\xi R\phi^{-2}g_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$ and $-(1/2)\eta\phi^{-2}R_{\mu\nu}\phi^{\mu}\phi^{\nu}$. In this model when consider in the most simplest form with no potential and no kinetic term, NMDC coupling term acts as dark matter at early stage and it is giving the power-law solution with $a \sim t^{2/3}$ for $\eta = -2\xi$ and accelerated expansion solution in the interval $0 < \xi < 1/3$ for $\eta = -\xi - 1$. In the present of potential, the model presents phantom behavior where effective EoS, $w_{\text{eff}} \to -1$ and behave close to the cosmological constant. The quantum gravity it allows to separate the two coupling parameters at low energy [113]. There are other forms of two coupling parameters with no re-scaling factor ϕ^{-2} [40, 41, 114] which is including of the Gauss-Bonnet 4D-invariance [115] which gives the future de Sitter solution or the Chaplygin gas [116] which gives rise to the Chaplygin gas solution.

3.1.3 Sushkov's Models

In Suskov's models, there are various forms of NMDC which give us more interested behaviors.

1. **Constant or Zero Potential**

   Sushkov has been proposed, in 2009[117], the model of a scalar field ϕ with nonminimal derivative coupling to curvature. There are two separated coupling constant, κ_1 and κ_2 in the form of $\kappa_1 R\phi_{,\mu}\phi^{,\mu}$ and $\kappa_2 R^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$. He was studied in a special case with $\kappa \equiv \kappa_2 = -2\kappa_1$ and this results give the field equation is in a form of Einstein tensor as $\kappa G_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$. A good point of one coupling constant κ is that it can be reduced the order of derivative of $g_{\mu\nu}$ and ϕ in field equation from third-order to second-order derivative.Therefore the Lagrangian is consisting of the Ricci scalar, R term, free kinetic term $g_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$ and a coupling Einstein tensor term $\kappa G_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$ with no potential $V(\phi)$. To study of the model with flat FLRW universe, at very early stage of the universe, there is an initial singularity stage for $\kappa < 0$ and quasi-de-Sitter stage for $\kappa > 0$. For any values of κ, it is giving the power-law solution, $a \propto t^{1/3}$, at very late time [117]. Another case of this model is that the model has an additional term of constant potential [118]. In any values of coupling parameters, besides the transition between different de Sitter stages, we can obtain various behaviors and fates of the universe, for example, a Big Bang, a Big Crunch, a Big Rip etc. [118].

2. **With Potential But Without Free Kinetic Term**

   From the Sushkov's model, in case of there is no free kinetic term, no $(1/2)g^{\mu\nu}\ \phi_{,\mu}\phi_{,\nu}$ term, and there is only the Einstein tensor coupling kinetic term, $\kappa G_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$. Gao in 2010 [119] found that in case of no potential and in absence of other matter sources or in the presence of pressureless matter, the scalar field acts as the dust dominate or pressureless matter and its sound speed is vanished. In the presence of potential and the values of EoS parameter, $-1 \le w \le 0$, suggests that the scalar field may behaves like both of dark energy and cold dark matter. If the kinetic term is coupling to more than one Einstein tensors [119], it was claimed not to be likely by [120] and the EoS parameter approches to -1 whether the potential is flat or not. There is the work in nonminimal derivative coupling curvaton which can be seen in [121].

3. **Having Purely Kinetic Coupling Term and a Matter Term**

The Sushkov's model cannot explain the phantom acceleration or no phantom crossing when there is no potential and no matter Lagrangians, $L_{\rm m} = 0$. To solve this problem and to allows phantom crossing, Gubitosi and Linder, in 2011 [122], proposed the most general Lagragians with purely kinetic term in the form of $(a_1\phi_{,\mu}\phi^{,\mu} + a_2\nabla^{\mu}\nabla_{\mu}\phi)R$ term, $a_3\phi_{,\mu}\phi_{,\nu}R^{\mu\nu}$ term and $a_4R^{\alpha\beta\gamma\delta}\Phi_{\alpha\beta\gamma\delta}(\phi_{,\mu})$ term where $\Phi_{\alpha\beta\gamma\delta}$ is a function of $\phi_{,\mu}$ and a matter term and a_i are dimensionless coefficients.

The model's action is at the lowest possible order of the Planck mass or equivalent to the Newton constant and it verifies the action of Sushkov [117]. In the case of purely kinetic approach with any potential, the model would worse at high energy quantum corrections and obeying shift symmetry. The model has a wide range of EoS parameter values and it possible ranging from stiff behavior ($w = 1$) to phantom crossing. It is possible to go through a quasistable loitering phase that is a cosmological constant-like phase, with no potential, before entering matter dominated phase. For the Sushkov's purely kinetic model include the matter Lagrangian is found to be the same as the action in the Fab Four theory [123]. The positive values of the coupling constant of the theory only gives the result in phantom crossing or inflation with graceful exit. The negative values of the coupling is possible but do not allow for inflation and may ghost state be appeared [123]. The investigations of the model without potential in blackhole spacetime can be found in [124, 125, 126, 127].

4. **Adding Potential Term with Matter Term**

For a model with purely kinetic term, when we add the potential into the model without any matter term. It is found that the potential requires to be less steep than quadratic potential [128], less than $V(\phi) \sim \phi^2$, in the case of to have inflation. In addition the matter term into the model with potential, it can be able to describe the transition from inflation to matter dominate epoch which is characterized by the decelerated expansion without reheating. Later the cosmological constant come into play, then the model can describes the transition from one to another phase of the de-Sitter and universe is at the beginning of the accelerated expansion epoch [42]. For the model with positive potential and positive coupling parameter, it gives unbound $\dot{\phi}$ value by using the dynamical analysis with restricted Hubble parameter [128]. When considering the positive value of coupling parameter with constant potential, inflationary phase is always possible and it depends only on the value of coupling parameter. During inflation, if the more strength of the NMDC couplings of either the inflaton field or to the particles to Einstein tensor is increased, the more decreasing of gravitational heavy particles are produced [129]. The study of perturbations and inflationary analysis of the inflation model with a constant potential, acts as a cosmological constant, can be found in [130] to confront observational results.

3.1.4 Model With Negative-Sign NMDC

The model is related to natural inflation where the inflaton is pseudo-Nambu-Goldstone boson [120] which has a naturally flat potential and related to the slowly rolling conditions to create inflation as well as related to three-form inflation [131]. The model is also related to Higgs inflation with the quartic potential, $V(\phi) \sim \lambda\phi^4$. In this model, Einstein gravity via NMDC coupling to the Standard Higgs field with a tree-level modification [132]. The Lagrangian of the model is looks like the Lagrangian in the Sushkov's model but the free kinetic term has the opposite sign to the coupling term, i.e. $g^{\mu\nu} - w^2 G^{\mu\nu}$.[5] The model gives a UV-protected inflation where the inflaton potential is obtained by quantum breaking symmetry and enhances friction of the field dynamics gravitationally at high energies [133]. The inflationary scenario in the framework of the NMDC model with quadratic potential, $V(\phi) \sim \varphi^2$, where $\varphi \equiv M_{\rm P}\phi$ and modifications of standard reheating was investigated by Sadjadi and Goodarzi in 2013 [134].

Tsujikawa in 2012 [43] was reported that the kinetic coupling with the Einstein tensor can cause the gravitational friction inflation, even with steep potentials i.e. $V(\phi) \sim \lambda\phi^4$. The class of inflationary models can be made compatible with the CMB observations. The particle production of the model with NMDC coupling to gravity after inflation is reported in [135] and one slow roll parameter is play a major role for describing the inflationary phase [136]. The NMDC coupling to Einstein tensor models, in the high-field friction limit, brings the energy scale in the inflationary models reduce to sub-Planckian and the models are more consistent to observations [137]. The model without free kinetic term is also investigated with various forms of potential for inflation [138]. As dark energy, this model with no matter and potential terms is impossible to give phantom crossing but for the model with matter term and a power-law potential is possible [139]. The quintessence model with the power-law potential $V(\phi) \propto \phi^n$ can be giving rise to the oscillatory dark energy. The oscillatory NMDC quintessence with power-law potential satisfies EoS observational value for $n < 2$ and in the high friction regime the universe can reenter the acceleration expansion mode in the future [140, 141] however inconsistencies are also reported in [142]. The results of the NMDC coupling term when we applying exponential and power-law potentials in the perturbation analysis with combined SN-Ia, BAO and CMB are very small effect on the late time acceleration of the universe if it is needed to satisfy instability

---

[5]This form is not a full Lagrangian form but just only the part of opposite signs between $g^{\mu\nu}$ and $G^{\mu\nu}$. For the full form of Lagrangian, we have something like [132],

$$L = \frac{R}{16\pi G} - \frac{1}{2}\left(g^{\mu\nu} - w^2 G^{\mu\nu}\right)\partial_\mu\phi\partial_\nu\phi - V(\phi).$$

avoidance. This means the coupling parameter needs to be small and is making a term of $9\kappa H^2$ in the Friedmann equation small. Therefore the model behaves like the quintessence model at late time as it is driven by the potential. However at early time the large H value [143] allow the NMDC coupling to driving the inflationary phase and at late time the potential becomes a major role to driving the universe acceleration. Phase space analysis of the model with the exponential potential was performed in [144].

3.2 Background Equations

In this work, we will test whether the model is valid by studying the EoS parameter. We assumed the universe is spatially flat FLRW and filled with a perfect fluid and scalar field ϕ with non-minimal derivative coupling (NMDC) to the curvature. We will consider the action [42]

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{8\pi G} - [\varepsilon g_{\mu\nu} + \kappa G_{\mu\nu}]\phi^{,\mu}\phi^{,\nu} - 2V(\phi)\right] + S_{\rm m}, \tag{3.1}$$

where $S_{\rm m}$ is the action of the matter filled in the universe, $V(\phi)$ is the scalar field potential, $g_{\mu\nu}$ is the tensor metric, R is the Ricci scalar, $G_{\mu\nu}$ is the Einstein tensor, ε is a parameter takes the value +1 (-1) for canonical (phantom) scalar field, and κ is the coupling parameter with the dimension of $(\mathit{length})^2$. By using the flat FLRW universe with the metric

$$ds^2 = -c^2dt^2 + a^2(t)d\mathbf{x}^2, \tag{3.2}$$

where $d\mathbf{x}^2$ is the Euclidian metric, $a(t)$ is the scale factor. Then we obtain the Friedmann equation [42],

$$3H^2 = 4\pi G\dot{\phi}^2(\varepsilon - 9\kappa H^2) + 8\pi G V(\phi) + 8\pi G\rho_{\rm m}, \tag{3.3}$$

where $\rho_{\rm m}$ is the ordinary matter energy density. The Hubble parameter is a function of time t and defined in a form $H = H(t) = \dot{a}(t)/a(t)$. The acceleration equation takes the form,

$$2\dot{H}+3H^2 = -4\pi G\dot{\phi}^2\left[\varepsilon + \kappa\left(2\dot{H} + 3H^2 + 4H\ddot{\phi}\dot{\phi}^{-1}\right)\right] + 8\pi G V(\phi) - 8\pi G p_{\rm m}, \tag{3.4}$$

where $p_{\rm m}$ is the pressure of matter. The Klein-Gordon equation or equation of motion (EoM) of the system is

$$\left(\varepsilon - 3\kappa H^2\right)\ddot{\phi} + \left(3\varepsilon H - 6\kappa H\dot{H} - 9\kappa H^3\right)\dot{\phi} = -V_{,\phi}, \tag{3.5}$$

where $V_{,\phi}$ is the derivative of a potential with respect to scalar filed, $\partial V/\partial\phi$. From above equation we rearrange to get

$$\begin{aligned}\left(\varepsilon - 3\kappa H^2\right)\ddot{\phi} &= -V_{,\phi} - \left(3\varepsilon H - 6\kappa H\dot{H} - 9\kappa H^3\right)\dot{\phi}, \\ \ddot{\phi} &= \frac{-V_{,\phi} - \left(3\varepsilon H - 6\kappa H\dot{H} - 9\kappa H^3\right)\dot{\phi}}{\left(\varepsilon - 3\kappa H^2\right)},\end{aligned}$$

$$
\begin{aligned}
&= -\frac{V_{,\phi}}{\varepsilon - 3\kappa H^2} - \frac{3H\dot{\phi}}{\varepsilon - 3\kappa H^2}\left(\varepsilon - 2\kappa \dot{H} - 3\kappa H^2\right), \\
\ddot{\phi} &= -3H\dot{\phi} - \frac{V_{,\phi}}{\varepsilon - 3\kappa H^2} + \frac{6\kappa H\dot{H}\dot{\phi}}{\varepsilon - 3\kappa H^2}. \qquad (3.6)
\end{aligned}
$$

Subtract Eq.(3.4) from Eq.(3.3), we obtain

$$
\begin{aligned}
2\dot{H} &= -4\pi G\dot{\phi}^2\left[\varepsilon + \kappa\left(2\dot{H} + 3H^2 + 4H\ddot{\phi}\dot{\phi}^{-1}\right)\right] + 8\pi G V(\phi) - 8\pi G p_{\mathrm{m}} \\
&\quad - 4\pi G\dot{\phi}^2(\varepsilon - 9\kappa H^2) - 8\pi G V(\phi) - 8\pi G\rho_{\mathrm{m}}, \\
&= -4\pi G\dot{\phi}^2\left[2\varepsilon + 2\kappa\dot{H} - 6\kappa H^2 + 4\kappa H\ddot{\phi}\dot{\phi}^{-1}\right] - 8\pi G\left[p_{\mathrm{m}} + \rho_{\mathrm{m}}\right], \\
&= -8\pi G\dot{\phi}^2\left[\varepsilon + \kappa\dot{H} - 3\kappa H^2 + 2\kappa H\ddot{\phi}\dot{\phi}^{-1} + p_{\mathrm{m}} + \rho_{\mathrm{m}}\right],
\end{aligned}
$$

finally,

$$
\dot{H} = -4\pi G\dot{\phi}^2\left[\varepsilon + \kappa\dot{H} - 3\kappa H^2 + 2\kappa H\ddot{\phi}\dot{\phi}^{-1} + p_{\mathrm{m}} + \rho_{\mathrm{m}}\right] \qquad (3.7)
$$

From the Friedmann equation, Eq. (3.3), we can rearrange and compare with the general form of the Friedmann equation in a flat FLRW universe

$$
H^2 = \frac{8\pi G}{3}(\rho_\phi + \rho_{\mathrm{m}}). \qquad (3.8)
$$

Therefore,

$$
\begin{aligned}
\frac{8\pi G}{3}\left[\frac{1}{2}(\varepsilon - 9\kappa H^2)\dot{\phi}^2 + V(\phi) + \rho_{\mathrm{m}}\right] &= \frac{8\pi G}{3}(\rho_\phi + \rho_{\mathrm{m}}), \\
\frac{1}{2}(\varepsilon - 9\kappa H^2)\dot{\phi}^2 + V(\phi) + \rho_{\mathrm{m}} &= \rho_\phi + \rho_{\mathrm{m}}.
\end{aligned}
$$

Then we can obtain the energy density of the scalar field in the NMDC model is

$$
\rho_\phi = \frac{1}{2}(\varepsilon - 9\kappa H^2)\dot{\phi}^2 + V(\phi). \qquad (3.9)
$$

Take time derivative to above equation,

$$
\begin{aligned}
\dot{\rho}_\phi &= \frac{1}{2}(2)\dot{\phi}\ddot{\phi}(\varepsilon - 9\kappa H^2) + \frac{1}{2}\dot{\phi}^2(-9\kappa(2)H\dot{H}) + V_{,\phi}\dot{\phi}, \\
&= \dot{\phi}\ddot{\phi}(\varepsilon - 9\kappa H^2) - 9\kappa H\dot{H}\dot{\phi}^2 + V_{,\phi}\dot{\phi}. \qquad (3.10)
\end{aligned}
$$

Consider the continuity equation of the scalar field,

$$
\dot{\rho}_\phi + 3H\rho_\phi(1 + w_\phi) = 0, \qquad (3.11)
$$

substituting Eq.(3.9) into continuity equation, Eq.(3.11), we obtain

$$
\begin{aligned}
\dot{\rho}_\phi + 3H\left(\frac{1}{2}(\varepsilon - 9\kappa H^2)\dot{\phi}^2 + V(\phi)\right)(1 + w_\phi) &= 0, \\
\dot{\rho}_\phi = -3H\left(\frac{1}{2}(\varepsilon - 9\kappa H^2)\dot{\phi}^2 + V(\phi)\right)(1 + w_\phi). \qquad (3.12)
\end{aligned}
$$

From Eq.(3.10) and Eq.(3.12) we can compare to each other and we obtain

$$\begin{aligned}
-3H\left(\frac{1}{2}(\varepsilon-9\kappa H^2)\dot{\phi}^2+V(\phi)\right)(1+w_\phi) &= \dot{\phi}\ddot{\phi}(\varepsilon-9\kappa H^2)-9\kappa H\dot{H}\dot{\phi}^2+V_{,\phi}\dot{\phi}, \\
1+w_\phi &= \frac{\dot{\phi}\ddot{\phi}(\varepsilon-9\kappa H^2)-9\kappa H\dot{H}\dot{\phi}^2+V_{,\phi}\dot{\phi}}{-3H\left(\frac{1}{2}(\varepsilon-9\kappa H^2)\dot{\phi}^2+V(\phi)\right)},
\end{aligned}$$

or the equation of state parameter,

$$\begin{aligned}
w_\phi &= -\frac{\dot{\phi}\ddot{\phi}(\varepsilon-9\kappa H^2)-9\kappa H\dot{H}\dot{\phi}^2+V_{,\phi}\dot{\phi}}{3H\left(\frac{1}{2}(\varepsilon-9\kappa H^2)\dot{\phi}^2+V(\phi)\right)}-1, \\
&= -\left[\frac{\dot{\phi}\ddot{\phi}(\varepsilon-9\kappa H^2)-9\kappa H\dot{H}\dot{\phi}^2+V_{,\phi}\dot{\phi}+3H\left(\frac{1}{2}(\varepsilon-9\kappa H^2)\dot{\phi}^2+V(\phi)\right)}{3H\left(\frac{1}{2}(\varepsilon-9\kappa H^2)\dot{\phi}^2+V(\phi)\right)}\right], \\
&= -\frac{\dot{\phi}\ddot{\phi}(\varepsilon-9\kappa H^2)-9\kappa H\dot{H}\dot{\phi}^2+V_{,\phi}\dot{\phi}+\frac{3H}{2}(\varepsilon-9\kappa H^2)\dot{\phi}^2+3HV(\phi)}{3H\rho_\phi}. \quad (3.13)
\end{aligned}$$

From Eq. (3.6), we multiply both sides by $\dot{\phi}(\varepsilon-9\kappa H^2)$, we obtain

$$\dot{\phi}\ddot{\phi}(\varepsilon-9\kappa H^2)=-3H(\varepsilon-9\kappa H^2)\dot{\phi}^2-\frac{V_{,\phi}\dot{\phi}(\varepsilon-9\kappa H^2)}{(\varepsilon-3\kappa H^2)}+\frac{6\kappa H\dot{H}\dot{\phi}^2(\varepsilon-9\kappa H^2)}{(\varepsilon-3\kappa H^2)}. \quad (3.14)$$

Substituting Eq. (3.14) into the EoS parameter, Eq. (3.13), we obtain

$$\begin{aligned}
w_\phi = -\frac{1}{3H\rho_\phi}\Bigg[&-3H(\varepsilon-9\kappa H^2)\dot{\phi}^2-\frac{V_{,\phi}\dot{\phi}(\varepsilon-9\kappa H^2)}{(\varepsilon-3\kappa H^2)}+\frac{6\kappa H\dot{H}\dot{\phi}^2(\varepsilon-9\kappa H^2)}{(\varepsilon-3\kappa H^2)} \\
&-9\kappa H\dot{H}\dot{\phi}^2+V_{,\phi}\dot{\phi}+\frac{3H}{2}(\varepsilon-9\kappa H^2)\dot{\phi}^2+3HV(\phi)\Bigg] \quad (3.15)
\end{aligned}$$

Comparing with the standard EoS parameter of scalar field $w_\phi = p_\phi/\rho_\phi$. We can extract the pressure of scalar field from above equation,

$$\begin{aligned}
p_\phi &= -\frac{1}{3H}\Bigg[-3H(\varepsilon-9\kappa H^2)\dot{\phi}^2-\frac{V_{,\phi}\dot{\phi}(\varepsilon-9\kappa H^2)}{(\varepsilon-3\kappa H^2)}+\frac{6\kappa H\dot{H}\dot{\phi}^2(\varepsilon-9\kappa H^2)}{(\varepsilon-3\kappa H^2)} \\
&\quad -9\kappa H\dot{H}\dot{\phi}^2+V_{,\phi}\dot{\phi}+\frac{3H}{2}(\varepsilon-9\kappa H^2)\dot{\phi}^2+3HV(\phi)\Bigg], \\
&= (\varepsilon-9\kappa H^2)\dot{\phi}^2\left[1-\frac{2\kappa\dot{H}}{\varepsilon-3\kappa H^2}+\frac{3\kappa\dot{H}}{\varepsilon-9\kappa H^2}-\frac{1}{2}\right] \\
&\quad +\frac{\dot{\phi}V_{,\phi}}{3H}\left[\frac{\varepsilon-9\kappa H^2}{\varepsilon-3\kappa H^2}-1\right]-V(\phi), \\
&= \frac{1}{2}(\varepsilon-9\kappa H^2)\dot{\phi}^2\left[1-2\kappa\dot{H}\left(\frac{2\varepsilon-18\kappa H^2-3\varepsilon+9\kappa H^2}{(\varepsilon-3\kappa H^2)(\varepsilon-9\kappa H^2)}\right)\right] \\
&\quad +\frac{\dot{\phi}V_{,\phi}}{3H}\left[\frac{-6\kappa H^2}{\varepsilon-3\kappa H^2}\right]-V(\phi),
\end{aligned}$$

finally, the pressure of scalar field is

$$p_\phi = \frac{1}{2}(\varepsilon - 9\kappa H^2)\dot{\phi}^2\left[1 + \frac{2\kappa\dot{H}(\varepsilon + 9\kappa H^2)}{(\varepsilon - 3\kappa H^2)(\varepsilon - 9\kappa H^2)}\right] - \frac{2\kappa H\dot{\phi}V_{,\phi}}{\varepsilon - 3\kappa H^2} - V(\phi). \quad (3.16)$$

Therefore we can write the EoS parameter from the pressure, Eq.(3.16), and energy density, Eq.(3.9), of scalar field in this form

$$w_\phi = \frac{\frac{1}{2}(\varepsilon - 9\kappa H^2)\dot{\phi}^2\left[1 + \frac{2\kappa\dot{H}(\varepsilon+9\kappa H^2)}{(\varepsilon-3\kappa H^2)(\varepsilon-9\kappa H^2)}\right] - \frac{2\kappa H\dot{\phi}V_{,\phi}}{\varepsilon-3\kappa H^2} - V(\phi)}{\frac{1}{2}(\varepsilon - 9\kappa H^2)\dot{\phi}^2 + V(\phi)}. \quad (3.17)$$

We can find the EoS parameter in the general kinematical form, by using the Friedmann and acceleration equations, we find

$$\begin{aligned} w_\phi &= \frac{p_\phi}{\rho_\phi}, \\ &= \frac{-\frac{3H^2}{8\pi G} - \frac{\dot{H}}{4\pi G} - p_{\rm m}}{\frac{3H^2}{8\pi G} - \rho_{\rm m}}, \\ w_\phi(H, \dot{H}, \rho_{\rm m}) &= -\left[\frac{3H^2 + 2\dot{H}}{3H^2 - 8\pi G\rho_{\rm m}}\right], \end{aligned} \quad (3.18)$$

where the pressure of matter is zero, $p_{\rm m} = 0$. We see that this form of EoS parameter is independent on the scalar field model but depends only on the form of expansion function. This EoS parameter equation is the same as Eq.(2.62) in the case of flat space, $k = 0$ is replaced. To find the kinetic term, $\dot{\phi}^2$, we take time derivative to the Friedmann equation, Eq. (3.3),

$$\begin{aligned} 6H\dot{H} &= 8\pi G\dot{\phi}\ddot{\phi}(\varepsilon - 9\kappa H^2) + 4\pi G\dot{\phi}^2(-18\kappa H\dot{H}) + 8\pi GV_{,\phi}\dot{\phi} + 8\pi G\dot{\rho_{\rm m}}, \\ &= -8\pi G\left[-\dot{\phi}\ddot{\phi}(\varepsilon - 9\kappa H^2) + 9\kappa H\dot{H}\dot{\phi}^2 - V_{,\phi}\dot{\phi} - \dot{\rho}_{\rm m}\right], \\ &= -\frac{4\pi G}{3H}\left[-\dot{\phi}\ddot{\phi}(\varepsilon - 9\kappa H^2) + 9\kappa H\dot{H}\dot{\phi}^2 - V_{,\phi}\dot{\phi} - \dot{\rho}_{\rm m}\right]. \end{aligned} \quad (3.19)$$

Substituting $\ddot{\phi}$ from Eq.(3.6) and the continuity equation of matter, $\dot{\rho}_{\rm m} = -3H\rho_{\rm m}$, where $w_{\rm m} = 0$ into above equation, we obtain

$$\begin{aligned} \dot{H} &= -4\pi G\Bigg[-\frac{\dot{\phi}(\varepsilon - 9\kappa H^2)}{3H}\left(-3H\dot{\phi} - \frac{V_{,\phi}}{\varepsilon - 3\kappa H^2} + \frac{6\kappa H\dot{H}\dot{\phi}}{\varepsilon - 3\kappa H^2}\right) \\ &\quad + \frac{9\kappa H\dot{H}\dot{\phi}^2}{3H} - \frac{\dot{\phi}V_{,\phi}}{3H} + \frac{3H\rho_{\rm m}}{3H}\Bigg], \\ &= -4\pi G\Bigg[(\varepsilon - 9\kappa H^2)\dot{\phi}^2 + \left(\frac{\varepsilon - 9\kappa H^2}{\varepsilon - 3\kappa H^2}\right)\left(\frac{V_{,\phi}\dot{\phi}}{3H}\right) - \frac{2(\varepsilon - 9\kappa H^2)\kappa\dot{H}\dot{\phi}^2}{\varepsilon - 3\kappa H^2} \\ &\quad + 3\kappa\dot{H}\dot{\phi}^2 - \frac{V_{,\phi}\dot{\phi}}{3H} + \rho_{\rm m}\Bigg], \end{aligned}$$

$$\begin{aligned}\dot{H} &= -4\pi G\Bigg[\left((\varepsilon-9\kappa H^2)-\frac{2(\varepsilon-9\kappa H^2)\kappa\dot{H}}{\varepsilon-3\kappa H^2}+3\kappa\dot{H}\right)\dot{\phi}^2 \\ &\quad+\left(\frac{\varepsilon-9\kappa H^2}{\varepsilon-3\kappa H^2}-1\right)\frac{V_{,\phi}\dot{\phi}}{3H}+\rho_{\rm m}\Bigg], \\ &= -4\pi G\Bigg[\left((\varepsilon-9\kappa H^2)-\frac{2(\varepsilon-9\kappa H^2)\kappa\dot{H}}{\varepsilon-3\kappa H^2}+3\kappa\dot{H}\right)\dot{\phi}^2 \\ &\quad-\frac{2\kappa H V_{,\phi}}{(\varepsilon-3\kappa H^2)}\dot{\phi}+\rho_{\rm m}\Bigg]. \end{aligned} \tag{3.20}$$

Then rearrange to obtain the kinetic term in the form of

$$\dot{\phi}^2 = \frac{\frac{2\kappa H V_{,\phi}}{(\varepsilon-3\kappa H^2)}\dot{\phi}-\frac{\dot{H}}{4\pi G}-\rho_{\rm m}}{(\varepsilon-9\kappa H^2)-\frac{2(\varepsilon-9\kappa H^2)\kappa\dot{H}}{\varepsilon-3\kappa H^2}+3\kappa\dot{H}}. \tag{3.21}$$

Let consider the denominator of above equation,

$$(\varepsilon-9\kappa H^2)-\frac{2(\varepsilon-9\kappa H^2)\kappa\dot{H}}{\varepsilon-3\kappa H^2}+3\kappa\dot{H}$$

$$\begin{aligned} &= (\varepsilon-9\kappa H^2)\left[1-\frac{2\kappa\dot{H}}{\varepsilon-3\kappa H^2}+\frac{3\kappa\dot{H}}{\varepsilon-9\kappa H^2}\right], \\ &= (\varepsilon-9\kappa H^2)\left[1+\left(\frac{-2\kappa\dot{H}(\varepsilon-9\kappa H^2)+3\kappa\dot{H}(\varepsilon-3\kappa H^2)}{(\varepsilon-3\kappa H^2)(\varepsilon-9\kappa H^2)}\right)\right], \\ &= (\varepsilon-9\kappa H^2)\left[1+\frac{\kappa\dot{H}(\varepsilon+9\kappa H^2)}{(\varepsilon-3\kappa H^2)(\varepsilon-9\kappa H^2)}\right]. \end{aligned} \tag{3.22}$$

Therefore the kinetic term, Eq.(3.21), becomes

$$\dot{\phi}^2 = \frac{\frac{2\kappa H V_{,\phi}}{(\varepsilon-3\kappa H^2)}\dot{\phi}-\frac{\dot{H}}{4\pi G}-\rho_{\rm m}}{(\varepsilon-9\kappa H^2)\left[1+\frac{\kappa\dot{H}(\varepsilon+9\kappa H^2)}{(\varepsilon-3\kappa H^2)(\varepsilon-9\kappa H^2)}\right]}. \tag{3.23}$$

3.3 NMDC with Power-Law Cosmology

In this section, $\varepsilon = 1$ and we will use the information of the canonical power-law from Subsection (2.2.1) to deriving the form of background equations in the NMDC power-law cosmology. We also used the zero and constant potentials to simplify the background equations as well. Therefore the kinetic term of the model from Eq.(3.21)

$$\dot{\phi}^2 = \frac{\frac{2\kappa(\alpha/t)V_{,\phi}}{(1-3\kappa(\alpha/t)^2)}\dot{\phi}-\frac{(-\alpha/t^2)}{4\pi G}-\rho_{\rm m,0}\left(\frac{t_0}{t}\right)^{3\alpha}}{(1-9\kappa(\alpha/t)^2)-\frac{2(1-9\kappa(\alpha/t)^2)\kappa(-\alpha/t^2)}{1-3\kappa(\alpha/t)^2}+3\kappa(-\alpha/t^2)},$$

$$\begin{aligned}
\dot{\phi}^2 &= \frac{\frac{2\kappa\alpha V_{,\phi}}{t(1-3\kappa\alpha^2/t^2)}\dot{\phi} + \frac{\alpha}{4\pi G t^2} - \rho_{\rm m,0}\left(\frac{t_0}{t}\right)^{3\alpha}}{(1-\frac{9\kappa\alpha^2}{t^2}) + \frac{2\kappa\alpha}{t^2}\frac{(1-9\kappa\alpha^2/t^2)}{(1-3\kappa\alpha^2/t^2)} - \frac{3\kappa\alpha}{t^2}}, \\
&= \frac{\frac{1}{t^2}\left[\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} + \frac{\alpha}{4\pi G} - \rho_{\rm m,0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}}\right]}{\frac{1}{t^2}\left[(t^2-9\kappa\alpha^2) + 2\kappa\alpha\frac{(t^2-9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)} - 3\kappa\alpha\right]}, \\
\dot{\phi}^2 &= \frac{\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} - \rho_{\rm m,0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}}{(t^2-9\kappa\alpha^2) + 2\kappa\alpha\frac{(t^2-9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)} - 3\kappa\alpha}. \qquad (3.24)
\end{aligned}$$

Let consider the denominator of above equation,

$$(t^2-9\kappa\alpha^2) + 2\kappa\alpha^2\frac{(t^2-9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)} - 3\kappa\alpha$$

$$\begin{aligned}
&= (t^2-9\kappa\alpha^2)\left[1 + \frac{2\kappa\alpha}{(t^2-3\kappa\alpha^2)} - \frac{3\kappa\alpha}{(t^2-9\kappa\alpha^2)}\right], \\
&= (t^2-9\kappa\alpha^2)\left[1 + \left(\frac{2\kappa\alpha(t^2-9\kappa\alpha^2) - 3\kappa\alpha(t^2-3\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right)\right], \\
&= (t^2-9\kappa\alpha^2)\left[1 - \frac{\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right]. \qquad (3.25)
\end{aligned}$$

Therefore the kinetic term, Eq.(3.24), can be rewritten as

$$\dot{\phi}^2 = \frac{\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} - \rho_{\rm m,0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}}{(t^2-9\kappa\alpha^2)\left[1 - \frac{\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right]} \qquad (3.26)$$

For the equation of state parameter, Eq.(3.17), we obtain

$$\begin{aligned}
w_\phi &= \frac{\frac{1}{2}(1-9\kappa H^2)\dot{\phi}^2\left[1 + \frac{2\kappa\dot{H}(1+9\kappa H^2)}{(1-3\kappa H^2)(1-9\kappa H^2)}\right] - \frac{2\kappa H\dot{\phi}V_{,\phi}}{1-3\kappa H^2} - V(\phi)}{\frac{1}{2}(1-9\kappa H^2)\dot{\phi}^2 + V(\phi)}, \\
&= \frac{\frac{1}{2}(1-9\kappa(\alpha/t)^2)\dot{\phi}^2\left[1 + \frac{2\kappa(-\alpha/t^2)(1+9\kappa(\alpha/t)^2)}{(1-3\kappa(\alpha/t)^2)(1-9\kappa(\alpha/t)^2)}\right] - \frac{2\kappa(\alpha/t)\dot{\phi}V_{,\phi}}{1-3\kappa(\alpha/t)^2} - V(\phi)}{\frac{1}{2}(1-9\kappa(\alpha/t)^2)\dot{\phi}^2 + V(\phi)}, \\
&= \frac{\frac{1}{2t^2}\left[(t^2-9\kappa\alpha^2)\dot{\phi}^2\left[1 - \frac{2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right] - \frac{4\kappa\alpha\dot{\phi}t^3V_{,\phi}}{(t^2-3\kappa\alpha^2)} - 2t^2V(\phi)\right]}{\frac{1}{2t^2}\left[(t^2-9\kappa\alpha^2)\dot{\phi}^2 + 2t^2V(\phi)\right]}, \\
w_\phi &= \frac{(t^2-9\kappa\alpha^2)\dot{\phi}^2\left[1 - \frac{2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right] - \frac{4\kappa\alpha\dot{\phi}t^3V_{,\phi}}{(t^2-3\kappa\alpha^2)} - 2t^2V(\phi)}{(t^2-9\kappa\alpha^2)\dot{\phi}^2 + 2t^2V(\phi)}. \qquad (3.27)
\end{aligned}$$

Let consider a first term of numerator of above equation,

$$(t^2-9\kappa\alpha^2)\dot{\phi}^2\left[1 - \frac{2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right]$$

$$
\begin{aligned}
&= (t^2 - 9\kappa\alpha^2)\dot{\phi}^2 \left[\frac{(t^2 - 3\kappa\alpha^2)(t^2 - 9\kappa\alpha^2) - 2\kappa\alpha t^2 + 9\kappa\alpha^2}{(t^2 - 3\kappa\alpha^2)(t^2 - 9\kappa\alpha^2)}\right], \\
&= \dot{\phi}^2 \left[\frac{(t^2 - 3\kappa\alpha^2)(t^2 - 9\kappa\alpha^2) - 2\kappa\alpha t^2 + 9\kappa\alpha^2}{(t^2 - 3\kappa\alpha^2)}\right], \\
&= \dot{\phi}^2 \left[(t^2 - 9\kappa\alpha^2) - 2\kappa\alpha\frac{t^2 + 9\kappa\alpha^2}{t^2 - 3\kappa\alpha^2}\right]. \qquad (3.28)
\end{aligned}
$$

Then substituting the scalar field kinetic term from Eq.(3.24) to above equation,

$$
\begin{aligned}
\dot{\phi}^2 \left[(t^2 - 9\kappa\alpha^2) - 2\kappa\alpha\frac{t^2 + 9\kappa\alpha^2}{t^2 - 3\kappa\alpha^2}\right] &= \left[\frac{\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} - \rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}}{(t^2 - 9\kappa\alpha^2) + 2\kappa\alpha\frac{(t^2-9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)} - 3\kappa\alpha}\right] \\
&\quad \times \left[(t^2 - 9\kappa\alpha^2) - 2\kappa\alpha\frac{t^2 + 9\kappa\alpha^2}{t^2 - 3\kappa\alpha^2}\right], \\
&= \frac{\left[\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} - \rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}\right]\left[(t^2 - 9\kappa\alpha^2) - 2\kappa\alpha\frac{t^2+9\kappa\alpha^2}{t^2-3\kappa\alpha^2}\right]}{(t^2 - 9\kappa\alpha^2) + 2\kappa\alpha\frac{(t^2-9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)} - 3\kappa\alpha}, \\
&= \frac{\left[\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} - \rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}\right]\left[1 - \frac{2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right]}{\left[1 + \frac{2\kappa\alpha(t^2-9\kappa\alpha^2)-3\kappa\alpha(t^2-3\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right]}, \\
&= \left[\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2 - 3\kappa\alpha^2)} - \rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}\right] \\
&\quad \times \left[\frac{(t^2 - 3\kappa\alpha^2)(t^2 - 9\kappa\alpha^2) - 2\kappa\alpha(t^2 + 9\kappa\alpha^2)}{(t^2 - 3\kappa\alpha^2)(t^2 - 9\kappa\alpha^2) - \kappa\alpha(t^2 + 9\kappa\alpha^2)}\right]. \qquad (3.29)
\end{aligned}
$$

Finally, the EoS parameter Eq.(3.27) becomes

$$
\begin{aligned}
w_\phi &= \frac{\left[\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} - \rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}\right]\left[\frac{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-\kappa\alpha(t^2+9\kappa\alpha^2)}\right] - \frac{4\kappa\alpha\dot{\phi}t^3V_{,\phi}}{(t^2-3\kappa\alpha^2)} - 2t^2V(\phi)}{(t^2 - 9\kappa\alpha^2)\left[\frac{\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} - \rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}}{(t^2-9\kappa\alpha^2)+2\kappa\alpha^2\frac{(t^2-9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)}-3\kappa\alpha}\right] + 2t^2V(\phi)}, \\
&= \frac{\left[\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} - \rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}\right]\left[\frac{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-\kappa\alpha(t^2+9\kappa\alpha^2)}\right] - \frac{4\kappa\alpha\dot{\phi}t^3V_{,\phi}}{(t^2-3\kappa\alpha^2)} - 2t^2V(\phi)}{(t^2 - 9\kappa\alpha^2)\left[\frac{\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} - \rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}}{(t^2-9\kappa\alpha^2)\left(1-\frac{\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right)}\right] + 2t^2V(\phi)}, \\
w_\phi &= \frac{\left[\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} - \rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}\right]\left[\frac{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-\kappa\alpha(t^2+9\kappa\alpha^2)}\right] - \frac{4\kappa\alpha\dot{\phi}t^3V_{,\phi}}{(t^2-3\kappa\alpha^2)} - 2t^2V(\phi)}{\frac{\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} - \rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}}{\left(1-\frac{\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right)} + 2t^2V(\phi)}.
\end{aligned}
$$

(3.30)

In the case of zero potential, $V(\phi) = 0$, the consequence of its derivative is $V_{,\phi} = 0$. Therefore the kinetic term reduce to

$$\begin{aligned} \dot{\phi}^2 &= \frac{-\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}}{(t^2-9\kappa\alpha^2) + 2\kappa\alpha^2\frac{(t^2-9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)} - 3\kappa\alpha}, \\ &= -\frac{\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} - \frac{\alpha}{4\pi G}}{(t^2-9\kappa\alpha^2) + 2\kappa\alpha^2\frac{(t^2-9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)} - 3\kappa\alpha}, \end{aligned} \tag{3.31}$$

and the EoS parameter also reduces to

$$\begin{aligned} w_\phi &= \frac{\left[-\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}\right]\left[\frac{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-\kappa\alpha(t^2+9\kappa\alpha^2)}\right]}{\frac{-\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}}+\frac{\alpha}{4\pi G}}{\left(1-\frac{\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right)}}, \\ &= \left[\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} - \frac{\alpha}{4\pi G}\right]\left[\frac{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-\kappa\alpha(t^2+9\kappa\alpha^2)}\right] \\ &\quad \times \left[\frac{\left(1-\frac{\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right)}{\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} - \frac{\alpha}{4\pi G}}\right], \\ &= \frac{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}, \\ w_\phi &= 1 - \frac{2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}. \end{aligned} \tag{3.32}$$

In the case of constant potential, we defined the scalar field potential to be the cosmological constant; that is,

$$V(\phi) \equiv \frac{\Lambda}{8\pi G}, \tag{3.33}$$

where Λ is the cosmological constant. The consequence of it derivative is $V_{,\phi} = 0$ and the kinetic term of scalar field reduce to

$$\dot{\phi}^2 = -\frac{\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} - \frac{\alpha}{4\pi G}}{(t^2-9\kappa\alpha^2) + 2\kappa\alpha^2\frac{(t^2-9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)} - 3\kappa\alpha}. \tag{3.34}$$

We see that the kinetic term is in the same form of the zero potential case, Eq.(3.31), because its depends only on the derivative of potential $V_{,\phi}$ not potential itself. Therefore the EoS parameter also reduce to

$$w_\phi = \frac{\left[-\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} + \frac{\alpha}{4\pi G}\right]\left[\frac{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-\kappa\alpha(t^2+9\kappa\alpha^2)}\right] - 2t^2\frac{\Lambda}{8\pi G}}{\frac{-\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}}+\frac{\alpha}{4\pi G}}{\left(1-\frac{\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right)} + 2t^2\frac{\Lambda}{8\pi G}},$$

$$= \frac{-\left\{\left[\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} - \frac{\alpha}{4\pi G}\right]\left[\frac{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-\kappa\alpha(t^2+9\kappa\alpha^2)}\right] + \frac{\Lambda}{4\pi G}t^2\right\}}{-\left\{\left[\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} - \frac{\alpha}{4\pi G}\right]\left[\frac{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-\kappa\alpha(t^2+9\kappa\alpha^2)}\right] - \frac{\Lambda}{4\pi G}t^2\right\}},$$

$$w_\phi = \frac{\left[\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} - \frac{\alpha}{4\pi G}\right]\left[\frac{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-\kappa\alpha(t^2+9\kappa\alpha^2)}\right] + \frac{\Lambda}{4\pi G}t^2}{\left[\rho_{\mathrm{m},0}\frac{t_0^{3\alpha}}{t^{3\alpha-2}} - \frac{\alpha}{4\pi G}\right]\left[\frac{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)-\kappa\alpha(t^2+9\kappa\alpha^2)}\right] - \frac{\Lambda}{4\pi G}t^2}. \quad (3.35)$$

3.4 NMDC with Phantom Power-Law Cosmology

In this section, we use the information about the phantom power-law from Subsection (2.2.2) to to deriving the form of background equations in the NMDC phantom power-law cosmology. Here we use $\varepsilon = -1$. We also used the zero and constant potentials to simplify the background equations as well. The kinetic term of the model with phantom power-law, Eq.(3.21), is

$$\dot{\phi}^2 = \frac{\frac{2\kappa(-\beta/(t_\mathrm{s}-t))V_{,\phi}\dot{\phi}}{(-1-3\kappa(-\beta/(t_\mathrm{s}-t))^2)} - \frac{(-\beta/(t_\mathrm{s}-t)^2)}{4\pi G} - \rho_{\mathrm{m},0}\left(\frac{t_\mathrm{s}-t_0}{t_\mathrm{s}-t}\right)^{3\beta}}{(-1-9\kappa(-\beta/(t_\mathrm{s}-t))^2) - \frac{2\kappa(-\beta/(t_\mathrm{s}-t)^2)(-1-9\kappa(-\beta/(t_\mathrm{s}-t))^2)}{-1-3\kappa(-\beta/(t_\mathrm{s}-t))^2} + 3\kappa(-\beta/(t_\mathrm{s}-t)^2)},$$

$$= \frac{\frac{2\kappa\beta V_{,\phi}\dot{\phi}/(t_\mathrm{s}-t)}{\frac{(t_\mathrm{s}-t)^2+3\kappa\beta^2}{(t_\mathrm{s}-t)^2}} + \frac{\beta}{4\pi G(t_\mathrm{s}-t)^2} - \rho_{\mathrm{m},0}\frac{(t_\mathrm{s}-t_0)^{3\beta}}{(t_\mathrm{s}-t)^{3\beta}}}{-\frac{(t_\mathrm{s}-t)^2+9\kappa\beta^2}{(t_\mathrm{s}-t)^2} + \frac{2\kappa\beta}{(t_\mathrm{s}-t)^2}\left(\frac{\frac{(t_\mathrm{s}-t)^2+9\kappa\beta^2}{(t_\mathrm{s}-t)^2}}{\frac{(t_\mathrm{s}-t)^2+3\kappa\beta^2}{(t_\mathrm{s}-t)^2}}\right) - \frac{3\kappa\beta}{(t_\mathrm{s}-t)^2}},$$

$$= \frac{\frac{2\kappa\beta V_{,\phi}\dot{\phi}(t_\mathrm{s}-t)^3}{(t_\mathrm{s}-t)^2+3\kappa\beta^2} + \frac{\beta}{4\pi G} - \rho_{\mathrm{m},0}\frac{(t_\mathrm{s}-t_0)^{3\beta}}{(t_\mathrm{s}-t)^{3\beta-2}}}{-((t_\mathrm{s}-t)^2+9\kappa\beta^2) + 2\kappa\beta\left(\frac{(t_\mathrm{s}-t)^2+9\kappa\beta^2}{(t_\mathrm{s}-t)^2+3\kappa\beta^2}\right) - 3\kappa\beta}. \quad (3.36)$$

Let consider the denominator of above equation,

$$-((t_\mathrm{s}-t)^2+9\kappa\beta^2) + 2\kappa\beta\left(\frac{(t_\mathrm{s}-t)^2+9\kappa\beta^2}{(t_\mathrm{s}-t)^2+3\kappa\beta^2}\right) - 3\kappa\beta$$

$$= -\left((t_\mathrm{s}-t)^2+9\kappa\beta^2\right)\left[1 - \frac{2\kappa\beta}{(t_\mathrm{s}-t)^2+3\kappa\beta^2} + \frac{3\kappa\beta}{(t_\mathrm{s}-t)^2+9\kappa\beta^2}\right],$$

$$= -\left((t_\mathrm{s}-t)^2+9\kappa\beta^2\right)\left[1 + \frac{-2\kappa\beta((t_\mathrm{s}-t)^2+9\kappa\beta^2)+3\kappa\beta((t_\mathrm{s}-t)^2+3\kappa\beta^2)}{((t_\mathrm{s}-t)^2+3\kappa\beta^2)((t_\mathrm{s}-t)^2+9\kappa\beta^2)}\right],$$

$$= -\left((t_\mathrm{s}-t)^2+9\kappa\beta^2\right)\left[1 + \frac{\kappa\beta((t_\mathrm{s}-t)^2-9\kappa\beta^2)}{((t_\mathrm{s}-t)^2+3\kappa\beta^2)((t_\mathrm{s}-t)^2+9\kappa\beta^2)}\right]. \quad (3.37)$$

Then the kinetic term, Eq.(3.36), becomes

$$\dot{\phi}^2 = \frac{\frac{2\kappa\beta V_{,\phi}\dot{\phi}(t_\mathrm{s}-t)^3}{(t_\mathrm{s}-t)^2+3\kappa\beta^2} + \frac{\beta}{4\pi G} - \rho_{\mathrm{m},0}\frac{(t_\mathrm{s}-t_0)^{3\beta}}{(t_\mathrm{s}-t)^{3\beta-2}}}{-\left((t_\mathrm{s}-t)^2+9\kappa\beta^2\right)\left[1 + \frac{\kappa\beta((t_\mathrm{s}-t)^2-9\kappa\beta^2)}{((t_\mathrm{s}-t)^2+3\kappa\beta^2)((t_\mathrm{s}-t)^2+9\kappa\beta^2)}\right]}. \quad (3.38)$$

The equation of state parameter, Eq.(3.17), is

$$
\begin{aligned}
w_\phi &= \left\{\frac{1}{2}\left(-1-9\kappa\left(\frac{-\beta}{(t_\mathrm{s}-t)}\right)^2\right)\dot{\phi}^2\left[1+\frac{2\kappa\left(\frac{-\beta}{(t_\mathrm{s}-t)^2}\right)\left(-1+9\kappa\left(\frac{-\beta}{(t_\mathrm{s}-t)}\right)^2\right)}{\left(-1-3\kappa(\left(\frac{-\beta}{(t_\mathrm{s}-t)}\right)^2\right)\left(-1-9\kappa\left(\frac{-\beta}{(t_\mathrm{s}-t)}\right)^2\right)}\right]\right.\\
&\left.-\frac{2\kappa(-\beta/(t_\mathrm{s}-t))\dot{\phi}V_{,\phi}}{-1-3\kappa\left(\frac{-\beta}{(t_\mathrm{s}-t)}\right)^2}-V(\phi)\right\}\Bigg/\left[\frac{1}{2}\left(-1-9\kappa\left(\frac{-\beta}{(t_\mathrm{s}-t)}\right)^2\right)\dot{\phi}^2+V(\phi)\right],\\
&= \frac{\left(\frac{(t_\mathrm{s}-t)^2+9\kappa\beta^2}{(t_\mathrm{s}-t)^2}\right)\dot{\phi}^2\left[1+\frac{\left(\frac{2\kappa\beta}{(t_\mathrm{s}-t)^2}\right)\left(\frac{(t_\mathrm{s}-t)^2-9\kappa\beta^2}{(t_\mathrm{s}-t)^2}\right)}{\left(\frac{(t_\mathrm{s}-t)^2+3\kappa\beta^2}{(t_\mathrm{s}-t)^2}\right)\left(\frac{(t_\mathrm{s}-t)^2+9\kappa\beta^2}{(t_\mathrm{s}-t)^2}\right)}\right]+\frac{4\kappa\beta\dot{\phi}V_{,\phi}}{(t_\mathrm{s}-t)\left(\frac{(t_\mathrm{s}-t)^2+3\kappa\beta^2}{(t_\mathrm{s}-t)^2}\right)}+2V(\phi)}{\left(\frac{(t_\mathrm{s}-t)^2+9\kappa\beta^2}{(t_\mathrm{s}-t)^2}\right)\dot{\phi}^2-2V(\phi)},\\
w_\phi &= \frac{\left((t_\mathrm{s}-t)^2+9\kappa\beta^2\right)\dot{\phi}^2\left[1+\frac{2\kappa\beta\left((t_\mathrm{s}-t)^2-9\kappa\beta^2\right)}{((t_\mathrm{s}-t)^2+3\kappa\beta^2)((t_\mathrm{s}-t)^2+9\kappa\beta^2)}\right]+\frac{4\kappa\beta\dot{\phi}V_{,\phi}(t_\mathrm{s}-t)^3}{((t_\mathrm{s}-t)^2+3\kappa\beta^2)}+2V(\phi)(t_\mathrm{s}-t)^2}{\left((t_\mathrm{s}-t)^2+9\kappa\beta^2\right)\dot{\phi}^2-2V(\phi)(t_\mathrm{s}-t)^2}.
\end{aligned}
\tag{3.39}
$$

Consider a first term of denominator of above equation and by substituting the kinetic term, Eq.(3.38), we obtain

$$
\begin{aligned}
\left((t_\mathrm{s}-t)^2+9\kappa\beta^2\right)\dot{\phi}^2 &= \left((t_\mathrm{s}-t)^2+9\kappa\beta^2\right)\\
&\times\left[\frac{\frac{2\kappa\beta V_{,\phi}\dot{\phi}(t_\mathrm{s}-t)^3}{(t_\mathrm{s}-t)^2+3\kappa\beta^2}+\frac{\beta}{4\pi G}-\rho_{\mathrm{m},0}\frac{(t_\mathrm{s}-t_0)^{3\beta}}{(t_\mathrm{s}-t)^{3\beta-2}}}{-\left((t_\mathrm{s}-t)^2+9\kappa\beta^2\right)\left[1+\frac{\kappa\beta((t_\mathrm{s}-t)^2-9\kappa\beta^2)}{((t_\mathrm{s}-t)^2+3\kappa\beta^2)((t_\mathrm{s}-t)^2+9\kappa\beta^2)}\right]}\right],\\
&= -\frac{\frac{2\kappa\beta V_{,\phi}\dot{\phi}(t_\mathrm{s}-t)^3}{(t_\mathrm{s}-t)^2+3\kappa\beta^2}+\frac{\beta}{4\pi G}-\rho_{\mathrm{m},0}\frac{(t_\mathrm{s}-t_0)^{3\beta}}{(t_\mathrm{s}-t)^{3\beta-2}}}{1+\frac{\kappa\beta((t_\mathrm{s}-t)^2-9\kappa\beta^2)}{((t_\mathrm{s}-t)^2+3\kappa\beta^2)((t_\mathrm{s}-t)^2+9\kappa\beta^2)}}.
\end{aligned}
\tag{3.40}
$$

Therefore the EoS parameter Eq.(3.39) becomes

$$
\begin{aligned}
w_\phi &= \left\{\left[-\frac{\frac{2\kappa\beta V_{,\phi}\dot{\phi}(t_\mathrm{s}-t)^3}{(t_\mathrm{s}-t)^2+3\kappa\beta^2}+\frac{\beta}{4\pi G}-\rho_{\mathrm{m},0}\frac{(t_\mathrm{s}-t_0)^{3\beta}}{(t_\mathrm{s}-t)^{3\beta-2}}}{1+\frac{\kappa\beta((t_\mathrm{s}-t)^2-9\kappa\beta^2)}{((t_\mathrm{s}-t)^2+3\kappa\beta^2)((t_\mathrm{s}-t)^2+9\kappa\beta^2)}}\right]\right.\\
&\times\left[1+\frac{2\kappa\beta\left((t_\mathrm{s}-t)^2-9\kappa\beta^2\right)}{\left((t_\mathrm{s}-t)^2+3\kappa\beta^2\right)\left((t_\mathrm{s}-t)^2+9\kappa\beta^2\right)}\right]\\
&\left.+\frac{4\kappa\beta\dot{\phi}V_{,\phi}(t_\mathrm{s}-t)^3}{((t_\mathrm{s}-t)^2+3\kappa\beta^2)}+2V(\phi)(t_\mathrm{s}-t)^2\right\}\\
&\Bigg/\left[-\left(\frac{\frac{2\kappa\beta V_{,\phi}\dot{\phi}(t_\mathrm{s}-t)^3}{(t_\mathrm{s}-t)^2+3\kappa\beta^2}+\frac{\beta}{4\pi G}-\rho_{\mathrm{m},0}\frac{(t_\mathrm{s}-t_0)^{3\beta}}{(t_\mathrm{s}-t)^{3\beta-2}}}{1+\frac{\kappa\beta((t_\mathrm{s}-t)^2-9\kappa\beta^2)}{((t_\mathrm{s}-t)^2+3\kappa\beta^2)((t_\mathrm{s}-t)^2+9\kappa\beta^2)}}\right)-\ 2V(\phi)(t_\mathrm{s}-t)^2\right],
\end{aligned}
$$

$$
\begin{aligned}
&= \Bigg\{ -\left[\frac{2\kappa\beta V_{,\phi}\dot{\phi}(t_{\rm s}-t)^3}{(t_{\rm s}-t)^2+3\kappa\beta^2}+\frac{\beta}{4\pi G}-\rho_{\rm m,0}\frac{(t_{\rm s}-t_0)^{3\beta}}{(t_{\rm s}-t)^{3\beta-2}}\right] \\
&\times\left[\frac{\frac{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)+2\kappa\beta\left((t_{\rm s}-t)^2-9\kappa\beta^2\right)}{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)}}{\frac{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\rm s}-t)^2-9\kappa\beta^2)}{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)}}\right]+\frac{4\kappa\beta\dot{\phi}V_{,\phi}(t_{\rm s}-t)^3}{((t_{\rm s}-t)^2+3\kappa\beta^2)} \\
&+2V(\phi)(t_{\rm s}-t)^2\Bigg\}\Bigg/\Bigg[-\left(\frac{\frac{2\kappa\beta V_{,\phi}\dot{\phi}(t_{\rm s}-t)^3}{(t_{\rm s}-t)^2+3\kappa\beta^2}+\frac{\beta}{4\pi G}-\rho_{\rm m,0}\frac{(t_{\rm s}-t_0)^{3\beta}}{(t_{\rm s}-t)^{3\beta-2}}}{\frac{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\rm s}-t)^2-9\kappa\beta^2)}{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)}}\right) \\
&-2V(\phi)(t_{\rm s}-t)^2\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
w_\phi &= \Bigg\{\left[\frac{2\kappa\beta V_{,\phi}\dot{\phi}(t_{\rm s}-t)^3}{(t_{\rm s}-t)^2+3\kappa\beta^2}+\frac{\beta}{4\pi G}-\rho_{\rm m,0}\frac{(t_{\rm s}-t_0)^{3\beta}}{(t_{\rm s}-t)^{3\beta-2}}\right] \\
&\times\left[\frac{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)+2\kappa\beta\left((t_{\rm s}-t)^2-9\kappa\beta^2\right)}{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\rm s}-t)^2-9\kappa\beta^2)}\right] \\
&-\frac{4\kappa\beta\dot{\phi}V_{,\phi}(t_{\rm s}-t)^3}{((t_{\rm s}-t)^2+3\kappa\beta^2)}-2V(\phi)(t_{\rm s}-t)^2\Bigg\} \\
&\Bigg/\Bigg\{\left[\frac{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)}{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\rm s}-t)^2-9\kappa\beta^2)}\right] \\
&\times\left[\frac{2\kappa\beta V_{,\phi}\dot{\phi}(t_{\rm s}-t)^3}{(t_{\rm s}-t)^2+3\kappa\beta^2}+\frac{\beta}{4\pi G}-\rho_{\rm m,0}\frac{(t_{\rm s}-t_0)^{3\beta}}{(t_{\rm s}-t)^{3\beta-2}}\right]+2V(\phi)(t_{\rm s}-t)^2\Bigg\}.
\end{aligned}
\tag{3.41}
$$

In the case of zero potential, $V(\phi)=0$, and its derivative is zero, $V_{,\phi}=0$. Therefore the kinetic term, Eq.(3.38), can be reduced to

$$
\begin{aligned}
\dot{\phi}^2 &= \frac{\frac{\beta}{4\pi G}-\rho_{\rm m,0}\frac{(t_{\rm s}-t_0)^{3\beta}}{(t_{\rm s}-t)^{3\beta-2}}}{-\left((t_{\rm s}-t)^2+9\kappa\beta^2\right)\left[1+\frac{\kappa\beta((t_{\rm s}-t)^2-9\kappa\beta^2)}{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)}\right]}, \\
&= \frac{\rho_{\rm m,0}\frac{(t_{\rm s}-t_0)^{3\beta}}{(t_{\rm s}-t)^{3\beta-2}}-\frac{\beta}{4\pi G}}{\frac{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\rm s}-t)^2-9\kappa\beta^2)}{((t_{\rm s}-t)^2+3\kappa\beta^2)}}, \\
&= -\frac{\left(\rho_{\rm m,0}\frac{(t_{\rm s}-t_0)^{3\beta}}{(t_{\rm s}-t)^{3\beta-2}}-\frac{\beta}{4\pi G}\right)((t_{\rm s}-t)^2+3\kappa\beta^2)}{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\rm s}-t)^2-9\kappa\beta^2)},
\end{aligned}
\tag{3.42}
$$

and the EoS parameter, Eq.(3.41), can be reduced to

$$
\begin{aligned}
w_\phi &= \Bigg\{\left[\frac{\beta}{4\pi G}-\rho_{\rm m,0}\frac{(t_{\rm s}-t_0)^{3\beta}}{(t_{\rm s}-t)^{3\beta-2}}\right] \\
&\times\left[\frac{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)+2\kappa\beta\left((t_{\rm s}-t)^2-9\kappa\beta^2\right)}{((t_{\rm s}-t)^2+3\kappa\beta^2)((t_{\rm s}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\rm s}-t)^2-9\kappa\beta^2)}\right]\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
& \Bigg/ \left\{ \left[\frac{\beta}{4\pi G} - \rho_{\mathrm{m},0}\frac{(t_{\mathrm{s}}-t_0)^{3\beta}}{(t_{\mathrm{s}}-t)^{3\beta-2}}\right] \right.\\
& \left. \times \left[\frac{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)}{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\mathrm{s}}-t)^2-9\kappa\beta^2)}\right]\right\},\\
= & \frac{\left[\frac{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+2\kappa\beta\left((t_{\mathrm{s}}-t)^2-9\kappa\beta^2\right)}{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\mathrm{s}}-t)^2-9\kappa\beta^2)}\right]}{\left[\frac{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)}{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\mathrm{s}}-t)^2-9\kappa\beta^2)}\right]},\\
= & \frac{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+2\kappa\beta\left((t_{\mathrm{s}}-t)^2-9\kappa\beta^2\right)}{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)},\\
w_\phi = & 1+\frac{2\kappa\beta\left((t_{\mathrm{s}}-t)^2-9\kappa\beta^2\right)}{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)}, \qquad (3.43)
\end{aligned}
$$

this equation will recover $w_\phi = 1$ when there is no coupling constant $\kappa = 0$.

In the case of constant potential, we defined the potential is a cosmological as Eq.(3.33) and its derivative is zero, $V_{,\phi} = 0$. Therefore the kinetic term, Eq.(3.38), can be reduced to

$$
\begin{aligned}
\dot{\phi}^2 = & \frac{\frac{\beta}{4\pi G} - \rho_{\mathrm{m},0}\frac{(t_{\mathrm{s}}-t_0)^{3\beta}}{(t_{\mathrm{s}}-t)^{3\beta-2}}}{-\left((t_{\mathrm{s}}-t)^2+9\kappa\beta^2\right)\left[1+\frac{\kappa\beta((t_{\mathrm{s}}-t)^2-9\kappa\beta^2)}{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)}\right]},\\
= & \frac{\rho_{\mathrm{m},0}\frac{(t_{\mathrm{s}}-t_0)^{3\beta}}{(t_{\mathrm{s}}-t)^{3\beta-2}} - \frac{\beta}{4\pi G}}{\left((t_{\mathrm{s}}-t)^2+9\kappa\beta^2\right)\left[\frac{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\mathrm{s}}-t)^2-9\kappa\beta^2)}{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)}\right]},\\
= & -\frac{\left(\rho_{\mathrm{m},0}\frac{(t_{\mathrm{s}}-t_0)^{3\beta}}{(t_{\mathrm{s}}-t)^{3\beta-2}} - \frac{\beta}{4\pi G}\right)\left((t_{\mathrm{s}}-t)^2+3\kappa\beta^2\right)}{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\mathrm{s}}-t)^2-9\kappa\beta^2)}. \qquad (3.44)
\end{aligned}
$$

We see that this equation is in the same form of Eq.(3.42). The EoS parameter, Eq.(3.41),

$$
\begin{aligned}
w_\phi = & \left\{ \left[\frac{\beta}{4\pi G} - \rho_{\mathrm{m},0}\frac{(t_{\mathrm{s}}-t_0)^{3\beta}}{(t_{\mathrm{s}}-t)^{3\beta-2}}\right] \right.\\
& \times \left[\frac{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+2\kappa\beta\left((t_{\mathrm{s}}-t)^2-9\kappa\beta^2\right)}{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\mathrm{s}}-t)^2-9\kappa\beta^2)}\right]\\
& \left. -2\left(\frac{\Lambda}{8\pi G}\right)(t_{\mathrm{s}}-t)^2 \right\} \Bigg/ \left\{ \left[\frac{\beta}{4\pi G} - \rho_{\mathrm{m},0}\frac{(t_{\mathrm{s}}-t_0)^{3\beta}}{(t_{\mathrm{s}}-t)^{3\beta-2}}\right] \right.\\
& \times \left[\frac{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)}{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\mathrm{s}}-t)^2-9\kappa\beta^2)}\right]\\
& \left. +2\left(\frac{\Lambda}{8\pi G}\right)(t_{\mathrm{s}}-t)^2 \right\},
\end{aligned}
$$

finally, it can be reduced to

$$
\begin{aligned}
w_\phi \;=\; & \left\{ \left[\frac{\beta}{4\pi G} - \rho_{\mathrm{m},0}\frac{(t_{\mathrm{s}}-t_0)^{3\beta}}{(t_{\mathrm{s}}-t)^{3\beta-2}}\right] \right. \\
& \times \left[\frac{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+2\kappa\beta\left((t_{\mathrm{s}}-t)^2-9\kappa\beta^2\right)}{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\mathrm{s}}-t)^2-9\kappa\beta^2)}\right] \\
& \left. - \left(\frac{\Lambda}{4\pi G}\right)(t_{\mathrm{s}}-t)^2 \right\} \Bigg/ \left\{ \left[\frac{\beta}{4\pi G} - \rho_{\mathrm{m},0}\frac{(t_{\mathrm{s}}-t_0)^{3\beta}}{(t_{\mathrm{s}}-t)^{3\beta-2}}\right] \right. \\
& \times \left[\frac{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)}{((t_{\mathrm{s}}-t)^2+3\kappa\beta^2)((t_{\mathrm{s}}-t)^2+9\kappa\beta^2)+\kappa\beta((t_{\mathrm{s}}-t)^2-9\kappa\beta^2)}\right] \\
& \left. + \left(\frac{\Lambda}{4\pi G}\right)(t_{\mathrm{s}}-t)^2 \right\}. \qquad (3.45)
\end{aligned}
$$

Now we obtained all of the background equations using in the next chapter, to constrain the cosmological values by using the observational datasets.

CHAPTER IV

RESULTS AND DISCUSSIONS

In this chapter, we show the derived cosmological parameters and the results from our investigated in the previous two chapters. We show the derived cosmological parameters from WMAP7 and WMAP7+BAO+H_0 combined datasets in a first section. We also show the parametric plots and the results from the tachyonic with power-law cosmology scenario in this section. In the second section, we show the derived cosmological parameters observed by WMAP9 (combined WMAP9+eCMB+BAO+H_0) dataset and PLANCK satellite datasets. Including the cosmological constants derived from the NMDC model with power-law cosmology by using those observational parameters with some parametric plots.

4.1 Tachyonic Power-Law Cosmology

The derived cosmological parameters from WMAP7 and WMAP7+BAO $+H_0$ are shown in Table 1. We will set the present scale factor to unity, $a_0 = a(t_0) = 1$, and consider the flat FLRW universe where the curvature $k = 0$ throughout (but kept k in the formulae for completeness). Our universe is composed of a pressureless matter or dust and a scalar field, ϕ, acting as a tachyon field. The present energy density of matter is defined as

$$\rho_{\rm m,0} \equiv \Omega_{\rm m,0}\rho_{\rm c,0}, \tag{4.1}$$

where Ω is the dimensionless parameter called density parameter and $\rho_{\rm c,0}$ is the present critical density defined as

$$\rho_{\rm c,0} = \frac{3H_0^2}{8\pi G}. \tag{4.2}$$

In this case $\Omega_{\rm m,0}$ is the density parameter of matter at present time t_0. The total matter fluid energy density at present is sum of that of all dust matter types

$$\Omega_{\rm m,0} = \Omega_{\rm CDM,0} + \Omega_{\rm b,0} \tag{4.3}$$

where $\Omega_{\rm CDM,0}$ is the density parameter of cold dark matter in the universe and $\Omega_{\rm b,0}$ is the density parameter of barotropic fluid at present. We take the maximum likelihood value assuming spatially flat case. Although in deriving the present cosmic time t_0, the ΛCDM model is assumed with the cosmic microwave background (CMB) data, however one can estimably use t_0 since $w_{\rm DE}$ is very close to -1. In SI units, the reduced Planck mass is

$$\begin{aligned} M_{\rm P} &= \sqrt{\frac{\hbar c}{8\pi G}}, \\ &\approx 4.341 \times 10^{-9}{\rm kg} = 2.435 \times 10^{18}{\rm GeV/c^2}, \end{aligned} \tag{4.4}$$

Table 1: Combined WMAP7+BAO+H_0 and WMAP7 derived parameters (maximum likelihood) from Refs. [13] and [14]. Here we also calculate (with error analysis) $\Omega_{\rm m,0} = \Omega_{\rm b,0} + \Omega_{\rm CDM,0}$, critical density: $\rho_{\rm c,0} = 3H_0^2/8\pi G$ and matter density: $\rho_{\rm m,0} = \Omega_{\rm m,0}\rho_{\rm c,0}$. The space is flat and a_0 is set to unity.

Parameter	WMAP7+BAO+H_0	WMAP7
t_0	13.76 ± 0.11 Gyr or	13.79 ± 0.13 Gyr or
	$(4.34 \pm 0.03) \times 10^{17}$ sec	$(4.35 \pm 0.04) \times 10^{17}$ sec
H_0	70.4 ± 1.4 km/s/Mpc	70.3 ± 2.5 km/s/Mpc
	$(2.28 \pm 0.04) \times 10^{-18}$ sec^{-1}	$(2.28 \pm 0.08) \times 10^{-18}$ sec^{-1}
$\Omega_{\rm b,0}$	0.0455 ± 0.0016	0.0451 ± 0.0028
$\Omega_{\rm CDM,0}$	0.226 ± 0.015	0.226 ± 0.027
$\Omega_{\rm m,0}$	$0.271(5) \pm 0.015(1)$	$0.271(1) \pm 0.027(1)$
$\rho_{\rm m,0}$	$(2.52(49)^{+0.18(24)}_{-0.16(64)}) \times 10^{-27}$ kg/m^3	$(2.52(12)^{+0.30(97)}_{-0.30(61)}) \times 10^{-27}$ kg/m^3
$\rho_{\rm c,0}$	$(9.29(99)^{+0.32(92)}_{-0.32(35)}) \times 10^{-27}$ kg/m^3	$(9.29(99)^{+0.66(41)}_{-0.64(12)}) \times 10^{-27}$ kg/m^3

and it is related to Planck mass with factor $1/\sqrt{8\pi}$; that is,

$$\begin{aligned} m_{\rm P} &= \sqrt{\frac{\hbar c}{G}} = \sqrt{8\pi} M_{\rm P}, \\ &\approx 1.2209 \times 10^{19} {\rm GeV/c}^2 = 2.17651(13) \times 10^{-8} {\rm kg}. \end{aligned} \tag{4.5}$$

In this work, we give the corrections to errors on the future singularity time, $t_{\rm s}$ or the phantom power-law case. We also improve the values of the present equation of state (EoS) parameter, $w_{\phi,0}$, of the phantom power-law case in the work done by Chakkrit, Burin and Saridakis [78] and of the usual power-law case reported earlier [77].

Typically, the astrophysical tests of the power-law cosmology are indicating the value of α are performed by observing the Hubble parameter as a function of redshift, $H(z)$ data of SNIa or high-redshift objects such as distant globular clusters [107, 108, 109]. To specific the value of α one can also use gravitational lensing statistics [87], compact-radio source [145] or using X-ray gas mass fraction measurements of galaxy clusters [146, 147, 148]. The values of α can be found in Table 2.

Table 2: The values of the power-law exponent α from various sources.

Sources	α
Angular size to redshift z [a]	1.0 ± 0.3
WMAP5 dataset [b]	1.01
X-ray mass fraction [c]	$2.3^{+1.4}_{-0.7}$
SNLS+$H(z)$ [d]	$1.62^{+0.10}_{-0.09}$
$H(z)$ data [e] [107]	$1.07^{+0.11}_{-0.09}$
$H(z)$ data [e] [77, 109]	$1.11^{+0.21}_{-0.14}$

[a] Study of angular size to redshift z relation of a large sample of milli-arcsecond compact radio sources in flat FLRW universe at 68 % C.L. [145].

[b] For closed geometry [76].

[c] X-ray mass fraction data of galaxy clusters in flat geometry [146] and this procedures of measurement give large value of α.

[d] Joint test using Supernova Legacy Survey (SNLS) and $H(z)$ data in flat geometry [107].

[e] When α is found to be independent of curvature procedure (i.e. with neither SNIa nor cluster X-ray mass fraction) or in flat case, α is nearly equal to unity.

From Table 2, we can notice that the assumption of non-zero spatial curvature ($k = \pm 1, 0$) is assumed in these results in evaluating of α except in the WMAP5 of which the result puts also constraint on the spatial curvature. Short review of recent α values can be found in Ref. [77]. Here we can calculate the values of α from the present of H_0 and t_0 from $\alpha = H_0 t_0$.

At present, we set the cosmic time $t = t_0$ and the effective EoS parameter is followed Eq.(2.109), $w_{\rm eff,0} = -1 + 2/(3\alpha)$. In Table 3, we show that the values of the power-law exponent, α, the EoS parameters at present derived in the power-law cosmology (true for both tachoynic and quintessence) do not match observational data. Therefore our results of $w_{\phi,0}$ and $w_{\rm eff,0}$ found to be much greater than observational (spatially flat) WMAP derived results as shown in Table 4. We can conclude here that the power-law expansion universe with quintessential scalar field [77] or tachyonic field is neither viable.

Results presented in Table 5 are the phantom power-law exponent β, the future big-rip time $t_{\rm s}$ and the equation of state parameters at present. For phantom power-law cosmology driven by tachyonic field (also true for phantom quintessence), the resulting value is

$$
\begin{aligned}
w_{\phi,0} &= -1.49^{+11.64}_{-4.08} \qquad \text{(using WMAP7} + \text{BAO+}H_0\text{)},\\
w_{\phi,0} &= -1.51^{+3.89}_{-6.72} \qquad \text{(using WMAP7)}.
\end{aligned}
$$

Table 3: Power-law cosmology exponent and its prediction of equation of state parameters. The value does not match the WMAP7 results.

Parameter	WMAP7+BAO+H_0	WMAP7
α	$0.98(95) \pm 0.01(87)$	$0.99(18) \pm 0.03(60)$
$w_{\phi,0}$ (with power-law cosmology)	$-0.44(79)^{+0.01(66)}_{-0.01(54)}$	$-0.44(98)^{+0.02(97)}_{-0.02(82)}$
$w_{\text{eff},0}$ (with power-law cosmology)	$-0.32(63) \pm 0.01(25)$	$-0.32(78) \pm 0.02(35)$

Table 4: The present values of the EoS parameter of scalar field, $w_{\phi,0}$, obtained from the WMAP7 observational probe and its combined with other datasets.

Sources	$w_{\phi,0}$
WMAP7 [a]	$-1.12^{+0.42}_{-0.43}$ (68 % CL)
WMAP7+BAO+H_0 combined [b]	$-1.10^{+0.14}_{-0.14}$ (68 % CL)
WMAP7+BAO+H_0+SN [c]	$-1.34^{+1.74}_{-0.36}$ (68 % CL)
WMAP7+BAO+ H_0+SN with time delay distance information correction [d]	$-1.31^{+1.67}_{-0.38}$ (68 % CL)

[a] flat geometry, constant $w_{\phi,0}$ (Section (4.2.5) of Ref. [13]),
[b] flat geometry, constant $w_{\phi,0}$ (Section (5.1) of Ref. [14]),
[c] flat geometry, time varying dark energy EoS, $w_\phi(a) = w_0 + w_a(1-a)$ with $w_0 = -0.93 \pm 0.13, w_a = -0.41^{+0.72}_{-0.71}$ (Section (5.3) of Ref. [14]),
[d] flat geometry, time varying dark energy EoS, $w_\phi(a) = w_0 + w_a(1-a)$ with $w_0 = -0.93 \pm 0.12, w_a = -0.38^{+0.66}_{-0.65}$ (Section (5.3) of Ref. [14]).

These do not much differ from results from the observational data [14]

$$\begin{aligned} w_{\phi,0} &= -1.34^{+1.74}_{-0.36} \qquad (68\%\ \text{CL})^6, \\ w_{\phi,0} &= -1.31^{+1.67}_{-0.38} \qquad (68\%\ \text{CL})^7. \end{aligned}$$

Using observational data in Tables 1 and 5 we derive the EoS parameter as a

[6] WMAP7+BAO+H_0+SN data (flat, varying dark energy EoS).

[7] WMAP7+BAO+H_0+SN +time delay distance correction data (flat varying dark energy EoS).

Table 5: Phantom power-law cosmology exponent and its prediction of equation of state parameters. The equation of state lies in acceptable range of values given by WMAP7 results. Large error bar of $w_{\phi,0}$ is an effect of large error bar in $t_{\rm s}$.

Parameter	**WMAP7+BAO+H_0**	**WMAP7**
β	$-7.81(08)^{+11.71(8)}_{-4.56(1)}$	$-6.50(72)^{+3.91(92)}_{-5.09(96)}$
$t_{\rm s}$ (Gyr)	$122.30(0)^{+162.83(7)}_{-63.36(0)}$	$104.21(5)^{+54.37(3)}_{-70.79(9)}$
$w_{\phi,0}$ (with phantom power-law)	$-1.48(99)^{+11.64(46)}_{-4.08(45)}$	$-1.51(26)^{+3.89(23)}_{-6.71(90)}$
$w_{\rm eff,0}$ (with phantom power-law)	$-1.08(54)^{+0.25(60)}_{-0.11(98)}$	$-1.10(24)^{+0.15(52)}_{-0.37(12)}$

function of β exponent for data from WMAP7+BAO+H_0 and WMAP7 as

$$w_{\phi,0} = -\left[\frac{1-2/(3\beta)}{1-(16.60/\beta^2)}\right], \tag{4.6}$$

$$w_{\phi,0} = -\left[\frac{1-2/(3\beta)}{1-(11.47/\beta^2)}\right]. \tag{4.7}$$

With these, we show parametric plots of the present EoS parameter $w_{\phi,0}$ versus the exponent of power-law α in Fig. 1 and β in Fig. 2. We see that, in Fig. 1, there is no values of α from our model match with the observational value, see Table 2 and Table 3. In the case of phantom model, Fig. 2, the values measured for β and $w_{\phi,0}$ from WMAP7+BAO+H_0 and WMAP7 are the purple cross and yellow spot, respectively. We see that the values of β is lies in a range $-\infty < \beta \lesssim -6$ and the EoS parameter $w_{\phi,0}$ lies in the range $(-1,-2)$. These values of β are viable for the phantom model to be a good candidate for dark energy responsible for present accelerating expansion of our universe. The more error bar when β is increasing due to the effect from the future big-rip time which has more error itself. Fig. 3 shows evolution of the EoS parameter $w(z)$ in late phantom power-law universe from $0 < z < 0.45$, i.e. $t = 8.48$ Gyr (both datasets) till present era (this is to avoid singularity in w_ϕ at $z = 0.492$ (WMAP7+BAO+H_0) and at $z = 0.484$ (WMAP7))[8].

When the tachyonic field is phantom ($\varepsilon = -1$) and is the dominant component, therefore Eq.(2.58) for flat FLRW space which we can neglect the matter term, hence

$$\dot{\phi}^2 = \frac{2\dot{H}}{3H^2} = -\frac{2}{3\beta}. \tag{4.8}$$

[8]These are equivalent to the past 5.28 Gyr ago (WMAP7+BAO+H_0) and the past 5.31 Gyr ago (WMAP7).

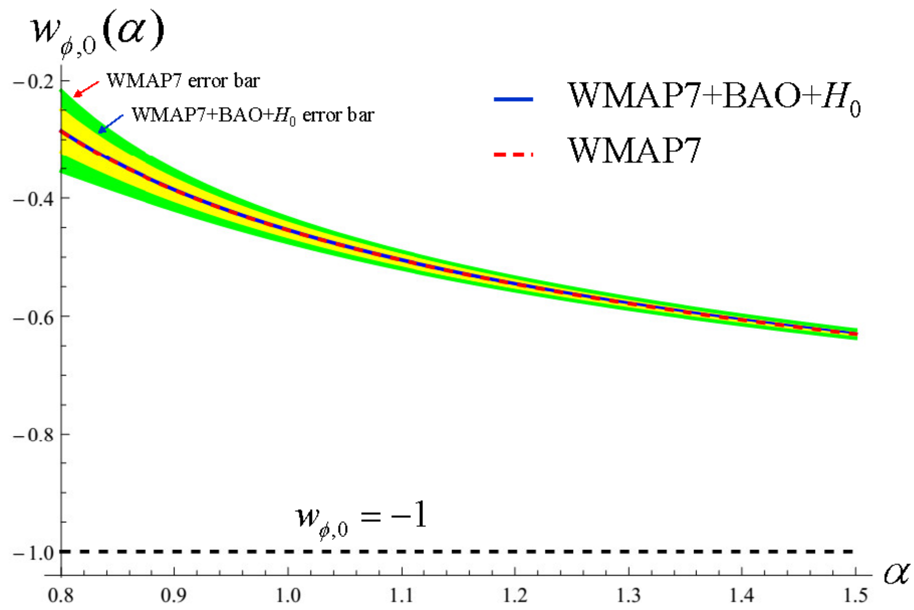

Figure 1: Present value of canonical tachyonic dark energy equation of state plotted versus α. Their error bar results from the error bar in α. This is the same for quintessence case.

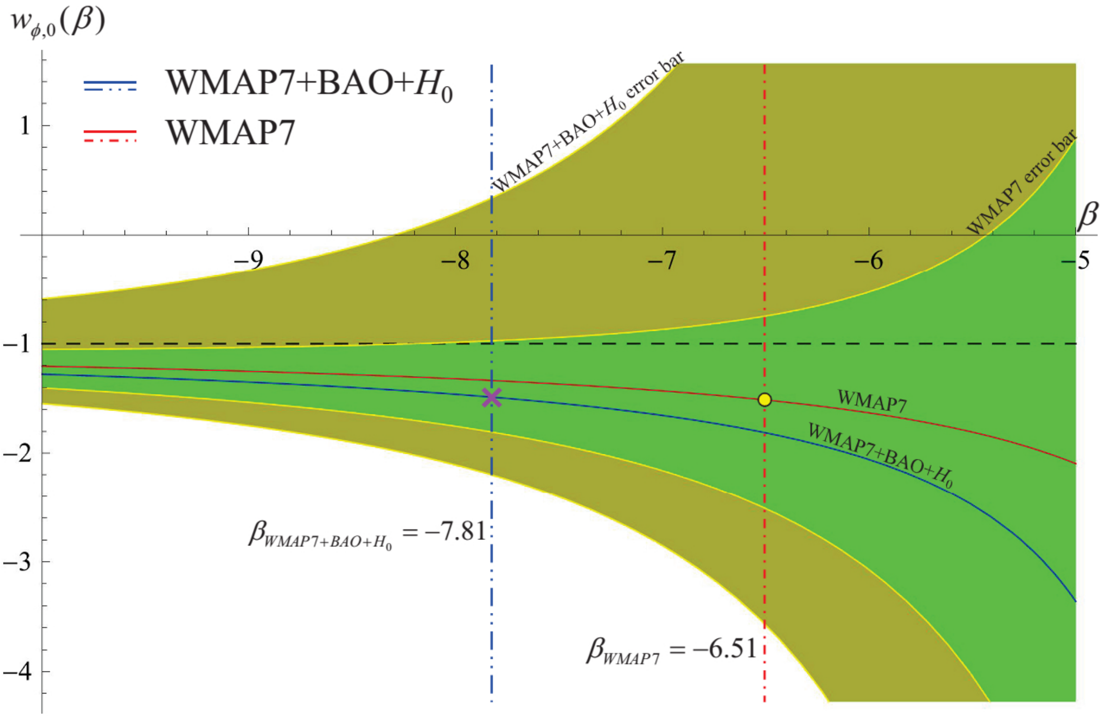

Figure 2: Present value of phantom tachyonic dark energy equation of state plotted versus β. Their error bar results from the error bar in β. This is the same for quintessence case [149].

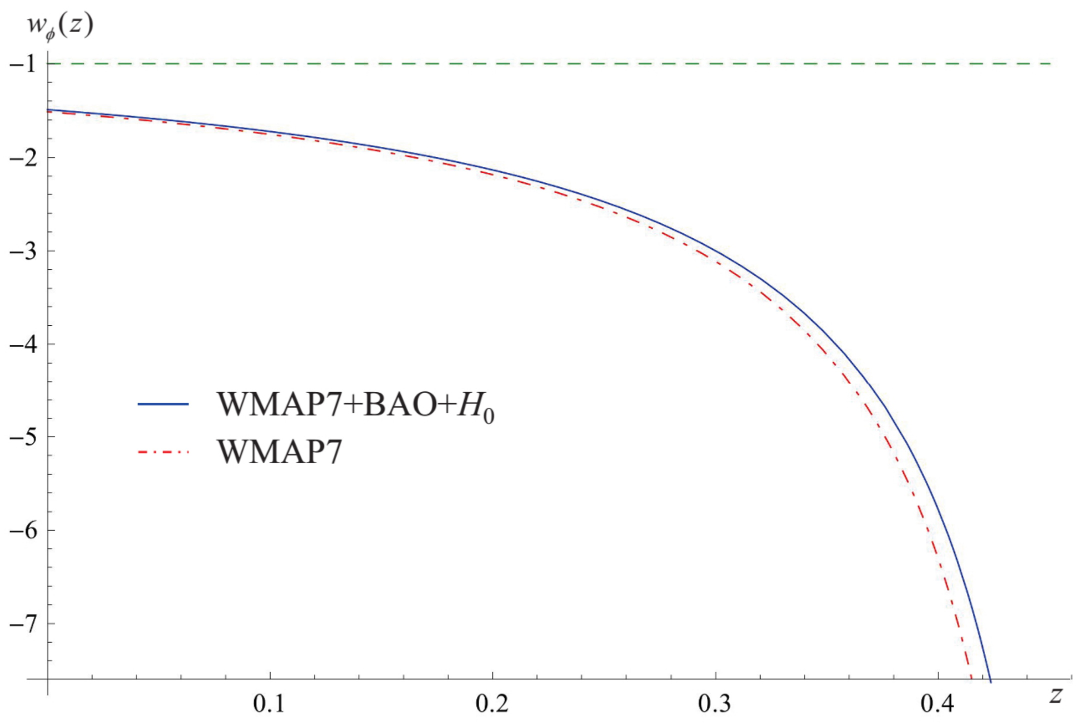

Figure 3: Phantom tachyonic (and quintessence) dark energy equation of state versus z [149].

Integrating above equation from t to t_{s}, and choosing positive solution, we can imply that the scalar field as a function of cosmic time is in form of

$$\phi(t) = \sqrt{\frac{2}{3|\beta|}}\,(t_{\mathrm{s}} - t)\,, \tag{4.9}$$

where the scalar field at a big-rip time $\phi_{\mathrm{s}} = \phi(t_{\mathrm{s}}) = 0$. Since $\beta < 0$ hence we can be written it as $-\beta = |\beta|$. From Eq.(2.61) the tachyonic potential is

$$\begin{aligned} V(\phi) &= \frac{3c^2H^2}{\kappa}\sqrt{1-(-1)\left(-\frac{2}{3\beta}\right)}, \\ &= \frac{3c^2}{\kappa}\frac{\beta^2}{(t_{\mathrm{s}}-t)^2}\sqrt{1+\frac{2}{3|\beta|}}, \\ &= \frac{2c^2|\beta|}{\kappa}\frac{3|\beta|}{2(t_{\mathrm{s}}-t)^2}\sqrt{1+\frac{2}{3|\beta|}}, \\ &= \frac{2c^2|\beta|}{\kappa\phi^2}\sqrt{1+\frac{2}{3|\beta|}}, \end{aligned} \tag{4.10}$$

where $\kappa \equiv 8\pi G$. With parameters in Table 5, the potential is plotted in Fig. 4 which is no surprised as it was found earlier [48] regardless of the expansion is either normal power-law or phantom power-law. The steepness of the potential is typically determined by a dimensionless variable Γ defined as

$$\Gamma \equiv \frac{V''V}{V'^2}, \tag{4.11}$$

where $'$ denotes the total derivative with respect to scalar field, $\mathrm{d}/\mathrm{d}\phi$. For the potential from Eq.(4.10), it is found that the steepness

$$\begin{aligned} \Gamma &= \frac{(6A\phi^{-4})\,(A\phi^{-2})}{(-2A\phi^{-3})^2}, \\ &= \frac{6A^2\phi^{-6}}{4A^2\phi^{-6}}, \\ \therefore \Gamma &= \frac{3}{2}, \end{aligned} \tag{4.12}$$

where $A = \frac{2c^2|\beta|}{\kappa}\sqrt{1+\frac{2}{3|\beta|}}$ is a constant.

Considering Eq.(2.58) for flat FLRW universe and $\varepsilon = -1$ then the kinetic term becomes

$$\dot{\phi}^2 = \frac{2\dot{H} + 8\pi G\rho_{\mathrm{m}}}{3H^2 - 8\pi G\rho_{\mathrm{m}}}. \tag{4.13}$$

We can approximate that the dust term is much less contributive compared to the $\dot{H}$ and H^2 terms therefore we can neglect the ρ_{m} term and we obtain

$$\dot{\phi}^2 \approx \frac{2\dot{H}}{3H^2} = -\frac{2}{3\beta}, \quad \phi(t) \approx \sqrt{\frac{2}{3|\beta|}}\,(t_{\mathrm{s}} - t) \tag{4.14}$$

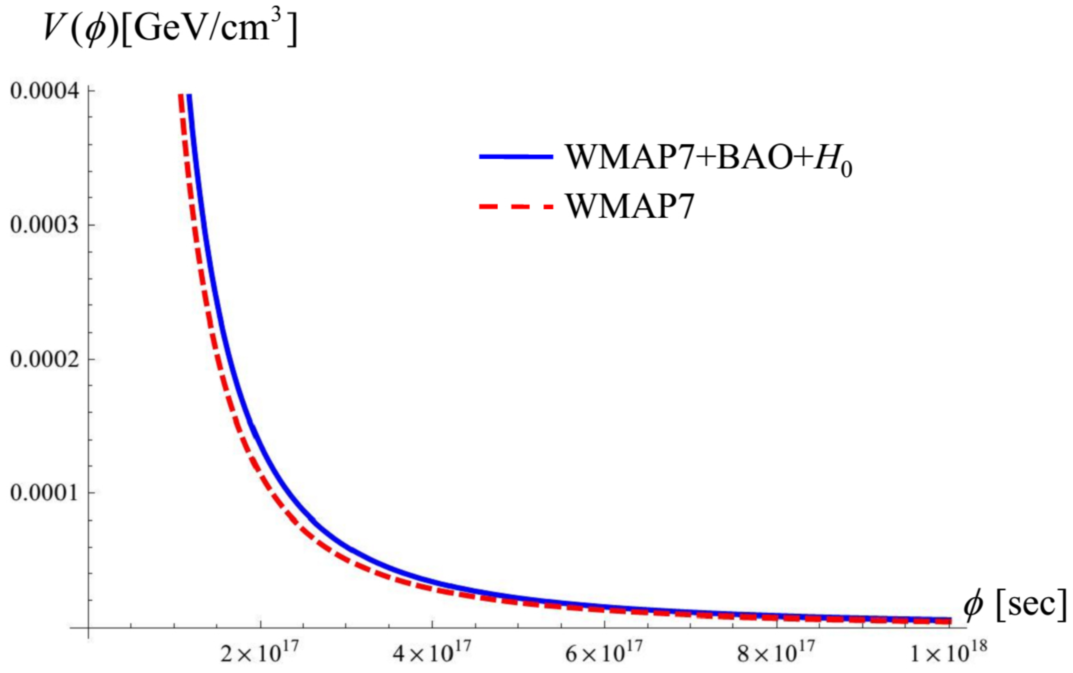

Figure 4: Potential versus field using WMAP7+BAO+H_0, WMAP7 for the case of tachyonic field domination ($V \propto \phi^{-2}$) [149].

Now we will use the solution of the scalar field $\phi(t)$ with tachyonic field dominant approximation to find the tachyonic potential. Actually, this is not an exact way to deriving the potential which has also contribution of baryonic matter density. However the approximation which we made here does not much alter the result and it could be roughly acceptable. Let $B \equiv \sqrt{3|\beta|/2} = constant$, hence we can be rewritten the future big-rip time $t_{\rm s} - t = B\phi$. By using Eq.(2.61), we find that

$$
\begin{aligned}
V(\phi) &= \left[\frac{3c^2H^2}{8\pi G} - \rho_{\rm m}c^2\right]\sqrt{\frac{3H^2+2\dot{H}}{3H^2-8\pi G\rho_{\rm m}}}, \\
&= \left[\frac{3c^2\beta^2}{\kappa(t_{\rm s}-t)^2} - \rho_{\rm m,0}c^2\left(\frac{t_{\rm s}-t_0}{t_{\rm s}-t}\right)^{3\beta}\right]\sqrt{\frac{1+\frac{2\dot{H}}{3H^2}}{1-\frac{\kappa\rho_{\rm m,0}}{3\beta^2}(t_{\rm s}-t)^2\left(\frac{t_{\rm s}-t_0}{t_{\rm s}-t}\right)^{3\beta}}}, \\
&\approx \left[\frac{3c^2\beta^2}{\kappa(B\phi)^2} - \rho_{\rm m,0}\,c^2\left(\frac{t_{\rm s}-t_0}{B\phi}\right)^{3\beta}\right] \\
&\quad \times \left[\frac{1-2/(3\beta)}{1-\rho_{\rm m,0}[\kappa/(3\beta^2)]\,(B\phi)^{2-3\beta}\,(t_{\rm s}-t_0)^{3\beta}}\right]^{1/2}. \qquad (4.15)
\end{aligned}
$$

Note that the term $1-2/(3\beta)$ is just $-w_{\rm eff,0}$. Furthermore, we can rearrange the potential in form of cosmological observables $H_0, \Omega_{\rm m,0}$ and q,

$$\begin{aligned} V(\phi) &= \left[\frac{3c^2\beta^2}{\kappa(t_{\rm s}-t)^2} - \rho_{\rm m,0}c^2\left(\frac{t_{\rm s}-t_0}{t_{\rm s}-t}\right)^{3\beta}\right]\left[\frac{1+\frac{2\dot{H}}{3H^2}}{1-\frac{\kappa\rho_{\rm m,0}}{3\beta^2}(t_{\rm s}-t)^2\left(\frac{t_{\rm s}-t_0}{t_{\rm s}-t}\right)^{3\beta}}\right]^{1/2}, \\ &= \frac{c^2}{\kappa}\left[\frac{(2|\beta|)(3|\beta|)}{2(t_{\rm s}-t)^2} - \frac{\kappa\rho_{\rm m,0}}{3H_0^2}(3H_0^2)\left(\frac{(-\beta/H_0)^{3\beta}}{(3|\beta|/2)^{3\beta/2}\phi^{3\beta}}\right)\right] \\ &\times\left[\frac{1+\frac{2}{3|\beta|}}{1-\frac{\kappa\rho_{\rm m,0}}{3H_0^2}\frac{H_0^2}{\beta^2}\left(\frac{3|\beta|}{2}\right)^{(2-3\beta)/2}\left(\frac{-\beta}{H_0}\right)^{3\beta}\phi^{2-3\beta}}\right]^{1/2}, \\ &= \frac{c^2}{\kappa}\left[\frac{2|\beta|}{\phi^2} - 3\Omega_{\rm m,0}\left(\frac{3|\beta|}{2}\right)^{-3\beta/2}\phi^{-3\beta}(-\beta)^{3\beta}H_0^{2-3\beta}\right] \\ &\times\left[\frac{1+2/(3|\beta|)}{1-\Omega_{\rm m,0}H_0^{2-3\beta}\left(\frac{3|\beta|}{2}\right)^{1-3\beta/2}(-\beta)^{3\beta-2}\phi^{2-3\beta}}\right]^{1/2}, \\ &\approx \frac{c^2}{\kappa}\left[\frac{2|\beta|}{\phi^2} - 3\left(\frac{3}{2|\beta|}\right)^{\frac{3|\beta|}{2}}\Omega_{\rm m,0}\,H_0^{2+3|\beta|}\phi^{3|\beta|}\right] \\ &\times\left[\frac{1+2/(3|\beta|)}{1-\left(\frac{3}{2}\right)^{1+\frac{3|\beta|}{2}}\Omega_{\rm m,0}\,(H_0\phi)^{2+3|\beta|}|\beta|^{-1-\frac{3|\beta|}{2}}}\right]^{1/2}, \end{aligned} \tag{4.16}$$

where $\beta = \beta(q) = (1+q)^{-1}$. This is plotted in Fig. 5 where the field values at present $z = 0$ and at $z = 0.45$ are shown in Table 6.

Table 6: The scalar field values at present $z = 0$ and at $z = 0.45$.

Parameter	**WAMP7+BAO+H_0**	**WAMP7**
$\phi\|_{z=0}$	1.268×10^{17} sec	1.392×10^{17} sec
$\phi\|_{z=0.45}$	7.803×10^{16} sec	8.555×10^{16} sec

In order to account for the late acceleration, the tachyonic potential should not be steeper than the potential $V \propto \phi^{-2}$ [48, 49]. To check whether our derived tachyonic potential could fit in this criteria, i.e. whether it is shallower than $V \propto \phi^{-2}$, we use dimensionless variable, Γ, and its values is one-half, $\Gamma = 3/2$, as in Eq.(4.12). Hence in general the potential with $\Gamma < 3/2$ still satisfies this criteria. Considering the potential from Eq.(4.16), we use both derived datasets to compute its dynamical slope $\Gamma(\phi)$ which is in very complicated form and we plot this in Fig. 6. We found that by using our data with the field value at present, for both WMAP7+BAO+H_0 and for WMAP7 we found $\Gamma(\phi(z=0)) = 1.500$ up to

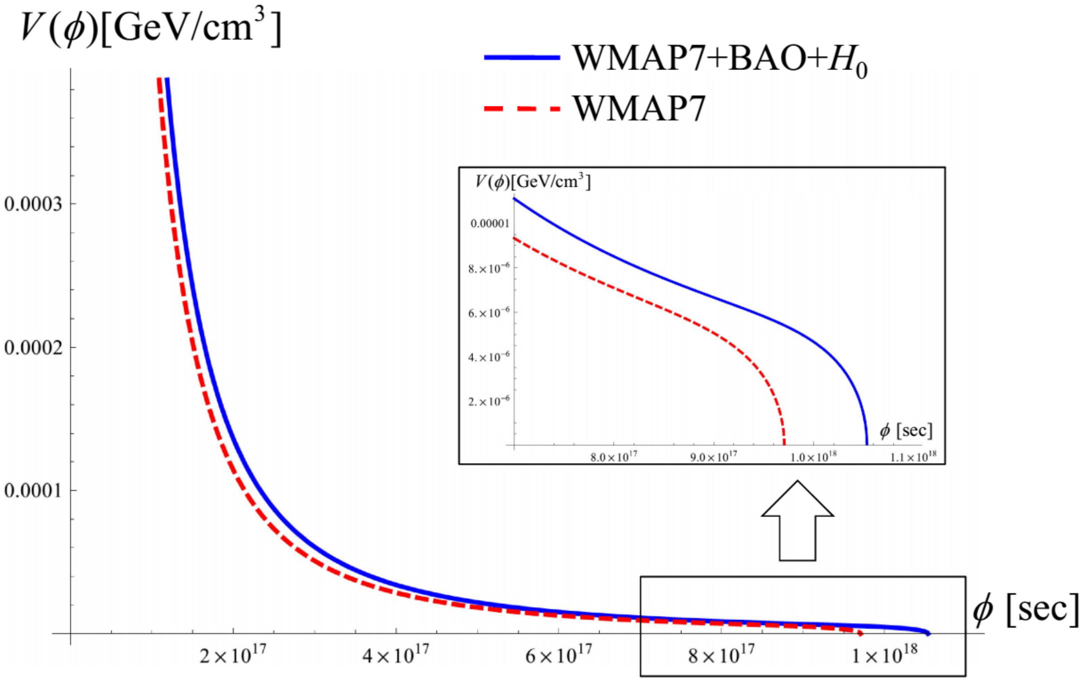

Figure 5: Approximated potential versus field using WMAP7+BAO $+H_0$, WMAP7 for the case of mixed tachyonic field with barotropic dust [149].

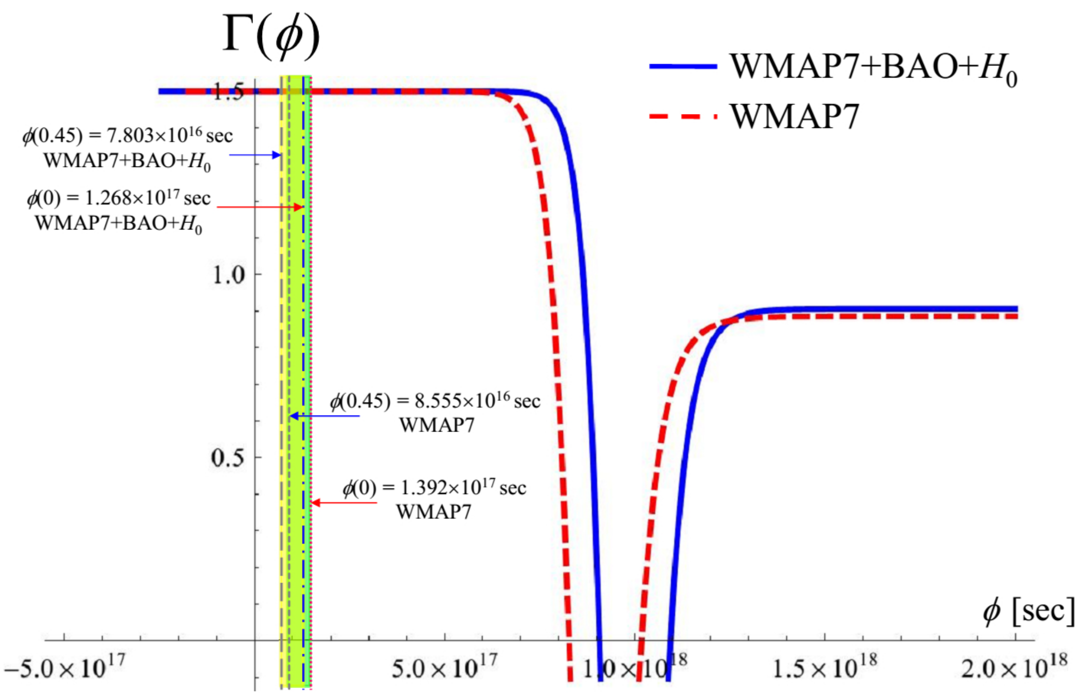

Figure 6: Dimensionless variable Γ plotted versus field using WMAP7+BAO $+H_0$ and WMAP7. The considered region for late universe $z < 0.45$ lies in the bars. This is for the case of mixed tachyonic field with barotropic dust [149].

three decimal digits. These values at present are approximately the same as that of the values derived from $V \propto \phi^{-2}$ where this potential is found when the universe is filled with tachyon field as a single component. Indeed from our derived potential Eq.(4.16), in the limit of $\Omega_{\rm m,0} \to 0$ our potential becomes $V \propto \phi^{-2}$. The other forms of the tachyonic potentials such as $V = V_0/[\cosh(a\phi/2)]$ and $V = V_0 e^{(1/2)m^2\phi^2}$ have $\Gamma = 1 - \text{csch}^2(a\phi/2)$ and $1 + (m\phi)^{-2}$ respectively. These examples are typical tachyonic potentials which also have dynamical slopes. In Fig. 6, $\Gamma(\phi)$ diverges twice however, in the region we consider where the redshift from $z = 0.45$ to $z = 0$, the value of Γ stays approximately at 1.5.

4.2 NMDC with Power-Law Cosmology

The cosmological parameters derived from WMAP9 (combined WMAP9 +eCMB+BAO+H_0) dataset [93], PLANCK+WP dataset [94] and PLANCK including polarization and other external parameters (TT, TE, EE+lowP+Lensing +ext.) dataset [95] are shown in Table 7 for NMDC with canonical power-law and in Table 8 for NMDC with phantom power-law. In this section, we are starting with applying the canonical power-law where $a = a_0(t/t_0)^\alpha$ as shown in Subsection (2.2.1) with $\varepsilon = +1$. Here a_0 is scale factor at a present time and we will set to unity, t_0 is age of the universe at present and α is constant exponent. The power-law expansion has been widely considered in astrophysical observations, for example in [76, 77, 108, 109] and also in [150] for constraints. It is found that the attractor solution of a canonical scalar field evolving under exponential potential [100] and also the same for the solution of barotropic fluid-dominant universe. In this model, the universe is under acceleration phase if $\alpha > 1$. We consider constant α in a range $0 < \alpha < \infty$. Hence, to calculate α at the present we use the details from Subsection (2.2.1) for Hubble parameter, H, its time derivative, $\dot{H}$, dust energy density, $\rho_{\rm m}$, and $\alpha = H_0 t_0$.

In the scenario of phantom power-law function for which $a \sim (t_{\rm s} - t)^\beta$ and $\varepsilon = -1$. Here $t_{\rm s}$ is the future singularity Big-Rip time defined as in [110] and β is a constant. In this case we use details from Subsection (2.2.2) for Hubble parameter, its time derivative, dust energy density and $\beta = H_0(t_0 - t_{\rm s})$ to calculate the β exponent. At present, $t = t_0$, the Big-Rip time $t_{\rm s}$ can be estimated from Eq.(2.98); that is,

$$t_{\rm s} \approx t_0 - \frac{2}{3(1 + w_{\rm DE})} \frac{1}{H_0\sqrt{1 - \Omega_{\rm m,0}}}$$

Here, the EoS parameter of dark energy $w_{\rm DE}$ must be less than -1. Above expression can be derived by assuming the flat geometry and constant dark energy EoS parameter [101, 102]. This type of expansion function with phantom scalar field was considered in [151].

Considering the case with constant potential where the potential is in a form of cosmological constant, $V(\phi) = \Lambda/(8\pi G)$ and our universe filled with dust and scalar field term (where it is including both free kinetic term and the NMDC term), the Friedmann equation, Eq.(3.3), can be written as

Table 7: Derived parameters from the combined WMAP9 (WMAP9+eCMB +BAO+H_0), PLANCK+WP and *TT,TE,EE*+lowP+Lensing +external data.

Parameters	WMAP9+eCMB+BAO+H_0 [93]	PLANCK+WP [94]	TT,TE,EE+lowP+Lensing+ext. [95]
t_0	$(4.346(4) \pm 0.018(6)) \times 10^{17}$ sec	$(4.360(6) \pm 0.015(1)) \times 10^{17}$ sec	$(4.354(9) \pm 0.006(6)) \times 10^{17}$ sec
	13.772 ± 0.059 Gyr	13.817 ± 0.048 Gyr	13.799 ± 0.021 Gyr
H_0	$(2.245(9) \pm 0.025(9)) \times 10^{-18}\text{sec}^{-1}$	$(2.18(1) \pm 0.03(8)) \times 10^{-18}\text{sec}^{-1}$	$(2.195(1) \pm 0.014(9)) \times 10^{-18}\text{sec}^{-1}$
	69.32 ± 0.80 km/s/Mpc	67.3 ± 1.2 km/s/Mpc	67.74 ± 0.46 km/s/Mpc
$\Omega_{\text{m},0}$	$0.2865^{+0.0096}_{-0.0095}$	$0.315^{+0.016}_{-0.018}$	0.3089 ± 0.0062
$\rho_{\text{c},0}$	$(9.019(6) \pm 0.208(8)) \times 10^{-27}\text{kg/m}^3$	$(8.50(6) \pm 0.14(8)) \times 10^{-27}\text{kg/m}^3$	$(8.618(6) \pm 0.117(0)) \times 10^{-27}\text{kg/m}^3$
$\rho_{\text{m},0}$	$(2.584(1)^{+0.146(4)}_{-0.145(5)}) \times 10^{-27}\text{kg/m}^3$	$(2.67(9)^{+0.18(3)}_{-0.19(9)}) \times 10^{-27}\text{kg/m}^3$	$(2.662(3) \pm 0.089(6)) \times 10^{-27}\text{kg/m}^3$
w_{DE} (of wCDM)	$-1.073^{+0.090}_{-0.089}$	$-1.49^{+0.65}_{-0.57}$	$-1.019^{+0.075}_{-0.080}$

Table 8: Expansion derived parameters from the three datasets.

Parameters	WMAP9+eCMB+BAO+H_0	PLANCK+WP	TT,TE,EE+lowP+Lensing+ext.
α	$0.9761(6) \pm 0.0154(3)$	$0.951(0) \pm 0.019(9)$	$0.9559(4) \pm 0.0079(4)$
$q_{\mathrm{power-law}}$	$0.0244(2) \pm 0.0161(9)$	$0.0515(2) \pm 0.0220(0)$	$0.04613(4) \pm 0.00868(9)$
t_{s}	$(5.248(1)^{+6.056(1)}_{-5.990(1)}) \times 10^{18}$ sec	$(1.19(0)^{+1.03(2)}_{-0.91(1)}) \times 10^{18}$ sec	$(1.96(6)^{+7.62(8)}_{-8.13(4)}) \times 10^{19}$ sec
	$166.2(9)^{191.8(9)}_{189.8(0)}$ Gyr	$37.7(1)^{+32.7(0)}_{-28.8(7)}$ Gyr	$622.9(4)^{+2416.9(8)}_{-2577.3(1)}$ Gyr
β	$-10.81(1)^{+13.73(1)}_{-13.58(1)}$	$-1.64(4)^{+2.28(2)}_{-2.01(8)}$	$-42.1(9)^{+167.8(8)}_{-178.9(9)}$
q_{phantom}	$-1.0925(0)^{+0.1174(8)}_{-0.1162(0)}$	$-1.6082(7)^{+0.8443(3)}_{-0.7440(6)}$	$-1.0237(0)^{+0.0943(1)}_{-0.1005(5)}$

$$\begin{aligned} 3H^2 &= 8\pi G\left[\frac{1}{2}\dot{\phi}^2(\varepsilon - 9\kappa H^2) + \rho_{\rm m} + V\right], \\ &= 8\pi G\left[\frac{1}{2}\dot{\phi}^2(\varepsilon - 9\kappa H^2) + \rho_{\rm m} + \frac{\Lambda}{8\pi G}\right]. \end{aligned} \tag{4.17}$$

In this case, we can find $\dot{\phi}^2$ from Eq.(3.5) by rearrange to

$$\begin{aligned} [(\varepsilon - 3\kappa H^2)\dot{\phi}]\dot{} + 3H(\varepsilon - 3\kappa H^2)\dot{\phi} &= 0, \\ \frac{d((\varepsilon - 3\kappa H^2)\dot{\phi})}{(\varepsilon - 3\kappa H^2)\dot{\phi}} &= -3\frac{da}{a}, \end{aligned} \tag{4.18}$$

and then take the integrate on both sides,

$$\begin{aligned} (\varepsilon - 3\kappa H^2)\dot{\phi} &= Ca^{-3}, \\ \dot{\phi} &= \frac{Ca^{-3}}{(\varepsilon - 3\kappa H^2)}. \end{aligned} \tag{4.19}$$

Here it is freely to choose the values of the constant of integration, therefore we let $C = \varepsilon\sqrt{2\mu}$. Then we have

$$\dot{\phi} = \frac{\varepsilon\sqrt{2\mu}}{a^3(\varepsilon - 3\kappa H^2)}. \tag{4.20}$$

Substituting Eq.(4.20) into Eq.(4.17) and rewritten in a form of density parameter Ω as

$$\begin{aligned} 3H^2 &= 8\pi G\left[\frac{1}{2}\left(\frac{\varepsilon\sqrt{2\mu}}{a^3(\varepsilon - 3\kappa H^2)}\right)^2(\varepsilon - 9\kappa H^2) + \rho_{\rm m} + \frac{\Lambda}{8\pi G}\right], \\ &= 8\pi G\left[\frac{1}{2}\frac{2\varepsilon^2\mu}{a^6(\varepsilon - 3\kappa H^2)^2}(\varepsilon - 9\kappa H^2) + \rho_{\rm m} + \frac{\Lambda}{8\pi G}\right], \\ H^2 &= \frac{8\pi G}{3H_0^2}H_0^2\left[\frac{\mu(\varepsilon - 9\kappa H^2)}{a^6(\varepsilon - 3\kappa H^2)^2} + \rho_{\rm m} + \frac{\Lambda}{8\pi G}\right], \\ &= H_0^2\left[\frac{\mu(\varepsilon - 9\kappa H^2)}{\rho_{\rm c}a^6(\varepsilon - 3\kappa H^2)^2} + \frac{\rho_{\rm m,0}}{\rho_{\rm c}a^3} + \frac{\Lambda}{8\pi G\rho_{\rm c}}\right], \\ &= H_0^2\left[\Omega_{\Lambda,0} + \frac{\Omega_{\rm m,0}}{a^3} + \frac{\Omega_{\phi,0}(\varepsilon - 9\kappa H^2)}{a^6(\varepsilon - 3\kappa H^2)^2}\right], \end{aligned} \tag{4.21}$$

where $\rho_{\rm c} = 3H_0^2/8\pi G$ is the critical density and Ω_i are density parameters of the $i^{\rm th}$ component of cosmic fluids and defined

$$\Omega_{\rm m,0} = \frac{\rho_{\rm m,0}}{\rho_{\rm c}},\ \ \Omega_{\phi,0} = \frac{\mu}{\rho_{\rm c}},\ \ \Omega_{\Lambda,0} = \frac{\Lambda}{8\pi G\rho_{\rm c}}. \tag{4.22}$$

In the case of our system come from Eqs.(3.3), (3.4) and (3.5) with zero potential and barotropic fluid is a closed autonomous dynamical system. The interesting particular solution of this system is when we set $\dot{\phi} = \psi(t)$ and $\dot{\psi}_{\rm p} = 0 = \ddot{\phi}$ hence $\psi_{\rm p} = \dot{\phi} =$ *constant*. As it is found in [37] that the solution is a de-Sitter type. For the case of two coupling constants corresponding to each other through the condition $\kappa \equiv \kappa_2 = -2\kappa_1$, as of Sushkov's model, the solution gives,

$$H^2 = \frac{\Lambda_{\rm NMDC}}{3}\,. \tag{4.23}$$

The effective cosmological constant is defined as

$$\Lambda_{\rm NMDC} = \frac{\varepsilon}{\kappa}. \tag{4.24}$$

The solution is found as $\psi_{\rm p} = \dot{\phi} = 1/\sqrt{\kappa}$, which is

$$\phi_{\rm p} \;=\; \frac{t}{\sqrt{\kappa}} + \phi_0, \tag{4.25}$$

from this solution, it is suggesting that the coupling constant should take a positive value and the effective cosmological constant, $\Lambda_{\rm NMDC}$ should be positive. However the general consideration in [42, 117, 118], the coupling NMDC term, κ is strong at early time hence gives new inflation mechanism that made our universe transition from a quasi-de-Sitter phase to power-law phase happens naturally. At late time, the system having constant potential $V = \Lambda/(8\pi G)$, the transition will change from one quasi-de-Sitter to another de-Sitter phase is also possible. The particular solution suggests that $\Lambda_{\rm NMDC} > 0$. Therefore, in presence of the usual cosmological constants, both of Λ (from a constant V) and $\Lambda_{\rm NMDC}$ (effective cosmological constant) can be contributed both at late time. In the case of having enough inflation, κ is estimated to 10^{-74} sec^2 [42]. Hence $\Lambda_{\rm NMDC} \approx 10^{74}$ sec^{-2} and it seems to be large, therefore the NMDC coupling term is suppressed by its multiplication with curvature which is very small at late time. Fig. 7 and Fig. 8 are the plot of the effective cosmological constant versus the usual cosmological constant that come from a constant potential. To plot $\Lambda_{\rm eff}$ versus Λ we have to find the effective as a function of a usual one, $\Lambda_{\rm eff} = \Lambda_{\rm eff}(\Lambda)$. The results that give us two roots of function, therefore we denote those two roots with number 1 and 2, respectively. Those two roots of function give the same but opposite form of the plot to each other and this behaviors on both canonical and phantom plots.

From the Table 7 and Table 8, we see that the value of H_0 is kinematically hence it is model-independent. The value of EoS parameter of dark energy $w_{\rm DE}$ is of the wCDM model obtained from observational data. The barotropic density contributes to power-law expansion shape while the NMDC and Λ contributes to de-Sitter expansion. In combination of NMDC with constant potential (Λ term), the expansion function is a mixing between those two. For the phantom case, the free kinetic part of the Lagrangian has negative kinetic energy, $-g_{\mu\nu}\phi^{;\mu}\phi^{;\nu}$, therefore the combined effect to the expansion should be the phantom-power law (super acceleration) mixing with the de-Sitter expansion. We will calculate the cosmological constant, Λ, of the model using observed value of $w_{\rm DE}$ and using suggested value of $\kappa \approx 10^{-74}$ sec^2 as required by the end of inflation [42]. Therefore

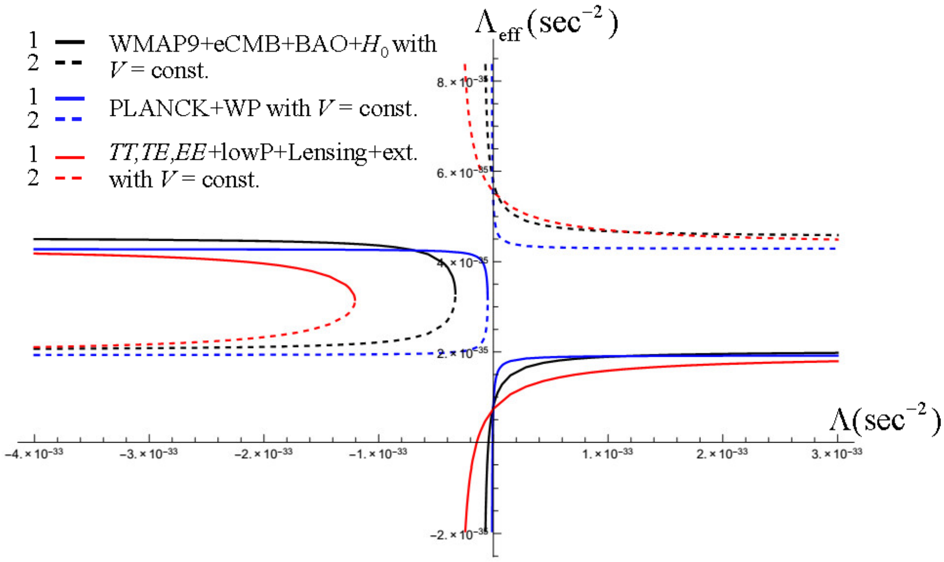

Figure 7: The canonical plots of Λ_{eff} versus a usual Λ coming from the constant potential. There are two roots of function which we denoted with number 1 and 2, respectively.

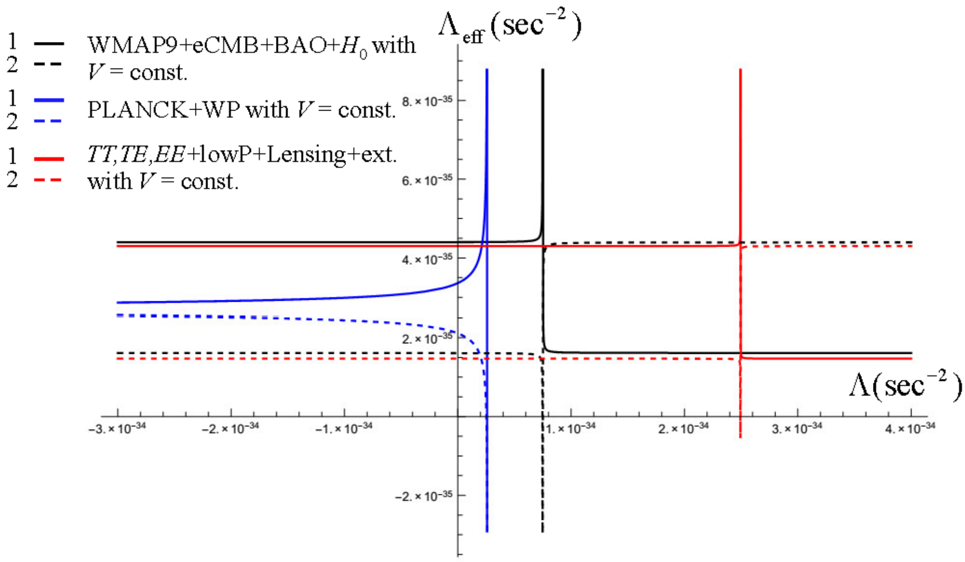

Figure 8: The phantom plots of Λ_{eff} versus a usual Λ coming from the constant potential. There are two roots of function which we denoted with number 1 and 2, respectively.

the coupling constant, κ, is regarded as a constant in data analysis as suggested. Fig. 9 and Fig. 10 are the evolutionary plot of the cosmological constant versus the EoS parameter for both canonical and phantom cases. Both canonical and phantom plots are look alike but up-side-down to each other and there are two singularity points around (approximately) $w_\phi = -1$. Furthermore, we also plot a cosmological as a function of redshift, z, by using $t = t_0/(1+z)^{1/\alpha}$ and $t_\mathrm{s} - t = (t_\mathrm{s} - t_0)/(1+z)^{1/\beta}$ for canonical and phantom respectively. The plots of Λ versus z are shown in Fig. 11 and in Fig. 12 where the range of plot are from present $z = 0$ to $z = 2$ and this rage is near our range of consideration in the model, $z \lesssim 0.45$. In the canonical plot of $\Lambda(z)$, the cosmological values are starting from the negative value and increasing to the positive part. While in the phantom plot, they are starting from positive value and increasing as well. From the plots, we can estimate the present values of Λ, at $z = 0$, and those values are the same as the calculation directly from the equation. Values of cosmological constant in this model using three datasets are shown in Table 9. We show plots of Λ versus varying value of the exponents α and β in Fig. 13 and Fig. 14 respectively.

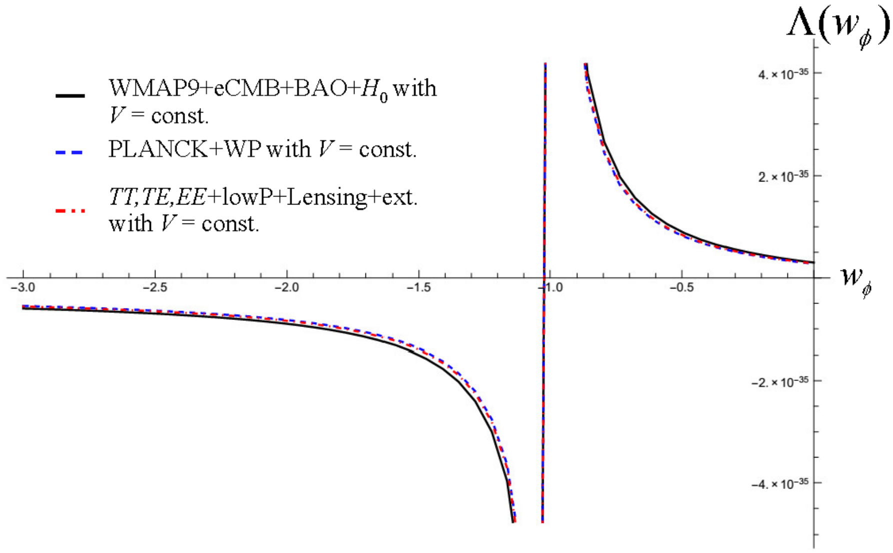

Figure 9: The canonical plots of Λ versus the EoS parameter w_ϕ. There are two singularity points around (approximately) $w_\phi = -1$.

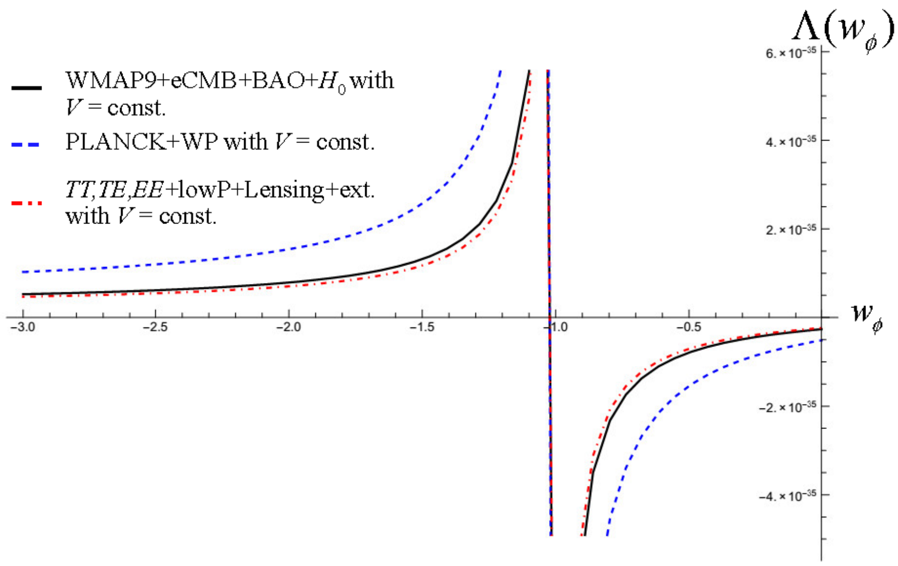

Figure 10: The phantom plots of Λ versus the EoS parameter w_ϕ. There are two singularity points around (approximately) $w_\phi = -1$.

Table 9: Value of the cosmological constant with power-law expansion (using $\varepsilon = +1$) and phantom power-law expansion (using $\varepsilon = -1$) for each of observational data.

Parameters	WMAP9+eCMB+BAO+H_0	PLANCK+WP	TT, TE, EE+lowP+Lensing+ext.
$\Lambda_{(\varepsilon=+1)}$ (sec^{-2})	$-8.5194(5)^{+43.5168(8)}_{-4.5365(9)} \times 10^{-35}$	$-1.3997(8)^{+4.5685(7)}_{-0.6130(8)} \times 10^{-35}$	$-2.9833(7)^{+3.9590(8)}_{-2.3899(2)} \times 10^{-34}$
$\Lambda_{(\varepsilon=-1)}$ (sec^{-2})	$7.4792(3)^{+38.2550(6)}_{-21.2910(5)} \times 10^{-35}$	$2.6114(3)^{+8.8452(5)}_{-32.4699(7)} \times 10^{-35}$	$2.4939(1)^{+3.3466(3)}_{-2.0660(4)} \times 10^{-34}$

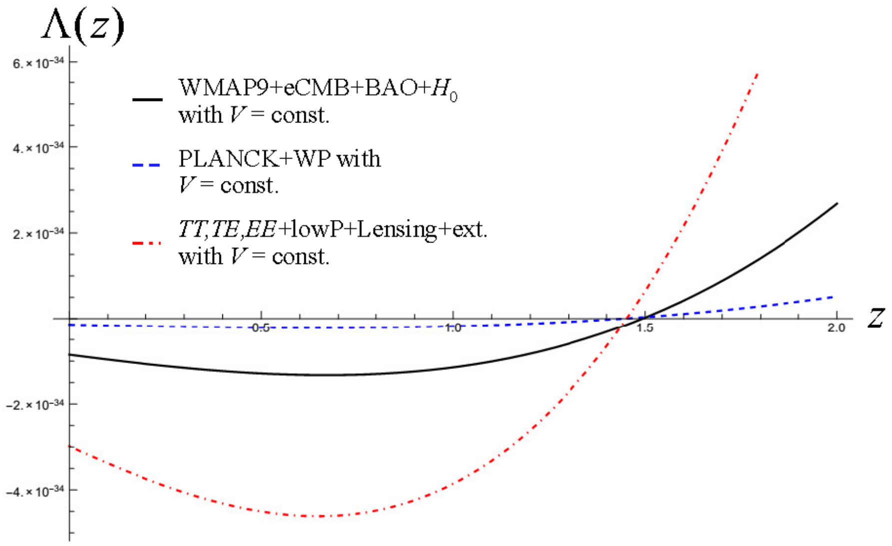

Figure 11: The canonical plots of Λ versus redshift z. The values of Λ are starting from the negative values and then increasing to positive values.

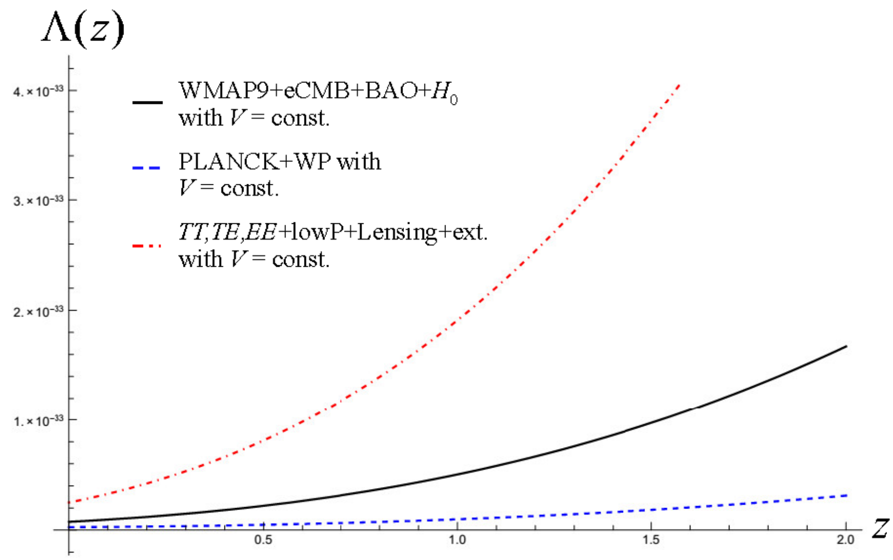

Figure 12: The canonical plots of Λ versus redshift z. The values of Λ are starting from the positive values and then increasing to positive values.

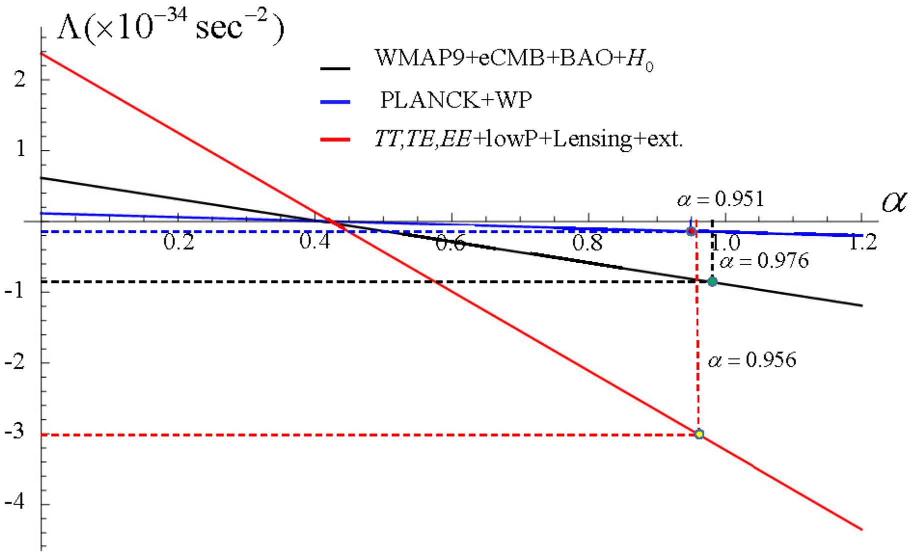

Figure 13: Parametric plots of Λ versus α in a power-law expansion [152].

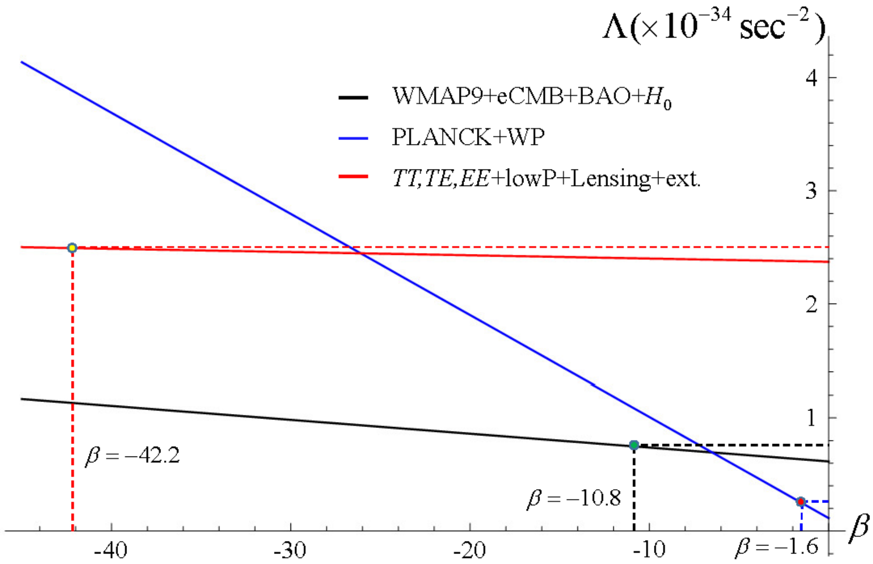

Figure 14: Parametric plots of Λ versus β in a phantom power-law expansion [152].

CHAPTER V

CONCLUSIONS AND OUTLOOKS

In this chapter, we conclude our works both tachyonic model and NMDC model with canonical and phantom power-law scenarios. In a first section, the tachyonic (phantom) power-law are concluded. A second section is the conclusions of the NMDC with power-law cosmology. Finally, a last section is the outlook and possibility of the future work of the NMDC model.

5.1 Tachyonic (Phantom) Power-Law Cosmology

The model of tachyonic-driven universe are investigated in the scenario of canonical power-law cosmology, $a \sim t^{\alpha}$, and phantom power-law cosmology, $a \sim (t_{\rm s} - t)^{\beta}$. In our works, the universe is assumed a flat FLRW geometry ($k = 0$) and filled with tachyonic scalar field and pressureless matter. We consider late universe, nearly present time, when dark energy has dominated in a short rage of redshift $z \lesssim 0.45$ to avoiding the singularity. WMAP7 and its combined with BAO and $H(z)$ at present, at $z = 0$, (WMAP7+BAO+H_0) derived datasets are used to constrain the equation of state (EoS) parameter in this study. We find the exponents of canonical and phantom power-law expansions and other cosmological observable parameters. We also want to know whether the power-law is still valid in the scenario of tachyonic scalar field.

We find that, in general, the equation of state parameter of the tachyonic scalar field in terms of Hubble parameter H, its time derivative $\dot{H}$ and the matter density $\rho_{\rm m}$; that is, $w_{\phi}(H, \dot{H}, \rho_{\rm m})$ are the same as the equation of state parameter obtained from the quintessence scalar field although the forms of potential and the field solution are different for both of them [77, 78]. Therefore it can be said that, for quintessence and tachyonic field, the equation of state does not depend on type of the scalar field but depends only on form of expansion function of the scale factor. Results from canonical power-law cosmology with tachyonic scalar field, the present values of dark energy equation of state are shown that their values do not match both WMAP7 and combined WMAP7 datasets as shown in Table 3 comparing with the observational data in Table 4comparing with the observational data in Table 4. In the case of phantom power-law cosmology with tachyonic scalar field, the values of equation of state we obtained do not much differ from observational results as shown in Table 5 comparing with the observational data in Table 4. Therefore we conclude that for the canonical power-law cosmology model with tachyonic scalar field are excluded by these observational data.

From parametric plot in Fig. 2, we see that the values of $\beta \lesssim -6$ are staying within the expected range $(-2, -1)$ of the EoS parameter at present, $w_{\phi,0}$. We reconstruct the tachyonic potential as the function of the observable parameters, i.e. the present Hubble parameter, H_0, the dimensionless density parameter of matter at present, $\Omega_{\rm m,0}$, and the deceleration parameter q, see Eq.(4.16). From the new form of tachyonic potential, we find that the dimensionless slope variable

Γ required for determine the steepness of the potential of our derived potential at present and it is about 1.5 matched with our standard requirement comparing with the steepness of the potential $V(\phi) \sim \phi^{-2}$. For the phantom-power-law cosmology with tachyonic scalar field, the potential found here can be reduced to $V = V_0\phi^{-2}$ in the limit of the dimensionless density parameter of matter at present approaches to zero, $\Omega_{\mathrm{m},0} \to 0$.

5.2 NMDC with Power-Law Cosmology

First of all, we give a brief review of the non-minimum derivative coupling (NMDC) of the canonical scalar field, $\partial_\mu\phi\partial_\nu\phi$ to the curvature in cosmology or to Einstein tensor, $G_{\mu\nu}$, as seen in Section (3.1). In our work, we are interested in and starting with the action in Suskov's model [42, 117] and we assumed the flat FLRW universe filled with usual scalar field and pressureless matter. Our usual scalar field are in the non-minimal derivative coupling to Einstein tensor with the coupling constant κ. We are investigate the NMDC model in the scenario of canonical and phantom power-law cosmology. We consider the case when the potential is constant and in the form of cosmological constant, $V(\phi) = \Lambda/(8\pi G)$, and the coupling constant is positive. We assumed that the universe kinematically expands with power-law or super acceleration only from very recent redshift $z \lesssim 0.45$ or when dark energy has dominated. We use the derived observational data from the combined WMAP9 dataset (WMAP9+eCMB+BAO+H_0), PLANCK+WP dataset and PLANCK including polarization and other external parameters (TT, TE, EE+lowP+Lensing +ext.) dataset to find cosmological constant of the theory.

Our derived cosmological parameters from those three sources are shown in Table 7 and in Table 8 for using in the NMDC with canonical and phantom power-law respectively. The NMDC coupling term behaves like an effective cosmological constant and it is in a form of inverse proportional to effective cosmological constant as $\Lambda_{\mathrm{NMDC}} = \varepsilon/\kappa$. Hence the NMDC term, $\kappa G_{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, together with the free kinetic term, $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, contributes to de-Sitter like acceleration to the dynamics in the slow-roll regime at early time, i.e. inflation. At late time, the NMDC contribution is very little due to small curvature and in presence of the pressureless dust matter term and cosmological constant, Λ , modeled with canonical and phantom power-law(super-acceleration) expansion functions. The results of the cosmological constant values for power-law expansion are shown in Table 9. We see that the results are in the same order of ΛCDM model but with the negative sign. Hence in this model, the cosmological constant have to be negative in order to have power-law expansions. Therefore the canonical power-law expansion is not suitable for modeling NMDC cosmology. For the phantom power-law expansion (super-acceleration), the results are in the same order and the same sign of ΛCDM model as shown in Table 9. The values of the cosmological constant for both canonical and phantom power-law scenario are very sensitive to the value of a future big-rip time t_{s} which give us with large error bar.

5.3 Outlooks

We study the NMDC model with Palatini formalism by defining the connection $\Gamma^{\sigma}_{\mu\nu}$ and metric tensor $g^{\mu\nu}$ as the independent field. In other words the connection is not the Levi-Civita connection of metric $g_{\mu\nu}$ [153, 154, 155, 156]. There are two separated coupling constant with non-zero potential, one for the Ricci scalar R and another one for Ricci tensor $R_{\mu\nu}$. We can write the NMDC action with Palatini formalism in a form [157]

$$\begin{aligned} S(g,\Gamma) &= \int d^4x\sqrt{-g}\Bigg\{\frac{\tilde{R}(\Gamma)}{8\pi G} - \left[\varepsilon g_{\mu\nu} + \kappa_1 g_{\mu\nu}\tilde{R}(\Gamma) + \kappa_2\tilde{R}_{\mu\nu}(\Gamma)\right]\phi^{,\mu}\phi^{,\nu} \\ &\quad - 2V(\phi)\Bigg\} + \tilde{S}_{\rm m}(h_{\mu\nu},\Psi), \end{aligned} \tag{5.1}$$

where $\tilde{R}_{\mu\nu}(\Gamma)$ is the Ricci tensor with Ricci scalar $\tilde{R}(\Gamma)$ in Palatini formalism. The Ricci tensor defined as

$$\tilde{R}_{\mu\nu}(\Gamma) = \tilde{R}^{\lambda}_{\mu\lambda\nu}(\Gamma) = \partial_\lambda\Gamma^{\lambda}{}_{\mu\nu} - \partial_\nu\Gamma^{\lambda}{}_{\mu\lambda} + \Gamma^{\lambda}{}_{\sigma\lambda}\Gamma^{\sigma}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\sigma\nu}\Gamma^{\sigma}{}_{\mu\lambda}, \tag{5.2}$$

and the Ricci scalar

$$\tilde{R} = \tilde{R}(\Gamma) = g^{\mu\nu}\tilde{R}_{\mu\nu}(\Gamma). \tag{5.3}$$

Therefore we can define the Einstein tensor in Palatini formalism

$$\tilde{G}_{\mu\nu}(\Gamma) = \tilde{R}_{\mu\nu}(\Gamma) - \frac{1}{2}g_{\mu\nu}\tilde{R}(\Gamma). \tag{5.4}$$

Moreover, we may use the action of NMDC as usual,

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{8\pi G} - \left(g^{\mu\nu} + \kappa G^{\mu\nu}\right)\phi_{,\mu}\phi_{,\nu} - 2V(\phi)\right]. \tag{5.5}$$

Then we may consider the Higgs-like potential of scalar field [158],

$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \phi_0^2\right)^2, \tag{5.6}$$

to investigate the Higgs inflation model by study the dynamics of the scalar field.

On the other hands, we may investigate how the different forms of potential effect to the model by consider the different potential forms i.e. the power-law potential $V(\phi) = V_0\phi^m$, the exponential potential $V(\phi) = V_0 e^{m\phi}$, the effective potential in the open string theory $V(\phi) = V_0/\cosh(\phi/\phi_0)$. Finally, in the future work, I will continue work on the NMDC model with both Palatini formalism which give the difference dynamics compared with my previous works and metric formalism with various forms of potential.

REFERENCES

REFERENCES

[1] Amanullah, R., Lidman, C., Rubin, D., Aldering, G., Astier, P., Barbary, K. et al. (2010). Spectra and Light Curves of Six Type Ia Supernovae at $0.511 < z < 1.12$ and the Union2 Compilation. **Astrophys. J.**, 716, 712-738. [arXiv:1004.1711 [astro-ph.CO]].

[2] Astier, P., Guy, J., Regnault, N., Pain, R., Aubourg, E., Balam, D. et al. (SNLS Collaboration). (2006). The Supernova Legacy Survey: Measurement of Omega_M, Omega_Lambda and w from the First Year Data Set. **Astron. Astrophys.**, 447(1), 31-48. [arXiv:astro-ph/0510447].

[3] Goldhaber, G., Groom, D. E., Kim, A., Aldering, G., Astier, P., Conley, A. et al. (The Supernova Cosmology Project Collaboration). (2001). Timescale Stretch Parameterization of Type Ia Supernova B-Band Light Curves. **Astrophys. J.**, 558(1), 359-368. [arXiv:astro-ph/0104382].

[4] Perlmutter, S., Aldering, G., Della Valle, M., Deustua, S., Ellis, R.S., Fabbro, S. et al. (Supernova Cosmology Project Collaboration). (1998). Discovery of a supernova explosion at half the age of the Universe and its cosmological implications. **Nature** 391, 51-54. [arXiv:astro-ph/9712212].

[5] Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R.A., Nugent, P., Castro, P.G. et al. (Supernova Cosmology Project Collaboration). (1999). Measurements of Omega and Lambda from 42 High-Redshift Supernovae. **Astrophys. J.** 517, 565-586. [arXiv:astro-ph/9812133].

[6] Riess, A.G., Filippenko, A.V., Challis, P., Clocchiattia, A., Diercks, A., Garnavich, P.M. et al. (Supernova Search Team Collaboration). (1998). Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. **Astron. J.** 116, 1009-1038. [arXiv:astro-ph/9805201].

[7] Riess, A. G. (1999). Peculiar Velocities from Type Ia Supernovae. **Cosmic Flows Workshop, Victoria, Canada**. [arXiv:astro-ph/9908237].

[8] Riess, A.G., Strolger, L.-G., Tonry, J., Casertano, S., Ferguson, H.C., Mobasher, B. et al. (Supernova Search Team Collaboration). (2004). Type Ia Supernova Discoveries at $z > 1$ From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution. **Astrophys. J.** 607, 665-687. [arXiv:astro-ph/0402512].

[9] Riess, A.G., Strolger, L.-G., Casertano, S., Ferguson, H.C., Mobasher, B., Gold, B. et al. (2007). New Hubble Space Telescope Discoveries of

Type Ia Supernovae at $z > 1$: Narrowing Constraints on the Early Behavior of Dark Energy. **Astrophys. J.** 659, 98-121. [arXiv:astro-ph/0611572].

[10] Tonry, J.L., Schmidt, B.P., Barris, B., Candia, P., Challis, P., Clocchiatti, A. et al. (Supernova Search Team Collaboration). (2003). Cosmological Results from High-z Supernovae. **Astrophys. J.** 594, 1-24. [arXiv:astro-ph/0305008].

[11] Scranton, R., Connolly, A. J., Nichol, R. C. Sttebbins, A., Szapudi, I., Eisenstein, D. J., et al. (SDSS Collaboration). (2003). **Physical Evidence for Dark Energy**. Retrieved June 19, 2012, from http://arxiv.org/abs/astro-ph/0307335

[12] Tegmark, K., Strauss, M., Blanton, M., Abazajian, K., Dodelson, S., Sandvik, H., et al. (SDSS Collaboration). (2004). Cosmological parameters from SDSS and WMAP. **Phys. Rev. D** 69, 103501. [arXiv:astro-ph/0310723]

[13] Larson, D., Dunkley, J., Hinshaw, G., Komatsu, E., Nolta, M. R., Bennett, C. L. et al. (2011). Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Power Spectra and WMAP-Derived Parameters. **Astrophys. J. Suppl.** 192, 16. [arXiv:1001.4635 [astro-ph.CO]].

[14] Komatsu, E., Smith, K. M., Dunkley, J., Bennett, C. L., Gold, B., Hinshaw, G. et al. (WMAP Collaboration). (2011). Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. **Astrophys. J. Suppl.** 192, 18. [arXiv:1001.4538 [astro-ph.CO]].

[15] Hu, J.-W., Cai, R.-G., Guo, Z.-K., Hu, B. (2014). Cosmological parameter estimation from CMB and X-ray clusters after Planck. **JCAP** 1405, 020 [arXiv:1401.0717 [astro-ph.CO]].

[16] Masi, S., Ade, P.A.R., Bock, J.J., Bond, J.R., Borrill, J., Boscaleri, A. et al. (2002). The BOOMERanG experiment and the curvature of the universe. **Prog. Part. Nucl. Phys.** 48, 243-261. [arXiv:astro-ph/0201137].

[17] Allen, S.W., Schmidt, R.W., Ebeling, H., Fabian, A.C., Speybroeck, L. van (2004). Constraints on dark energy from Chandra observations of the largest relaxed galaxy clusters. **Mon. Not. Roy. Astron. Soc.** 353, 457. [arXiv:astro-ph/0405340].

[18] Rapetti, D., Allen, S.W., Weller, J. (2005). Constraining Dark Energy with X-ray Galaxy Clusters, Supernovae and the Cosmic Microwave Background. **Mon. Not. Roy. Astron. Soc.** 360, 555-564. [arXiv:astro-ph/0409574].

[19] Copeland, E.J., Sami, M., Tsujikawa, S. (2006). Dynamics of dark energy. **Int. J. Mod. Phys. D** 15, 1753-1936. [arXiv:hep-th/0603057].

[20] Padmanabhan, T. (2005). Dark Energy: the Cosmological Challenge of the Millennium. **Curr. Sci.** 88, 1057. [arXiv:astro-ph/0411044].

[21] Padmanabhan, T. (2006). Dark Energy: Mystery of the Millennium **AIP Conf. Proc.** 861, 179-196. [arXiv:astro-ph/0603114].

[22] Amendola, L., Tsujikawa, S. (2010). **Dark Energy: Theory and Observations**. New York, United State of America: Cambridge University Press.

[23] Starobinsky, A. A. (1980). A new type of isotropic cosmological models without singularity. **Phys. Lett. B** 91, 99-102.

[24] Guth, A. H. (1981). Inflationary universe: A possible solution to the horizon and flatness problems. **Phys. Rev. D** 23, 347.

[25] Sato, K. (1981). First Order Phase Transition of a Vacuum and Expansion of the Universe. **Mon. Not. Roy. Astro. Soc.** 195, 467-479.

[26] Albrecht, A. and Steinhardt, P.J. (1982). Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking. **Phys. Rev. Lett.** 48, 1220.

[27] Linde, A.D. (1982). A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems. **Phys. Lett. B** 108, 389-393.

[28] Dutta, S., Saridakis, E. N., Scherrer, R. J. (2009). Dark energy from a quintessence (phantom) field rolling near a potential minimum (maximum). **Phys. Rev. D** 79, 103005. [arXiv:0903.3412 [astro-ph.CO]].

[29] Liddle, A. R. and Scherrer, R. J. (1999) Classification of scalar field potentials with cosmological scaling solutions. **Phys. Rev. D** 59, 023509. [arXiv:astro-ph/9809272].

[30] Ratra, B. and Peebles, P. J. E. (1988). Cosmological consequences of a rolling homogeneous scalar field. **Phys. Rev. D** 37, 3406.

[31] Wetterich, C. (1988). Cosmology and the fate of dilatation symmetry. **Nucl. Phys. B** 302, 668-696.

[32] Armendariz-Picon, C., Mukhanov, V.F., Steinhardt, P.J. (2000). A Dynamical Solution to the Problem of a Small Cosmological Constant and Late-time Cosmic Acceleration. **Phys. Rev. Lett.** 85, 4438-4441. [arXiv:astro-ph/0004134].

[33] Armendariz-Picon, C., Mukhanov, V.F., Steinhardt, P.J. (2001). Essentials of k essence. **Phys. Rev. D** 63, 103510. [arXiv:astro-ph/0006373].

[34] De Felice, A. and Tsujikawa, S. (2010). f(R) Theories. **Living Rev. Relativity** 13, 3. [arXiv:1002.4928 [gr-qc]].

[35] Carroll, S. M., Duvvuri, V., Trodden, M., Turner, M. S. (2004). Is cosmic speed-up due to new gravitational physics? **Phys. Rev. D** 70, 043528. [arXiv:astro-ph/0306438].

[36] Amendola, L. (1993). Cosmology with Nonminimal Derivative Couplings. **Phys. Lett. B** 301, 175-182. [arXiv:gr-qc/9302010].

[37] Capozziello, S. and Lambiase, G. (1999). Nonminimal Derivative Coupling and the Recovering of Cosmological Constant. **Gen. Rel. Grav.** 31, 1005-1014. [arXiv:gr-qc/9901051].

[38] Capozziello, S., Lambiase, G., Schmidt, H. J. (2000). Nonminimal Derivative Couplings and Inflation in Generalized Theories of Gravity. **Annalen Phys.** 9(1), 39-48. [arXiv:gr-qc/9906051].

[39] Daniel, S. F., Caldwell, R. R. (2007). Consequences of a cosmic scalar with kinetic coupling to curvature. **Class. Quant. Grav.** 24, 5573-5580. [arXiv:0709.0009 [gr-qc]].

[40] Granda, L. N. and Cardona, W. (2010). General Non-minimal Kinetic coupling to gravity. **JCAP** 1007, 021. [arXiv:1005.2716 [hep-th]].

[41] Granda, L. N. (2011). Non-minimal kinetic coupling and the phenomenology of dark energy. **Class. Quant. Grav.** 28, 025006. [arXiv:1009.3964 [hep-th]].

[42] Sushkov, S. V. (2012). Realistic cosmological scenario with non-minimal kinetic coupling. **Phys.Rev. D** 85, 123520. [arXiv:1204.6372 [gr-qc]].

[43] Tsujikawa, S. (2012). Observational tests of inflation with a field derivative coupling to gravity, **Phys. Rev. D** 85, 083518. [arXiv:1201.5926 [astro-ph.CO]].

[44] Garousi, M.R. (2000). Tachyon couplings on nonBPS D-branes and Dirac-Born-Infeld action. **Nucl. Phys. B** 584, 284-299. [arXiv:hep-th/0003122].

[45] Sen, A. (2002a). Rolling tachyon. **JHEP** 0204, 048. [arXiv:hep-th/0203211].

[46] Sen, A. (2002b). Tachyon matter. **JHEP** 0207, 065. [arXiv:hep-th/0203265].

[47] Garousi, M.R., Sami, M., Tsujikawa, S. (2004). Cosmology from a rolling massive scalar field on the anti-D3 brane of de Sitter vacua. **Phys. Rev. D** 70, 043536. [arXiv:hep-th/0402075].

[48] Padmanabhan, T. (2002). Accelerated expansion of the universe driven by tachyonic matter. **Phys. Rev. D** 66, 021301. [arXiv:hep-th/0204150].

[49] Bagla, J.S., Jassal, H.K., Padmanabhan, T. (2003). Cosmology with tachyon field as dark energy. **Phys. Rev. D** 67, 063504. [arXiv:astro-ph/0212198].

[50] Kutasov, D. and Niarchos, V. (2003). Tachyon effective actions in open string theory. **Nucl. Phys. B** 666, 56-70. [arXiv:hep-th/0304045].

[51] Abramo, L.R.W. and Finelli, F. (2003). Cosmological dynamics of the tachyon with an inverse power-law potential. **Phys. Lett. B** 575, 165-171. [arXiv:astro-ph/0307208].

[52] Aguirregabiria, J.M. and Lazkoz, R. (2004). Tracking solutions in tachyon cosmology. **Phys. Rev. D** 69, 123502. [arXiv:hep-th/0402190].

[53] Copeland, E.J., Garousi, M.R., Sami, M., Tsujikawa, S. (2005). What is needed of a tachyon if it is to be the dark energy? **Phys. Rev. D** 71, 043003. [arXiv:hep-th/0411192].

[54] Brans, C. and Dicke, R. H. (1961). Mach's Principle and a Relativistic Theory of Gravitation. **Phys. Rev. D** 124, 925.

[55] Dirac, P. A. M. (1974). Cosmological Models and the Large Numbers Hypothesis. **Proc. Roy. Soc.** A338, 439.

[56] Birrel, N. D. and Davies, P. C. W. (1982). **Quantum Fields in Curved Space**, Melbourne, Australia: Cambridge University Press.

[57] Zee, A. (1979). Broken-Symmetric Theory of Gravity. **Phys. Rev. Lett.** 42, 417.

[58] Applequist, T. and Chodos, A. (1983). Quantum Effects in Kaluza-Klein Theories. **Phys. Rev. Lett.** 50, 141.

[59] Randjbar-Daemi, S., Salam, A. and Strathdee, J. (1984). On Kaluza-Klein cosmology. **Phys. Lett. B** 135, 388-392.

[60] Accetta, F. S., Zoller, D. J. and Turner, M. S. (1985). Induced-gravity inflation. **Phys. Rev. D** 31, 3046.

[61] Maeda, K. (1986). Stability and attractor in a higher-dimensional cosmology. I. **Class. Quant. Grav.** 3(2), 233-247.

[62] La, D. and Steinhardt, P. J. (1989). Extended Inflationary Cosmology. **Phys. Rev. Lett.** 62, 376.

[63] Accetta, F. S. and Trester, J. J. (1989). Extended inflation with induced gravity. **Phys. Rev. D** 39, 2854.

[64] Wang, Y. (1990). Higher-order curvature terms and extended inflation. **Phys. Rev. D** 42, 2541.

[65] Amendola, L., Capozziello, S., Litterio, M. and Occhionero, F. (1992). Coupling first-order phase transitions to curvature-squared inflation. **Phys. Rev. D** 45, 417.

[66] Futamase, T. and Maeda, K. (1989). Chaotic inflationary scenario of the Universe with a nonminimally coupled "inflaton" field. **Phys. Rev. D** 39, 399.

[67] Kasper, U. (1989). On the cosmological constant problem and inflation in the framework of nonminimal coupling between a scalar field and the gravitation. **Il Nuovo Cimento B** 103(3), 291-310.

[68] Amendola, L., Bellisai, D. and Occhionero, F. (1993). Inflationary attractors and perturbation spectra in generally coupled gravity. **Phys. Rev. D** 47, 4267. [arXiv:gr-qc/9303023].

[69] Chiba, T. (1999). Quintessence, the gravitational constant, and gravity. **Phys. Rev. D** 60, 083508. [arXiv:gr-qc/9903094].

[70] Uzan, J-P. (1999). Cosmological scaling solutions of nonminimally coupled scalar fields. **Phys. Rev. D** 59, 123510. [arXiv:gr-qc/9903004].

[71] Holden, D. J. and Wands, D. (2000). Self-similar cosmological solutions with a nonminimally coupled scalar field. **Phys. Rev. D** 61, 043506. [arXiv:gr-qc/9908026].

[72] Easson, D. A. (2007). The Accelerating Universe and a Limiting Curvature Proposal. **JCAP** 0702, 004. [arXiv:astro-ph/0608034].

[73] Liu, H. and Tseytlin, A. A. (1998). D = 4 superYang-Mills, D = 5 gauged supergravity, and D = 4 conformal supergravity. **Nucl. Phys. B** 533, 88-108. [arXiv:hep-th/9804083].

[74] Nojiri, S. and Odintsov, S. D. (1998). Conformal anomaly for dilaton coupled theories from AdS/CFT correspondence. **Phys. Lett. B** 444, 92-97. [arXiv:hep-th/9810008].

[75] Magnano, G., Ferraris, F. and Francaviglia, M. (1987). Nonlinear gravitational Lagrangians. **Gen. Relativ. Grav.** 19, 465.

[76] Gumjudpai, B., Thepsuriya, K. (2012). Scalar field power-law cosmology with spatial curvature and dark energy-dark matter interaction. **Astrophys. Space Sci.** 342, 537-547. [arXiv:1207.2920 [astro-ph.CO]].

[77] Gumjudpai, B. (2013). Quintessential power-law cosmology: dark energy equation of state. **Mod. Phys. Lett. A.** 28, 1350122. [arXiv:1307.4552 [astro-ph.CO]].

[78] Kaeonikhom, C., Gumjudpai, B., Saridakis, E.N. (2011). Observational constraints on phantom power-law cosmology. **Phys. Lett. B** 695, 45-54. [arXiv:1008.2182 [astro-ph.CO]].

[79] Wei, Y.-H. (2004). **The power-law expansion universe and the transition**. Retrieved June 19, 2012, from http://arxiv.org/abs/astro-ph/0405368

[80] Kaplinghat, M., Steigman, G., Tkachev, I., Walker, T.P. (1999). Observational constraints on power law cosmologies. **Phys. Rev. D** 59, 043514. [arXiv:astro-ph/9805114].

[81] Kaplinghat, M., Steigman, G., Walker, T.P. (2000). Nucleosynthesis in power law cosmologies. **Phys. Rev. D** 61, 103507. [arXiv:astro-ph/9911066].

[82] Kolb, E.W. (1989). A coasting cosmology. **Astrophys. J.** 344, 543-550.

[83] Peebles, P.J.E. (1993). **Principles of Physical Cosmology**, United States of America: Princeton University Press.

[84] Lohiya, D., Mahajan, S., Mukherjee, A., Batra, A. (1996). **Nucleosynthesis in a simmering universe**. Retrieved June 20, 2012, from http://arxiv.org/abs/astro-ph/9606082

[85] Sethi, M., Batra, A., Lohiya, D. (1999). On "observational constraints on power-law cosmologies". **Phys. Rev. D** 60, 108301. [arXiv:astro-ph/9903084].

[86] Dev, A., Sethi, M., Lohiya, D. (2001). Linear coasting in cosmology and SNe Ia. **Phys. Lett. B** 504, 207-212.

[87] Dev, A., Safonova, M., Jain, D., Lohiya, D. (2002). Cosmological tests for a linear coasting cosmology. **Phys. Lett. B** 548, 12-18. [arXiv:astro-ph/0204150].

[88] Allen, R.E. (1999). Four testable predictions of instanton cosmology. **AIP Conf. Proc.** 478, 204. [arXiv:astro-ph/9902042].

[89] Mannheim, P. D. (1990). Conformal Cosmology With No Cosmological Constant. **Gen. Rel. Grav.** 22, 289-298.

[90] Lohiya, D. and Sethi, M. (1999). A Program for a problem free cosmology within a framework of a rich class of scalar tensor theories. **Class. Quant. Grav.** 16, 1545-1563. [arXiv:gr-qc/9803054].

[91] Melia, F. and Shevchuk, A. S. H. (2012). The R_h = ct Universe. **Mon. Not. Roy. Astron. Soc.** 419, 2579-2586. [arXiv:1109.5189 [astro-ph.CO]].

[92] Bilicki, M. and Seikel, M. (2012). We do not live in the R_h = c t universe. **Mon. Not. Roy. Astron. Soc.** 425, 1664-1668. [arXiv:1206.5130 [astro-ph.CO]].

[93] Hinshaw, G., Larson, D., Komatsu, E., Spergel, D. N., Bennett, C. L., Dunkley, J. et al. (WMAP Collaboration). (2012). Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results. **Astrophys. J. Suppl.** 208(2), 19. [arXiv:1212.5226 [astro-ph.CO]].

[94] Ade, P. A. R., Aghanim, N., Armitage-Caplan, C., Arnaud, M., Ashdown, M., Atrio-Barandela, F. et al. (Planck Collaboration). (2014). Planck 2013 results. XVI. Cosmological parameters. **Astron. Astrophys.** 571, A16. [arXiv:1303.5076 [astro-ph.CO]].

[95] Ade, P. A. R., Aghanim, N., Arnaud, M., Ashdown, M., Aumont, J., Baccigalupi, C. et al. [Planck Collaboration]. (2015). **Planck 2015 results. XIII. Cosmological parameters**. Retrieved February 7, 2015, from http://arxiv.org/abs/1502.01589

[96] Dolgov, A.D. (1982). **The Very Early Universe**. ed. Gibbons, G., Hawking, S.W., Tiklos, S.T. United States of America: Cambridge University Press.

[97] Ford, L.H. (1987). Cosmological-constant damping by unstable scalar fields. **Phys. Rev. D** 35, 2339.

[98] Fujii, Y. and Nishioka, T. (1990). Model of a decaying cosmological constant. **Phys. Rev. D** 42, 361.

[99] Dolgov, A.D. (1997). Higher spin fields and the problem of the cosmological constant. **Phys. Rev. D** 55, 5881. [arXiv:astro-ph/9608175].

[100] Lucchin, F. and Matarrese, S. (1985). Power-law inflation. **Phys. Rev. D** 32, 1316.

[101] Caldwell, R.R. (2002). A Phantom Menace? Cosmological consequences of a dark energy component with super-negative equation of state. **Phys. Lett. B** 545, 23-29. [arXiv:astro-ph/9908168].

[102] Caldwell, R.R., Kamionkowski, M., Weinberg, N.N. (2003). Phantom energy and cosmic doomsday. **Phys. Rev. Lett.** 91, 071301. [arXiv:astro-ph/0302506].

[103] Gibbons, G. W. (2002). Cosmological Evolution of the Rolling Tachyon. **Phys. Lett. B** 537, 1-4. [arXiv:hep-th/0204008].

[104] Kim, C., Kim, H. B., Kim, Y., Kwon, O-K. (2003). **Proceedings of Pac Memorial Symposium on Theoretical Physics**, 209-239. Seoul, South Korea: Chungbum Publishing House. [arXiv:hep-th/0301142].

[105] Dyer, E. and Hinterbichler, K. (2009). Boundary terms, variational principles, and higher derivative modified gravity. **Phys. Rev. D** 79, 024028. [arXiv:0809.4033 [gr-qc]].

[106] Gumjudpai, B. (2015). **Cosmology: A Prologue** (in Thai). Thailand: The Institute for Fundamental Study "The Tah Poe Academia Institute", Naresuan University.

[107] Dev, A., Jain, D., Lohiya, D. (2008). **Power law cosmology - a viable alternative**. Retrieved June 20, 2012, from http://arxiv.org/abs/0804.3491

[108] Sethi, G., Dev, A., Jain, D. (2005). Cosmological constraints on a power law universe. **Phys. Lett. B** 624, 135-140. [arXiv:astro-ph/0506255].

[109] Kumar, S. (2012). Observational constraints on Hubble constant and deceleration parameter in power-law cosmology. **Mon. Not. Roy. Astron. Soc.** 422, 2532-2538. [arXiv:1109.6924 [gr-qc]].

[110] Coles, P. and Lucchin, F. (2002). **Cosmology: The Origin and Evolution of Cosmic Structure** (2^{nd} ed.) West Sussex, England: John Wiley & Sons.

[111] Granda, L. N. (2010). Non-Minimal Kinetic Coupling To Gravity And Dark Energy. **Rev. Col. Fís** 42, 257-261.

[112] Granda, L. N. (2010). Non-minimal Kinetic coupling to gravity and accelerated expansion. **JCAP** 1007, 006. [arXiv:0911.3702 [hep-th]].

[113] Donoghue, J. F. (1994). General relativity as an effective field theory: The leading quantum corrections. **Phys. Rev. D** 50, 3874-3888. [arXiv:gr-qc/9405057].

[114] Granda, L. N. (2011). Inflation driven by scalar field with non-minimal kinetic coupling with Higgs and quadratic potentials. **JCAP** 1104, 016. [arXiv:1104.2253 [hep-th]].

[115] Granda, L. N. (2012). Dark energy from scalar field with Gauss Bonnet and non-minimal kinetic coupling. **Mod. Phys. Lett. A** 27, 1250018. [arXiv:1108.6236 [hep-th]].

[116] Granda, L. N., Torrente-Lujan, E. and Fernandez-Melgarejo, J. J. (2011). Non-minimal kinetic coupling and Chaplygin gas cosmology. **Eur. Phys. J. C** 71, 1704. [arXiv:1106.5482 [hep-th]].

[117] Sushkov, S. V. (2009). Exact cosmological solutions with nonminimal derivative coupling. **Phys. Rev. D** 80, 103505. [arXiv:0910.0980 [gr-qc]].

[118] Saridakis, E. N. and Sushkov, S. V. (2010). Quintessence and phantom cosmology with non-minimal derivative coupling. **Phys. Rev. D** 81, 083510. [arXiv:1002.3478 [gr-qc]].

[119] Gao, C. (2010). When scalar field is kinetically coupled to the Einstein tensor. **JCAP** 1006, 023. [arXiv:1002.4035 [gr-qc]].

[120] Germani, C. and Kehagias, A. (2011). UV-Protected Inflation. **Phys. Rev. Lett.** 106, 161302. [arXiv:1012.0853 [hep-ph]].

[121] Feng, K. and Qiu, T. (2014). Curvaton with nonminimal derivative coupling to gravity: Full perturbation analysis. **Phys. Rev. D** 90(12), 123508. [arXiv:1409.2949 [hep-th]].

[122] Gubitosi G. and Linder E. V. (2011). Purely Kinetic Coupled Gravity. **Phys. Lett. B** 703, 113. [arXiv:1106.2815 [astro-ph.CO]].

[123] Bruneton, J. P., Rinaldi, M., Kanfon, A., Hees, A., Schlogel S. and Fuzfa, A. (2012). Fab Four: When John and George play gravitation and cosmology. **Adv. Astron.** 2012, 430694. [arXiv:1203.4446 [gr-qc]].

[124] Chen, S. and Jing, J. (2010). Greybody factor for a scalar field coupling to Einstein's tensor. **Phys. Lett. B** 691, 254-260. [arXiv:1005.5601 [gr-qc]].

[125] Chen, S. and Jing, J. (2010). Dynamical evolution of a scalar field coupling to Einstein's tensor in the Reissner-Nordstróm black hole spacetime. **Phys. Rev. D** 82, 084006. [arXiv:1007.2019 [gr-qc]].

[126] Ding, C., Liu, C., Jing, J. and Chen, S. (2010). Extension of non-minimal derivative coupling theory and Hawking radiation in black-hole spacetime. **JHEP** 1011, 146. [arXiv:1007.4240 [gr-qc]].

[127] Kolyvaris, T., Koutsoumbas, G., Papantonopoulos E. and Siopsis, G. (2012). Scalar Hair from a Derivative Coupling of a Scalar Field to the Einstein Tensor. **Class. Quant. Grav.** 29, 205011. [arXiv:1111.0263 [gr-qc]].

[128] Skugoreva, M. A., Sushkov S. V. and Toporensky, A. V. (2013). Cosmology with nonminimal kinetic coupling and a power-law potential. **Phys. Rev. D** 88, 083539. [**Phys. Rev. D** 88(10), 109906 Erratum]. [arXiv:1306.5090 [gr-qc]].

[129] Koutsoumbas, G., Ntrekis K. and Papantonopoulos, E, (2013). Gravitational Particle Production in Gravity Theories with Non-minimal Derivative Couplings. **JCAP** 1308, 027. [arXiv:1305.5741 [gr-qc]].

[130] Darabi, F. and Parsiya, A. (2015). Cosmology with non-minimal coupled gravity: inflation and perturbation analysis. **Class. Quantum Grav.** 32, 155005. [arXiv:1312.1322 [gr-qc]].

[131] Germani, C. and Kehagias, A. (2009). P-nflation: generating cosmic Inflation with p-forms. **JCAP** 0903, 028. [arXiv:0902.3667 [astro-ph.CO]].

[132] Germani, C. and Kehagias, A (2010). New Model of Inflation with Non-minimal Derivative Coupling of Standard Model Higgs Boson to Gravity. **Phys. Rev. Lett.** 105, 011302. [arXiv:1003.2635 [hep-ph]].

[133] Germani, C. and Watanabe, Y. (2011) UV-protected (Natural) Inflation: Primordial Fluctuations and non-Gaussian Features. **JCAP** 1107, 031. [arXiv:1106.0502 [astro-ph.CO]].

[134] Sadjadi, H. M. and Goodarzi, P. (2013). Reheating in nonminimal derivative coupling model. **JCAP** 1302, 038. [arXiv:1203.1580 [gr-qc]].

[135] Ema, Y., Jinno, R., Mukaida K. and Nakayama, K. (2015). Particle Production after Inflation with Non-minimal Derivative Coupling to Gravity. **JCAP** 10, 020. [arXiv:1504.07119 [gr-qc]].

[136] Ghalee, A. (2014). **Notes on a scalar field with a kinetic term non-minimally coupled to gravity**. Retrieved March 29, 2014, from http://arxiv.org/abs/1402.6798

[137] Yang, N., Gao, Q. and Gong, Y. (2015). **Inflationary models with non-minimally derivative coupling**. Retrieved March 29, 2014, from http://arxiv.org/abs/1504.05839

[138] Myung, Y. S., Moon, T. and Lee, B. H. (2015). Revisiting slow-roll inflation in nonminimal derivative coupling with potentials. **JCAP** 10, 007. [arXiv:1505.04027 [gr-qc]].

[139] Sadjadi, H. M. (2011). Super-acceleration in non-minimal derivative coupling model. **Phys. Rev. D** 83, 107301. [arXiv:1012.5719 [gr-qc]].

[140] Sadjadi, H. M. and Goodarzi, P. (2014). Oscillatory inflation in non-minimal derivative coupling model. **Phys. Lett. B** 732, 278. [arXiv:1309.2932 [astro-ph.CO]].

[141] Sadjadi, H. M. (2014). Rapid Oscillatory quintessence with variable equation of state parameter in non minimal derivative coupling model. **Gen. Rel. Grav.** 46(11), 1817. [arXiv:1312.1151 [gr-qc]].

[142] Jinno, R., Mukaida, K. and Nakayama, K. (2014). The universe dominated by oscillating scalar with non-minimal derivative coupling to gravity. **JCAP** 1401(01), 031. [arXiv:1309.6756 [astro-ph.CO]].

[143] Dent, J. B., Dutta, S., Saridakis, E. N. and Xia, J. Q. (2013). Cosmology with non-minimal derivative couplings: perturbation analysis and observational constraints. **JCAP** 1311, 058. [arXiv:1309.4746 [astro-ph.CO]].

[144] Huang, Y., Gao, Q. and Gong, Y. (2015). The Phase-space analysis of scalar fields with non-minimally derivative coupling. **Eur. Phys. J. C** 75(4), 143. [arXiv:1412.8152 [gr-qc]].

[145] Jain, D., Dev, A., Alcaniz, J.S. (2003). The angular size - redshift relation in power-law cosmologies. **Class. Quant. Grav.** 20, 4485-4494.

[146] Zhu, Z.H., Hu, M., Alcaniz, J.S., Liu, Y.X. (2008). Testing power-law cosmology with galaxy clusters. **Astron. and Astrophys.** 483, 15-18. [arXiv:0712.3602 [astro-ph]].

[147] Allen, S.W., Schmidt, R.W., Fabian, A.C. (2002). Cosmological constraints from the X-ray gas mass fraction in relaxed lensing clusters observed with Chandra. **Mon. Not. Roy. Astro. Soc.** 334, L11. [arXiv:astro-ph/0205007].

[148] Allen, S.W., Schmidt, R.W., Fabian, A.C., Ebeling, H. (2003). Cosmological constraints from the local X-ray luminosity function of the most X-ray luminous galaxy clusters. **Mon. Not. Roy. Astro. Soc.** 342, 287. [arXiv:astro-ph/0208394].

[149] Rangdee, R., Gumjudpai, B. (2014). Tachyonic (phantom) power-law cosmology. **Astrophys. Space Sci.** 349, 975-984. [arXiv:1210.5550 [astro-ph.CO]].

[150] Rani, S., Altaibayeva, A., Shahalam, M., Singh, J. K. and Myrzakulov, R. (2015). Constraints on cosmological parameters in power-law cosmology. **JCAP** 1503(03), 031. [arXiv:1404.6522 [gr-qc]].

[151] Kaeonikhom, C., Gumjudpai, B. and Saridakis, E. N. (2011). Observational constraints on phantom power-law cosmology. **Phys. Lett. B** 695, 45-54. [arXiv:1008.2182 [astro-ph.CO]].

[152] Gumjudpai, B., Rangdee, P. (2015). Non-minimal derivative coupling gravity in cosmology. **Gen. Rel. Grav.** 47, 140. [arXiv:1511.00491 [gr-qc]].

[153] Misner, C. W., Thorne, K. S. and Wheeler, J. A.. (1973). **Gravitation** United States of America: W. H. Freeman and Company.

[154] Ferraris, M., Francaviglia, I. and Volovich, I. (1994). The Universality of Einstein Equations. **Class. Quant. Grav.** 11, 1505-1517.

[155] Magnano. G. and Sokolowski, L. M. (1994). Physical equivalence between nonlinear gravity theories and a general-relativistic self-gravitating scalar field. **Phys. Rev. D** 50, 5039. [arXiv:gr-qc/9312008].

[156] Capozziello, S. and Faraoni, V. (2010). **Beyond Einstein Gravity** N.P.: Springer.

[157] Luo, X., Wu, P. and Yu, H. (2014). Non-minimal derivatively coupled quintessence in the Palatini formalism. **Astrophys. Space. Sci.** 350, 831-837.

[158] Matsumoto, J., Suskov, S. V. (2015). Cosmology with nonminimal kinetic coupling and a Higgs-like potential. **JCAP** 11, 047. [arXiv:1510.03264 [gr-qc]].

APPENDIX

APPENDIX F ERRORS ANALYSIS

In calculating of the accumulated errors, we follow the procedure here. If f is valued of answer in the form

$$f = f(x_1, x_2, \ldots, x_n) \tag{F.1}$$

and f_0 is the value when x_i is set to their measured values, then the value of f_i is defined as

$$f_i = f(x_1, \ldots, x_i + \sigma_i, \ldots, x_n) \tag{F.2}$$

This value of f is the value with effect of error in variable x_i, that is σ_i. One can find square of the accumulated error from

$$\sigma_f^2 = \sum_i^n (f_i - f_0)^2 \tag{F.3}$$

Hence giving the error of f from accumulating effect from errors of x_i. Here, it is assuming that the error in x_i is independent of the error in other variables, x_j.

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RESEARCH ARTICLE

Non-minimal derivative coupling gravity in cosmology

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Abstract We give a brief review of the non-minimal derivative coupling (NMDC) scalar field theory in which there is non-minimal coupling between the scalar field derivative term and the Einstein tensor. We assume that the expansion is of power-law type or super-acceleration type for small redshift. The Lagrangian includes the NMDC term, a free kinetic term, a cosmological constant term and a barotropic matter term. For a value of the coupling constant that is compatible with inflation, we use the combined WMAP9 (WMAP9 + eCMB + BAO + H_0) dataset, the PLANCK + WP dataset, and the PLANCK TT, TE, EE + lowP + Lensing + ext datasets to find the value of the cosmological constant in the model. Modeling the expansion with power-law gives a negative cosmological constants while the phantom power-law (super-acceleration) expansion gives positive cosmological constant with large error bar. The value obtained is of the same order as in the ΛCDM model, since at late times the NMDC effect is tiny due to small curvature.

Keywords Non-minimum derivative coupling to gravity · Cosmological constant · Power-law expansion

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1 Introduction

Recently, cosmic accelerating expansion has been confirmed by astrophysical observations. Amongst these are supernova type Ia (SNIa) [1–10], large-scale structure surveys [11,12], cosmic microwave background (CMB) anisotropies [13–16] and X-ray luminosity from galaxy clusters [15,17,18]. The acceleration is responsible by an unknown energy form called dark energy [19–21] which is typically in form of either cosmological constant or scalar field [19–22]. There are many scalar field models proposed to explain the accelerating expansion of the universe, for example, quintessence [23] and classes of k-essence type models [24–26]. Modifications of gravity, for instance, braneworlds, $f(R)$ and others are as well possible answers of present acceleration (see e.g. [27,28]). Acquiring the acceleration needs the effective equation of state of matter species, especially a dynamical scalar field evolving under its potential, to be $p < -\rho c^2/3$.

It is possible to have a non-minimal coupling (NMC) between scalar field to Ricci scalar in GR in form of $\sqrt{-g}\, f(\phi)R$. The NMC is motivated by scalar-tensor theories in the Jordan-Brans-Dicke models [29,30], re-normalizing term of quantum field in curved space [31] or supersymmetries, superstring and induced gravity theories [32–36]. It was applied to extended inflations with first-order phase transition and other inflationary models [37–43]. In context of quintessence field driving present acceleration, non-minimal coupling to curvature has been studied as in [44–47]. In strong coupling regime, power-law and de-Sitter expansions are found as late time attractor [48] and moreover the NMC term could also behave as effective cosmological constant [49].

First cosmological consideration of the non-minimal curvature coupling to the derivative term of scalar field was proposed by Amendola in 1993 [50]. Therein the coupling function is in form of $f(\phi, \phi_{,\mu}, \phi_{,\mu\nu}, \ldots)$. This type of derivative coupling is required in scalar quantum electrodynamics to satisfy U(1) invariance of the theory and is required in models of which the gravitational constant is function of the mass density of the gravitational source. The non-minimal derivative coupling-NMDC terms are commonly found as lower energy limits of higher dimensional theories which makes quantum gravity possible to be studied perturbatively. They are also found in Weyl anomaly in $\mathcal{N} = 4$ conformal supergravity [51,52]. With simplest NMDC term, $R\phi_{,\mu}\phi^{;\mu}$, class of inflationary attractors is enlarged from the previous NMC model of [43] and the NMDC renders non-scale invariant spectrum without requirement of multiple scalar fields. Moreover it is possible to realize double inflation without adding more fields to the theory [50]. However conformation transformation can not transform the NMDC theory into the standard field equation in Einstein frame. The conformal (metric) re-scaling transformation needs to be generalized to Legendre transformation in order to recover the Einstein frame equations [50,53]. There are various versions of the NMDC proposed in order to match plausible theory and to predict observation results as will be seen in the next section.

We give a brief review of the NMDC gravity models in this paper and we consider a model in which the Einstein tensor couples to the kinetic scalar field term with a free kinetic term and a constant potential (considered as a cosmological constant). In setups of power-law or phantom power-law (super) acceleration expansions and using inflation-estimated value of the coupling constant, we evaluate value of the cosmological constant and show a parametric plots of the cosmological constant versus the power-law exponents. Cosmological parameters given by WMAP9 (combined WMAP9 + eCMB + BAO + H_0) dataset [54] and PLANCK satellite dataset [55,56] are used here.

2 Non-minimal derivative coupling theory

2.1 Capozziello, Lambiase and Schmidt's result

Capozziello, Lambiase and Schmidt [57] found in 2000 that all other possible coupling Lagrangian terms are not necessary in scalar-curvature coupling theory, leaving only $R\phi_{,\mu}\phi^{;\mu}$ and $R^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ terms in the Lagrangian without losing its generality, hence motivating cosmological study in the case of having both terms. One character of the two new terms is to modulate gravitational strength with a free canonical kinetic term without either scalar field potential $V(\phi)$ or Λ. This results in an effective cosmological constant and hence effectively giving de-Sitter expansion [58]. The conditions for which de-Sitter expansion is a late time attractor are given in [57]. When considering only $R\phi_{,\mu}\phi^{;\mu}$ with free Ricci scalar, free kinetic term, potential and matter terms, the equation of state, in absence of $V(\phi)$, goes to -1 at late time. When assuming slowly-rolling field and power-law expansion, $V(\phi)$ is found directly [59]. Another case is to consider only the $R^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ term as extra term to standard scalar field cosmology, i.e. a free Ricci scalar with a free kinetic scalar term and a potential, the field equation contains third-order derivatives of ϕ and the continuity equation of the scalar field contains third-order derivative of $g_{\mu\nu}$. This model is tightly constrained in weakly coupling regime, i.e. solar system constraint puts limit of the pressure, $p_\phi < 10^{-6}\rho_c c^2$, where ρ_c is critical density hence it can not play a role of quintessence. If the coupling is strong with negative sign, the coupling term can flattens the slope of the inflationary potential [60].

2.2 Granda's two coupling constant model

Another modification of the NMDC model is proposed by Granda in 2010 [61]. The model contains the usual Einstein-Hilbert term, a scalar field kinetic term, a potential term and two separated dimensionless couplings, κ, η re-scaled by $1/\phi^2$ in form of $-(1/2)\kappa R\phi^{-2}g_{\mu\nu}\phi^{;\mu}\phi^{;\nu}$ and $-(1/2)\eta\phi^{-2}R_{\mu\nu}\phi^{\mu}\phi^{\nu}$. In this model when there is no free kinetic scalar term (i.e. strictly NMDC) and no potential term, NMDC term takes a role of dark matter at early stage giving the power-law dust solution, $a \sim t^{2/3}$ for $\eta = -2\kappa$ and accelerating solution for $\eta = -\kappa - 1$ where $0 < \kappa < 1/3$. Acceleration at present time is assured if including the potential into the Lagrangian. Motivation of such two separated couplings comes from an attempt to approach quantum gravity perturbatively [62]. This gives ideas of the other versions of two coupling models without the $1/\phi^2$ re-scaling factor [63–65] such as inclusion of Gauss-Bonnet invariance [66] or in context of Chaplygin gas [67].

2.3 Sushkov's model

2.3.1 Constant or zero potential

Sushkov, in 2009, [68] considered a special case $\kappa_1 R\phi_{,\mu}\phi^{,\mu}$ and $\kappa_2 R^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ with $\kappa \equiv \kappa_2 = -2\kappa_1$. This results in combination of the two NMDC terms into one Einstein tensor coupling to kinetic scalar field part, $\kappa G_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$. The chosen coupling constant κ renders good dynamical theory, that is to say, the field equations contain terms with second-order derivative of $g_{\mu\nu}$ and ϕ at most so that the Lagrangian contains only divergence free tensors. Hence it consists of the R term, free kinetic scalar $g_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$ and $\kappa G_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$ in absence of $V(\phi)$. Cosmological study of the model for flat FLRW universe yields, for $\kappa > 0$, quasi-de-Sitter at very early stage but, for $\kappa < 0$, initial singularity at very early stage. For any sign of the coupling, $a \propto t^{1/3}$ at very late time [68]. A direct modification of this model is to have a constant potential with possibility of phantom behavior of the free kinetic term [69]. In a range of coupling constant values, this modification enables the model to transit from de-Sitter phase to other types of expansions giving various fates and various origins of the universe [69].

2.3.2 With potential but without free kinetic term

Inspired by Sushkov's model, in case of without free kinetic term, $(1/2)g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$, but having Einstein tensor coupling kinetic term alone (strictly NMDC), Gao in 2010 [70], found that for $V(\phi) = 0$, the scalar field behaves like dust in absence of other matters or in presence of pressureless matter. Its value of the equation of state parameter suggests that it could be a candidate of dark energy and dark matter. However the model is not viable due to superluminal sound speed. When adding more than one Einstein tensor coupling to the kinetic term [70], it was claimed not to be likely by [71]. Strictly NMDC term in curvaton model can also be seen in the work by [72].

2.3.3 Purely kinetic coupling term and a matter term

The Sushkov's model, in absence of potential and absence of matter Lagrangians, is not able to explain phantom acceleration, i.e. no phantom crossing. In order to fix the purely kinetic Lagrangian to allow phantom crossing, in 2011, Gubitosi and Linder proposed most general Lagragians with purely kinetic term obeying shift symmetry. These are the $(a_1\phi_{,\mu}\phi^{,\mu} + a_2\nabla^2\phi)R$ term, $\phi_{,\mu}\phi_{,\nu}R^{\mu\nu}$ term and $R^{\alpha\beta\gamma\delta} f_{\alpha\beta\gamma\delta}(\phi_{,\mu})$ term where $f_{\alpha\beta\gamma\delta}$ is a function of $\phi_{,\mu}$ and a matter term [73]. Absence of potential helps avoiding high energy quantum correction. Their model is at lowest possible order of Planck mass and it verifies Sushkov's action [68]. The model achieve wide range of w values from stiff ($w = 1$) to phantom crossing and is possible to result in loitering cosmological constant-like phase before entering matter domination phase. Sushkov's purely kinetic model with matter Lagrangian is found to be a special case of the Fab Four theory. Only positive coupling constant of the theory could result in phantom crossing however it also gives non-causal scalar and tensor perturbation, hence making the purely-kinetic model discarded for inflation [74]. Investigations of this model for $V(\phi) = 0$ in blackhole spacetime are presented in [75–78].

2.3.4 Adding potential term with matter term

As another way out of problem in purely kinetic model, potential is added into the theory (without matter term). In order to have inflation, it is found that the potential needs to be less steep than quadratic potential [79]. With constant potential and matter term in the model, it is able to describe transition from inflation to matter domination epoch without reheating and later it describes the transit to late de-Sitter epoch. The derivative coupling to curvature is strong at early time to drive inflation since the coupling constant acts as another cosmological constant Λ_{NMDC}. At late time the scalar field behaves like dark matter and the cosmological constant (or the constant potential) together with the NMDC term (with little effect) drives the present acceleration [80]. Dynamical analysis shows that for positive potential, the positive coupling gives unbound $\dot{\phi}$ value with restricted Hubble parameter [79]. Indeed when considering constant potential and positive coupling, inflationary phase is always possible and the inflation depends solely on the value of coupling constant. During inflation, gravitational heavy particles are less produced, if having stronger NMDC couplings to the inflaton field or to the particles [81]. Perturbations analysis and inflationary analysis of the model with a constant potential considered as a cosmological constant was performed in [82] to confront observational data.

2.4 Model with negative-sign NMDC

The model is related (by Germani and Kehagias in 2011 [71]) to natural inflation of which pseudo-Nambu-Goldstone boson slowly rolling to create inflation as well as related to three-form inflation [83]. The model is related to Higgs inflation with $V(\phi) \sim \lambda\phi^4$ which is a NMDC coupling to gravity modification at tree-level of Higgs field [84]. The Lagrangian looks similar to Sushkov's action but the free kinetic term and the NMDC term have opposite sign to each other, i.e. $g^{\mu\nu} - G^{\mu\nu}/M^2$. The model gives a UV-protected inflation and enhances friction of the field dynamics gravitationally [85]. Inflationary scenario of the model with quadratic potential and modifications of standard reheating by the NMDC term is found by Sadjadi and Goodarzi in 2013 [86]. Tsujikawa in 2012 showed that, due to gravitational friction produced by the NMDC, even with steep potentials, a class of inflationary potentials is compatible with observation [87]. Particle production of this action after inflation is reported in [88] and one slow roll parameter is necessary for describing inflation [89]. The NMDC coupling contributes to high-field friction making the energy scale reduce to sub-Planckian therefore more consistent to observation [90]. The model is also investigated without free kinetic term for inflation [91]. As dark energy, this model with matter term and a power-law potential is possible to give phantom crossing [92]. Power-law quintessence potential $V_0\phi^n$ gives rise to oscillatory dark energy. The oscillatory NMDC quintessence satisfies EoS observational value for $n < 2$ [93,94] however inconsistencies are also reported in [95]. Applying exponential and power-law potentials, perturbation analysis with combined SN Ia, BAO and CMB shows that NMDC coupling term has very small effect on late acceleration if it is needed to satisfy instability avoidance. This suggests that the coupling needs to be small, making $9\kappa H^2$

term in the Friedmann equation small. Hence it behaves like quintessence at late time as it is driven by the potential. However at early time the NMDC coupling plays major role in driving the acceleration due to large H value at inflation [96]. Phase space analysis for the case of exponential potential was performed in [97].

3 Equations of motion

In this work, we consider the Sushkov's model which takes the action [68,80],

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{8\pi G} - \left(\varepsilon g_{\mu\nu} + \kappa G_{\mu\nu}\right)\phi^{,\mu}\phi^{,\nu} - 2V(\phi)\right] + S_{\rm m}, \tag{1}$$

where R is the Ricci scalar, g is the determinant of metric tensor $g_{\mu\nu}$, G is the universal gravitational constant, $G_{\mu\nu}$ is the Eintein tensor, ϕ is the scalar field, $V(\phi)$ is the scalar field potential, $S_{\rm m}$ is ordinary matter action, ε is a constant with values $+1(-1)$ for canonical (and phantom) scalar field, $\kappa > 0$ is the coupling constant as in [68,80]. Our universe is assumed to be a spatially flat FLRW, with the metric

$$ds^2 = -c^2 dt^2 + a^2(t) d\mathbf{x}^2, \tag{2}$$

where $a(t)$ is the scale factor and $d\mathbf{x}^2$ is Euclidian metric. Varying the action in Eq. (1) with respect to metric tensor $g_{\mu\nu}$ using line element in Eq. (2) we obtain

$$3H^2 = 4\pi G\dot{\phi}^2(\varepsilon - 9\kappa H^2) + 8\pi G V(\phi) + 8\pi G\rho_{\rm m}, \tag{3}$$

where H is the Hubble parameter and $\rho_{\rm m}$ is the energy density of matter. The Hubble parameter is a function of time t and defined in a form $H = H(t) = \dot{a}(t)/a(t)$. The acceleration equation takes the form,

$$2\dot{H} + 3H^2 = -4\pi G\left[\varepsilon + \kappa\left(2\dot{H} + 3H^2 + 4H\ddot{\phi}\dot{\phi}^{-1}\right)\right] + 8\pi G V(\phi) - 8\pi G p_{\rm m}, \tag{4}$$

where $p_{\rm m}$ is the pressure of matter. The scalar field equation is

$$\varepsilon(\ddot{\phi} + 3H\dot{\phi}) - 3\kappa(H^2\ddot{\phi} + 2H\dot{H}\dot{\phi} + 3H^3\dot{\phi}) = -V_{,\phi} \tag{5}$$

where $V_{,\phi} \equiv dV/d\phi$. The Eqs. (3), (4) and (5) are the dynamical system of the field equations. We can write

$$\ddot{\phi} = -\frac{V_{,\phi}}{\varepsilon - 3\kappa H^2} - \frac{3}{\varepsilon - 3\kappa H^2}\left(\varepsilon H - 2\kappa H\dot{H} - 3\kappa H^3\right)\dot{\phi}, \tag{6}$$

or

$$\ddot{\phi} = -3H\dot{\phi} - \frac{V_{,\phi}}{\varepsilon - 3\kappa H^2} + \frac{6\kappa H\dot{H}\dot{\phi}}{\varepsilon - 3\kappa H^2}. \tag{7}$$

Subtracting Eq. (4) with (3), we obtain

$$\dot{H} = -4\pi G\left[\dot{\phi}^2\left(\varepsilon + \kappa\dot{H} - 3\kappa H^2 + 2\kappa H\ddot{\phi}\dot{\phi}^{-1}\right) + p_{\rm m} + \rho_{\rm m}\right]. \tag{8}$$

From above equations, energy density and pressure of the scalar field is found to be

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2(\varepsilon - 9\kappa H^2) + V(\phi), \tag{9}$$

and

$$p_\phi = \frac{1}{2}\dot{\phi}^2(\varepsilon - 9\kappa H^2)\left[1 + \frac{2\kappa\dot{H}(\varepsilon + 9\kappa H^2)}{(\varepsilon - 3\kappa H^2)(\varepsilon - 9\kappa H^2)}\right] - \frac{2\kappa H\dot{\phi}V_{,\phi}}{\varepsilon - 3\kappa H^2} - V(\phi). \tag{10}$$

Therefore we find the equation of state parameter as follow

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2(\varepsilon - 9\kappa H^2)\left(1 + \frac{2\kappa\dot{H}(\varepsilon+9\kappa H^2)}{(\varepsilon-3\kappa H^2)(\varepsilon-9\kappa H^2)}\right) - \frac{2\kappa H\dot{\phi}V_{,\phi}}{\varepsilon-3\kappa H^2} - V(\phi)}{\frac{1}{2}\dot{\phi}^2(\varepsilon - 9\kappa H^2) + V(\phi)}. \tag{11}$$

Using the Friedmann equation, the potential is found as

$$V(\phi) = \frac{3H^2}{8\pi G} - \frac{1}{2}(\varepsilon - 9\kappa H^2)\dot{\phi}^2 - \rho_{\rm m}, \tag{12}$$

One can check if this is correct by substituting the scalar field potential in to Eq. (9) to obtain the usual Friedmann equation, $\rho_\phi + \rho_{\rm m} = 3H^2/8\pi G$. From Eq. (8), we see that

$$\rho_\phi + p_\phi = \dot{\phi}^2(\varepsilon + \kappa\dot{H} - 3\kappa H^2 + 2\kappa H\ddot{\phi}\dot{\phi}^{-1}). \tag{13}$$

Using Friedmann equation and Eq. (13), hence Eq. (8) recovers its general kinematical form,

$$\dot{H} = -4\pi G\left[\left(3H^2/8\pi G\right) + p_{\rm m} + p_\phi\right] \tag{14}$$

and the equation of state parameter also recovers general kinematical form,

$$w_\phi(H, \dot{H}, \rho_{\rm m}) = -\frac{3H^2 + 2\dot{H} + 8\pi G p_{\rm m}}{3H^2 - 8\pi G\rho_{\rm m}}. \tag{15}$$

Taking time derivative to the Friedmann equation (3), hence

$$\dot{H} = -\frac{4\pi G}{3H}\left[-\dot{\phi}\ddot{\phi}(\varepsilon - 9\kappa H^2) + 9\kappa H\dot{H}\dot{\phi}^2 - V_{,\phi}\dot{\phi} - \dot{\rho}_{\rm m}\right]. \tag{16}$$

Using the continuity equation of matter, $\dot{\rho} = -3H\rho$, with dust matter ($w_{\rm m} = 0$) to Eqs. (7) and (16) becomes

$$\dot{H} = -4\pi G\left[\left\{(\varepsilon - 9\kappa H^2) - 2\kappa\dot{H}\frac{(\varepsilon - 9\kappa H^2)}{(\varepsilon - 3\kappa H^2)} + 3\kappa\dot{H}\right\}\dot{\phi}^2 - \frac{2\kappa H V_{,\phi}\dot{\phi}}{\varepsilon - 3\kappa H^2} + \rho_{\rm m}\right]. \tag{17}$$

Rearrange to obtain the kinetic term,

$$\dot{\phi}^2 = \frac{\frac{2\kappa H V_{,\phi}\dot{\phi}}{\varepsilon - 3\kappa H^2} - \rho_{\rm m} - \frac{\dot{H}}{4\pi G}}{(\varepsilon - 9\kappa H^2) - 2\kappa\dot{H}\left(\frac{\varepsilon - 9\kappa H^2}{\varepsilon - 3\kappa H^2}\right) + 3\kappa\dot{H}}. \tag{18}$$

Considering the case with constant potential, or equivalently a cosmological constant term, $V(\phi) = \Lambda/(8\pi G)$ in the system, with dust and scalar field term (both free kinetic term and the NMDC term), the Friedmann equation can be written as

$$H^2 = H_0^2\left[\Omega_{\Lambda,0} + \frac{\Omega_{\rm m,0}}{a^3} + \frac{\Omega_{\phi,0}(\varepsilon - 9\kappa H^2)}{a^6(\varepsilon - 3\kappa H^2)^2}\right] \tag{19}$$

where Ω are density parameters of each component of cosmic fluids. The system (3), (4) and (5) with $\dot{\phi} = \psi(t)$ in absence of potential and barotropic fluid is a closed autonomous dynamical system. An interesting particular solution of this system is when $\dot{\psi}_{\rm p} = 0 = \ddot{\phi}$ where $\psi \equiv \dot{\phi}$ hence $\psi_{\rm p} = \dot{\phi} = $ constant. As found in [58], that the solution is a de-Sitter type. For the case of $\kappa \equiv \kappa_2 = -2\kappa_1$, as of Sushkov's model, the solution gives,

$$H^2 = \frac{\Lambda_{\rm NMDC}}{3}. \tag{20}$$

The effective cosmological constant is defined as

$$\Lambda_{\rm NMDC} = \frac{\varepsilon}{\kappa} \tag{21}$$

The solution is found as $\psi_{\rm p} = \dot{\phi} = 1/\sqrt{\kappa}$ which is

$$\phi_{\rm p} = \frac{t}{\sqrt{\kappa}} + \phi_0 \tag{22}$$

suggesting that the coupling constant should take a positive value and the effective cosmological constant, $\Lambda_{\rm NMDC}$ should be positive. However general consideration in [68,69,80] the NMDC term is strong at early time hence gives new inflation mechanism that transition from a quasi-de-Sitter phase to power-law phase happens naturally. Having constant $V = \Lambda/(8\pi G)$, at late time, the transition from quasi-de-Sitter to de-Sitter phase is also possible. The particular solution suggests that $\Lambda_{\rm NMDC} > 0$.

Therefore, in presence of the usual cosmological constant (or constant V), both Λ and Λ_{NMDC} contribute both at late time. In order to have enough inflation, κ is estimated to $10^{-74}\ s^2$. Although $\Lambda_{\text{NMDC}} \approx 10^{74}\ s^{-2}$ seems to be large, the NMDC term is suppressed by its multiplication with curvature which is very small at late time.

4 Results

We estimate that the present universe in very recent range of z evolves as power-law $a = a_0\,(t/t_0)^{\alpha}$ for $\varepsilon = +1$. Here a_0 is scale factor at a present time, t_0 is age of the universe and α is constant exponent. The power-law expansion has been considered widely in astrophysical observations, see e.g. [98–101] (see also [102] for constraints). It is realized as an attractor solution of a canonical scalar field evolving under exponential potential [103] and solution of a barotropic fluid-dominant universe. Space is under acceleration if $\alpha > 1$. We consider constant α in a range $0 < \alpha < \infty$. Hence, $\dot{a} = \alpha a/t$, and the acceleration is $\ddot{a} = \alpha(\alpha - 1)a/t^2$. The Hubble parameter and its time derivative are $H = \dot{a}/a = \alpha/t$, and $\dot{H} = -\alpha/t^2$. The value of α can be evaluated with data from gravitational lensing statistics [104], compact radio source [105], X-ray gas mass fraction measurements of galaxy cluster [106]. Values of α from various observational data are listed in [101]. To calculate α at the present we use $\alpha = H_0 t_0$ and dust density is $\rho_{\text{m}} = \rho_{\text{m},0}\,(t_0/t)^{3\alpha}$, where $\rho_{\text{m},0}$ is the dust density at present.

In the scenario of super-acceleration, i.e. the phantom power-law function for which $\varepsilon = -1$, $a = a_0\,[(t_{\text{s}} - t)/(t_{\text{s}} - t_0)]^{\beta}$, where t_{s} is the future singularity-the Big-Rip time defined as in [107] $t_{\text{s}} \equiv t_0 + |\beta|/H(t_0)$, and β is a constant. In this case $\dot{a} = -a_0\beta(t_{\text{s}} - t)^{\beta-1}/(t_{\text{s}} - t_0)^{\beta} = -\beta a/(t_{\text{s}} - t)$, and cosmic acceleration is, $\ddot{a} = a_0\beta(\beta - 1)(t_{\text{s}} - t)^{\beta-2}/(t_{\text{s}} - t_0)^{\beta} = \beta(\beta - 1)a/(t_{\text{s}} - t)^2$. Acceleration requires $\beta < 0$. The Hubble parameter is $H = -\beta/(t_{\text{s}} - t)$, and $\dot{H} = -\beta/(t_{\text{s}} - t)^2$. At present, $\beta = H_0(t_0 - t_{\text{s}})$. Dust density in the phantom power-law case is $\rho_{\text{m}} = \rho_{\text{m},0}\,[(t_{\text{s}} - t_0)/(t_{\text{s}} - t)]^{3\beta}$. At present, $t = t_0$, the Big-Rip time t_{s} can be estimated from

$$t_{\text{s}} \approx t_0 - \frac{2}{3(1 + w_{\text{DE}})}\,\frac{1}{H_0\sqrt{1 - \Omega_{\text{m},0}}} \tag{23}$$

Here, w_{DE} must be less than -1. To derive the above expression the flat geometry and constant dark energy equation of state are assumed [108,109]. This type of expansion function with phantom scalar field was considered in [110]. We use cosmological parameters are from WMAP9 (combined WMAP9 + eCMB + BAO + H_0) dataset [54], PLANCK + WP dataset [55] and PLANCK including polarization and other external parameters (TT, TE, EE + lowP + Lensing + ext.) [56]. The value of w_{DE} is of the wCDM model obtained from observational data. The barotropic density contributes to power-law expansion shape while the NMDC and Λ contributes to de-Sitter expansion, in combination, the expansion function is a mixing between these two. For the phantom case, the free kinetic part of the Lagrangian has negative kinetic energy, therefore the combined effect to the expansion should be the phantom-power law (super

Table 1 Derived parameters from the combined WMAP9 (WMAP9 + eCMB + BAO + H_0), PLANCK + WP and *TT, TE, EE* + lowP + Lensing + external data

Parameters	WMAP9 + eCMB + BAO + H_0 [54]	PLANCK + WP [55]	*TT, TE, EE* + lowP + Lensing + ext. [56]
t_0	$(4.346(4) \pm 0.018(6)) \times 10^{17}$ s	$(4.360(6) \pm 0.015(1)) \times 10^{17}$ s	$(4.354(9) \pm 0.006(6)) \times 10^{17}$ s
	13.772 ± 0.059 Gyr	13.817 ± 0.048 Gyr	13.799 ± 0.021 Gyr
H_0	$(2.245(9) \pm 0.025(9)) \times 10^{-18}\text{s}^{-1}$	$(2.18(1) \pm 0.03(8)) \times 10^{-18}\text{s}^{-1}$	$(2.195(1) \pm 0.014(9)) \times 10^{-18}\text{s}^{-1}$
	69.32 ± 0.80 km/s/Mpc	67.3 ± 1.2 km/s/Mpc	67.74 ± 0.46 km/s/Mpc
$\Omega_{\text{m},0}$	$0.2865^{+0.0096}_{-0.0095}$	$0.315^{+0.016}_{-0.018}$	0.3089 ± 0.0062
$\rho_{\text{c},0}$	$(9.019(6) \pm 0.208(8)) \times 10^{-27}\text{kg/m}^3$	$(8.50(6) \pm 0.14(8)) \times 10^{-27}\text{kg/m}^3$	$(8.618(6) \pm 0.117(0)) \times 10^{-27}\text{kg/m}^3$
$\rho_{\text{m},0}$	$(2.584(1)^{+0.146(4)}_{-0.145(5)}) \times 10^{-27}\text{kg/m}^3$	$(2.67(9)^{+0.18(3)}_{-0.19(9)}) \times 10^{-27}\text{kg/m}^3$	$(2.662(3) \pm 0.089(6)) \times 10^{-27}\text{kg/m}^3$
w_{DE} (of wCDM)	$-1.073^{+0.090}_{-0.089}$	$-1.49^{+0.65}_{-0.57}$	$-1.019^{+0.075}_{-0.080}$

Table 2 Expansion derived parameters from the three datasets

Parameters	WMAP9 + eCMB + BAO + H_0	PLANCK + WP	*TT, TE, EE* + lowP + Lensing + ext.
α	$0.9761(6) \pm 0.0154(3)$	$0.951(0) \pm 0.019(9)$	$0.9559(4) \pm 0.0079(4)$
$q_{\text{power-law}}$	$0.0244(2) \pm 0.0161(9)$	$0.0515(2) \pm 0.0220(0)$	$0.04613(4) \pm 0.00868(9)$
t_s	$(5.248(1)^{+6.056(1)}_{-5.990(1)}) \times 10^{18}$ s	$(1.19(0)^{+1.03(2)}_{-0.91(1)}) \times 10^{18}$ s	$(1.96(6)^{+7.62(8)}_{-8.13(4)}) \times 10^{19}$ s
	$166.2(9)^{191.8(9)}_{189.8(0)}$ Gyr	$37.7(1)^{+32.7(0)}_{-28.8(7)}$ Gyr	$622.9(4)^{+2416.9(8)}_{-2577.3(1)}$ Gyr
β	$-10.81(1)^{+13.73(1)}_{-13.58(1)}$	$-1.64(4)^{+2.28(2)}_{-2.01(8)}$	$-42.1(9)^{+167.8(8)}_{-178.9(9)}$
q_{phantom}	$-1.0925(0)^{+0.1174(8)}_{-0.1162(0)}$	$-1.6082(7)^{+0.8443(3)}_{-0.7440(6)}$	$-1.0237(0)^{+0.0943(1)}_{-0.1005(5)}$

acceleration) mixing with the de-Sitter expansion. We will calculate the cosmological constant, Λ of the model using observed value of w_{DE} and using suggested value of $\kappa \approx 10^{-74}$ s^2 as required by inflation [80]. The coupling constant is regarded as a constant in data analysis. The derived parameters from observations are shown in Table 1 while Table 2 shows values of variables calculated from observations. Values of cosmological constant in this model using three datasets are shown in Table 3. We show plots of Λ versus varying value of the exponents α and β in Figs. 1 and 2.

5 Conclusions

In this work we give a brief review of the canonical scalar field model with non-minimum derivative coupling to curvature in cosmology. Of our interest in Sushkov's model [68,80], we consider the case when the potential is constant, i.e. $V = \Lambda/(8\pi G)$

Table 3 Value of the cosmological constant with power-law expansion (using $\varepsilon = +1$) and phantom power-law expansion (using $\varepsilon = -1$) for each of observational data

Parameters	WMAP9 + eCMB + BAO + H_0	PLANCK + WP	*TT, TE, EE* + lowP + Lensing + ext.
$\Lambda_{(\varepsilon=+1)}$ (s^{-2})	$-8.5194(5)^{+43.5168(8)}_{-4.5365(9)} \times 10^{-35}$	$-1.3997(8)^{+4.5685(7)}_{-0.6130(8)} \times 10^{-35}$	$-2.9833(7)^{+3.9590(8)}_{-2.3899(2)} \times 10^{-34}$
$\Lambda_{(\varepsilon=-1)}$ (s^{-2})	$7.4792(3)^{+38.2550(6)}_{-21.2910(5)} \times 10^{-35}$	$2.6114(3)^{+8.8452(5)}_{-32.4699(7)} \times 10^{-35}$	$2.4939(1)^{+3.3466(3)}_{-2.0660(4)} \times 10^{-34}$

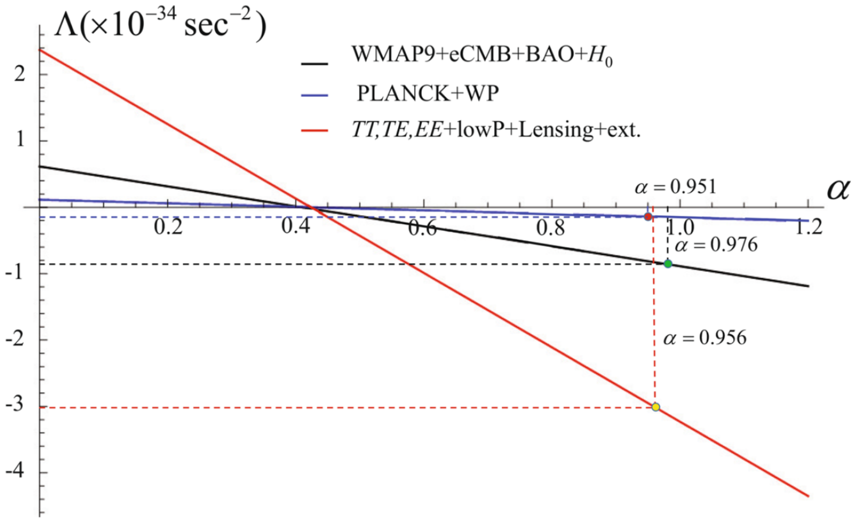

Fig. 1 Parametric plots of Λ versus α in a power-law expansion

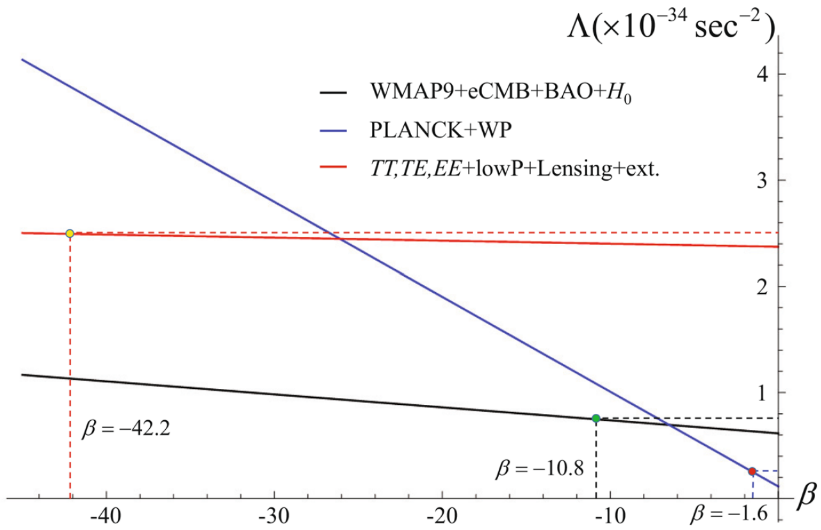

Fig. 2 Parametric plots of Λ versus β in a phantom power-law expansion

and the coupling constant is positive. The NMDC coupling term behaves like an effective cosmological constant, $\Lambda_{\rm NMDC} = \varepsilon/\kappa$. Hence the NMDC term together with the free kinetic term contributes to de-Sitter like acceleration to the dynamics in the slow-roll regime at early time, i.e. inflation. At late time the NMDC contribution is very little due to small curvature. At late time, in presence of barotropic matter term and cosmological constant, we use observational data from WMAP9 + eCMB + BAO + H_0, PLANCK + WP and *TT, TE, EE* + lowP + Lensing + external data to find cosmological constant of the theory, modeled with power-law and super-acceleration (phantom power-law) expansion functions. We estimate that the universe kinematically expands with power-law or super acceleration only from very recent redshifts. For power-law expansion, the results are $\Lambda = -8.52 \times 10^{-35}$ s^{-2} (combined WMAP9), -1.40×10^{-35} s^{-2} (PLANCK + WP) and $\Lambda = -2.98 \times 10^{-34}$ s^{-2} (*TT, TE, EE* + lowP + Lensing + external data). These are of the same order as of ΛCDM model but negative. Hence in this model, to have power-law expansion, the cosmological constant must be negative. Hence the power-law expansion is not suitable for modeling NMDC cosmology. For the super-acceleration (phantom) expansion, the results are $\Lambda = 7.48 \times 10^{-35}$ s^{-2} (combined WMAP9), $\Lambda = 2.61 \times 10^{-35}$ s^{-2} (PLANCK + WP) and $\Lambda = 2.49 \times 10^{-34}$ s^{-2} (*TT, TE, EE* + lowP + Lensing + external data). The value is very sensitive to $t_{\rm s}$ which has large error bar.

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6 Appendix 1: Equation of state parameter for power-law case

In this part, we apply the power-law expansion $a = a_0\,(t/t_0)^{\alpha}$ to the NMDC cosmology. The equation of state parameter in Eq. (11) takes the form

$$w_\phi = \frac{\dot{\phi}^2(t^2 - 9\kappa\alpha^2)\left[1 - \frac{2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right] - \frac{4\kappa\alpha\dot{\phi}V_{,\phi}t^3}{t^2-3\kappa\alpha^2} - 2V(\phi)t^2}{\dot{\phi}^2(t^2 - 3\kappa\alpha^2) + 2V(\phi)t^2}. \quad (24)$$

Eq. (18) takes the form,

$$\dot{\phi}^2 = \frac{F_1(t, \phi, \dot{\phi})}{(t^2 - 9\kappa\alpha^2)}. \quad (25)$$

Substituting Eq. (25) into the equation of state parameter, Eq. (24), we obtain

$$w_\phi = \frac{F_1(t, \phi, \dot{\phi})\left[1 - \frac{2\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}\right] - \frac{4\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-3\kappa\alpha^2)} - 2V(\phi)t^2}{F_1(t, \phi, \dot{\phi}) + 2V(\phi)t^2} \quad (26)$$

where

$$F_1(t,\phi,\dot{\phi}) = \frac{\frac{2\kappa\alpha V_{,\phi}\dot{\phi}t^3}{(t^2-9\kappa\alpha^2)} - \rho_{m,0}\frac{t_0^{3\alpha}}{(t^{3\alpha-2})} + \frac{\alpha}{(4\pi G)}}{1-\frac{\kappa\alpha(t^2+9\kappa\alpha^2)}{(t^2-3\kappa\alpha^2)(t^2-9\kappa\alpha^2)}} \tag{27}$$

7 Appendix 2: Equation of state parameter for phantom power-law case

Apply the phantom power-law expansion (super-acceleration), $a = a_0\left[(t_s-t)/(t_s-t_0)\right]^\beta$, The kinetic term can be written as

$$\dot{\phi}^2 = -\frac{F_2(t,\phi,\dot{\phi})}{\left[(t_s-t)^2+9\kappa\beta^2\right]} \tag{28}$$

The equation of state parameter of a phantom power-law expansion is

$$w_\phi = \frac{F_2(t,\phi,\dot{\phi})\left[1+\frac{2\kappa\beta\left[(t_s-t)^2-9\kappa\beta^2\right]}{\left[(t_s-t)^2+3\kappa\beta^2\right]\left[(t_s-t)^2+9\kappa\beta^2\right]}\right] - \frac{4\kappa\beta V_{,\phi}\dot{\phi}(t_s-t)^3}{\left[(t_s-t)^2+3\kappa\beta^2\right]} - 2V(\phi)(t_s-t)^2}{F_2(t,\phi,\dot{\phi})+2V(\phi)(t_s-t)^2} \tag{29}$$

where

$$F_2(t,\phi,\dot{\phi}) = \frac{\frac{2\kappa\beta V_{,\phi}\dot{\phi}(t_s-t)^3}{(t_s-t)^2+3\kappa\beta^2} - \rho_{m,0}\frac{(t_s-t_0)^{3\beta}}{(t_s-t)^{3\beta-2}} + \frac{\beta}{4\pi G}}{1+\frac{\kappa\beta\left[(t_s-t)^2-9\kappa\beta^2\right]}{\left(\left[(t_s-t)^2+3\kappa\beta^2\right]\left[(t_s-t)^2+9\kappa\beta^2\right]\right)}} \tag{30}$$

With constant potential in form of $V(\phi) = \Lambda/8\pi G$ hence $V_{,\phi} = 0$ for both cases.

References

1. Amanullah, R., et al.: Astrophys. J. **716**, 712 (2010)
2. Astier, P., et al.: (SNLS Collaboration): Astron Astrophys. **447**, 31 (2006)
3. Goldhaber, G., et al.: Astrophys. J. **558**, 359 (2001)
4. Perlmutter, S., et al.: (Supernova Cosmology Project Collaboration): Nature **391**, 51 (1998)
5. Perlmutter, S., et al.: (Supernova Cosmology Project Collaboration): Astrophys. J. **517**, 565 (1999)
6. Riess, A.G., et al.: (Supernova Search Team Collaboration): Astron. J. **116**, 1009 (1998)
7. Riess, A.G.: arXiv:astro-ph/9908237
8. Riess, A.G., et al.: (Supernova Search Team Collaboration): Astrophys. J. **607**, 665 (2004)
9. Riess, A.G., et al.: Astrophys. J. **659**, 98 (2007)
10. Tonry, J.L., et al.: (Supernova Search Team Collaboration): Astrophys. J. **594**, 1 (2003)
11. Scranton, R., *et al.* (SDSS Collaboration): arXiv:astro-ph/0307335
12. Tegmark, K., et al.: (SDSS Collaboration): Phys. Rev. D **69**, 103501 (2004)
13. Larson, D., et al.: Astrophys. J. Suppl. **192**, 16 (2011)
14. Komatsu, E., et al.: (WMAP Collaboration): Astrophys. J. Suppl. **192**, 18 (2011)
15. Hu, J.W., Cai, R.G., Guo, Z.K., Hu, B.: JCAP **1405**, 020 (2014)
16. Masi, S., et al.: Prog. Part. Nucl. Phys. **48**, 243 (2002)
17. Allen, S.W., et al.: Mon. Not. Roy. Astron. Soc. **353**, 457 (2004)

18. Rapetti, D., Allen, S.W., Weller, J.: Mon. Not. Roy. Astron. Soc. **360**, 555 (2005)
19. Copeland, E.J., Sami, M., Tsujikawa, S.: Int. J. Mod. Phys. D **15**, 1753 (2006)
20. Padmanabhan, T.: Curr. Sci. **88**, 1057 (2005)
21. Padmanabhan, T.: AIP Conf. Proc. **861**, 179 (2006)
22. Amendola, L., Tsujikawa, S.: Dark Energy: Theory and Observations. Cambridge University Press, Cambridge (2010)
23. Caldwell, R.R., Dave, R., Steinhardt, P.J.: Phys. Rev. Lett. **80**, 1582 (1998)
24. Chiba, T., Okabe, T., Yamaguchi, M.: Phys. Rev. D **62**, 023511 (2000)
25. Armendariz-Picon, C., Mukhanov, V.F., Steinhardt, P.J.: Phys. Rev. Lett. **85**, 4438 (2000)
26. Armendariz-Picon, C., Mukhanov, V.F., Steinhardt, P.J.: Phys. Rev. D **63**, 103510 (2001)
27. De Felice, A., Tsujikawa, S.: Liv. Rev. Rel. **13**, 3 (2010)
28. Carroll, S.M., Duvvuri, V., Trodden, M., Turner, M.S.: Phys. Rev. D **70**, 043528 (2004)
29. Brans, C., Dicke, R.H.: Phys. Rev. D **124**, 925 (1961)
30. Dirac, P.A.M.: Proc. Roy. Soc. **A338**, 439 (1974)
31. Birrel, N.D., Davies, P.C.W.: Quantum Fields in Curved Spaces. Cambridge University Press, Cambridge (1982)
32. Zee, A.: Phys. Rev. Lett. **42**, 417 (1979)
33. Applequist, T., Chodos, A.: Phys. Rev. Lett. **50**, 141 (1983)
34. Randjbar-Daemi, S., Salam, A., Strathdee, J.: Phys. Lett. B **135**, 388 (1984)
35. Accetta, F.S., Zoller, D.J., Turner, M.S.: Phys. Rev. D **31**, 3046 (1985)
36. Maeda, K.: Class. Quant. Grav. **3**, 233 (1986)
37. Futamase, T., Maeda, K.: Phys. Rev. D **39**, 399 (1989)
38. Kasper, U.: Nuovo Cimento **103**, 291 (1989)
39. La, D., Steinhardt, P.J.: Phys. Rev. Lett. **62**, 376 (1989)
40. Accetta, F.S., Trester, J.J.: Phys. Rev. D **39**, 2854 (1989)
41. Wang, Y.: Phys. Rev. D **42**, 2541 (1990)
42. Amendola, L., Capozziello, S., Litterio, M., Occhionero, F.: Phys. Rev. D **45**, 417 (1992)
43. Amendola, L., Bellisai, D., Occhionero, F.: Phys. Rev. D **47**, 4267 (1993)
44. Chiba, T.: Phys. Rev. D **60**, 083508 (1999)
45. Uzan, J.-P.: Phys. Rev. D **59**, 123510 (1999)
46. Holden, D.J., Wands, D.: Phys. Rev. D **61**, 043506 (2000)
47. Easson, D.A.: JCAP **0702**, 004 (2007)
48. Amendola, L.: Phys. Rev. D **60**, 043501 (1999)
49. Capozziello, S., de Ritis, R.: Gen. Relativ. Gravit. **29**, 1425 (1997)
50. Amendola, L.: Phys. Lett. B **301**, 175 (1993)
51. Liu, H., Tseytlin, A.A.: Nucl. Phys. B **533**, 88 (1998)
52. Nojiri, S., Odintsov, S.D.: Phys. Lett. B **444**, 92 (1998)
53. Magnano, G., Ferraris, F., Francaviglia, M.: Gen. Relativ. Gravit. **19**, 465 (1987)
54. Hinshaw, G., *et al.* (WMAP Collaboration): Cosmological Parameter Results," arXiv:1212.5226 [astro-ph.CO]
55. Ade, P. A. R., *et al.* (Planck Collaboration): arXiv:1303.5076 [astro-ph.CO]
56. Ade, P. A. R. *et al.* [Planck Collaboration]: arXiv:1502.01589 [astro-ph.CO]
57. Capozziello, S., Lambiase, G., Schmidt, H.J.: Annalen Phys. **9**, 39 (2000)
58. Capozziello, S., Lambiase, G.: Gen. Relativ. Gravit. **31**, 1005 (1999)
59. Granda, L.N.: Rev. Col. Fís. **42**, 257 (2010)
60. Daniel, S.F., Caldwell, R.R.: Class. Quant. Grav. **24**, 5573 (2007)
61. Granda, L.N.: JCAP **1007**, 006 (2010)
62. Donoghue, J.F.: Phys. Rev. D **50**, 3874 (1994)
63. Granda, L.N., Cardona, W.: JCAP **1007**, 021 (2010)
64. Granda, L.N.: Class. Quant. Grav. **28**, 025006 (2011)
65. Granda, L.N.: JCAP **1104**, 016 (2011)
66. Granda, L.N.: Mod. Phys. Lett. A **27**, 1250018 (2012)
67. Granda, L.N., Torrente-Lujan, E., Fernandez-Melgarejo, J.J.: Eur. Phys. J. C **71**, 1704 (2011)
68. Sushkov, S.V.: Phys. Rev. D **80**, 103505 (2009)
69. Saridakis, E.N., Sushkov, S.V.: Phys. Rev. D **81**, 083510 (2010)
70. Gao, C.: JCAP **1006**, 023 (2010)
71. Germani, C., Kehagias, A.: Phys. Rev. Lett. **106**, 161302 (2011)

72. Feng, K., Qiu, T.: Phys. Rev. D **90**(12), 123508 (2014)
73. Gubitosi, G., Linder, E.V.: Phys. Lett. B **703**, 113 (2011)
74. Bruneton, J.P., Rinaldi, M., Kanfon, A., Hees, A., Schlogel, S., Fuzfa, A.: Adv. Astron. **2012**, 430694 (2012)
75. Chen, S., Jing, J.: Phys. Lett. B **691**, 254 (2010)
76. Chen, S., Jing, J.: Phys. Rev. D **82**, 084006 (2010)
77. Ding, C., Liu, C., Jing, J., Chen, S.: JHEP **1011**, 146 (2010)
78. Kolyvaris, T., Koutsoumbas, G., Papantonopoulos, E., Siopsis, G.: Class. Quant. Grav. **29**, 205011 (2012)
79. Skugoreva, M.A., Sushkov S.V., Toporensky, A.V.: Phys. Rev. D **88**, (2013) 083539. [Phys. Rev. D **88**, no. 10, (2013) 109906 Erratum]
80. Sushkov, S.V.: Phys. Rev. D **85**, 123520 (2012)
81. Koutsoumbas, G., Ntrekis, K., Papantonopoulos, E.: JCAP **1308**, 027 (2013)
82. Darabi, F., Parsiya, A.: arXiv:1312.1322 [gr-qc]
83. Germani, C., Kehagias, A.: JCAP **0903**, 028 (2009)
84. Germani, C., Kehagias, A.: Phys. Rev. Lett. **105**, 011302 (2010)
85. Germani, C., Watanabe, Y.: JCAP **1107**, 031 (2011)
86. Sadjadi, H.M., Goodarzi, P.: JCAP **1302**, 038 (2013)
87. Tsujikawa, S.: Phys. Rev. D **85**, 083518 (2012)
88. Ema, Y., Jinno, R., Mukaida, K., Nakayama, K.: arXiv:1504.07119 [gr-qc]
89. Ghalee, A.: arXiv:1402.6798 [astro-ph.CO]
90. Yang, N., Gao, Q., Gong, Y.: arXiv:1504.05839 [gr-qc]
91. Myung, Y. S., Moon, T. and Lee, B. H.: arXiv:1505.04027 [gr-qc]
92. Sadjadi, H.M.: Phys. Rev. D **83**, 107301 (2011)
93. Sadjadi, H.M., Goodarzi, P.: Phys. Lett. B **732**, 278 (2014)
94. Sadjadi, H.M.: Gen. Relativ. Gravit. **46**(11), 2014 (1817)
95. Jinno, R., Mukaida, K., Nakayama, K.: JCAP **1401**(01), 031 (2014)
96. Dent, J.B., Dutta, S., Saridakis, E.N., Xia, J.Q.: JCAP **1311**, 058 (2013)
97. Huang, Y., Gao, Q., Gong, Y.: Eur. Phys. J. C **75**(4), 143 (2015)
98. Sethi, G., Dev, A., Jain, D.: Phys. Lett. B **624**, 135 (2005)
99. Kumar, S.: Mon. Not. Roy. Astron. Soc. **422**, 2532 (2012)
100. Gumjudpai, B., Thepsuriya, K.: Astrophys. Space Sci. **342**, 537 (2012)
101. Gumjudpai, B.: Mod. Phys. Lett. A **28**, 1350122 (2013)
102. Rani, S., Altaibayeva, A., Shahalam, M., Singh, J.K., Myrzakulov, R.: JCAP **1503**(03), 031 (2015)
103. Lucchin, F., Matarrese, S.: Phys. Rev. D **32**, 1316 (1985)
104. Dev, A., Safonova, M., Jain, D., Lohiya, D.: Phys. Lett. B **548**, 12 (2002)
105. Jain, D., Dev, A., Alcaniz, J.S.: Class. Quan. Grav. **20**, 4163 (2003)
106. Zhu, Z.H., Hu, M., Alcaniz, J.S., Liu, Y.X.: Astron. Astrophys. **483**, 15 (2008)
107. Coles, P., Lucchin, F.: Cosmology: The Origin and Evolution of Cosmic Structure, 2nd edn. Wiley, Chichester (2002)
108. Caldwell, R.R.: Phys. Lett. B **545**, 23 (2002)
109. Caldwell, R.R., Kamionkowski, M., Weinberg, N.N.: Phys. Rev. Lett. **91**, 071301 (2003)
110. Kaeonikhom, C., Gumjudpai, B., Saridakis, E.N.: Phys. Lett. B **695**, 45 (2011)

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ORIGINAL ARTICLE

Tachyonic (phantom) power-law cosmology

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Abstract Tachyonic scalar field-driven late universe with dust matter content is considered. The cosmic expansion is modeled with power-law and phantom power-law expansion at late time, i.e. $z \lesssim 0.45$. WMAP7 and its combined data are used to constraint the model. The forms of potential and the field solution are different for quintessence and tachyonic cases. Power-law cosmology model (driven by either quintessence or tachyonic field) predicts unmatched equation of state parameter to the observational value, hence the power-law model is excluded for both quintessence and tachyonic field. In the opposite, the phantom power-law model predicts agreeing valued of equation of state parameter with the observational data for both quintessence and tachyonic cases, i.e. $w_{\phi,0} = -1.49^{+11.64}_{-4.08}$ (WMAP7+BAO+H_0) and $w_{\phi,0} = -1.51^{+3.89}_{-6.72}$ (WMAP7). The phantom-power law exponent β must be less than about -6, so that the $-2 < w_{\phi,0} < -1$. The phantom power-law tachyonic potential is reconstructed. We found that dimensionless potential slope variable Γ at present is about 1.5. The tachyonic potential reduced to $V = V_0\phi^{-2}$ in the limit $\Omega_{\mathrm{m},0} \to 0$.

Keywords Power-law cosmology · Tachyonic dark energy

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1 Introduction

There have been clear evidences that the present universe is under accelerating expansion as observed in, e.g. the cosmic microwave background (CMB) (Masi et al. 2002; Larson et al. 2011; Komatsu et al. 2011), large-scale structure surveys (Scranton et al. 2003; Tegmark et al. 2004), supernovae type Ia (SNIa) (Perlmutter et al. 1998, 1999; Riess 1999; Riess et al. 1998, 2004, 2007; Goldhaber et al. 2001; Tonry et al. 2003; Astier et al. 2006; Amanullah et al. 2010) and X-ray luminosity from galaxy clusters (Allen et al. 2004; Rapetti et al. 2005). One prime explanation is that the acceleration is an effect of a scalar field evolving under its potential to acquire negative pressure with $p < -\rho c^2/3$ giving repulsive gravity. Form of energy with this negative pressure range is generally called dark energy (Padmanabhan 2005, 2006; Copeland et al. 2006). Scalar field is responsible for symmetry breaking mechanisms and super-fast expansion in inflationary scenario, resolving horizon and flatness problems as well as explaining the origin of structures (Starobinsky 1980; Guth 1981; Sato 1981; Albrecht and Steinhardt 1982; Linde 1982). Introducing a cosmological constant into the field equation is simplest way to have dark energy (Weinberg 1989; Ford 1987; Dolgov 1997), but it creates new problem on fine-tuning of energy density scales (Sahni and Starobinsky 2000; Peebles and Ratra 2003). For the cosmological constant to be viable, idea of varying cosmological constant needs to be installed (Sola and Stefancic 2005; Shapiro and Sola 2009). If dark energy is the scalar field, the field could have non-canonical kinetic part such as tachyon which is classified in a type of k-essence models (Armendariz-Picon et al. 2000, 2001). The tachyon field is a negative mass mode of an unstable non-BPS D3-brane in string theory (Garousi 2000; Sen 2002a, 2002b) or a massive scalar field on anti-D3 brane

(Garousi et al. 2004). It was found that the tachyonic field potential must not be too steep, i.e. less steep than $V(\phi) \propto \phi^{-2}$ in order to account for the late acceleration (Padmanabhan 2002; Bagla et al. 2003; Kutasov and Niarchos 2003; Abramo and Finelli 2003; Aguirregabiria and Lazkoz 2004; Copeland et al. 2005).

In this work, we considered dark energy in form of tachyonic scalar field in power-law cosmology of which the scale factor scaled as $a \propto t^{\alpha}$ with $0 \le \alpha \le \infty$, corresponding to acceleration if $\alpha > 1$ and in addition we also consider phantom power-law cosmology with $a \propto (t_s - t)^{\beta}$. In cosmic history, there were epoch when radiation or dust is dominant component in the universe for which the scale factor evolves as power-law $a \propto t^{1/2}$ and $a \propto t^{2/3}$. A universe with mixed combination of different cosmic ingredients can be modeled using power-law expansion with some approximately constant α during a brief period of cosmic time. Adjustability of expansion rate is characterized by only one parameter, α which is used widely in astrophysical observations. There are also other situations that one can obtain the power-law solution. These are such as non-minimally coupled scalar-tensor theory in which the scalar field couples to the curvature contributing to energy density that cancels out the vacuum energy (Dolgov 1982, 1997; Ford 1987; Fujii and Nishioka 1990) and simple inflationary model in which the power-law cosmology can avoid flatness and horizon problems and can give simple spectrum (Lucchin and Matarrese 1985). The power-law has proved to be a very good phenomenological description of the cosmic evolution, since it can describe radiation epoch, dark matter epoch, and dark energy epoch according to value of the exponent (Kolb 1989; Peebles 1993). Previously linear-coasting cosmology, $\alpha \approx 1$ was analyzed (Lohiya et al. 1996; Sethi et al. 1999; Dev et al. 2001, 2002) with motivation from SU(2) instanton cosmology (Allen 1999), higher order (Weyl) gravity (Manheim and Kazanas 1990), or from scalar-tensor theories (Lohiya and Sethi 1999). However the universe expanding with $\alpha = 1$ (Melia and Shevchuk 2012) was not able to agree with observational constraint from Type Ia supernovae, Hubble rate data from cosmic chronometers and BAO (Bilicki and Seikel 2012) which indicates that $H'(z)$=const and $q(z) = 0$ are not favored by the observations.

For a specific gravity or dark energy model, power-law cosmology is considered in $f(T)$ and $f(G)$ gravities (Rastkar et al. 2012; Setare and Darabi 2012) and in the case of which there is coupling between cosmic fluids (Cataldo et al. 2008). The power-law cosmology were also studied in context of scalar field cosmology (Gumjudpai and Thepsuriya 2012; Gumjudpai 2013), phantom scalar field cosmology (Kaeonikhom et al. 2011). There is also slightly different form of the power-law function which α can evolved with time so that it can parameterize cosmological observables (Wei 2004).

For the power-law to be valid throughout the cosmic evolution, it is not possible with constant exponent. For example, at big bang primordial nucleosynthesis (BBN), α is allowed to have maximum value at approximately 0.55 in order to be capable of light element abundances (Kaplinghat et al. 1999, 2000). The value is about $1/2$ at highly-radiation dominated era, about $2/3$ at highly-dust dominated era and greater than one at present. Low value of α results in much younger cosmic age and does not give acceleration. On the other hand $\alpha \ge 1$ value is needed to solve age problem in the CDM model (Kolb 1989) without flatness and horizon problems. In universe dominated with cold dark matter and dark energy, considering that the power-law expansion happens long after matter-radiation equality era, $z \ll 3196$ (value from Larson et al. 2011), the BBN constraint can be relaxed and large α can be allowed. We consider power-law cosmology with a brief period of recent cosmic era when dark energy began to dominate, i.e. from $z \lesssim 0.45$ to present using results from WMAP7 (Larson et al. 2011) and WMAP7+BAO+H_0 combined datasets (Komatsu et al. 2011). There are tachyonic scalar field evolving under potential $V(\phi)$ and dust barotropic fluid (cold dark matter and baryonic matter) as two major ingredients. We aim to test whether the power-law cosmology is still valid in the scenario of tachyonic scalar field by looking at value of the equation of state predicted by the power-law tachyonic cosmology and that of varying dark energy equation of state direct-observational result. The WMAP7 and WMAP7+BAO+H_0 data used here are presented in Table 1. We also consider when the field is phantom, i.e. having negative kinetic term with phantom power-law expansion (Caldwell 2002; Caldwell et al. 2003), $a \propto (t_s - t)^{\beta}$, $\beta < 0$ from $z \lesssim 0.45$ till present. We determine tachyonic field equation of state parameter, w_ϕ and we perform parametric plot versus exponent β. We then analyze the result and conclude this work.

2 Background cosmology and observational data

We consider standard FLRW universe containing dust matter (cold dark matter and baryonic matter) with tachyonic field with Lagrangian,

$$\mathcal{L}_{\text{tachyon}} = -V(\phi)\sqrt{1 - \partial_\mu \phi \partial^\mu \phi} \tag{1}$$

evolving under the background Friedmann equation,

$$H^2 = \frac{8\pi G}{3}(\rho_\phi + \rho_{\text{m}}) - \frac{kc^2}{a^2} \tag{2}$$

and acceleration rate,

$$\dot{H} = \frac{\ddot{a}}{a} - H^2 = -\frac{4\pi G}{c^2}\left(\rho_\phi c^2 + p_\phi + \rho_{\text{m}} c^2 + p_{\text{m}}\right) + \frac{kc^2}{a^2} \tag{3}$$

Table 1 Combined WMAP7+BAO+H_0 and WMAP7 derived parameters (maximum likelihood) from Larson et al. (2011) and Komatsu et al. (2011). Here we also calculate (with error analysis) $\Omega_{m,0} = \Omega_{b,0} + \Omega_{CDM,0}$, critical density: $\rho_{c,0} = 3H_0^2/8\pi G$ and matter density: $\rho_{m,0} = \Omega_{m,0}\rho_{c,0}$. The space is flat and a_0 is set to unity

Parameter	WMAP7+BAO+H_0	WMAP7
t_0	13.76 ± 0.11 Gyr or $(4.34 \pm 0.03) \times 10^{17}$ sec	13.79 ± 0.13 Gyr or $(4.35 \pm 0.04) \times 10^{17}$ sec
H_0	70.4 ± 1.4 km/s/Mpc	70.3 ± 2.5 km/s/Mpc
	$(2.28 \pm 0.04) \times 10^{-18}$ sec^{-1}	$(2.28 \pm 0.08) \times 10^{-18}$ sec^{-1}
$\Omega_{b,0}$	0.0455 ± 0.0016	0.0451 ± 0.0028
$\Omega_{CDM,0}$	0.226 ± 0.015	0.226 ± 0.027
$\Omega_{m,0}$	$0.271(5) \pm 0.015(1)$	$0.271(1) \pm 0.027(1)$
$\rho_{m,0}$	$(2.52(49)^{+0.18(24)}_{-0.16(64)}) \times 10^{-27}$ kg/m^3	$(2.52(12)^{+0.30(97)}_{-0.30(61)}) \times 10^{-27}$ kg/m^3
$\rho_{c,0}$	$(9.29(99)^{+0.32(92)}_{-0.32(35)}) \times 10^{-27}$ kg/m^3	$(9.29(99)^{+0.66(41)}_{-0.64(12)}) \times 10^{-27}$ kg/m^3

Tachyonic field energy density and pressure are

$$\rho_\phi c^2 = \frac{V(\phi)}{\sqrt{1-\epsilon\dot{\phi}^2}} \tag{4}$$

$$p_\phi = -V(\phi)\sqrt{1-\epsilon\dot{\phi}^2} \tag{5}$$

where $\epsilon = \pm 1$. The negative ϵ represents the case when kinetic term of the tachyon is phantom. The tachyonic fluid equation reads

$$\frac{\epsilon\ddot{\phi}}{1-\epsilon\dot{\phi}^2} + 3H\epsilon\dot{\phi} + \frac{V'}{V} = 0 \tag{6}$$

Using Eqs. (4), (5) and (6) in the (3) (dust pressure is zero), we obtain

$$\dot{H} = -\frac{4\pi G}{c^2}\left(\frac{V\epsilon\dot{\phi}^2}{\sqrt{1-\epsilon\dot{\phi}^2}} + \rho_m c^2\right) + \frac{kc^2}{a^2} \tag{7}$$

Using tachyonic density (4) in the Friedmann equation therefore

$$\frac{V}{\sqrt{1-\epsilon\dot{\phi}^2}} = \frac{3}{8\pi G/c^2}\left(H^2 + \frac{kc^2}{a^2}\right) - \rho_m c^2 \tag{8}$$

Substituting (8) into Eq. (7), we obtain

$$\dot{H} = -4\pi G\left[\frac{3\epsilon\dot{\phi}^2}{8\pi G}\left(H^2 + \frac{kc^2}{a^2}\right) - \rho_m\epsilon\dot{\phi}^2 + \rho_m\right] + \frac{kc^2}{a^2} \tag{9}$$

which can be rewritten as

$$\epsilon\dot{\phi}^2 = -\left[\frac{2\dot{H} - (2kc^2/a^2) + 8\pi G\rho_m}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_m}\right] \tag{10}$$

and hence

$$1 - \epsilon\dot{\phi}^2 = \frac{3H^2 + 2\dot{H} + (kc^2/a^2)}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_m} \tag{11}$$

We use the above expression in Eq. (8), as a result we can get tachyonic potential

$$V = \left[\frac{3}{8\pi G/c^2}\left(H^2 + \frac{kc^2}{a^2}\right) - \rho_m c^2\right] \times \sqrt{\frac{3H^2 + 2\dot{H} + (kc^2/a^2)}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_m}}. \tag{12}$$

Tachyonic potential of the phantom-power law is in different form from the quintessential potential of the normal power-law cosmology. The tachyonic equation of state, w_ϕ is, from (11),

$$w_\phi = \frac{p}{\rho c^2} = -(1 - \epsilon\dot{\phi}^2) = -\left[\frac{3H^2 + 2\dot{H} + (kc^2/a^2)}{3H^2 + (3kc^2/a^2) - 8\pi G\rho_m}\right] \tag{13}$$

This can be weighed with the dust-matter content to give effective equation of state, $w_{eff} = \rho_\phi w_\phi/(\rho_\phi + \rho_m)$. With all information above, w_{eff} is expressed as

$$w_{eff} = w_\phi\left[1 - \frac{8\pi G\rho_m/3}{H^2 + (kc^2/a^2)}\right] \tag{14}$$

We found that Eqs. (13) and (14) are the same for both quintessence scalar field (Gumjudpai 2013) and tachyonic field cases, albeit the $\dot{\phi}$ and $V(\phi)$ are expressed differently in both cases. That is for both quintessence and tachyonic cases, w_ϕ does not depend on the scalar field model but depends on the form of expansion function. This is also true for $w_{eff,0}$. The equation of state is also independent of the sign of ϵ which indicates negative kinetic energy. Using power-law expansion and phantom power-law expansion into (13), one can find the present value of the equation of state, $w_{\phi,0}$. This value is a (phantom) power-law prediction of the $w_{\phi,0}$. We can compare this predicted value to the $w_{\phi,0}$ (of varying equation of state) obtained from CMB observation.

The derived data from WMAP7+BAO+H_0 and WMAP7 are presented in Table 1. We will set $a_0 = 1$ and consider flat universe $k = 0$ throughout (but kept k in the formulae for completeness). Dust density is defined as $\rho_{m,0} = \Omega_{m,0}\rho_{c,0}$. Total dust fluid density at present is sum of that of all dust mater types $\Omega_{m,0} = \Omega_{CDM,0} + \Omega_{b,0}$. Present value of the critical density is $\rho_{c,0} = 3H_0^2/8\pi G$, and radiation density is negligible. We take the maximum likelihood value assuming spatially flat case. Although in deriving t_0, the ΛCDM model is assumed with the CMB data, however one can estimably use t_0 since w_{DE} is very close to -1. In SI units, the reduced Planck mass squared is $M_P^2 = \hbar c/8\pi G$. In this work, we also give correction to errors on future singularity time, t_s (phantom power-law case) reported previously in Kaeonikhom et al. (2011) and improve values of $w_{\phi,0}$ of the phantom power-law case in Kaeonikhom et al. (2011) and of the usual power-law case reported earlier (Gumjudpai 2013).

3 Power-law cosmology

Origin of power-law cosmology comes from a solution of the Friedmann equation with flat geometry and domination of dark energy, $H^2 = 8\pi G\rho_\phi/3$. For constant equation of state w_ϕ, the solution is well known as (see, for example, in p. 150 of Coles and Lucchin (2002))

$$a = a_0\left[1 + \frac{H(t_0)}{\alpha}(t - t_0)\right]^\alpha \tag{15}$$

where $\alpha = 2/[3(1+w_\phi)]$ is constant. For $-1/3 > w_\phi > -1$, the solution takes power-law form,

$$a(t) = a_0\left(\frac{t}{t_0}\right)^\alpha \tag{16}$$

Note that although the function is motivated by domination of constant w_ϕ scalar field in the flat Friedmann equation which gives $1 < \alpha < \infty$, here we will consider the range $0 < \alpha < \infty$ (constant value of α) and we will estimably use the power-law expansion in presence of barotropic dust fluid and varying w_ϕ in a short range of redshift $z \lesssim 0.45$ to present. Later section on phantom-power law (Sect. 4) is based on the same estimation as well. In the power-law cosmology, the speed is $\dot{a} = \alpha a/t$ and the acceleration is $\ddot{a} = \alpha(\alpha - 1)a/t^2$. The Hubble parameter is $H(t) = \dot{a}/a = \alpha/t$ with $\dot{H} = -\alpha/t^2$. The deceleration parameter in this scenario is $q \equiv -a\ddot{a}/\dot{a}^2 = (1/\alpha) - 1$, that is $\alpha = 1/(q + 1)$. As $\alpha \geq 0$ is required in power-law cosmology, hence $q \geq -1$ and $H_0 \geq 0$. To convert into redshift z, from $1 + z = a_0/a$ then $1 + z = (t_0/t)^\alpha$. Typically astrophysical tests for power-law cosmology indicating the value of α are performed by observing $H(z)$ data of SNIa or high-redshift objects such as distant globular clusters (Dev et al. 2008; Sethi et al. 2005; Kumar 2012). To indicate the value of α one can also use gravitational lensing statistics (Dev et al. 2002), compact-radio source (Jain et al. 2003) or using X-ray gas mass fraction measurements of galaxy clusters (Zhu et al. 2008; Allen et al. 2002, 2003). Study of angular size to z relation of a large sample of milliarcsecond compact radio sources in flat FLRW universe found that $\alpha = 1.0 \pm 0.3$ at 68 % C.L. (Jain et al. 2003). WMAP5 dataset gives $\alpha = 1.01$ for closed geometry (Gumjudpai and Thepsuriya 2012). Some procedures of measurement give large value of α such as $\alpha = 2.3^{+1.4}_{-0.7}$ (X-ray mass fraction data of galaxy clusters in flat geometry) (Zhu et al. 2008) and $\alpha = 1.62^{+0.10}_{-0.09}$ (joint test using Supernova Legacy Survey (SNLS) and $H(z)$ data in flat geometry) (Dev et al. 2008). Notice that assumption of non-zero spatial curvature $(\pm 1, 0)$ is assumed in these results in evaluating of α except in the WMAP5 of which the result puts also constraint on the spatial curvature. When α is found with curvature-independent procedure (i.e. with neither SNIa nor cluster X-ray gass mass fraction) or in flat case, α is near unity. For example, $H(z)$ data gives $\alpha = 1.07^{+0.11}_{-0.09}$ (Dev et al. 2008) and $\alpha = 1.11^{+0.21}_{-0.14}$ (Gumjudpai 2013; Kumar 2012). Short review of recent α values can be found in Gumjudpai (2013). Here α is calculated from value at present H_0, t_0 as $\alpha = H_0 t_0$. From (13) and (14), in case of power-law cosmology driven by tachyonic field, the equation of state of dark energy is

$$w_\phi = -\frac{[\frac{3\alpha^2}{t^2} - \frac{2\alpha}{t^2} + \frac{kc^2}{a_0^2}(\frac{t_0}{t})^{2\alpha}]}{[\frac{3\alpha^2}{t^2} + \frac{3kc^2}{a_0^2}(\frac{t_0}{t})^{2\alpha} - 8\pi G\rho_{m,0}(\frac{t_0}{t})^{3\alpha}]} \tag{17}$$

and

$$w_{eff} = w_\phi\left[1 - \frac{(8\pi G/3)\rho_{m,0}(t_0/t)^{3\alpha}}{(\alpha^2/t^2) + (kc^2/a_0^2)(t_0/t)^{2\alpha}}\right] \tag{18}$$

At present, $t = t_0$, $w_{eff,0} = -1 + 2/(3\alpha)$. In Table 2, values of equation of state parameters derived in the power-law cosmology (true for both tachyonic and quintessence) do not match observational data, i.e. $w_{\phi,0}$ and $w_{eff,0}$ found here are much greater than observational (spatially flat) WMAP derived results, for example WMAP7:[1] $w_{\phi,0} = -1.12^{+0.42}_{-0.43}$, WMAP7+BAO+$H_0$ combined:[2] $w_{\phi,0} = -1.10^{+0.14}_{-0.14}$ (68 % CL), WMAP7+BAO+H_0+SN:[3] $w_{\phi,0} = -1.34^{+1.74}_{-0.36}$ (68 % CL) and WMAP7+BAO+H_0+SN with time delay distance

[1] Flat geometry, constant $w_{\phi,0}$ (Sect. 4.2.5 of Larson et al. (2011)).

[2] Flat geometry, constant $w_{\phi,0}$ (Sect. 5.1 of Komatsu et al. (2011)).

[3] Flat geometry, time varying dark energy EoS, $w_\phi(a) = w_0 + w_a(1 - a)$ with $w_0 = -0.93 \pm 0.13$, $w_a = -0.41^{+0.72}_{-0.71}$ (Sect. 5.3 of Komatsu et al. (2011)).

Table 2 Power-law cosmology exponent and its prediction of equation of state parameters. The value does not match the WMAP7 results

Parameter	WMAP7+BAO+H_0	WMAP7
α	$0.98(95) \pm 0.01(87)$	$0.99(18) \pm 0.03(60)$
$w_{\phi,0}$ (with power-law cosmology)	$-0.44(79)^{+0.01(66)}_{-0.01(54)}$	$-0.44(98)^{+0.02(97)}_{-0.02(82)}$
$w_{\mathrm{eff},0}$ (with power-law cosmology)	$-0.32(63) \pm 0.01(25)$	$-0.32(78) \pm 0.02(35)$

information correction:[4] $w_{\phi,0} = -1.31^{+1.67}_{-0.38}$ (68 % CL). We conclude that the power-law expansion universe with quintessential scalar field (Gumjudpai 2013) or tachyonic field is neither viable.

4 Phantom power-law cosmology

In this section, we can check if phantom power-law could be a valid solution for the tachyonic-driven universe. From Eq. (15), with constant $w_\phi < -1$, the solution becomes phantom power-law,

$$a(t) = a_0\left(\frac{t_s - t}{t_s - t_0}\right)^{\beta} \tag{19}$$

with speed,

$$\dot{a} = -a_0\beta\frac{(t_s - t)^{\beta-1}}{(t_s - t_0)^{\beta}} = -\beta\frac{a}{(t_s - t)}$$

and acceleration,

$$\ddot{a} = a_0\beta(\beta - 1)\frac{(t_s - t)^{\beta-2}}{(t_s - t_0)^{\beta}} = \frac{\beta(\beta - 1)a}{(t_s - t)^2}$$

where $t_s \equiv t_0 + |\beta|/H(t_0)$ (Coles and Lucchin 2002) is future big-rip singularity time (Caldwell 2002; Caldwell et al. 2003) and we use β instead of α to distinct the two solutions. For both $\dot{a}$ and $\ddot{a}$ to be greater than zero, i.e. both expanding and accelerating, the condition $\beta < 0$ is needed. The Hubble parameter is therefore,

$$H = \frac{\dot{a}}{a} = -\frac{\beta}{t_s - t} \quad \text{hence} \quad \dot{H} = -\frac{\beta}{(t_s - t)^2} \tag{20}$$

At present, $\beta = H_0(t_0 - t_s)$. The deceleration parameter is $q \equiv -a\ddot{a}/\dot{a}^2 = (1/\beta) - 1$. The dust matter density, $\rho_m = \rho_{m,0}a_0^3/a^3$ is then

$$\rho_m = \rho_{m,0}\left(\frac{t_s - t_0}{t_s - t}\right)^{3\beta} \tag{21}$$

[4]Flat geometry, time varying dark energy EoS, $w_\phi(a) = w_0 + w_a(1 - a)$ with $w_0 = -0.93 \pm 0.12$, $w_a = -0.38^{+0.66}_{-0.65}$ (Sect. 5.3 of Komatsu et al. (2011)).

Substituting these equations into (13) and (14), we obtain,

$$w_\phi = -\left[\frac{\beta(3\beta - 2)}{(t_s - t)^2} + \left(\frac{kc^2}{a_0^2}\right)\left[\frac{(t_s - t_0)}{(t_s - t)}\right]^{2\beta}\right] \Big/ \left\{\frac{3\beta^2}{(t_s - t)^2} + \left(\frac{3kc^2}{a_0^2}\right)\left[\frac{(t_s - t_0)}{(t_s - t)}\right]^{2\beta} - 8\pi G\rho_{m,0}\left[\frac{(t_s - t_0)}{(t_s - t)}\right]^{3\beta}\right\} \tag{22}$$

$$w_{\mathrm{eff}} = w_\phi\left[1 - \left\{\frac{8\pi G\rho_{m,0}}{3}\left[\frac{(t_s - t_0)}{(t_s - t)}\right]^{3\beta} \Big/ \left(\frac{\beta^2}{(t_s - t)^2} + \left(\frac{kc^2}{a_0^2}\right)\left[\frac{(t_s - t_0)}{(t_s - t)}\right]^{2\beta}\right)\right\}\right] \tag{23}$$

To convert to redshift one can use $1 + z = a_0/a$ therefore $1 + z = [(t_s - t_0)/(t_s - t)]^{\beta}$ and $t_s - t = (t_s - t_0)(1 + z)^{-1/\beta}$. At present, $t = t_0$, $w_{\mathrm{eff},0} = -1 + 2/(3\beta)$. The big-rip time t_s, can be calculated from

$$t_s \approx t_0 - \frac{2}{3(1 + w_{\mathrm{DE}})}\frac{1}{H_0\sqrt{1 - \Omega_{m,0}}} \tag{24}$$

Here, w_{DE} must be less than -1 and in deriving this above expression flat geometry and constant dark energy equation of state is assumed (Caldwell 2002; Caldwell et al. 2003). We will estimably use t_s from this formula. In finding error bar of t_s, we exploit better procedure than that performed earlier in Kaeonikhom et al. (2011) by considering that the second order of error bar multiplications are too large to be neglected. We discuss this in the Appendix. Results presented in Table 3 are β, t_s and the equation of state. For phantom power-law cosmology driven by tachyonic field (also true for phantom quintessence), the resulting value is $w_{\phi,0} = -1.49^{+11.64}_{-4.08}$ (using WMAP7+BAO+H_0) and $-1.51^{+3.89}_{-6.72}$ (using WMAP7). These do not much differ from results from WMAP7+BAO+H_0+SN data (flat, varying dark energy EoS) which gives $w_{\phi,0} = -1.34^{+1.74}_{-0.36}$ (68 % CL) and WMAP7+BAO+H_0+SN+time delay distance correction data (flat varying dark energy EoS) which

Table 3 Phantom power-law cosmology exponent and its prediction of equation of state parameters. The equation of state lies in acceptable range of values given by WMAP7 results. Large error bar of $w_{\phi,0}$ is an effect of large error bar in t_s

Parameter	WMAP7+BAO+H_0	WMAP7
β	$-7.81(08)^{+11.71(8)}_{-4.56(1)}$	$-6.50(72)^{+3.91(92)}_{-5.09(96)}$
t_s	$122.30(0)^{+162.83(7)}_{-63.36(0)}$ Gyr	$104.21(5)^{+54.37(3)}_{-70.79(9)}$ Gyr
$w_{\phi,0}$ (with phantom power-law cosmology)	$-1.48(99)^{+11.64(46)}_{-4.08(45)}$	$-1.51(26)^{+3.89(23)}_{-6.71(90)}$
$w_{\rm eff,0}$ (with phantom power-law cosmology)	$-1.08(54)^{+0.25(60)}_{-0.11(98)}$	$-1.10(24)^{+0.15(52)}_{-0.37(12)}$

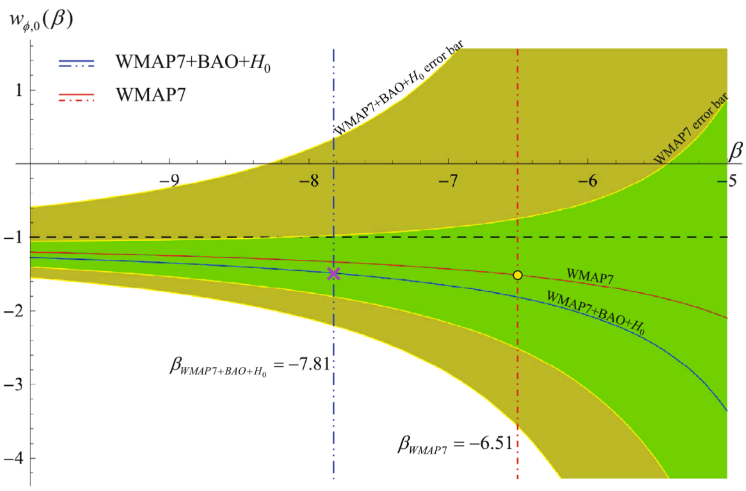

Fig. 1 Present value of phantom tachyonic dark energy equation of state plotted versus β. Their error bar results from the error bar in β. This is the same for quintessence case

gives $w_{\phi,0} = -1.31^{+1.67}_{-0.38}$ (68 % CL) (Komatsu et al. 2011). Using observational data in Tables 1 and 3 we derive

$$w_{\phi,0} = -\left[\frac{1-2/(3\beta)}{1-(16.60/\beta^2)}\right] \text{(WMAP7+BAO+}H_0\text{)} \quad (25)$$

$$w_{\phi,0} = -\left[\frac{1-2/(3\beta)}{1-(11.47/\beta^2)}\right] \text{(WMAP7)} \quad (26)$$

With these, we show parametric plots of the $w_{\phi,0}$ and β in Fig. 1. The values measured for β and $w_{\phi,0}$ are the purple cross (WMAP7+BAO+H_0) and yellow spot (WMAP7). For $-\infty < \beta \lesssim -6$, $w_{\phi,0}$ lies in the range $(-1, -2)$. Figure 2 shows evolution of $w(z)$ in late phantom power-law universe from $0 < z < 0.45$, i.e. $t = 8.48$ Gyr (both datasets) till present era (this is to avoid singularity in w_ϕ at $z = 0.492$ (WMAP7+BAO+H_0) and at $z = 0.484$ (WMAP7)). These are equivalent to the past 5.28 Gyr ago (WMAP7+BAO+H_0) and the past 5.31 Gyr ago (WMAP7).

5 Tachyonic potential for phantom power-law cosmology

5.1 Tachyonic field dominant case

When the field is phantom ($\epsilon = -1$) and is the dominant component, Eq. (10) for flat space hence

$$\dot{\phi}^2 = \frac{2\dot{H}}{3H^2} = -\frac{2}{3\beta} \quad (27)$$

Integrating from t to t_s, and choosing positive solution,

$$\phi(t) = \sqrt{\frac{2}{3|\beta|}}(t_s - t) \quad (28)$$

Since $\beta < 0$ hence $-\beta = |\beta|$. From (12) the tachyonic potential is

$$V(\phi) = \frac{2c^2|\beta|}{\kappa\phi^2}\sqrt{1+\frac{2}{3|\beta|}} \quad (29)$$

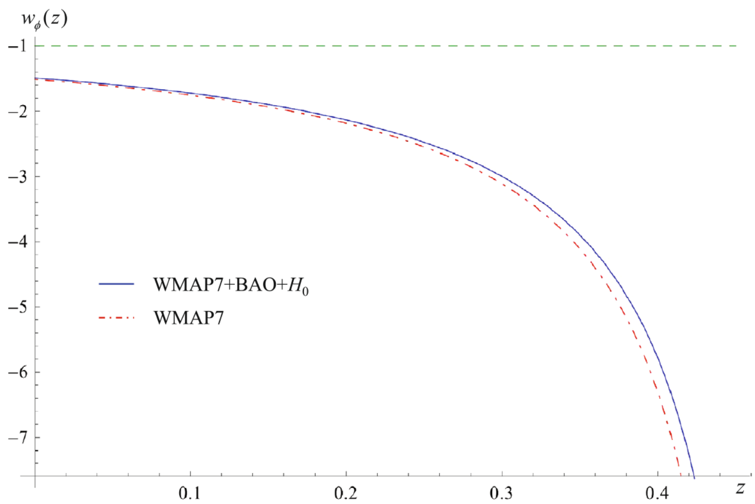

Fig. 2 Phantom tachyonic (and quintessence) dark energy equation of state versus z

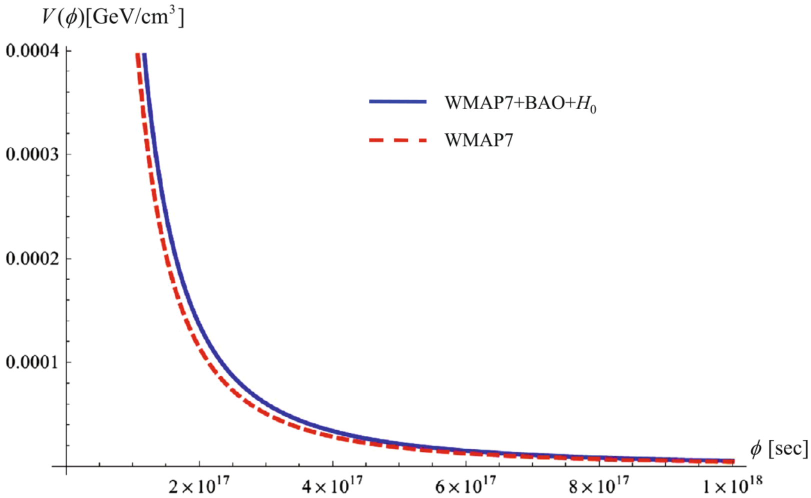

Fig. 3 Potential versus field using WMAP7+BAO+H_0, WMAP7 for the case of tachyonic field domination ($V \propto \phi^{-2}$)

where $\kappa \equiv 8\pi G$. With parameters in Table 3, the potential is plotted in Fig. 3 which is no surprised as it was found earlier (Padmanabhan 2002) regardless of the expansion is either normal power-law or phantom power-law. The steepness of the potential is typically determined by a dimensionless variable

$$\Gamma = \frac{V''V}{V'^2} \tag{30}$$

where $'$ denotes $\mathrm{d}/\mathrm{d}\phi$. For the potential (29), it is found that $\Gamma = 3/2$.

5.2 Using tachyonic field dominant solution to approximate $V(\phi)$ in mixed fluid universe

Considering Eq. (10) for flat space and $\epsilon = -1$ hence $\dot{\phi}^2 = (2\dot{H} + 8\pi G\rho_{\rm m})/(3H^2 - 8\pi G\rho_{\rm m})$. We approximate that the dust term is much less contributive compared to the $\dot{H}$ and H^2 terms therefore,

$$\dot{\phi}^2 \approx \frac{2\dot{H}}{3H^2} = -\frac{2}{3\beta}, \qquad \phi(t) \approx \sqrt{\frac{2}{3|\beta|}}(t_{\rm s} - t) \tag{31}$$

Now we will use this $\phi(t)$ solution found with tachyonic field dominant approximation to find the potential. This is

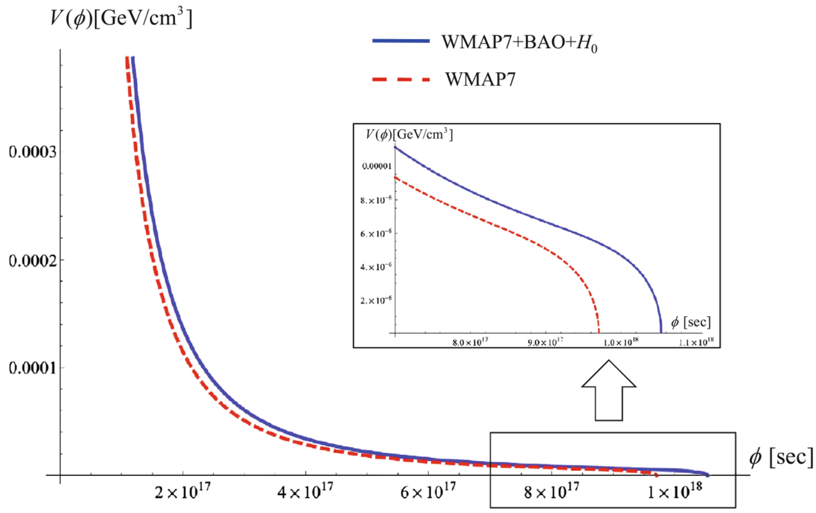

Fig. 4 Approximated potential versus field using WMAP7+BAO+H_0, WMAP7 for the case of mixed tachyonic field with barotropic dust

not exact way of deriving the potential which has also contribution of baryonic matter density. However the approximation made here does not much alter the result and could be roughly acceptable. Let $B \equiv \sqrt{3|\beta|/2}$, hence $t_s - t = B\phi$. Using Eq. (12) we find that

$$V(\phi) \approx \left[\frac{3c^2\beta^2}{\kappa(B\phi)^2} - \rho_{m,0}c^2\left(\frac{t_s - t_0}{B\phi}\right)^{3\beta}\right] \times \left[\frac{1-2/(3\beta)}{1-\rho_{m,0}[\kappa/(3\beta^2)](B\phi)^{2-3\beta}(t_s-t_0)^{3\beta}}\right]^{1/2} \tag{32}$$

Note that the term $1-2/(3\beta)$ is just $-w_{\mathrm{eff},0}$. We can rearrange the potential in form of cosmological observables H_0, $\Omega_{m,0}$ and q,

$$V \approx \frac{c^2}{\kappa}\left[\frac{2|\beta|}{\phi^2} - 3\left(\frac{3}{2|\beta|}\right)^{\frac{3|\beta|}{2}} \Omega_{m,0} H_0^{2+3|\beta|}\phi^{3|\beta|}\right] \times \left[\frac{1+2/(3|\beta|)}{1-(\frac{3}{2})^{1+\frac{3|\beta|}{2}}\Omega_{m,0}(H_0\phi)^{2+3|\beta|}|\beta|^{-1-\frac{3|\beta|}{2}}}\right]^{1/2} \tag{33}$$

where $\beta = \beta(q) = (1+q)^{-1}$. This is plotted in Fig. 4 where the field values at present and at $z = 0.45$ are

$\phi|_{z=0} = 1.268 \times 10^{17}$ sec and

$\phi|_{z=0.45} = 7.803 \times 10^{16}$ sec (WAMP7+BAO+H_0)

$\phi|_{z=0} = 1.392 \times 10^{17}$ sec and

$\phi|_{z=0.45} = 8.555 \times 10^{16}$ sec (WAMP7)

It has been known that in order for the tachyonic potential to account for the late acceleration, it should not be steeper than the potential $V \propto \phi^{-2}$ (Padmanabhan 2002; Bagla et al. 2003). To check if our derived tachyonic potential could fit in this criteria, i.e. shallower than $V \propto \phi^{-2}$, we use dimensionless variable, Γ. For the potential $V \propto \phi^{-2}$ in previous section, $\Gamma = 3/2$. Hence in general the potential with $\Gamma < 3/2$ satisfies this criteria. Considering the potential (33) we use both derived datasets to compute its dynamical slope $\Gamma(\phi)$ which is in very complicated form. We plot this in Fig. 5. We found that using our data with the field value at present, for WMAP7+BAO+H_0, $\Gamma(\phi(z=0)) = 1.500$ and for WMAP7, $\Gamma(\phi(z=0)) = 1.500$. Up to three decimal digits, these values are approximately the same as that of $V \propto \phi^{-2}$. Note that the $V \propto \phi^{-2}$ potential is found when the universe is filled with tachyon field as single component. Indeed in the limit $\Omega_{m,0} \to 0$, our derived potential (33) becomes $V \propto \phi^{-2}$. The other tachyonic potentials such as $V = V_0/[\cosh(a\phi/2)]$ and $V = V_0 e^{(1/2)m^2\phi^2}$ have $\Gamma = 1 - \mathrm{csch}^2(a\phi/2)$ and $1 + (m\phi)^{-2}$ respectively. These examples are typical tachyonic potentials which also have dynamical slopes. In Fig. 5, $\Gamma(\phi)$ diverges twice however, in the region we consider ($z = 0.45 \to z = 0$), the value of Γ stays approximately at 1.5.

6 Conclusion

In this work model of tachyonic-driven universe are investigated for normal power-law cosmology and phantom power-law cosmology. The universe is flat FLRW filled with tachyonic scalar field and dust. We consider late universe when dark energy has dominated, i.e. $z < 0.45$.

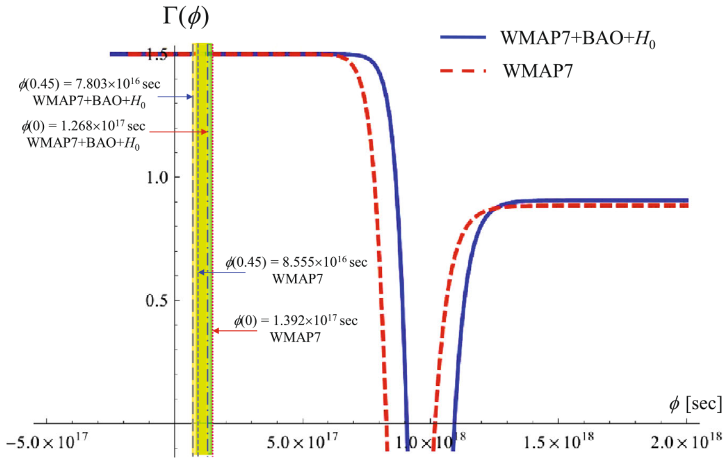

Fig. 5 Dimensionless variable Γ plotted versus field using WMAP7+BAO+H_0 and WMAP7. The considered region for late universe $z < 0.45$ lies in the bars. This is for the case of mixed tachyonic field with barotropic dust

WMAP7 data and its derived data when combined with BAO and $H(z)$ data are used in this study. We find exponents of power-law and phantom-power-law expansion and other cosmological observables. We improve data reported earlier in Kaeonikhom et al. (2011). We find that although the forms of potential and the field solution are different for quintessential scalar field (Gumjudpai 2013; Kaeonikhom et al. 2011) and tachyonic field, however the equation of state are identical for both quintessential scalar field and tachyonic field. This is to say that, for quintessence and tachyonic field, the equation of state does not depend on type of the scalar field but depends only on form of expansion function of the scale factor. The present value of dark energy equation of state predicted by quintessential and tachyonic normal power-law cosmology models do not match both WMAP7 datasets. We conclude that the usual power-law cosmology model with either quintessence or with tachyonic field are excluded by these observational data. When considering the other case, the phantom power-law cosmology, the model predicts values of equation of state not much differ from observational results (for both quintessence and tachyonic cases), i.e. $w_{\phi,0} = -1.49^{+11.64}_{-4.08}$ (phantom power-law using WMAP7+BAO+H_0) and $w_{\phi,0} = -1.51^{+3.89}_{-6.72}$ (phantom power-law using WMAP7) compared to $w_{\phi,0} = -1.34^{+1.74}_{-0.36}$ (WMAP7+BAO+H_0+SN): and $w_{\phi,0} = -1.31^{+1.67}_{-0.38}$ (WMAP7+BAO+H_0+SN+time delay distance correction) (Komatsu et al. 2011). From parametric plot in Fig. 1, at $\beta \lesssim -6$, $w_{\phi,0}$ is in the expected range $(-2,-1)$. We reconstruct the tachyonic potential in this scenario and we find that the dimensionless slope variable Γ of our derived potential at present time is about 1.5. The phantom-power-law tachyonic potential found here reduced to $V = V_0\phi^{-2}$ in the limit $\Omega_{\mathrm{m},0} \to 0$.

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Appendix: Errors analysis

In calculating of the accumulated errors, we follow the procedure here. If f is valued of answer in the form

$$f = f(x_1, x_2, \ldots, x_n) \tag{34}$$

and f_0 is the value when x_i is set to their measured values, then the value of f_i is defined as

$$f_i = f(x_1, \ldots, x_i + \sigma_i, \ldots, x_n) \tag{35}$$

This value of f is the value with effect of error in variable x_i, that is σ_i. One can find square of the accumulated error from

$$\sigma_f^2 = \sum_i^n (f_i - f_0)^2 \tag{36}$$

Hence giving the error of f from accumulating effect from errors of x_i.

References

Abramo, L.R.W., Finelli, F.: Phys. Lett. B **575**, 165 (2003)
Aguirregabiria, J.M., Lazkoz, R.: Phys. Rev. D **69**, 123502 (2004)
Albrecht, A., Steinhardt, P.J.: Phys. Rev. Lett. **48**, 1220 (1982)
Allen, R.E.: arXiv:astro-ph/9902042 (1999)
Allen, S.W., Schmidt, R.W., Fabian, A.C.: Mon. Not. R. Astron. Soc. **334**, L11 (2002)

Allen, S.W., Schmidt, R.W., Fabian, A.C., Ebeling, H.: Mon. Not. R. Astron. Soc. **342**, 287 (2003)
Allen, S.W., et al.: Mon. Not. R. Astron. Soc. **353**, 457 (2004)
Amanullah, R., et al.: Astrophys. J. **716**, 712 (2010)
Armendariz-Picon, C., Mukhanov, V.F., Steinhardt, P.J.: Phys. Rev. Lett. **85**, 4438 (2000)
Armendariz-Picon, C., Mukhanov, V.F., Steinhardt, P.J.: Phys. Rev. D **63**, 103510 (2001)
Astier, P., et al. (SNLS Collaboration): Astron. Astrophys. **447**, 31 (2006)
Bagla, J.S., Jassal, H.K., Padmanabhan, T.: Phys. Rev. D **67**, 063504 (2003)
Bilicki, M., Seikel, M.: Mon. Not. R. Astron. Soc. **425**, 1664 (2012)
Caldwell, R.R.: Phys. Lett. B **545**, 23 (2002)
Caldwell, R.R., Kamionkowski, M., Weinberg, N.N.: Phys. Rev. Lett. **91**, 071301 (2003)
Cataldo, M., Mella, P., Minning, P., Saavedra, J.: Phys. Lett. B **662**, 314 (2008)
Coles, P., Lucchin, F.: Cosmology, The Origin and Evolution of Cosmic Structure, 2nd edn. Wiley, New York (2002)
Copeland, E.J., Garousi, M.R., Sami, M., Tsujikawa, S.: Phys. Rev. D **71**, 043003 (2005)
Copeland, E.J., Sami, M., Tsujikawa, S.: Int. J. Mod. Phys. D **15**, 1753 (2006)
Dev, A., Sethi, M., Lohiya, D.: Phys. Lett. B **504**, 207 (2001)
Dev, A., Safonova, M., Jain, D., Lohiya, D.: Phys. Lett. B **548**, 12 (2002)
Dev, A., Jain, D., Lohiya, D.: arXiv:0804.3491 [astro-ph] (2008)
Dolgov, A.D.: In: Gibbons, G., Hawking, S.W., Tiklos, S.T. (eds.) The Very Early Universe. Cambridge University Press, Cambridge (1982)
Dolgov, A.D.: Phys. Rev. D **55**, 5881 (1997)
Ford, L.H.: Phys. Rev. D, Part. Fields **35**, 2339 (1987)
Fujii, Y., Nishioka, T.: Phys. Rev. D **42**, 361 (1990)
Garousi, M.R.: Nucl. Phys. B **584**, 284 (2000)
Garousi, M.R., Sami, M., Tsujikawa, S.: Phys. Rev. D **70**, 043536 (2004)
Goldhaber, G., et al. (The Supernova Cosmology Project Collaboration): Astrophys. J. **558**, 359 (2001)
Gumjudpai, B.: Mod. Phys. Lett. A **28**, 1350122 (2013)
Gumjudpai, B., Thepsuriya, K.: Astrophys. Space Sci. **342**, 537 (2012)
Guth, A.H.: Phys. Rev. D **23**, 347 (1981)
Jain, D., Dev, A., Alcaniz, J.S.: Class. Quantum Gravity **20**, 4163 (2003)
Kaeonikhom, C., Gumjudpai, B., Saridakis, E.N.: Phys. Lett. B **695**, 45 (2011)
Kaplinghat, M., Steigman, G., Tkachev, I., Walker, T.P.: Phys. Rev. D **59**, 043514 (1999)
Kaplinghat, M., Steigman, G., Walker, T.P.: Phys. Rev. D **61**, 103507 (2000)
Kolb, E.W.: Astrophys. J. **344**, 543 (1989)
Komatsu, E., et al. (WMAP Collaboration): Astrophys. J. Suppl. Ser. **192**, 18 (2011)
Kumar, S.: Mon. Not. R. Astron. Soc. **422**, 2532 (2012)
Kutasov, D., Niarchos, V.: Nucl. Phys. B **666**, 56 (2003)
Larson, D., et al.: Astrophys. J. Suppl. Ser. **192**, 16 (2011)
Linde, A.D.: Phys. Lett. B **108**, 389 (1982)
Lohiya, D., Sethi, M.: Class. Quantum Gravity **16**, 1545 (1999)
Lohiya, D., Mahajan, S., Mukherjee, A., Batra, A.: arXiv:astro-ph/9606082 (1996)
Lucchin, F., Matarrese, S.: Phys. Rev. D **32**, 1316 (1985)
Manheim, P., Kazanas, D.: Gen. Relativ. Gravit. **22**, 289 (1990)
Masi, S., et al.: Prog. Part. Nucl. Phys. **48**, 243 (2002)
Melia, F., Shevchuk, A.: Mon. Not. R. Astron. Soc. **419**, 2579 (2012)
Padmanabhan, T.: Phys. Rev. D **66**, 021301 (2002)
Padmanabhan, T.: Curr. Sci. **88**, 1057 (2005)
Padmanabhan, T.: AIP Conf. Proc. **861**, 179 (2006)
Peebles, P.J.E.: Principles of Physical Cosmology. Princeton University Press, Princeton (1993)
Peebles, P.J., Ratra, B.: Rev. Mod. Phys. **75**, 559 (2003)
Perlmutter, S., et al. (Supernova Cosmology Project Collaboration): Nature **391**, 51 (1998)
Perlmutter, S., et al. (Supernova Cosmology Project Collaboration): Astrophys. J. **517**, 565 (1999)
Rapetti, D., Allen, S.W., Weller, J.: Mon. Not. R. Astron. Soc. **360**, 555 (2005)
Rastkar, A.R., Setare, M.R., Darabi, F.: Astrophys. Space Sci. **337**, 487 (2012)
Riess, A.G.: arXiv:astro-ph/9908237 (1999)
Riess, A.G., et al. (Supernova Search Team Collaboration): Astron. J. **116**, 1009 (1998)
Riess, A.G., et al. (Supernova Search Team Collaboration): Astrophys. J. **607**, 665 (2004)
Riess, A.G., et al.: Astrophys. J. **659**, 98 (2007)
Sahni, V., Starobinsky, A.: Int. J. Mod. Phys. D **9**, 373 (2000)
Sato, K.: Mon. Not. R. Astron. Soc. **195**, 467 (1981)
Scranton, R., et al. (SDSS Collaboration): astro-ph/0307335 (2003)
Sen, A.: J. High Energy Phys. **0204**, 048 (2002a)
Sen, A.: J. High Energy Phys. **0207**, 065 (2002b)
Setare, M.R., Darabi, F.: Gen. Relativ. Gravit. **44**, 2521 (2012)
Sethi, M., Batra, A., Lohiya, D.: Phys. Rev. D **60**, 108301 (1999)
Sethi, G., Dev, A., Jain, D.: Phys. Lett. B **624**, 135 (2005)
Shapiro, I.L., Sola, J.: Phys. Lett. B **682**, 105 (2009)
Sola, J., Stefancic, H.: Phys. Lett. B **624**, 147 (2005)
Starobinsky, A.A.: Phys. Lett. B **91**, 99 (1980)
Tegmark, K., et al. (SDSS Collaboration): Phys. Rev. D **69**, 103501 (2004)
Tonry, J.L., et al. (Supernova Search Team Collaboration): Astrophys. J. **594**, 1 (2003)
Wei, Y.-H.: astro-ph/0405368 (2004)
Weinberg, S.: Rev. Mod. Phys. **61**, 1 (1989)
Zhu, Z.H., Hu, M., Alcaniz, J.S., Liu, Y.X.: Astron. Astrophys. **483**, 15 (2008)

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Publications

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[2] Burin Gumjudpai (IF and ICTP) and **Phongsaphat Rangdee** (IF), "Non-minimal derivative coupling gravity in cosmology", General Relativity and Gravitation **47** (2015) 140. [arXiv:1511.00491 [gr-qc]]

Presentations

[1] **Rachan Rangdee** and Burin Gumjudpai (IF), ORAL "Non-minimal Derivative Coupling with Power-Law", RGJ-Ph.D. Congress XVI "ASEAN: Emerging Research Oppotunities", Pattaya, Chonburi, Thailand (2015).

[2] **Rachan Rangdee** (IF), ORAL "Aspects of non-minimal derivative coupling", the 7th Aegean Summer School Beyond Einstein's Theory of Gravity, Parikia, Island of Paros, Greece, (2013).

[3] **Rachan Rangdee** (IF), ORAL "Tachyonic Phantom Power-Law Cosmology", the Workshop on Heavy Ion and High Energy Physics 2012 (HIHEP2012), Naresuan University, Phitsanulok, Thailand (2012).

[4] **Rachan Rangdee** (IF), ORAL "Tachyonic Power-Law Cosmology", the Siam Physics Congress 2012 (SPC2012): Past, Present and Future of Physics, Phranakhon Si Ayutthaya, Thailand (2012).

[5] **Rachan Rangdee** (TPTP), POSTER "TACHYONIC DARK ENERGY MODEL: DYNAMICS AND EVOLUTION", the 11th Asian-Pacific Regional IAU Meeting (APRIM2011), Chiang Mai, Thailand (2011).