Reheating in three-form inflation

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We consider the three-form field, which has been considered as a candidate for realizing inflation, coupled to a scalar field which models the relativistic matter particles produced during the reheating epoch. We have investigated the stability conditions for this theory and found that introducing such a coupling does not lead to any ghosts or Laplacian instabilities. We have also investigated the reheating temperature and the production of particles due to parametric resonances. We have found that this process is more efficient in this theory compared to the result of the standard-scalar-field inflationary scenario.

I. INTRODUCTION

The study of cosmology in the last few years has become more and more interesting and fundamental because available data have started giving us a nontrivial picture of the Universe at large scales. For example, data led to the unexpected result of the acceleration of the Universe [1–7]. In the coming years, we may expect new surprises coming also in the field of inflation and inflationary non-Gaussianities [8].

In the context of early cosmology, the theoretical predictions have strong connections with fundamental physics. However, we still do not have a clear picture of high-energy physics at energies above the electroweak scale. Therefore, in inflation, many different models have been introduced. People have recently tried to search for alternative models of inflation which, on the one hand, would still account for the standard inflationary results, and, on the other hand, would leave peculiar and unique-to-the-model imprints in the data.

Since data are getting more and more precise, some of these models have already been excluded, whereas others are still viable. In fact, different models would give different values for each inflationary observable, so that, in principle, as more data become available, the parameter space in model space will consistently reduce.

In the light of high-energy physics and in order to explore the parameter space of models which are not built only of fundamental scalar fields, the three-form inflationary field has been introduced [9–11]. Dynamical models for the three-form in cosmological backgrounds have been studied [12–19]. In particular, the background evolution during inflation was found to be similar to the one of the standard scalar-field case. However, at the level of perturbations, things change considerably [13,16]. First, besides the gravitational wave modes—which have standard features—only one scalar field does propagate. Second, the potential for the three-form must be chosen such that this scalar mode degree of freedom does not become a ghost. Third, the speed of propagation of such a scalar field is not equal to one, and for a class of potentials it may become even negative (leading to a Laplacian instability). In order to prevent this

latter possibility from happening, further conditions on the potential must be chosen. Finally, because of this nontrivial speed of propagation, these models can—depending on the form of the potential—lead to large values of non-Gaussianities, giving rise to a whole new phenomenological picture for this three-form inflation theory [16,17].

In the standard picture of the early universe, after inflation, the Universe passes through the reheating era. It is assumed that, at that time, all elementary particles are created and then the Universe enters the high temperature phase. In order to generate particles during the reheating time, the inflaton field must couple to some matter field and, consequently, decay to generate particles. This mechanism has been widely studied in the context of modeling the matter fields via a scalar field [20–25] (for more recent reviews see, e.g., Refs. [26–28]) and $f(R)$ theory [29–32].

In this paper we investigate the role that the three-form field can play in order to reheat the Universe. The model, its definition, and the coupling with matter fields are described in Sec. II. Then, in Sec. III, we study the conditions for the theory to be stable against ghosts (i.e., fields with a negative kinetic energy) and Laplacian instabilities (i.e., $c_s^2 < 0$) when such a three-form is coupled with a matter field. Section IV is devoted to the study of the background approximate solution during the reheating phase. We finally find an estimated expression for the reheating temperature and discuss the preheating phase in comparison to other already known and studied models.

II. THE LAGRANGIAN

We will study the Lagrangian for a three-form field, $A_{\mu\nu\rho}$ [13–16], coupled to a scalar field, ϕ, given as follows:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{48} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta} \right.$$
$$- V(A_{\alpha\beta\gamma} A^{\alpha\beta\gamma}) - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - \frac{1}{2} m_\phi^2 \phi^2$$
$$\left. - \frac{\lambda}{6} E^{\mu\alpha\beta\gamma} A_{\alpha\beta\gamma} \phi \nabla_\mu \phi \right], \tag{1}$$

where M_{Pl} is the Planck mass, $E_{\alpha\beta\gamma\delta}$ is the Levi-Civita antisymmetric tensor on curved backgrounds, which on Minkowski reduces to $\epsilon_{\alpha\beta\gamma\delta}$ (with $\epsilon_{0123} = 1 = -\epsilon^{0123}$). Then we also have $E^{\alpha\beta\gamma\delta} = \epsilon^{\alpha\beta\gamma\delta}/\sqrt{-g}$. The last term is a coupling term between the three-form and the scalar field (which mimics a relativistic matter field) into which the three-form decays. This coupling term is one of the simplest ones we may think of, and it is the lowest dimensional analytical one (in fact here λ is a dimensionless parameter), which can introduce a decay of the three-form field into two scalar-field particles (for a more realistic and particle physics approach to reheating can be seen e.g., in Ref. [33]. Other possible coupling terms may arise but we expect their coupling constants to be suppressed by the cutoff scale of the theory. By integration by parts, we have that the considered coupling term can be rewritten as

$$\frac{\lambda}{2}\phi^2\nabla_\mu B^\mu, \qquad (2)$$

where B^μ is the vector dual to the three-form, that is, $B^\mu = E^{\mu\alpha\beta\gamma}A_{\alpha\beta\gamma}/3!$. The coupling term can be rewritten as

$$-\frac{\lambda}{6}E^{\mu\alpha\beta\gamma}A_{\alpha\beta\gamma}\phi\nabla_\mu\phi$$
$$= -\frac{\lambda\phi}{\sqrt{-g}}[-A_{123}\partial_0\phi + A_{023}\partial_1\phi - A_{013}\partial_2\phi + A_{012}\partial_3\phi],$$
$$(3)$$

which explicitly shows the coupling between the scalar field and the four independent components of the three-form.

We assume a flat Friedmann-Lemaître-Robertson-Walker manifold

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2, \qquad (4)$$

and, on this background, we will set the the components of the three-form $A_{\alpha\beta\gamma}$ compatibly with the background symmetries and with the field equations of motion, as in

$$A_{0ij} = 0, \qquad A_{ijk} = a^3\epsilon_{ijk}X, \qquad (5)$$

where ϵ_{ijk} is the three-dimensional Levi-Civita symbol (with $\epsilon_{123} = 1$). The Friedmann equation of motion reads as

$$E_1 \equiv 3M_{\text{Pl}}^2 H^2 - \rho_X - \rho_\phi = 0, \qquad (6)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, and

$$\rho_X = \frac{1}{2}\dot{X}^2 + V + \frac{9}{2}H^2X^2 + 3HX\dot{X} = \frac{1}{2}Y^2 + V, \quad (7)$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m_\phi^2\phi^2, \qquad (8)$$

$$Y = \dot{X} + 3HX. \qquad (9)$$

We also have the second Einstein equation as

$$E_2 \equiv M_{\text{Pl}}^2(2\dot{H} + 3H^2) + p_X + p_\phi = 0, \qquad (10)$$

where p_X is the three-form effective pressure defined as follows:

$$p_X = -\left(\frac{1}{2}\dot{X}^2 + V + 3HX\dot{X} + \frac{9}{2}H^2X^2 - 12V_{,y}X^2\right), \quad (11)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m_\phi^2\phi^2. \qquad (12)$$

The equation of motion for X can be written as

$$E_X \equiv \ddot{X} + 3(H\dot{X} + X\dot{H}) + 12V_{,y}X - \lambda\phi\dot{\phi} = 0. \quad (13)$$

Finally, the equation of motion for the scalar matter can be written as

$$E_\phi \equiv \ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi + \lambda\phi Y = 0. \qquad (14)$$

The Bianchi identities lead to

$$\dot{E}_1 + 3H(E_1 - E_2) + YE_X + \dot{\phi}E_\phi = 0. \qquad (15)$$

III. PERTURBATION THEORY

In order to study the no-ghost conditions, from the action approach, it is convenient, though not necessary, to choose a gauge for which the spatial metric is diagonal (i.e., flat gauge) [34], so that the metric tensor can be written as

$$ds^2 = -(1 + 2\alpha)dt^2 + 2\partial_i\psi dt dx^i + a^2(1 + 2\Phi)d\mathbf{x}^2. \quad (16)$$

As for the 3-form, we can use a time gauge to fix its scalar perturbations as [13,16]

$$A_{0ij} = a\epsilon_{ijk}\partial_k\beta(t, \mathbf{x}), \qquad A_{ijk} = a^3\epsilon_{ijk}X(t). \quad (17)$$

Finally we will perturb the matter scalar field as $\phi = \phi(t) + \delta\phi$. By expanding the action at second order we find

$$S = \int dt a^3 \Big\{ \frac{(\partial^2 \beta)^2}{2a^4} + 6V_{,y} \frac{(\partial \beta)^2}{a^2} + [\lambda \phi \delta \phi + 12 V_{,y} X \psi + Y(\alpha + 3\Phi)] \frac{\partial^2 \beta}{a^2} + 6 V_{,y} X^2 \frac{(\partial \psi)^2}{a^2}$$

$$- [2M_{\rm Pl}^2(H\alpha - \dot{\Phi}) - \dot{\phi}\delta\phi] \frac{\partial^2 \psi}{a^2} - \frac{1}{2}(6M_{\rm Pl}^2 H^2 - Y^2 - \dot{\phi}^2)\alpha^2 - 3M_{\rm Pl}^2 \dot{\Phi}^2 + \frac{1}{2}\dot{\delta\phi}^2$$

$$+ \left[6M_{\rm Pl}^2 H\dot{\Phi} - 2M_{\rm Pl}^2 \frac{\partial^2 \Phi}{a^2} + 3(Y^2 + 12V_{,y}X^2)\Phi - \dot{\phi}\,\dot{\delta}\,\phi - m_\phi^2 \phi \delta \phi \right]\alpha + M_{\rm Pl}^2 \frac{(\partial \Phi)^2}{a^2}$$

$$- \frac{9}{2}(12V_{,y}X^2 - Y^2 + 144V_{,yy}X^4)\Phi^2 + 3[\dot{\phi}\,\dot{\delta}\,\phi - m_\phi^2 \phi \delta \phi]\Phi - \frac{1}{2}(\lambda Y + m_\phi^2)\delta\phi^2 - \frac{1}{2}\frac{(\partial \delta\phi)^2}{a^2} \Big\}, \qquad (18)$$

where we have defined $y = A_{\alpha\beta\gamma}A^{\alpha\beta\gamma} = 6X^2$ on the background. By integrating out the auxiliary fields one finds two no-ghost conditions for the remaining two propagating fields (Φ and $\delta\phi$). The independent no-ghost conditions for the kinetic matrix can then be written as

$$A_{22} = \frac{1}{2} \frac{M_{\rm Pl}^2 a^3 H[6a^2(2HM_{\rm Pl}^2 + 3X^2 H - 2XY)V_{,y} + M_{\rm Pl}^2 H k^2]}{M_{\rm Pl}^4 H^2 k^2 - 3a^2(X^2\dot{\phi}^2 - 4H^2 M_{\rm Pl}^4 + 4M_{\rm Pl}^2 XHY - 6M_{\rm Pl}^2 X^2 H^2)V_{,y}}, \qquad (19)$$

$$\det A = A_{11}A_{22} - A_{12}^2 = \frac{3M_{\rm Pl}^4 a^8 V_{,y} Y^2}{M_{\rm Pl}^4 H^2 k^2 - 3a^2(X^2\dot{\phi}^2 - 4H^2 M_{\rm Pl}^4 + 4M_{\rm Pl}^2 XHY - 6M_{\rm Pl}^2 X^2 H^2)V_{,y}}. \qquad (20)$$

Since

$$4H^2 M_{\rm Pl}^4 - 4M_{\rm Pl}^2 XHY + 6M_{\rm Pl}^2 X^2 H^2 = \frac{4M_{\rm Pl}^2}{3}\left(\frac{1}{2}\dot{X}^2 + V + \rho_\phi\right), \qquad (21)$$

we can change the denominator of the previous two expressions as

$$\det A = \frac{3M_{\rm Pl}^4 a^8 V_{,y} Y^2}{M_{\rm Pl}^4 H^2 k^2 + a^2 V_{,y}[4M_{\rm Pl}^2(\frac{1}{2}\dot{X}^2 + V + \rho_\phi) - 3X^2\dot{\phi}^2]} > 0, \qquad (22)$$

$$A_{22} = \frac{1}{2} \frac{a^3[4M_{\rm Pl}^2 a^2(\frac{1}{2}\dot{X}^2 + V + \rho_\phi)V_{,y} + M_{\rm Pl}^4 H^2 k^2]}{M_{\rm Pl}^4 H^2 k^2 + a^2 V_{,y}[4M_{\rm Pl}^2(\frac{1}{2}\dot{X}^2 + V + \rho_\phi) - 3X^2\dot{\phi}^2]} > 0. \qquad (23)$$

These results show that the coupling does not directly contribute to the ghost condition. Condition (22), for high k's, is satisfied when

$$V_{,y} > 0, \qquad (24)$$

which corresponds to the no-ghost condition already found in the vacuum case (i.e., in the absence of the ϕ field) [16]. Furthermore, we now also require

$$4M_{\rm Pl}^2\left(\frac{1}{2}\dot{X}^2 + V + \rho_\phi\right) - 3X^2\dot{\phi}^2$$
$$= 2M_{\rm Pl}^2(\dot{X}^2 + 2V) + (2M_{\rm Pl}^2 - 3X^2)\dot{\phi}^2 + 2M_{\rm Pl}^2 m_\phi^2 \phi^2 > 0. \qquad (25)$$

A sufficient condition for Eq. (25) to be verified is imposing $V \geq 0$ (the other condition found in the vacuum case [16]), and $|X/M_{\rm Pl}| \leq \sqrt{2/3}$. During reheating (i.e., after inflation ends), this condition is satisfied as, in general, $X/M_{\rm Pl} \to 0$.

In order to check whether this condition is violated or not also during inflation, we change the variables to the dimensionless variables as

$$x = \frac{X}{M_{\rm Pl}}, \qquad w^2 = \frac{Y^2}{6H^2 M_{\rm Pl}^2}, \qquad z^2 = \frac{V}{3H^2 M_{\rm Pl}^2},$$
$$u^2 = \frac{\dot{\phi}^2}{6H^2 M_{\rm Pl}^2}, \qquad v^2 = \frac{m_\phi^2 \phi^2}{6H^2 M_{\rm Pl}^2}. \qquad (26)$$

Therefore, the no-ghost condition becomes

$$(\sqrt{2} - \sqrt{3}x)^2 + (2 - 3x^2)u^2 + 2z^2 + 2v^2 > 0, \qquad (27)$$

where we have used $w^2 \sim 1$ which is the requirement for inflation [16]. According to the Friedmann equation, (6), this requirement implies that $z^2 \ll 1$ when the inflation is supposed to be driven only by the three-form field. Moreover, since the contribution of the scalar field ϕ to the dynamics of the inflationary universe is subdominant, the values of u^2 and v^2 need to be smaller than z^2. By taking into account both the first and the second terms, it

is clear that the no-ghost condition is satisfied when $x \lesssim \sqrt{2/3}$ and $x \gg \sqrt{2/3}$. However, for $x \gtrsim \sqrt{2/3}$ these terms will take a negative value. In order to check if this negative value violates the no-ghost condition or not, one can find the minimum of the function $f(x)$, by keeping u constant, as in $f(x) = (\sqrt{2} - \sqrt{3}x)^2 + (2 - 3x^2)u^2$. This minimum value takes the form $f_{\min} = -2u^4/(1 - u^2) \approx -2u^4$. Now it is clear that the no-ghost condition is satisfied for all ranges of x since $f_{\min} + 2z^2 + 2v^2 > 0$, as $u^2 \ll z^2 \ll 1$.

A. Speeds of propagation

In order to find the two speeds of propagation, we can proceed as follows. We perform the following field redefinition:

$$\Phi = kq_1, \tag{28}$$

$$\delta\phi = -\frac{A_{12}}{A_{22}}kq_1 + q_2$$
$$= \frac{\dot{\phi}[6a^2(2M_{\mathrm{Pl}}^2 H + 3HX^2 - XY)V_{,y} + M_{\mathrm{Pl}}^2 Hk^2]}{H[6a^2(2M_{\mathrm{Pl}}^2 H + 3HX^2 - 2XY)V_{,y} + M_{\mathrm{Pl}}^2 Hk^2]}kq_1 + q_2. \tag{29}$$

In this case, the second order action, after a few integrations by parts, reduces to

$$S = \int dt[Q_{ij}\dot{q}_i\dot{q}_j - C_{ij}q_iq_j - B_{ij}q_i\dot{q}_j], \tag{30}$$

where the matrix Q_{ij} is diagonal (without approximations, due to the proposed field redefinition) and its diagonal elements, for large k's are only background dependent as they reduce to

$$Q_{11} \simeq \frac{6a^5 V_{,y}Y^2}{H^2}, \qquad Q_{12} = 0, \qquad Q_{22} = \frac{a^3}{2}. \tag{31}$$

Along the same lines, by using the equations of motion, we can prove that at order k^2, the matrix elements C_{ij} reduce to

$$C_{11} \simeq \frac{6a^3 Y^2(V_{,y} + 12V_{,yy}X^2)k^2}{H^2}, \qquad C_{12} \simeq 0, \qquad C_{22} \simeq \frac{ak^2}{2}. \tag{32}$$

The antisymmetric matrix B_{ij}, by using the equations of motion, is of order $\mathcal{O}(k^{-1})$ and can be neglected, as long as we consider only the high-k behavior of the theory.

Therefore, the two speeds of propagation are

$$c_X^2 = 1 + \frac{12V_{,yy}X^2}{V_{,y}}, \qquad c_\phi^2 = 1. \tag{33}$$

This result is consistent with the result in vacuum [16] and with what we would naively expect, due to the form of the coupling we have chosen. Since after the end of inflation the ghost-free potentials studied in Ref. [16] all reduce to

$V \approx \frac{1}{2}m^2X^2 = \frac{1}{12}m^2y$, then during reheating there are no Laplacian instabilities as $c_X^2 \to 1$.

IV. REHEATING PHASE

Let us discuss about the behavior of the three-form field during the reheating phase. For simplicity, we will neglect the backreaction effects of the scalar-matter field on the leading term in the background expansion. Because of this approximation, we can neglect the interaction term in Eq. (13) to find a solution of this equation. By differentiating Eq. (13), one can write this equation in terms of Y as

$$\ddot{Y} + 3H\dot{Y} + m^2Y = 0. \tag{34}$$

Here, we have used the potential form $V = \frac{1}{2}m^2X^2$ for simplicity. This potential form is free from any ghost and Laplacian instabilities during inflation [16], since the field X is always less than $\sqrt{2/3}M_{\mathrm{Pl}}$. Moreover, when $X/M_{\mathrm{Pl}} < 1$, as already stated above, all potentials which are free from ghosts and Laplacian instabilities at the end of inflation approximately take this quadratic form.

From Eq. (34), it follows that $Y^2 \propto H^2 \gg |\dot{H}|$ when $m^2 \ll H^2$, so that the three-form field can indeed drive inflation. During inflation, H slowly decreases since $\dot{H} < 0$, fulfilling in this way the no-ghost condition, up to the end of inflation, at scales around $m^2 \sim H^2$. After the end of inflation, the reheating phase starts as both fields, X and Y, begin to oscillate at scales $m^2 \gg H^2$. The evolution of Y and X during the oscillating phase can be studied by changing variable $Y = a^{-3/2}\bar{Y}$, so that Eq. (34) becomes

$$\ddot{\bar{Y}} + \left(m^2 - \frac{9}{4}H^2 - \frac{3}{2}\dot{H}\right)\bar{Y} = 0. \tag{35}$$

Using the approximations $m^2 \gg H^2$ and $m^2 \gg |\dot{H}|$, the solution for Eq. (35) can be written as

$$\bar{Y} = C\sin(mt), \tag{36}$$

where C is a constant. Thus, the solution of Y can be written as

$$Y = Ca^{-3/2}\sin(mt). \tag{37}$$

By neglecting the contribution from ϕ in Eqs. (6) and (10) and combining them together, one obtains

$$2M_{\mathrm{Pl}}^2\left[\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2\right] = Y^2 = C^2a^{-3}\sin^2(mt). \tag{38}$$

The solution for this equation is

$$a(t) = \left[c_2 t + 3c_1 + \frac{3C^2[2m^2t^2 + \cos(2mt)]}{16m^2M_{\mathrm{Pl}}^2}\right]^{1/3}, \tag{39}$$

where c_1 and c_2 are integration constants. During the oscillating phase, $mt \gg 1$, so that the scale factor can be approximated as

$$a(t) \simeq \left(\frac{3C^2t^2}{8M_{\text{Pl}}^2}\right)^{1/3}. \tag{40}$$

By substituting this approximated solution into Eq. (37), one obtains

$$Y = \sqrt{\frac{8}{3}}M_{\text{Pl}}\frac{\sin(mt)}{t}. \tag{41}$$

On using the solution for a in Eq. (40), the expression for H becomes

$$H = \frac{2m\{8c_2mM_{\text{Pl}}^2 - 3C^2[\sin(2mt) - 2mt]\}}{48m^2(c_2t + 3c_1)M_{\text{Pl}}^2 + 9C^2[2m^2t^2 + \cos(2mt)]}. \tag{42}$$

Therefore, the approximated solution during oscillating phase, $mt \gg 1$, reads

$$H \simeq \frac{2}{3t}. \tag{43}$$

Note that this estimated value for H agrees with the numerical result in Ref. [16] since $H^2 \propto a^{-3} = t^{-2}$. Substituting H into Eq. (13) and using the relation in Eq. (9), the solution for X can be written as

$$X = \sqrt{\frac{8}{3}}\frac{M_{\text{Pl}}}{(mt)^2}[\sin(mt) - mt\cos(mt)]. \tag{44}$$

Substituting X, Y, and H into Eq. (7), one finds that, at a first approximation, $\rho_X \propto a^{-3}$. This implies, as expected, that during oscillating phase, the three-form field behaves as dust. This behavior of the field also agrees with the result found in vacuum [14].

A. Quantum production of particles

After knowing how the background evolves for the fields in the theory, we now want to find an (approximate) expression for the value of the reheating temperature. During the inflationary period, all matter fields are diluted away due to the exponentially accelerating expansion of the Universe. At the beginning of the oscillating phase, all matter fields can then be assumed to start off in their vacuum state. Hence, it is convenient to consider the interaction between the classical background field, X, which is driving inflation and the quantum scalar field ϕ with the Lagrangian in the action (1). The quantum scalar field ϕ can be decomposed, due to the symmetries of the Friedmann-Lemaître-Robertson-Walker manifold, in the Heisenberg picture, as

$$\hat{\phi}(t, \boldsymbol{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3k(\hat{a}_k\phi_k(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}_k^{\dagger}\phi_k^*(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}), \tag{45}$$

where $\hat{a}_k$ and $\hat{a}_k^{\dagger}$ are the annihilation and creation operators, respectively, and $\boldsymbol{k}$ represents the three-dimensional wave vector. The Fourier mode $\phi_k(t)$ obeys the classical equation of motion:

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \left(\frac{k^2}{a^2} + m_{\phi}^2 + \lambda Y\right)\phi_k = 0. \tag{46}$$

By comparing this equation of motion to the standard scalar inflaton field φ, with the four-legs coupling interaction $g^2\varphi^2\phi^2$ with matter fields, and the three-legs interaction $2g^2\sigma\varphi\phi^2$ (which arises when the field acquires a vacuum expectation value and consequently symmetry gets broken), the modification consists of replacing λY by $g^2\varphi^2$ and $2g^2\sigma\varphi$, respectively, where $\varphi = (2M_{\text{Pl}}/\sqrt{3}) \times \sin(m_{\varphi}t)/(m_{\varphi}t)$ [20–25]. On the other hand, as for the $f(R)$ models and Starobinsky inflation, defined by $f(R) = R + R^2/(6m^2)$ [35], the modification in Eq. (46) consists of replacing λY by ξR where $R = -4m\sin(mt)/t$ [30,31]. It should be pointed out that considering X as a classical field implies that the coupling term effectively contributes to the (time-dependent) mass term of the modes ϕ_k as in $\delta m^2 = \lambda Y$.

B. Reheating temperature

In order to find an expression for the reheating temperature in our model, we will use a strategy analogous to the one used in $f(R)$ gravity [30,31]. First, we introduce a new variable $u_k = a\phi_k$ and use the conformal time $\eta = \int a^{-1}dt$ as the time variable. The equation of motion for the modes of the produced particle field can be rewritten as

$$\frac{d^2u_k}{d\eta^2} + \left[k^2 + a^2m_{\phi}^2 - \frac{1}{6}a^2R + \lambda a^2Y\right]u_k = 0, \tag{47}$$

where $R = 6a^{-3}d^2a/d\eta^2$ is the Ricci scalar. According to the coupling we have introduced, a heavy particle X at rest decays into two particles ϕ in which both will have a total energy typically much larger than their rest mass. In other words, the produced particles will be relativistic, that is, $(k/a)^2 \gg m_{\phi}^2$. In this case, we can ignore the second term in the parentheses compared to the first one. Since $R \propto H^2 \sim \dot{H}$, the third term in the parentheses is also, in general, negligible when it is compared to the fourth one. In fact, this approximation is valid for large t, since $H^2 \propto t^{-2}$ and $Y \propto M_{\text{Pl}}t^{-1}$. Therefore, Eq. (47) will be written as

$$\frac{d^2u_k}{d\eta^2} + k^2u_k = Uu_k, \tag{48}$$

where $U = -\lambda a^2Y$. For the mode deep inside the Hubble radius, $k^2 \gg U$, we can choose, as usual, the initial vacuum as the state with positive-frequency solution, that is $u_k^{(i)} = e^{-ik\eta}/\sqrt{2k}$. The solution of Eq. (48) can then be written as [36]

$$u_k = u_k^{(i)} + \frac{1}{k}\int_0^{\eta}U(\eta')\sin[k(\eta - \eta')]u_k(\eta')d\eta'. \tag{49}$$

In curved spacetime, the choice of the decomposition of ϕ into $\hat{a}_k$ and $\hat{a}_k^{\dagger}$ is not unique. It is possible to use another

decomposition $\hat{A}_k$ and $\hat{A}_k^\dagger$ which can be written in terms of $\hat{a}_k$ and $\hat{a}_k^\dagger$ as $\hat{A}_k = \alpha_k \hat{a}_k + \beta_k^* \hat{a}_k^\dagger$. The coefficients α_k and β_k coincide with the coefficients of the Bogoliubov transformation for the ladder operators. This transformation is chosen as to diagonalize the Hamiltonian of the field $\hat{\phi}$ at each slicing time η. Normalization provides the following condition for the coefficients α_k and β_k:

$$|\alpha_k|^2 - |\beta_k|^2 = 1. \qquad (50)$$

If $\beta_k = 0$, we get $\alpha_k = 1$ and then we recover the standard creation and annihilation operators. In general, β will describe the produced particle due to the expansion of the Universe and can be written as [37]

$$\beta_k = \frac{-i}{2k} \int_0^\eta U(\eta') e^{-2ik\eta'} d\eta'. \qquad (51)$$

The energy density of the produced particle in comoving coordinate η is defined by [36]

$$\begin{aligned} \rho_\eta &= \frac{1}{2\pi^2} \int_0^\infty dk k^2 \cdot k |\beta_k|^2, \\ &= \frac{1}{8\pi^2} \int_0^\infty U(\eta') d\eta' \int_0^\infty U(\eta) d\eta \int_0^\infty dk k e^{2ik(\eta'-\eta)}. \end{aligned} \qquad (52)$$

By using $\int_0^\infty dk k e^{ikx} = -1/x^2$ and the fact that $U \to 0$ both at early and late times, we obtain

$$\rho_\eta = \frac{1}{32\pi^2} \int_0^\infty \frac{dU(\eta)}{d\eta} d\eta \int_0^\infty \frac{U(\eta')}{\eta' - \eta} d\eta'. \qquad (53)$$

From $Y \propto a^{-3/2}(t) \sin(mt) \propto a^{-3/2}(\eta) \sin(\int_0^\eta ma d\bar{\eta})$, we can estimate U as $U = -\lambda a^2 Y \propto -\lambda a^{1/2}(\eta) \sin(\int_0^\eta ma d\bar{\eta})$. Thus, we can write U in terms of conformal time as

$$U(\eta) = C a^{1/2}(\eta) \sin\left(\int_0^\eta \omega d\bar{\eta}\right), \qquad (54)$$

where C is an overall constant factor and $\omega = ma$. Using the approximation $a^{-1} da/d\eta \ll \omega$, valid during the rapid-oscillations phase, we obtain

$$\frac{dU(\eta)}{d\eta} \cong C a^{1/2}(\eta) \omega \cos\left(\int_0^\eta \omega d\bar{\eta}\right). \qquad (55)$$

By taking the limit $\int_0^\eta \omega d\bar{\eta} \gg 1$ and using the relation $\lim_{k\to\infty} \sin(kx)/x = \pi\delta(x)$, we obtain

$$\rho_\eta \cong \frac{1}{32\pi} \int_0^\infty C^2 a \omega \cos^2\left(\int_0^\eta \omega d\bar{\eta}\right) d\eta. \qquad (56)$$

Note that we have cut out the infinite contribution from our integration. Shifting the phase of the oscillating factor by $\pi/2$ and differentiating the above equation, one obtains

$$\frac{d\rho_\eta}{dt} = \frac{1}{a} \frac{d\rho_\eta}{d\eta} \cong \frac{mU^2}{32\pi} = \frac{ma^4 \lambda^2 Y^2}{32\pi}. \qquad (57)$$

The physical energy density of the matter field ϕ relates to the comoving energy density by $\rho_\phi = \rho_\eta/a^4$. By taking into account the effect of the total relativistic matter degrees of freedom, the energy density of the total radiation produced in the reheating process takes the following form:

$$\rho_r = \frac{g_*}{a^4} \rho_\eta = \frac{g_* m \lambda^2}{32\pi a^4} \int_{t_{\rm os}}^t a^4 Y^2 dt, \qquad (58)$$

where $t_{\rm os}$ is a time when the oscillating phase begins. In the regime $m(t - t_{\rm os}) \gg 1$, the behavior of the scale factor and the averaged expression for Y^2 can be written as

$$a \simeq a_0 (t - t_{\rm os})^{2/3}, \qquad \langle Y^2 \rangle \simeq \frac{4 M_{\rm Pl}^2}{3(t - t_{\rm os})^2}. \qquad (59)$$

Substituting these expressions into Eq. (58), we finally obtain

$$\rho_r \simeq \frac{g_* \lambda^2}{40\pi} \frac{m M_{\rm Pl}^2}{(t - t_{\rm os})}. \qquad (60)$$

From the evolution of the scale factor, one can find the evolution of the Hubble parameter

$$H^2 \simeq \frac{4}{9(t - t_{\rm os})^2}. \qquad (61)$$

From Eqs. (60) and (61), we find that the energy density ρ_r decreases slower than H^2. Therefore, ρ_r becomes, at some time, the dominant contribution to the total energy density. We can estimate the duration time of the reheating process by using the Friedmann equation $3 M_{\rm Pl}^2 H^2 \simeq \rho_r$. This provides $(t - t_{\rm os})$ and, consequently, the final ρ_r as

$$t - t_{\rm os} \simeq \frac{160\pi}{3 g_* \lambda^2 m}, \qquad \rho_r \simeq \frac{3 g_*^2 \lambda^4 m^2 M_{\rm Pl}^2}{6400 \pi^2}. \qquad (62)$$

The energy density of the produced particles is converted to the standard expression for the energy density of a radiation gas as in $g_* \pi^2 T_{\rm rh}^4/30$. Therefore, the reheating temperature can be estimated as

$$T_{\rm rh} \lesssim \lambda \left(\frac{9 g_*}{640 \pi^4}\right)^{1/4} M_{\rm Pl} \left(\frac{m}{M_{\rm Pl}}\right)^{1/2}. \qquad (63)$$

For the $f(R)$ gravity model $f(R) = R + R^2/(6m^2)$, the reheating temperature instead takes the form

$$T_{\rm rh} \lesssim \left(\frac{g_*}{2560 \pi^4}\right)^{1/4} M_{\rm Pl} \left(\frac{m}{M_{\rm Pl}}\right)^{3/2}. \qquad (64)$$

By comparing these two results, we find that the reheating temperature of the three-form field is larger than the one of the $f(R)$ gravity model by $M_{\rm Pl}/m \sim 10^5$. Note that we estimated the value of the three-form mass m by using the power spectrum of the curvature perturbation found in Ref. [13] and using the observation data from seven-year WMAP [4]. For the simple scalar-field model with a three-leg interaction, the reheating temperature takes the form

$$T_{\rm rh} \lesssim g^2 \sigma \left(\frac{9g_*}{40\pi^4}\right)^{1/4} \left(\frac{M_{\rm Pl}}{m}\right)^{1/2} \sim \sigma. \qquad (65)$$

The result from this last calculation is comparable to the result calculated by using the decay rate obtained from particle theory [21–23,38].

C. Parametric resonance

We investigate here the contribution of parametric resonances to the total energy density of produced matter particles in the three-form inflation discussed here. The majority of the energy for the thee-form field, at the end of inflation, is stored in the $k = 0$ mode of the field. When the three-form field oscillates around the minimum of the potential, its energy undergoes coherent oscillations. This dynamics is the same as the one for a standard scalar inflaton field. In order to take into account the coherent nature of the three-form field at the end of inflation, we follow the standard procedure by investigating the parametric resonance of the system [23–25]. We will begin by introducing a new variable $y_k = a^{3/2}\phi_k$. Therefore, Eq. (46) can be rewritten as

$$\ddot{y}_k + \left(\frac{k^2}{a^2} + m_\phi^2 + \lambda Y - \frac{9}{4}H^2 - \frac{3}{2}\dot{H}\right)y_k = 0. \quad (66)$$

In the limit $k^2 \gg H^2 \sim \dot{H}$, one can neglect the last two terms of the above equation. Note that we still neglect the backreaction of the three-form quantum field in the following. Thus, we can still use the solution of Y as in Eq. (41). Substituting the solution of Y into Eq. (66), we obtain

$$\ddot{y}_k + \left(\frac{k^2}{a^2} + m_\phi^2 + \lambda \frac{\sqrt{8}M_{\rm Pl}\sin[m(t - t_{\rm os})]}{\sqrt{3}(t - t_{\rm os})}\right)y_k = 0. \quad (67)$$

The parametric resonance occurs due to the oscillating term. In order to see this behavior, we will introduce a new variable z defined by $m(t - t_{\rm os}) = 2z - \pi/2$. Then Eq. (67) is changed to the Mathieu equation [39] as follows:

$$\frac{d^2 y_k}{dz^2} + (A_k - 2q\cos(2z))y_k = 0, \qquad (68)$$

where

$$A_k = \frac{4k^2}{a^2 m^2} + \frac{4m_\phi^2}{m^2}, \qquad q = \frac{4\sqrt{8}\lambda M_{\rm Pl}}{\sqrt{3}m^2(t - t_{\rm os})}. \quad (69)$$

According to the Mathieu equation, there are instability bands in which the perturbation y_k grows exponentially with the growth index $\mu_k = q/2$. These instability bands depend on the parameters A_k and q. For a broad resonance, these parameters must satisfy the conditions $A_k \simeq l^2$ and $q \gg 1$ where $l^2 = 1, 2, 3, \ldots$. For narrow resonance, these parameter must satisfy the conditions $A_k \simeq l^2$ and $q \ll 1$ where $l^2 = 1, 2, 3, \ldots$. To guarantee enough efficiency for the production of particles, the Mathieu equation's parameters should satisfy the broad-resonance condition. However,

in general, the parameter q decreases in time. Thus, q must take a large enough initial value.

For the $f(R)$ gravity model, the parameters take the form [31]

$$A_k = \frac{4k^2}{a^2 m^2} + \frac{4m_\phi^2}{m^2}, \qquad q = \frac{8\xi}{m(t - t_{\rm os})}. \quad (70)$$

In order to get a broad resonance, $q \gg 1$, the coupling constant ξ must take a large value. However, for the three-form model, the coupling constant does not need to be large, since $q \sim \lambda M_{\rm Pl}/m$.

For a standard scalar inflaton field with four-legs interaction, the parameters take the form [23–25]

$$A_k = \frac{k^2}{a^2 m^2} + \frac{4m_\phi^2}{m^2} + 2q, \quad q = \frac{2g^2}{3m^2(t - t_{\rm os})^2}\frac{M_{\rm Pl}^2}{m^2}. \quad (71)$$

This model naturally gives large values for q, initially. However, it decreases faster than the three-form model, so that the broad resonance tends to disappear more quickly.

Since the parameter q is initially very large, the field passes through many instability bands. This behavior leads to a stochastic change in the growth index μ_k. Therefore, in this situation, we need to analyze the system as a stochastic resonance [25]. While the above discussion is based on the assumption that the backreaction is negligible, it will be of interest to take into account the effect of backreaction for a more realistic model. The exponential growth of the field also provides the nonadiabaticity in the change of the frequency $\omega_k^2 = k^2/a^2 + m_\phi^2 + \sqrt{8/3}\lambda M_{\rm Pl}\sin[m(t - t_{\rm os})]/(t - t_{\rm os})$. It should be noticed that, during the parametric resonance regime, the produced particles are far away from equilibrium. The study of the thermalization at the end of the parametric resonance regime is also of interest, and we leave it for a future project.

V. CONCLUSIONS

The inflationary paradigm has joined cosmology with high energy particle physics. In particular, high-energy theories may leave their footprint in the data due to peculiar properties of the field which drives inflation.

In this paper we have investigated the properties of reheating due to the presence of a 3-form field coupled with a matter field. We have found that the existence of a coupled matter field does not change the conditions that are necessary for ghost-free and Laplacian stabilities. However, to avoid a ghost field from appearing, the matter field has to be subdominant during inflation, but this condition is naturally fulfilled for quite a large variety of dynamics. The subdominance of a matter field is not required after inflation so that reheating may occur without ghost and Laplacian instabilities, leading to a successful production of relativistic particles at a scale:

$$T_{\rm rh} \lesssim \lambda \left(\frac{9g_*}{640\pi^4}\right)^{1/4} M_{\rm Pl}\left(\frac{m}{M_{\rm Pl}}\right)^{1/2}. \qquad (72)$$

We have also investigated the production of particles due to parametric resonances. In particular, we have found that this process can still be modeled by the Mathieu equation. Furthermore, compared to the standard minimally coupled scalar inflaton field scenario, reheating is more efficient, as broad resonances typically survive longer, without the need of choosing extremely large values for the coupling constant which models the interaction between the three-form and the (relativistic) ordinary matter fields. Finally, we have shown that, indeed, the production of particles at the end of inflation, due to the peculiar phenomenology of reheating, gives us a possibility to distinguish experimentally such a model from other alternative and viable models of inflation.

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[1] A. G. Riess *et al.* (Supernova Search Team Collaboration), Astron. J. **116,** 1009 (1998); S. Perlmutter *et al.* (Supernova Cosmology Project Collaboration), Astrophys. J. **517,** 565 (1999).

[2] M. Hicken, W. M. Wood-Vasey, S. Blondin, P. Challis, S. Jha, P. L. Kelly, A. Rest, and R. P. Kirshner, Astrophys. J. **700,** 1097 (2009).

[3] D. N. Spergel *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **148,** 175 (2003).

[4] E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **192,** 18 (2011).

[5] D. J. Eisenstein *et al.* (SDSS Collaboration), Astrophys. J. **633,** 560 (2005).

[6] W. J. Percival *et al.*, Mon. Not. R. Astron. Soc. **401,** 2148 (2010).

[7] N. Suzuki *et al.*, Astrophys. J. **746,** 85 (2012).

[8] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **536,** A1 (2011).

[9] A. Aurilia, G. Denardo, F. Legovini, and E. Spallucci, Nucl. Phys. **B252,** 523 (1985); A. Aurilia, G. Denardo, F. Legovini, and E. Spallucci, Nucl. Phys. **B147,** 258 (1984).

[10] C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 03 (2009) 028.

[11] T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 09 (2009) 092.

[12] C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 11 (2009) 005.

[13] T. S. Koivisto and N. J. Nunes, Phys. Rev. D **80,** 103509 (2009).

[14] T. S. Koivisto and N. J. Nunes, Phys. Lett. B **685,** 105 (2010).

[15] T. Ngampitipan and P. Wongjun, J. Cosmol. Astropart. Phys. 11 (2011) 036.

[16] A. De Felice, K. Karwan, and P. Wongjun, Phys. Rev. D **85,** 123545 (2012).

[17] D. J. Mulryne, J. Noller, and N. J. Nunes, arXiv:1209.2156; N. Bartolo, S. Matarrese, M. Peloso, and A. Ricciardone, arXiv:1210.3257.

[18] T. S. Koivisto and F. R. Urban, Phys. Rev. D **85,** 083508 (2012).

[19] F. R. Urban and T. K. Koivisto, J. Cosmol. Astropart. Phys. 09 (2012) 025.

[20] A. D. Dolgov and A. D. Linde, Phys. Lett. **116B,** 329 (1982).

[21] L. F. Abbott, E. Farhi, and M. B. Wise, Phys. Lett. **117B,** 29 (1982).

[22] A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, Phys. Rev. Lett. **48,** 1437 (1982).

[23] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **73,** 3195 (1994).

[24] Y. Shtanov, J. H. Traschen, and R. H. Brandenberger, Phys. Rev. D **51,** 5438 (1995).

[25] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. D **56,** 3258 (1997).

[26] J. Martin, Lect. Notes Phys. **669,** 199 (2005).

[27] B. A. Bassett, S. Tsujikawa, and D. Wands, Rev. Mod. Phys. **78,** 537 (2006).

[28] R. Allahverdi, R. Brandenberger, F.-Y. Cyr-Racine, and A. Mazumdar, Annu. Rev. Nucl. Part. Sci. **60,** 27 (2010).

[29] S. A. Appleby, R. A. Battye, and A. A. Starobinsky, J. Cosmol. Astropart. Phys. 06 (2010) 005.

[30] M. B. Mijic, M. S. Morris, and W.-M. Suen, Phys. Rev. D **34,** 2934 (1986).

[31] A. De Felice and S. Tsujikawa, Living Rev. Relativity **13**, 3 (2010), http://relativity.livingreviews.org/Articles/lrr-2010-3/.

[32] H. Motohashi and A. Nishizawa, Phys. Rev. D **86,** 083514 (2012).

[33] A. Mazumdar and J. Rocher, Phys. Rep. **497,** 85 (2011).

[34] J. M. Bardeen, Phys. Rev. D **22,** 1882 (1980); H. Kodama and M. Sasaki, Prog. Theor. Phys. Suppl. **78,** 1 (1984); V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215,** 203 (1992); B. A. Bassett, S. Tsujikawa, and D. Wands, Rev. Mod. Phys. **78,** 537 (2006); K. A. Malik and D. Wands, Phys. Rep. **475,** 1 (2009).

[35] A. A. Starobinsky, Phys. Lett. **91B,** 99 (1980).

[36] Y. .B. Zeldovich and A. A. Starobinsky, Zh. Eksp. Teor. Fiz. **61,** 2161 (1971) [Sov. Phys. JETP **34,** 1159 (1972)].

[37] A. A. Starobinsky, in *Proceedings of Quantum Gravity, Moscow, 1981* (INR Press, Moscow, 1982), pp 103–128.

[38] N. Bouatta, A.-C. Davis, R. H. Ribeiro, and D. Seery, J. Cosmol. Astropart. Phys. 09 (2010) 011.

[39] N. W. McLachlan, *Theory and Application of Mathieu Functions* (Dover, New York, 1961).