

Observational constraints on quintessence: Thawing, tracker, and scaling modelsTakeshi Chiba,¹ Antonio De Felice,^{2,3} and Shinji Tsujikawa⁴¹*Department of Physics, College of Humanities and Sciences, Nihon University, Tokyo 156-8550, Japan*²*ThEP's CRL, NEP, The Institute for Fundamental Study, Naresuan University, Phitsanulok 65000, Thailand*³*Thailand Center of Excellence in Physics, Ministry of Education, Bangkok 10400, Thailand*⁴*Department of Physics, Faculty of Science, Tokyo University of Science, 1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan*

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For two types of quintessence models having thawing and tracking properties, there exist analytic solutions for the dark energy equation of state w expressed in terms of several free parameters. We put observational bounds on the parameters in such scenarios by using the recent type Ia supernovae, cosmic microwave background, and baryon acoustic oscillations data. The observational constraints are quite different depending on whether or not the recent baryon acoustic oscillations data from BOSS are taken into account. With the BOSS data the upper bounds of today's values of w ($= w_0$) in thawing models is very close to -1 , whereas without this data the values of w_0 away from -1 can still be allowed. The tracker equation of state $w_{(0)}$ during the matter era is constrained to be $w_{(0)} < -0.949$ at 95% C.L. even without the BOSS data, so that the tracker models with w away from -1 are severely disfavored. We also study observational constraints on scaling models in which w starts to evolve from 0 in the deep matter era and show that the transition to the equation of state close to $w = -1$ needs to occur at an early cosmological epoch. In the three classes of quintessence models studied in this paper, the past evolution of the Hubble parameters in the best-fit models shows only less than a 2.5% difference compared to Λ CDM.

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I. INTRODUCTION

Independent observational data—such as type Ia supernovae (SN Ia) [1,2], the cosmic microwave background (CMB) [3,4], and baryon acoustic oscillations (BAO) [5,6]—suggest that about 70% of the energy density today consists of dark energy, which is responsible for cosmic acceleration. For the constant dark energy equation of state w the recent joint data analysis based on SN Ia, CMB, BAO, and the Hubble constant measurement shows that w is constrained to be $w = -1.013^{+0.068}_{-0.073}$ at 68% C.L. [7]. If we use the time-dependent parametrization $w(a) = w_0 + w_a(1 - a)$, where a is the scale factor normalized as $a = 1$ today, the two parameters w_0 and w_a are constrained to be $w_0 = -1.046^{+0.179}_{-0.170}$ and $w_a = 0.14^{+0.60}_{-0.76}$ [7].

One of the simplest candidates for dark energy is the cosmological constant characterized by the equation of state $w = -1$, which is consistent with the current observational data. However, if the cosmological constant originates from the vacuum energy associated with particle physics, there is a huge gap between the theoretical and observed values [8]. Instead, alternative dark energy models with a dynamically changing w —such as quintessence [9,10] and k -essence [11]—have been proposed (see Ref. [12] for reviews).

Quintessence is described by a canonical scalar field ϕ with a potential $V(\phi)$. In the framework of particle physics it is generally difficult to accommodate a very light scalar field with a mass of the order of the Hubble parameter $H_0 \approx 10^{-33}$ eV today [13,14]. However, there have been theoretical attempts to construct viable quintessence models in particle physics, especially in supersymmetric

theories [15]. For example, the pseudo-Nambu-Goldstone boson [16] or axions [17] have a potential of the form $V(\phi) = \Lambda^4[1 \pm \cos(\phi/f)]$ with suppressed quantum corrections.

Caldwell and Linder [18] classified quintessence models into two classes, depending on the evolution of w . The first class corresponds to thawing models, in which the field is nearly frozen by a Hubble friction during the early cosmological epoch and starts to evolve once the field mass m_ϕ drops below the Hubble rate H . In this case the evolution of w is characterized by growth from -1 . The representative potential of this class is the hilltop potential, such as $V(\phi) = \Lambda^4[1 + \cos(\phi/f)]$.

The second class consists of freezing models, in which the evolution of the field gradually slows down because of the shallowness of the potential at late times. For the inverse-power-law potential $V(\phi) = M^{4+p}\phi^{-p}$ ($p > 0$) [10] there is a so-called tracker solution characterized by a nearly constant field equation of state $w = -2/(p + 2)$ during the matter era [19]. In this case solutions with different initial conditions approach a common trajectory (tracker) first, which is followed by the decrease of w toward -1 .

In addition to tracking freezing models there is another sub-class of freezing models associated with scaling solutions [20]. In this case the field equation of state scales as that of the background fluid during most of the matter era ($w \approx 0$). The representative potential of this class is $V(\phi) = V_1 e^{-\lambda_1 \phi/M_{\text{pl}}} + V_2 e^{-\lambda_2 \phi/M_{\text{pl}}}$, where M_{pl} is the reduced Planck mass, and λ_1 and λ_2 are constants with $\lambda_1 \gg 1$ and $\lambda_2 \lesssim 1$ [21]. In the early matter era the potential

is approximated as $V(\phi) \simeq V_1 e^{-\lambda_1 \phi/M_{\text{pl}}}$, which gives rise to the scaling solution characterized by $w = 0$ with the field density parameter $\Omega_\phi = 3/\lambda_1^2$. At late times the dominance of the potential $V_2 e^{-\lambda_2 \phi/M_{\text{pl}}}$ leads to the rapid decrease of w relative to the tracker case mentioned above. Note that the potential $V(\phi) = e^{-\lambda \phi/M_{\text{pl}}}[(\phi - B)^\alpha + A]$ proposed in Ref. [22] also exhibits a similar property to that of the double-exponential potential (see also Ref. [23]).

In this paper we place observational constraints on three types of quintessence models: (i) thawing models, (ii) tracking freezing models, and (iii) scaling freezing models (see Ref. [24] for related works). This analysis covers most of the quintessence potentials proposed in the literature.

For thawing models we employ the analytic expression of w derived in Refs. [25,26] under the approximation that $|w + 1| \ll 1$ (see also Ref. [27]). The likelihood analysis with the SN Ia and BAO data was carried out in Ref. [28] (see also Ref. [25]). We update the analysis by using the latest SN Ia data (Union 2.1 data set [7]) and by adding the data of CMB shift parameters measured by WMAP7 [4]. We also take into account the recent BAO data of the BOSS experiment [29].¹

Note that the observational constraints on thawing models were carried out by using a multiparameter extension of the exponential potential [31] and by introducing a statefinder hierarchy [32]. Our study based on the analytic solution of w is more convenient in that it covers any quintessence potential having thawing properties and that w is expressed in terms of three parameters without the need of introducing more free parameters.

For tracking freezing models one of the present authors obtained the approximate analytic formula of w expressed in terms of two free parameters [33] (see also Ref. [34]). The likelihood analysis based on the SN Ia and BAO data was performed in Ref. [33]. We show that adding the CMB and BOSS data further strengthens the constraints on the tracker equation of state $w_{(0)}$. Wang *et al.* [35] placed observational bounds on a number of quintessence potentials having tracker properties. Our study based on the analytic formula of w is general enough to cover such potentials. Moreover, we show that the inclusion of the BOSS BAO data further strengthens the bounds on $w_{(0)}$ previously derived in the literature.

For scaling freezing models it is difficult to derive an analytic expression of w , so we resort to numerical simulations to find a viable parameter space.

This paper is organized as follows. In Sec. II we briefly review the procedure for deriving approximate analytic expressions of w in thawing and tracking freezing models.

¹Two months after the initial submission of this paper, new BAO data in the Ly α forest appeared in the redshift range $2.1 \leq z \leq 3.5$ [30]. We do not take this new data into account in our likelihood analysis.

The accuracy of those approximations is also discussed by solving the equations of motion numerically. For scaling freezing models we show that in some cases it is possible to fit the evolution of w by using a specific parametrization. In Sec. III we first explain the method of our likelihood analysis based on the SN Ia, CMB, and BAO data, and then we proceed to observational constraints on three classes of quintessence models. Section IV is devoted to conclusions.

II. PARAMETRIZATIONS OF QUINTESSENCE

Quintessence [9] is described by a minimally coupled scalar field ϕ with a potential $V(\phi)$. In addition to the field ϕ we take into account nonrelativistic matter with an energy density ρ_m . The action in such a system is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + S_m, \quad (1)$$

where g is the determinant of the metric $g_{\mu\nu}$, R is the Ricci scalar, and S_m is the action for nonrelativistic matter. In the flat Friedmann-Lemaître-Robertson-Walker background with the scale factor $a(t)$, the dynamical equations of motion are

$$3H^2 M_{\text{pl}}^2 = \rho_\phi + \rho_m, \quad (2)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \quad (3)$$

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (4)$$

where $H = \dot{a}/a$ is the Hubble parameter, a dot represents a derivative with respect to cosmic time t , $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$, and $V_{,\phi} = dV/d\phi$. The pressure of the field is given by $P_\phi = \dot{\phi}^2/2 - V(\phi)$. We introduce the equation of state $w = P_\phi/\rho_\phi$ and the density parameter $\Omega_\phi = \rho_\phi/(3H^2 M_{\text{pl}}^2)$ of dark energy.

From Eqs. (2)–(4) we obtain the following equations for w and Ω_ϕ [25,27] (see also Refs. [20,36]):

$$w' = (1 - w) \left[-3(1 + w) + \lambda \sqrt{3(1 + w)\Omega_\phi} \right], \quad (5)$$

$$\Omega'_\phi = -3w\Omega_\phi(1 - \Omega_\phi), \quad (6)$$

$$\lambda' = -\sqrt{3(1 + w)\Omega_\phi}(\Gamma - 1)\lambda^2, \quad (7)$$

where $\lambda = -M_{\text{pl}} V_{,\phi}/V$, $\Gamma = V V_{,\phi\phi}/V_{,\phi}^2$, and a prime represents a derivative with respect to $N = \ln a$. Depending on the field potential and the initial conditions, there are several different cases for the evolution of w [18].

In the following we shall discuss possible analytic solutions of w for three different cases: (i) thawing, (ii) tracking freezing, and (iii) scaling freezing models. The presence of analytic solutions is useful in that some general properties

of physical parameters can be extracted without studying a host of quintessence potentials separately. Moreover, if w can be analytically expressed in terms of the redshift z with several free parameters, we do not need to integrate the background equations of motion with arbitrary initial conditions. This greatly simplifies the likelihood analysis carried out in Sec. III.

A. Thawing models

For thawing models of quintessence the field ϕ is nearly frozen in the early matter era because of the Hubble friction, so that w is close to -1 . One can regard $w = -1$ as the fixed point of Eq. (5). For $\lambda \neq 0$ such a point is not stable, and hence w starts to grow at the late cosmological epoch.

If we assume that λ is nearly constant, one can express w in terms of Ω_ϕ by using Eqs. (5) and (6) under the

approximation $|1 + w| \ll 1$ [27]. This neglects the effect of the field mass squared $V_{,\phi\phi}$, but it is possible to derive a more elaborate form of w with the mass term taken into account [25,26]. In doing so, the potential is expanded around the initial field value ϕ_i up to second order, i.e., $V(\phi) = \sum_{n=0}^2 V^{(n)}(\phi_i)(\phi - \phi_i)^n/n!$.

Provided that $|w + 1| \ll 1$, the evolution of the scale factor can be approximated by that of the Λ CDM model, i.e., $a(t) = [(1 - \Omega_{\phi 0})/\Omega_{\phi 0}]^{1/3} \sinh^{2/3}(t/t_\Lambda)$, where $t_\Lambda = 2M_{\text{pl}}/\sqrt{3V(\phi_i)}$ and $\Omega_{\phi 0}$ is today's density parameter of quintessence. On using this solution, Eq. (3) is integrated to give the field in terms of the function of t , and hence the field equation of state $w(t) \simeq -1 + \dot{\phi}^2/V(\phi_i)$ is known. This process leads to the following analytic expression of w [25,26]:

$$w(a) = -1 + (1 + w_0)a^{3(K-1)} \left[\frac{(K - F(a))(F(a) + 1)^K + (K + F(a))(F(a) - 1)^K}{(K - \Omega_{\phi 0}^{-1/2})(\Omega_{\phi 0}^{-1/2} + 1)^K + (K + \Omega_{\phi 0}^{-1/2})(\Omega_{\phi 0}^{-1/2} - 1)^K} \right]^2, \quad (8)$$

where w_0 is the value of w today, and

$$K = \sqrt{1 - \frac{4M_{\text{pl}}^2 V_{,\phi\phi}(\phi_i)}{3V(\phi_i)}}, \quad (9)$$

$$F(a) = \sqrt{1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}}. \quad (10)$$

The solution (8) is valid for $K^2 > 0$. The equation of state (8) is expressed in terms of the three parameters w_0 , $\Omega_{\phi 0}$, and K .

As a concrete example, let us consider the hilltop potential

$$V(\phi) = \Lambda^4 [1 + \cos(\phi/f)]. \quad (11)$$

In this case the parameter (9) is given by

$$K = \left[1 + \frac{4}{3} \left(\frac{M_{\text{pl}}}{f} \right)^2 \frac{\cos(\phi_i/f)}{1 + \cos(\phi_i/f)} \right]^{1/2}. \quad (12)$$

If $0 < \phi_i/f < \pi/2$ and $\pi/2 < \phi_i/f < \pi$, one has $K > 1$ and $K < 1$, respectively. In the former case the potential is approximately given by $V(\phi) \approx 2\Lambda^4 [1 - \phi^2/(4f^2)]$ around $\phi = 0$, whereas in the latter case it is approximated as $V(\phi) \approx \Lambda^4 (\phi - \pi f)^2/(2f^2)$ around $\phi = \pi f$.

In Fig. 1 we plot the numerical evolution of w versus a for $K > 1$ with several different values of f and ϕ_i . The bald dashed curves correspond to the results derived by the analytic expression (8), which show good agreement with the numerically integrated solutions for $w_0 \lesssim -0.3$. For K larger than 10, the initial displacement of the field is required to be close to 0 to avoid its rapid roll down. In such cases the field mass is largely negative, which leads to the tachyonic instability of field perturbations. If the field reaches the potential minimum by today and it starts to oscillate, numerical simulations show that Eq. (8) is no

longer reliable. We set the prior $K < 10$ in the likelihood analysis of Sec. III A.

The analytic estimation (8) starts to lose accuracy for K smaller than 1. This reflects the fact that the field is initially located away from the potential maximum. Then the Taylor expansion around $\phi = \phi_i$ tends to be more inaccurate

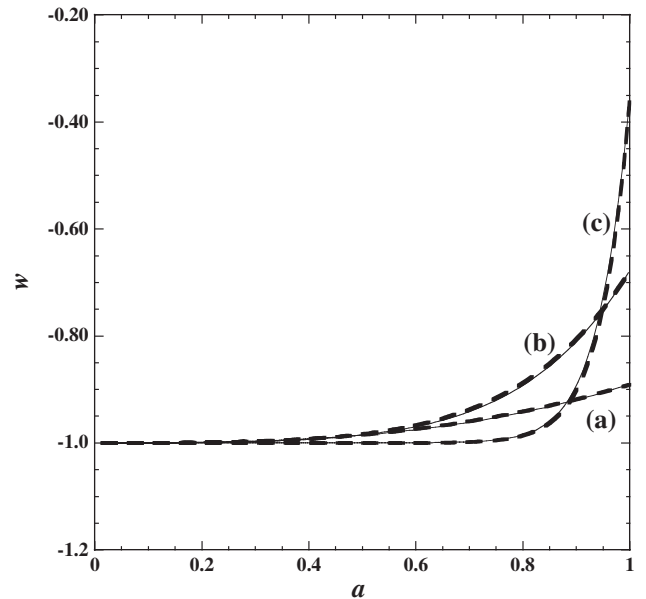


FIG. 1. The quintessence equation of state w versus a for the potential (11) with (a) $f/M_{\text{pl}}=0.5$, $\phi_i/f=0.5$ ($K=1.9$), (b) $f/M_{\text{pl}}=0.3$, $\phi_i/f=0.25$ ($K=2.9$), and (c) $f/M_{\text{pl}}=0.1$, $\phi_i/f=7.6 \times 10^{-4}$ ($K=8.2$). These cases correspond to $V_{,\phi\phi}(\phi_i) < 0$, so that $K > 1$. The solid curves show numerical solutions, whereas the bald dashed curves describe the results derived from the parametrization (8) with $\Omega_{\phi 0} = 0.73$.

because of the rapid variation of the field. Numerically we find that the analytic solution (8) is reliable for $0.5 \lesssim K < 1$ and $w_0 \lesssim -0.8$.

B. Tracking freezing models

In Eq. (5) there is another fixed point given by

$$\Omega_\phi = \frac{3(1+w)}{\lambda^2}, \quad (13)$$

along which w is constant. This corresponds to the tracker that attracts the solutions with different initial conditions to a common trajectory. The condition under which the tracking occurs is [19]²

$$\Gamma > 1. \quad (14)$$

In this case the variable λ approaches 0.

From Eq. (13) it follows that $\Omega'_\phi/\Omega_\phi = -2\lambda'/\lambda$. Using this relation with Eqs. (6) and (7) in the regime $\Omega_\phi \ll 1$, the field equation of state along the tracker is given by

$$\begin{aligned} w(a) &= w_{(0)} + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} w_{(0)} (1 - w_{(0)}^2)}{1 - (n+1)w_{(0)} + 2n(n+1)w_{(0)}^2} \left(\frac{\Omega_\phi(a)}{1 - \Omega_\phi(a)} \right)^n \\ &= w_{(0)} + \frac{(1 - w_{(0)}^2)w_{(0)}}{1 - 2w_{(0)} + 4w_{(0)}^2} \Omega_\phi(a) + \frac{(1 - w_{(0)}^2)w_{(0)}^2 (8w_{(0)} - 1)}{(1 - 2w_{(0)} + 4w_{(0)}^2)(1 - 3w_{(0)} + 12w_{(0)}^2)} \Omega_\phi(a)^2 \\ &\quad + \frac{2(1 - w_{(0)}^2)w_{(0)}^3 (4w_{(0)} - 1)(18w_{(0)} + 1)}{(1 - 2w_{(0)} + 4w_{(0)}^2)(1 - 3w_{(0)} + 12w_{(0)}^2)(1 - 4w_{(0)} + 24w_{(0)}^2)} \Omega_\phi(a)^3 + \dots, \end{aligned} \quad (18)$$

where in the second and third lines we carried out the expansion $(\Omega_\phi(a)/(1 - \Omega_\phi(a)))^n = \Omega_\phi(a)^n \times (\sum_{m=0}^{\infty} \Omega_\phi(a)^m)^n$. The equation of state (18) is expressed in terms of the two parameters $w_{(0)}$ and $\Omega_{\phi 0}$. Numerically we confirm that the approximated formula (18) tends to approach the full numerical solution by adding higher-order terms of $\Omega_\phi(a)$.

C. Scaling freezing models

The scaling solution [20] is a special case of a tracker along which Ω_ϕ in Eq. (13) is constant with $0 < \Omega_\phi < 1$. From Eq. (6) it then follows that $w = 0$ and hence $\Omega_\phi = 3/\lambda^2$ during the matter era. Since λ is constant, one has $\Gamma = 1$ from Eq. (7). This corresponds to the exponential potential $V(\phi) = V_0 e^{-\lambda\phi/M_{\text{pl}}}$, where V_0 is a constant.³ In this case, however, the field equation of state is the same

²A similar condition for the k-essence Lagrangian $P(\phi, X) = V(\phi)W(X)$ with $X = \dot{\phi}^2/2$ is given by $\Gamma > 3/2$ [37].

³For the k-essence Lagrangian $P(\phi, X)$ (where $X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi/2$) the condition for the existence of scaling solutions restricts the Lagrangian in the form $P = Xg(Y)$, where g is an arbitrary function in terms of $Y = X e^{\lambda\phi/M_{\text{pl}}}$ [38]. Quintessence with the exponential potential corresponds to the choice $g(Y) = 1 - V_0/Y$.

$$w = w_{(0)} \equiv -\frac{2(\Gamma - 1)}{2\Gamma - 1}. \quad (15)$$

For the potential $V(\phi) = M^{4+p} \phi^{-p}$ ($p > 0$) one has $\Gamma = 1 + 1/p$, and hence $w_{(0)} = -2/(p + 2)$ [19,34].

The result (15) was derived by neglecting the contribution of Ω_ϕ , but its effect can be accommodated by dealing with Ω_ϕ as a perturbation to the zeroth-order solution (15) [33]. We consider the first-order perturbation δw around $w_{(0)}$ and then approximate $\Omega_\phi(a)$ by the zeroth-order solution

$$\Omega_\phi(a) = \frac{\Omega_{\phi 0} a^{-3w_{(0)}}}{\Omega_{\phi 0} a^{-3w_{(0)}} + 1 - \Omega_{\phi 0}}. \quad (16)$$

For the models in which Γ is nearly constant, we obtain the following analytic solution [33]:

as that of the background fluid ($w = 0$), so that the system does not enter the phase of cosmic acceleration.

This problem can be circumvented for the following model [21]:

$$V(\phi) = V_1 e^{-\lambda_1 \phi/M_{\text{pl}}} + V_2 e^{-\lambda_2 \phi/M_{\text{pl}}}, \quad (19)$$

where λ_i and V_i ($i = 1, 2$) are constants. If $\lambda_1 \gg 1$ and $\lambda_2 \lesssim 1$, then the solution first enters the scaling regime characterized by $\Omega_\phi = 3(1 + w_m)/\lambda_1^2$, where w_m is the equation of state of the background fluid. From the bound coming from big bang nucleosynthesis [39] there is a constraint $\Omega_\phi < 0.045$ (95% C.L.) during the radiation era ($w_m = 1/3$), which translates into the condition $\lambda_1 > 9.4$. The scaling matter era ($\Omega_\phi = 3/\lambda_1^2$, $w = 0$) is followed by the dark energy-dominated epoch driven by the presence of the potential $V_2 e^{-\lambda_2 \phi/M_{\text{pl}}}$. If $\lambda_2^2 < 3$, the solution approaches another attractor characterized by $\Omega_\phi = 1$ and $w = -1 + \lambda_2^2/3$ [20]. The cosmic acceleration occurs for $\lambda_2^2 < 2$.

The onset of the transition from the scaling matter era to the dark energy-dominated epoch depends on the parameters λ_1 , λ_2 , and V_2/V_1 . Numerically we find that the transition redshift is not very sensitive to the choice of V_2/V_1 , so we study the case $V_2 = V_1$. In Fig. 2 we plot

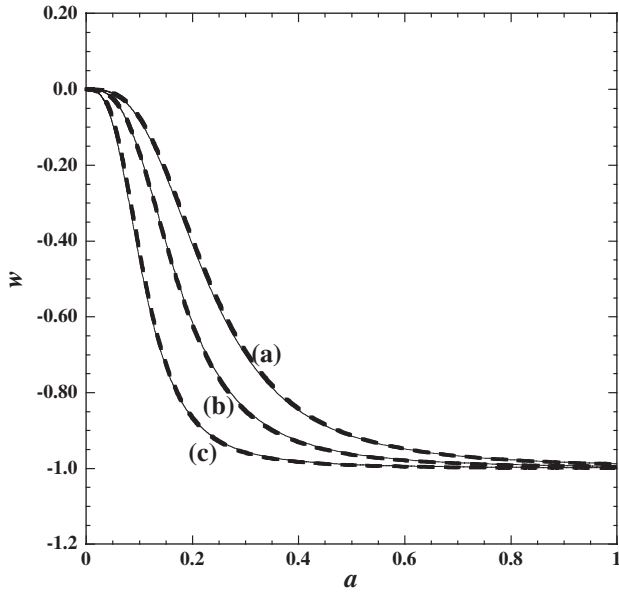


FIG. 2. The quintessence equation of state w versus a for the potential (19) with (a) $\lambda_1 = 10$, $\lambda_2 = 0$, (b) $\lambda_1 = 15$, $\lambda_2 = 0$, and (c) $\lambda_1 = 30$, $\lambda_2 = 0$. The solid curves show the numerical solutions, whereas the dashed curves represent the results derived from the parametrization (20) with $w_p = 0$ and $w_f = -1$. Each dashed curve corresponds to (a) $a_t = 0.23$, $\tau = 0.33$, (b) $a_t = 0.17$, $\tau = 0.33$, and (c) $a_t = 0.11$, $\tau = 0.32$.

the numerical evolution of w for $\lambda_2 = 0$ with three different values of λ_1 . For larger λ_1 the transition to the dark energy-dominated epoch occurs earlier.

It is possible to accommodate the above variation of w analytically by using the parametrization proposed by Linder and Huterer [40] (see also Ref. [41]),

$$w(a) = w_f + \frac{w_p - w_f}{1 + (a/a_t)^{1/\tau}}, \quad (20)$$

where w_p and w_f are asymptotic values of w in the past and future, respectively, a_t is the scale factor at the transition, and τ describes the width of the transition. The scaling solution during the matter era corresponds to $w_p = 0$. For $\lambda_2 = 0$ one has $w_f = -1$, so that the parametrization (20) reduces to $w(a) = -1 + [1 + (a/a_t)^{1/\tau}]^{-1}$. Figure 2 shows that the parametrization (20) can fit the numerical evolution of w very well for appropriate choices of a_t and τ . For $\lambda_2 = 0$ the transition width is around $\tau \approx 0.33$, while a_t depends on the values of λ_1 . In this case one can carry out the likelihood analysis by fixing $\tau = 0.33$ and find the constraints on a_t (see Ref. [42] for a related work).

If $\lambda_2 \neq 0$, then the field equation of state finally approaches the value $w = -1 + \lambda_2^2/3$. Numerically we find that w tends to have a minimum for larger λ_2 before the solutions reach the attractor. If $\lambda_1 = 20$, for example, the minimum appears for $\lambda_2 \gtrsim 0.3$ (see Fig. 3). In order to place observational bounds on λ_2 in such cases, we need to resort to numerical simulations without using the parametrization (20).

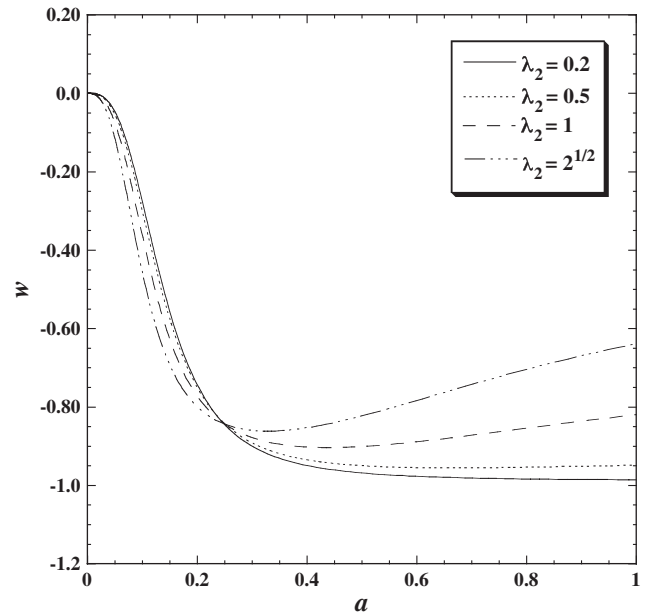


FIG. 3. The quintessence equation of state w versus a for the potential (19) with $\lambda_1 = 20$ and $\lambda_2 = 0.2, 0.5, 1, \sqrt{2}$.

III. OBSERVATIONAL CONSTRAINTS

In this section we place observational constraints on the three types of quintessence models separately. We use the recent SN Ia data (the Union 2.1 data set [7]), the shift parameters provided by WMAP7 [4], and the BAO distance measured by SDSS7 [6] and by BOSS [29]. In order to make the analysis simpler, we fix—for all the models under consideration—today's radiation density parameter to be equal to that of the Λ CDM model.

In SN Ia observations the luminosity distance $d_L(z) = (1+z) \int_0^z H^{-1}(\tilde{z}) d\tilde{z}$ is measured by the difference (distance modulus) of the apparent magnitude $m(z)$ and the absolute magnitude M as

$$\mu(z) \equiv m(z) - M = 5 \log_{10}[d_L(z)/10 \text{ pc}]. \quad (21)$$

For the observed distance modulus $\mu_{\text{obs}}(z_i)$ with the errors $\sigma_{\mu,i}$, the chi square of the SN Ia measurement is given by

$$\chi_{\text{SN Ia}}^2 = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma_{\mu,i}^2}, \quad (22)$$

where $\mu_{\text{th}}(z_i)$ is the theoretical value of $\mu(z_i)$ known for a given dark energy model.

The position of the CMB acoustic peaks is determined by the following parameter [43–45]:

$$l_a = \frac{\pi d_a^{(c)}(z_*)}{r_s(z_*)}, \quad (23)$$

where z_* is the redshift at the decoupling epoch, $d_a^{(c)}(z_*) = \mathcal{R}/(H_0 \sqrt{\Omega_{m0}})$ is the comoving angular diameter distance to the last scattering surface (Ω_{m0} is the matter density parameter today), and

$$\mathcal{R} = \sqrt{\Omega_{m0}} \int_0^{z_*} \frac{dz}{H(z)/H_0}. \quad (24)$$

The sound horizon $r_s(z_*)$ is defined by

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{dz}{H(z) \sqrt{3\{1 + 3\Omega_{b0}/[4\Omega_{\gamma0}(1+z)]\}}}. \quad (25)$$

Here Ω_{b0} and $\Omega_{\gamma0}$ are today's density parameters of baryons and photons, respectively. For the redshift z_* we use the fitting formula of Hu and Sugiyama [46].⁴ The chi square associated with the WMAP7 measurement is

$$\chi_{\text{CMB}}^2 = \mathbf{X}_{\text{CMB}}^T \mathbf{C}_{\text{CMB}}^{-1} \mathbf{X}_{\text{CMB}}, \quad (26)$$

where $\mathbf{X}_{\text{CMB}}^T = (l_a - 302.09, \mathcal{R} - 1.725, z_* - 1091.3)$, and the covariance matrix is given by [4]

$$\mathbf{C}_{\text{CMB}} = \begin{pmatrix} 0.58269 & 0.00274801 & 0.318613 \\ 0.00274801 & 0.000338358 & 0.0122901 \\ 0.318613 & 0.0122901 & 0.824753 \end{pmatrix}, \quad (27)$$

and the inverse covariance matrix is

$$\mathbf{C}_{\text{CMB}}^{-1} = \begin{pmatrix} 2.305 & 29.698 & -1.333 \\ 29.698 & 6825.27 & -113.18 \\ -1.333 & -113.18 & 3.414 \end{pmatrix}. \quad (28)$$

In BAO observations the ratio $r_{\text{BAO}}(z) \equiv r_s(z_d)/D_V(z)$ is measured, where $r_s(z_d)$ is the sound horizon at which the baryons are released from the Compton drag of photons and $D_V(z)$ is the effective BAO distance defined by $D_V(z) \equiv [(\int_0^z H^{-1}(\bar{z})d\bar{z})^2 z/H(z)]^{1/3}$ [5]. For the redshift z_d we use the fitting formula of Eisenstein and Hu [47]. The chi square of the SDSS7 measurement is given by

$$\chi_{\text{BAO,SDSS7}}^2 = \mathbf{X}_{\text{BAO}}^T \mathbf{C}_{\text{BAO}}^{-1} \mathbf{X}_{\text{BAO}}, \quad (29)$$

where $\mathbf{X}_{\text{BAO}}^T = (r_{\text{BAO}}(0.2) - 0.1905, r_{\text{BAO}}(0.35) - 0.1097)$. The covariance matrix is given by [6]

$$\mathbf{C}_{\text{BAO}} = \begin{pmatrix} 3.7436 \times 10^{-5} & 7.4148 \times 10^{-6} \\ 7.4148 \times 10^{-6} & 1.2966 \times 10^{-5} \end{pmatrix}, \quad (30)$$

and the inverse covariance matrix is

$$\mathbf{C}_{\text{BAO}}^{-1} = \begin{pmatrix} 30124 & -17227 \\ -17227 & 86977 \end{pmatrix}. \quad (31)$$

We also use the BAO data from the WiggleZ and 6dFGS surveys, for which $A_{\text{WiggleZ}}(z = 0.6) = 0.452 \pm 0.018$ [48],

⁴We note that, for fixed l_a , two parameters \mathcal{R} and z_* depend primarily on Ω_{m0} and Ω_{b0} , respectively. While l_a characterizes the position of the CMB acoustic peaks, the parameters Ω_{m0} and Ω_{b0} are mostly related to the amplitudes of the peaks [46]. It is possible to employ the parameter sets $(l_a, \Omega_{m0}, \Omega_{b0})$ in the likelihood analysis (as in Ref. [44]), but we use the parameter sets (l_a, \mathcal{R}, z_*) because those are the parameters that the WMAP team provides an approximate covariance matrix for.

and $A_{6\text{dFGS}}(z = 0.106) = 0.526 \pm 0.028$ [49], where $A(z)$ is defined as $A_{\text{th}}(z) = D_V(z) \sqrt{\Omega_{m0} H_0^2}/z$.

Finally, we use the latest and most precise data point from the BOSS experiment [29], for which

$$1/r_{\text{BAO}}(z = 0.57) = 13.67 \pm 0.22. \quad (32)$$

Note that the error bar of this data is less than 1.7%. This is the most precise distance measurement ever constrained from a galaxy survey. Moreover, as we will see below, this data puts a severe upper bound of w close to -1 at $z = 0.57$.

The total chi square from the three data sets is

$$\chi^2 = \chi_{\text{SN Ia}}^2 + \chi_{\text{CMB}}^2 + \chi_{\text{BAO}}^2, \quad (33)$$

where the best fit corresponds to the lowest value of χ^2 .

A. Thawing models

Let us study the observational constraints on thawing models given by the equation of state (8). Since K is weakly constrained in the three-parameter analysis, we first study this model by fixing the value of K and vary the two parameters w_0 and $\Omega_{\phi 0}$. This is the approach taken by Dutta and Scherrer [25]. However, because of a mild dependence (i.e., degeneracy) of the χ^2 as a function of the parameter K , we will also perform the data analysis by marginalizing the χ^2 over K itself. This last procedure allows us, by hiding the information of K , to easily understand the measured parameters $\Omega_{\phi 0}$ and w_0 .

Although quintessence corresponds to the case $w_0 > -1$, we also extend to the regime $w_0 < -1$ in the likelihood analysis. In fact, it was shown in Ref. [50] that the equation of state of a phantom scalar field can be accommodated by the analytic formula (8). The likelihood results are quite different depending on whether the recent BAO data from BOSS [29] are included or not, so we present two constraints with/without the BOSS data.

Let us first focus our analysis on some particular values of K , which allow us to make comparisons with the results given in Ref. [25]. Our numerical analysis gives that, in the absence of the BOSS BAO data, the dark energy equation of state today is constrained to be $-1.219 < w_0 < -0.930$ (at 95% C.L. for $K = 1.01$). Dutta and Scherrer [25] showed that even the value $w_0 = -0.7$ is allowed from the SN Ia data alone. Adding the WMAP7 and SDSS7 data gives rise to much tighter bounds on w_0 . This is the case for $\Omega_{\phi 0}$ as well. We obtained the bound $0.7078 < \Omega_{\phi 0} < 0.740$ (95% C.L. for $K = 1.01$), whereas the values $0.68 < \Omega_{\phi 0} < 0.8$ are allowed in Ref. [25].

We also study the same case *with* the latest BOSS data. Having so far fixed K , by studying the χ^2 over the parameter space of the remaining two parameters, we find that w_0 and $\Omega_{\phi 0}$ are constrained to be $-1.242 < w_0 < -0.995$ and $0.705 < \Omega_{\phi 0} < 0.734$, respectively, at 95% C.L. for $K = 1.01$. Rather surprisingly, the allowed parameter space in the regime $w_0 \geq -1$ is very narrow. In particular,

the Λ CDM model, which corresponds to $w_0 = -1$, is outside the 1σ observational contour. This comes from the fact that the BOSS data (32) does not allow a large parameter space with $w > -1$ at the redshift $z = 0.57$.

We have also studied the case $K = 4$ and derived bounds for w_0 and Ω_{ϕ_0} . Without the BOSS data we obtain the bounds $-1.408 < w_0 < -0.855$ and $0.711 < \Omega_{\phi_0} < 0.740$ (95% C.L., $K = 4$), whereas with the BOSS data these parameters are constrained to be $-1.460 < w_0 < -0.982$ and $0.709 < \Omega_{\phi_0} < 0.734$ (95% C.L., $K = 4$), respectively. In the former case the upper bound of w_0 gets larger than that for $K = 1.01$. This reflects the fact that w changes more rapidly at late times for larger K . In the presence of the BOSS data, however, the allowed parameter space in the regime $w_0 > -1$ is still very tiny. This same qualitative behavior holds for all the different values of K considered in our analysis, as we have found that the χ^2 does not vary significantly as a function of K .

We also vary the three parameters w_0 , Ω_{ϕ_0} , and K in the likelihood analysis with the prior $0.1 < K < 10$, updating the analysis made in Ref. [28]. We have fixed this prior for K because, for $K > 10$, the analytic expression (8) is not completely reliable due to the rapid rolling down of the field along the potential with carefully chosen initial conditions. With the BOSS data taken into account, the best-fit model parameters are found to be $w_0 = -1.102$, $\Omega_{\phi_0} = 0.71955$, and $K = 0.1$ with $\chi^2_{\min} = 568.57$. Under the prior $w_0 \geq -1$ the best-fit parameters reduce to those in the Λ CDM model, as χ^2 has a minimum at $w_0 = -1$.

Although the case $w_0 < -1$ is plagued by a ghost problem, the dynamics of w —given phenomenologically by Eq. (8)—is able to fit the data quite well for $w_0 \approx -1.2$. In fact, with two parameters more than those in the Λ CDM, according to the Akaike Information Criterion (AIC) (where we should add twice the number of free parameters k to the original χ^2) [51]⁵ the best-fit corresponds to $\tilde{\chi}^2 = 568.57 + 2 \times 3 = 574.57$, whereas in the Λ CDM $\tilde{\chi}^2_{\Lambda\text{CDM}} = 573.89 + 2 = 575.89$. Therefore the model with $w_0 < -1$, even with three parameters, can compete with the Λ CDM.

In Fig. 4 we show observational constraints in the (w_0, K) plane marginalized over Ω_{ϕ_0} . In the regime $0.1 < K < 1$ the constraints on w_0 are practically independent of K , i.e., $-1.212 < w_0 < -1.003$ (95% C.L.) with the BOSS data, which should be compared with the previous result of Ref. [33] (slightly updating the result in Ref. [28]), $-1.14 < w_0 < -0.92$ for $K < 2$. In the presence of the

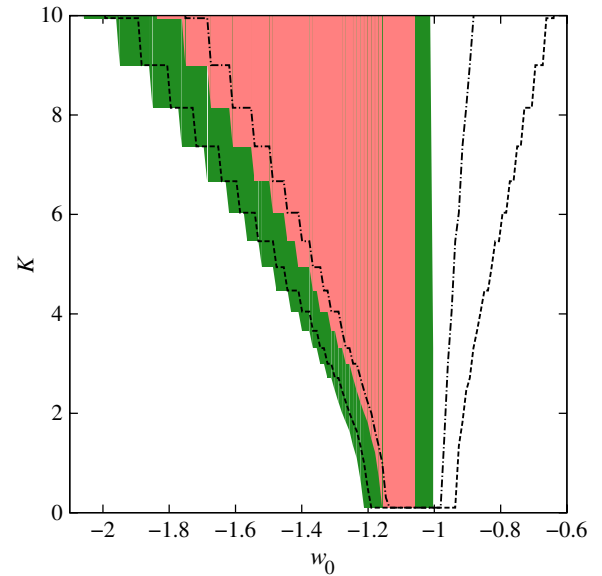


FIG. 4 (color online). 1σ (red) and 2σ (green) observational contours in the (w_0, K) plane marginalized over Ω_{ϕ_0} . We set the prior $0.1 < K < 10$. The dot-dashed and dotted curves correspond to the 1σ and 2σ constraints without the BOSS data, respectively.

BOSS data the allowed region shifts toward an w less than -1 , as can be seen in Fig. 4.

For $K > 1$ the lower bound on w gets smaller with increasing K , whereas the upper bound on w is practically unchanged. If $K = 9.95$, for example, w_0 is constrained to be $-2.059 < w_0 < -1.014$ (95% C.L.). For K larger than the order of 1 the field equation of state can rapidly increase in low redshifts, but such rapid growth of w is strongly disfavored from the BOSS data. If we do not take into account the BOSS data, the growth of w away from $w_0 = -1$ can still be allowed. For $K > 10$ the analytic expression (8) is not completely reliable because of the rapid rolling down of the field along the potential with carefully chosen initial conditions.

Finally, in Fig. 5 we plot observational constraints in the (w_0, Ω_{ϕ_0}) plane marginalized over K with the prior $0.1 < K < 10$. Also, in this case we find the same trend already mentioned above, namely, in the presence of the BOSS data the allowed region shifts toward the values of w less than -1 , as can be seen in Fig. 5. After the marginalization over K we obtain the bounds $-2.18 < w_0 < -0.893$ and $0.70265 < \Omega_{\phi_0} < 0.73515$ (95% C.L.). If we put the prior $w_0 > -1$, we find that w_0 is constrained to be $w_0 < -0.849$ (68% C.L.) and $w_0 < -0.695$ (95% C.L.).

B. Tracking freezing models

Let us proceed to the observational constraints on tracker solutions whose equation of state is given by Eq. (18). Although $w_{(0)}$ is theoretically larger than -1 for quintessence, we do not put the prior $w_{(0)} \geq -1$ in the actual likelihood analysis. In Fig. 6 we show the 1σ and 2σ observational contours in the $(w_{(0)}, \Omega_{\phi_0})$ plane.

⁵AIC assumes an infinite number of data points. For a finite number of data points (n), AIC should be modified to the Sugiura's criterion: $\chi^2 + 2nk/(n - k - 1)$, where k is the number of free parameters [52]. Since in our case n is large ($n > 500$) and k is of the order of 1, the difference between the two criteria is very small.

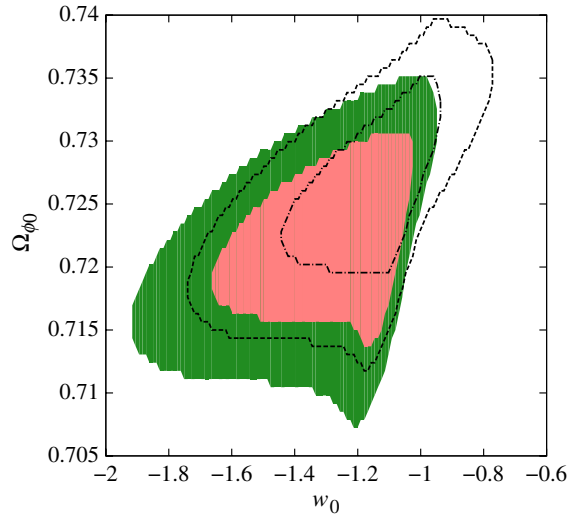


FIG. 5 (color online). 1σ (red) and 2σ (green) observational contours in the (w_0, Ω_{ϕ_0}) plane marginalized over K . We set the prior $0.1 < K < 10$. The dot-dashed and dotted curves correspond to the 1σ and 2σ constraints without the BOSS data, respectively.

Without the BAO data, the tracker equation of state is constrained to be $-1.188 < w_{(0)} < -0.949$ (95% C.L.). Meanwhile, the analysis of Ref. [33] based on the Constitution SN Ia and the SDSS BAO data gives the bound $-1.19 < w_{(0)} < -0.90$ (95% C.L.). Hence the upper bound of $w_{(0)}$ becomes tighter by including the WMAP7 data of the CMB shift parameters. For the potential $V(\phi) = M^{4+p}\phi^{-p}$ ($p > 0$) the 2σ constraint $w_{(0)} < -0.949$ translates into $p < 0.107$.

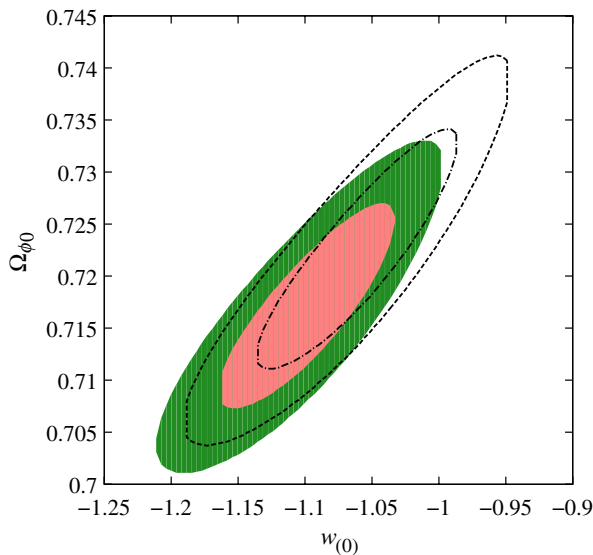


FIG. 6 (color online). 1σ (red) and 2σ (green) observational contours on tracking freezing models in the $(w_{(0)}, \Omega_{\phi_0})$ plane. The dot-dashed and dotted curves correspond to the 1σ and 2σ constraints without the BOSS data, respectively.

If we include the BOSS data in the analysis, the best-fit model parameters are found to be $w_{(0)} = -1.097$, and $\Omega_{\phi_0} = 0.717$ with $\chi^2_{\min} = 568.39$. In this case the Akaike criterion gives $\tilde{\chi}^2 = 568.39 + 2 \times 2 = 572.39$, which is smaller than the Λ CDM value $\tilde{\chi}^2_{\Lambda\text{CDM}} = 575.89$, with the difference being greater than 2. The 2σ observational bounds are found to be $-1.211 < w_{(0)} < -0.998$ and $0.701 < \Omega_{\phi_0} < 0.733$ (95% C.L.). The upper bound of $w_{(0)}$ is very close to -1 , which shows that the tracking quintessence away from -1 is strongly disfavored from the data.

If we put the prior $w_{(0)} \geq -1$ in the analysis with the BOSS data, we find that the best-fit is obtained for $w_{(0)} = -1$ and the model coincides with the standard Λ CDM case. We then obtain the upper bound $w_{(0)} < -0.964$ (95% C.L.).

C. Scaling freezing models

For the scaling models characterized by the potential (19) with $\lambda_2 = 0$, we already showed in Sec. II C that the evolution of w can be well approximated by Eq. (20). This parametrization admits an exact solution of H/H_0 in terms of a and the other four free parameters, which is numerically convenient for analyzing the data [40]. In Fig. 7 we plot observational bounds in the (a_t, Ω_{m0}) plane derived by using the parametrization (20) with $w_p = 0$, $w_f = -1$, and $\tau = 0.33$. If the BOSS data are taken into account in the analysis, the transition redshift is constrained to be $a_t < 0.23$ (95% C.L.). The minimum of χ^2 is found to be $a_t = 0$ with $\Omega_{m0} = 0.27$, i.e., the Λ CDM limit. The case (a) shown in Fig. 2 corresponds to the marginal one in which

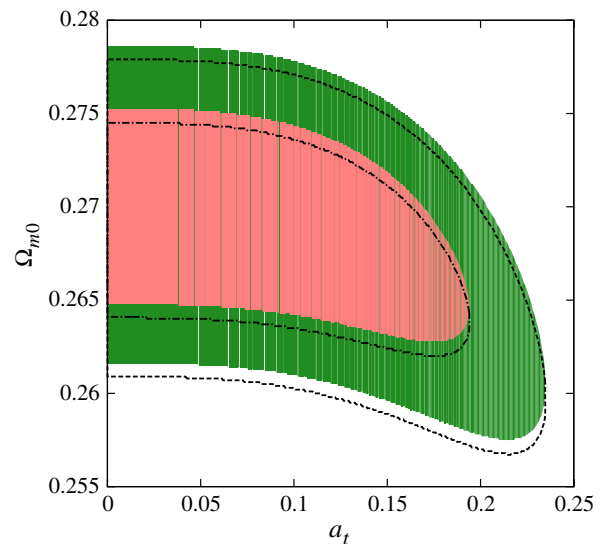


FIG. 7 (color online). 1σ (red) and 2σ (green) observational contours on a_t and Ω_{m0} for the parametrization (20) of the scaling solution with $w_p = 0$, $w_f = -1$, and $\tau = 0.33$. The dot-dashed and dotted curves correspond to the 1σ and 2σ constraints without the BOSS data, respectively.

the model is within the 2σ observational contour. This means that w needs to approach -1 in an early cosmological epoch ($w < -0.8$ for the redshift $z < 2$). Even without the BOSS data the upper bound on a_t is practically unchanged, which reflects the fact that w is close to -1 at low redshifts.

For $\lambda_2 = 0$ we also carry out the likelihood analysis without resorting to the parametrization (20). Numerically we solve the background equations of motion by tuning initial conditions to find the evolution that gives the desired values of Ω_{m0} and Ω_{r0} (radiation density parameter) for the potential $V(\phi) = V_1 e^{-\lambda_1 \phi/M_{\text{pl}}} + V_2$. We put the prior $\lambda_1 > 9.4$ coming from the constraint of big bang nucleosynthesis. As long as $V_1 e^{-\lambda_1 \phi/M_{\text{pl}}} \gg V_2$ the solutions approach the scaling fixed point $x = y = \sqrt{6}/(2\lambda_1)$ during the matter era, so the initial conditions of x and y are irrelevant to the likelihood analysis. In Fig. 8 we show observational bounds on the parameters λ_1 and Ω_{m0} . The parameter λ_1 is constrained to be $\lambda_1 > 13$ (95% C.L.), which is consistent with the results presented in Fig. 7. For larger λ_1 the transition from $w = 0$ to $w = -1$ occurs earlier, so that the models are favored by the data.

For nonzero values of λ_2 we do not have an analytic expression of w , so we solve the background equations of motion numerically with the priors $\lambda_1 > 9.4$ and $\lambda_2 > 10^{-4}$. We set the latter prior because the $\lambda_2 = 0$ case was already discussed above. Varying the three parameters λ_1 , λ_2 , and Ω_{m0} , we find that the best-fit model parameters are $\lambda_1 = 54.94$, $\lambda_2 = 10^{-4}$, and $\Omega_{m0} = 0.269$, with $\chi^2_{\text{min}} = 574.18$. Then the Akaike criterion gives $\tilde{\chi}^2 = 574.18 +$

$2 \times 3 = 580.18$, which is larger than the Λ CDM value $\tilde{\chi}^2_{\Lambda\text{CDM}} = 575.89$, with the difference being greater than 4. We also obtain the following bounds:

$$0.262 < \Omega_{m0} < 0.276 \quad (68\% \text{ CL}), \quad (34)$$

$$0.256 < \Omega_{m0} < 0.279 \quad (95\% \text{ CL}), \quad (35)$$

$$\lambda_1 > 16.3 \quad (68\% \text{ CL}), \quad (36)$$

$$\lambda_1 > 11.7 \quad (95\% \text{ CL}), \quad (37)$$

$$\lambda_2 < 0.361 \quad (68\% \text{ CL}), \quad (38)$$

$$\lambda_2 < 0.539 \quad (95\% \text{ CL}). \quad (39)$$

Figure 9 shows the observational constraints in the (λ_1, λ_2) plane for $\Omega_{m0} = 0.269$. If the three parameters λ_1 , λ_2 , and Ω_{m0} are varied in the likelihood analysis, it is difficult to marginalize over Ω_{m0} in the range $0 < \Omega_{m0} < 1$ because the solutions are prone to numerical instabilities around the tail regions of Ω_{m0} . Hence we use the fixed density parameter $\Omega_{m0} = 0.269$, which corresponds to the best-fit value when the three parameters are varied.

If $\lambda_2 \geq 0.5$, the model is excluded at 95% C.L. This is associated with the fact that w possesses a minimum for larger λ_2 (see Fig. 3). Therefore, in the context of this model, the expansion of the Universe accelerates forever. If $\lambda_1 = 20$, for example, the parameter λ_2 is constrained to be $\lambda_2 \lesssim 0.3$ (95% C.L.). In fact, the minimum of w appears

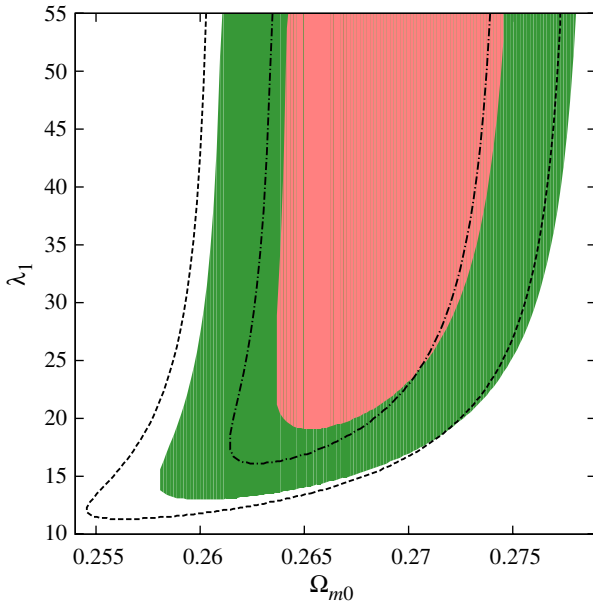


FIG. 8 (color online). 1σ (red) and 2σ (green) observational contours on λ_1 and Ω_{m0} for the potential (19) with $\lambda_2 = 0$. The dot-dashed and dotted curves correspond to the 1σ and 2σ constraints without the BOSS data, respectively.

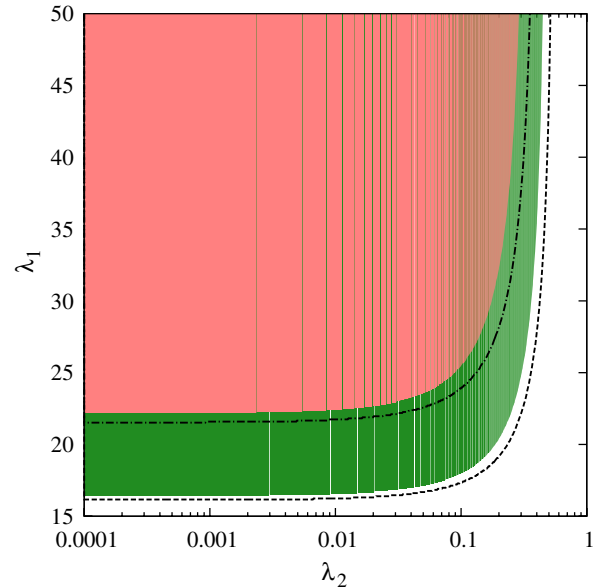


FIG. 9 (color online). 1σ (red) and 2σ (green) observational contours on λ_1 and λ_2 for the potential (19) with $\Omega_{m0} = 0.269$. The dot-dashed and dotted curves correspond to the 1σ and 2σ constraints without the BOSS data, respectively.

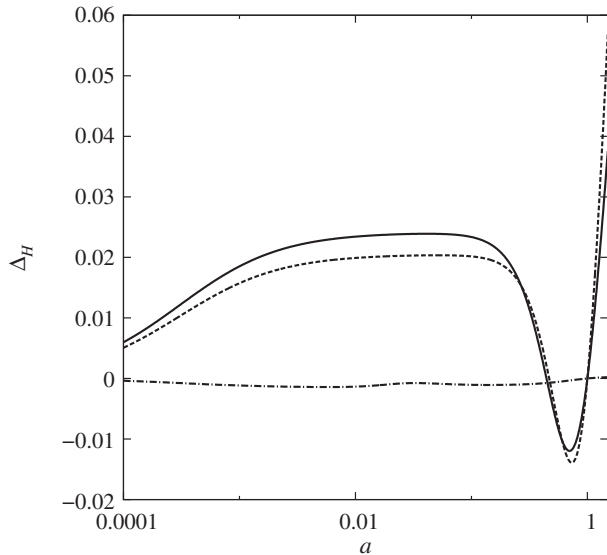


FIG. 10. Relative deviation of the Hubble parameter, $\Delta_H \equiv (H - H_{\Lambda\text{CDM}})/H_{\Lambda\text{CDM}}$, for the best-fit cases of (i) scaling (dot-dashed line), (ii) thawing (dotted line), and (iii) tracker (solid line) models. The relative deviations from the ΛCDM model are less than 2.5% for $a \leq 1$. The difference in the evolution among these models during dust domination is due to the fact that different model parameters lead to different values of Ω_{m0} for the best fit (in the ΛCDM we have $\Omega_{m0} = 0.2699$, whereas the Ω_{m0} 's for the other models exceed this value by about 4%).

for $\lambda_2 \gtrsim 0.3$, which leads to the deviation of w from -1 today. In summary, the model is within the 2σ observational contour provided that $\lambda_2 \lesssim 0.1$ and $\lambda_1 \gg 1$.

In Fig. 10 we plot the evolution of the relative deviation of the Hubble parameter from the ΛCDM as a function of a for the three best-fit models studied in this paper. The deviation Δ_H of the best-fit scaling model is very small, which reflects the fact that w quickly approaches -1 after the early transition from the scaling regime. Meanwhile, in the best-fit thawing and tracker models, the relative deviations from the ΛCDM can reach the level of 2%.

IV. CONCLUSIONS

In this paper we placed observational bounds on three types of quintessence models: (i) thawing, (ii) tracker, and (iii) scaling models. We used the recent data of SN Ia from Union 2.1, the CMB shift parameters from WMAP7, and BAO from SDSS7 and BOSS, by which the background cosmic expansion history is constrained from the distance measurements.

In thawing models where the field starts to evolve at the late cosmological epoch, the dark energy equation of state can be expressed as Eq. (8) with the three parameters w_0 , Ω_{ϕ_0} , and K . The parameter K is related to the field mass at the initial stage. We put observational bounds in the (w_0, Ω_{ϕ_0}) plane after marginalizing over K (see Fig. 5). If we include the BOSS data (32) in the likelihood analysis,

the upper bounds on w_0 are very close to -1 , independent of the values of K ranging in the region $0.1 < K < 10$ [under which the analytic formula (8) is reliable]. Without the BOSS data the deviation of w away from -1 today can still be allowed, as seen in Fig. 5.

In tracking freezing models where w is nearly constant ($w \approx w_{(0)}$) during the matter era, there is the analytic formula (18) derived by considering a homogeneous perturbation around the tracker. Without the BOSS data the tracker equation of state is constrained to be $-1.188 < w_{(0)} < -0.949$ (95% C.L.), whose upper bound is tighter than $w_{(0)} < -0.90$ derived by using SN Ia and SDSS7 data [33]. The inclusion of the BOSS data gives upper bounds of $w_{(0)}$ close to -1 . We find that without the prior on $w_{(0)}$ the constraint is $-1.211 < w_{(0)} < -0.998$ (95% C.L.), and with the quintessence prior $w_{(0)} > -1$ the upper bound is $w_{(0)} < -0.964$ (95% C.L.).

For the potential (19) with $\lambda_1 \gg 1$ and $\lambda_2 \ll 1$, w is close to 0 during the deep matter era because of the dominance of the steep potential $V_1 e^{-\lambda_1 \phi/M_{\text{pl}}}$. The field equation of state starts to decrease after the potential $V_2 e^{-\lambda_2 \phi/M_{\text{pl}}}$ dominates over $V_1 e^{-\lambda_1 \phi/M_{\text{pl}}}$. For larger λ_1 the exit from the scaling regime ($w = 0$) occurs earlier. When $\lambda_2 = 0$ we found that the evolution of w can be approximated by the parametrization (20) with $w_p = 0$, $w_f = -1$, and $\tau \approx 0.33$, where the transition scale factor a_t depends on λ_1 . Using this parametrization we derived the bound $a_t < 0.23$ (95% C.L.), which translates into the constraint $\lambda_1 > 13$. This is consistent with the bound shown in Fig. 7 derived by solving the background equations of motion numerically.

For the potential (19) with nonzero values of λ_2 , the field equation of state tends to have a minimum for larger λ_2 . As can be seen in Fig. 9, the models with $\lambda_2 \gtrsim 0.5$ are excluded at 95% C.L. The parameters $\lambda_2 \lesssim 0.1$ and $\lambda_1 > 17$ are allowed from the data, which shows that the early transition from the scaling regime to the regime close to $w = -1$ is favored. For the scaling models the observational constraints on λ_1 and λ_2 are not very sensitive to the inclusion of the BOSS data, because w evolves toward -1 at the late cosmological epoch.

As can be seen in Fig. 10, the difference of H between the three best-fit quintessence models and the ΛCDM model is only less than 2.5% in the past. In current observations there is no strong evidence that quintessence is favored over the ΛCDM from the statistical point of view. This property is made especially significant by including the BOSS BAO data.

By extending the analysis in the regime $w < -1$, we showed that some of the models studied in this paper can compete with the ΛCDM model according to the Akaike information criterion. We note that in many modified gravity models—such as $f(R)$ gravity [53], (extended) Galileons [54], and Lorentz-violating theories [55]—it is possible to realize $w < -1$ without having ghosts and

instabilities. It remains to be seen to what extent future high-precision observations can constrain dark energy models from the background expansion history, as well as from the cosmic growth of density perturbations.

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