Research & Knowledge

Volume 2, Number 1, January - June 2015

CONTENTS

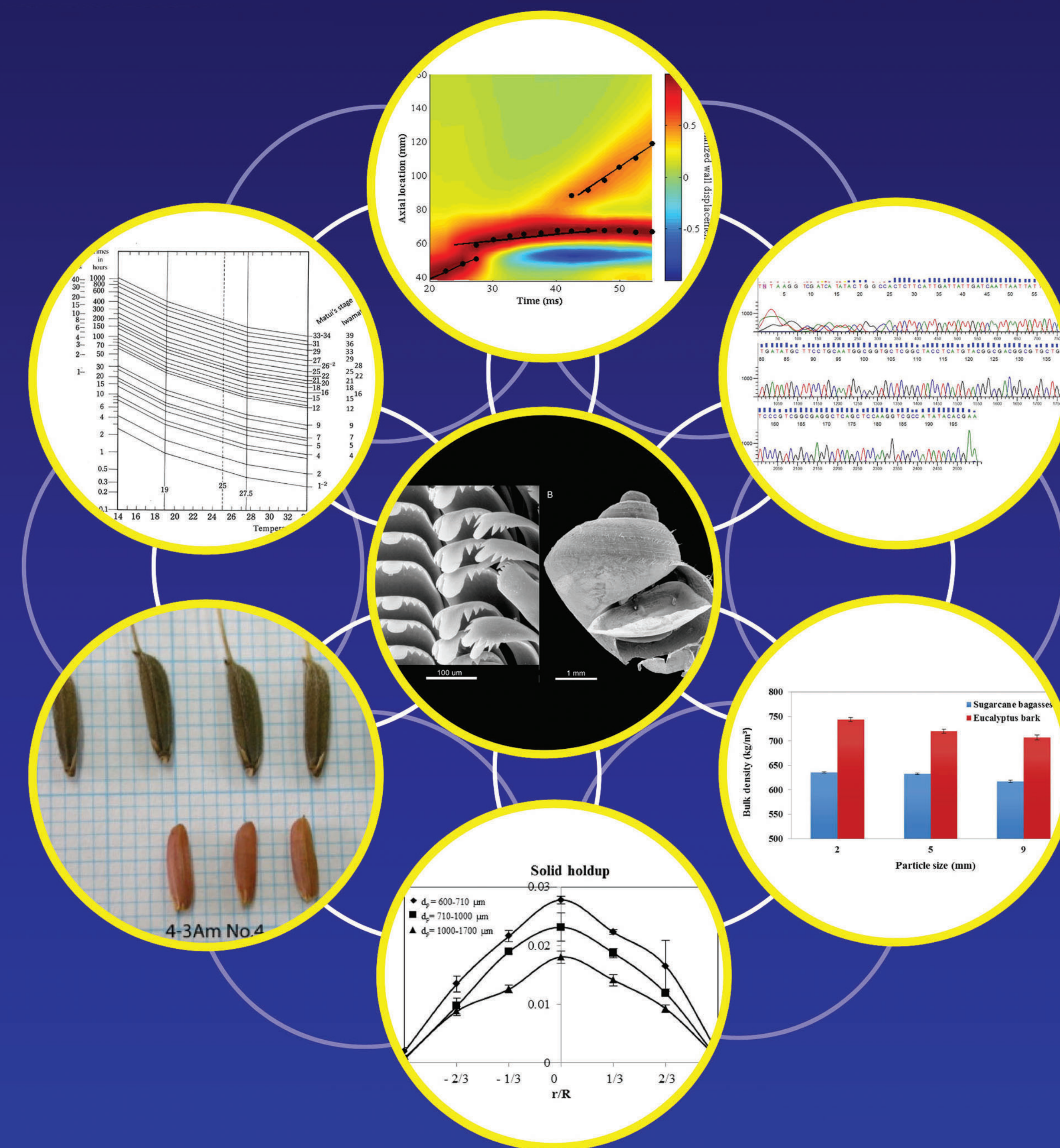
Research & Knowledge

Volume 2, Number 1, January - June 2016

Research & Knowledge

ISSN 2408-204X

Volume 2, Number 1, January - June 2016
https://rk.msu.ac.th



A peer-reviewed publication by
Mahasarakham University

Research & Knowledge

Owner Mahasarakham University (MSU)
Executive Editor Prof. Dr. Preecha Prathepa
Associate Executive Editor Prof. Dr. Pradit Terdtoon
Editor-in-Chief Prof. Dr. Wichian Magtoon

Periodicity Two issues per year

Distribution Open access and free distribution to a list of selected libraries in Thailand,
foreign university libraries, laboratories and by special request

Enquiry All inquiries should be directed to Research & Knowledge
Division of Research Facilitation and Dissemination (DRFD)
Mahasarakham University, Kantarawichai, Mahasarakham, 44150 Thailand
Phone: (66) 437-54322 ext. 1757/1173 Fax: (66) 437-54416
(66) 437-54416/437-54247 (66) 437-54247
E-mail: rk@msu.ac.th
Website: https://rk.msu.ac.th

Vol. 2 No. 1 page 35-40
DOI: 10.14456/randk.2016.4

RESEARCH & KNOWLEDGE

Research Article

Quantum theory of matter in bulk: Modern treatment

Edouard B. Manoukian* and Seckson Sukhasena

The Institute for Fundamental Study, "*The Tah Poe Academia Institute*",
Naresuan University, Phitsanulok 65000, Thailand

(Received 23 March 2016; accepted 24 May 2016)

Abstract - A systematic mathematical modern presentation is given, providing in a direct way the underlying technical details, to show how quantum theory, with the Pauli exclusion principle, has, over the years, solved the problem of why matter in bulk is stable and occupies so large a volume.

Keywords: Quantum theory of matter in bulk, fundamental role of the Pauli exclusion principle against collapse, large extension of matter

1. Introduction

One of the greatest problems that quantum mechanics has addressed over the years is the problem of why matter in bulk is stable, after realizing that classical theory fails to do so. That is, why matter, around us in our world, does not simply collapse. This paper deals in a *direct* mathematically rigorous way the fundamental role that quantum theory has played, over the years, in one of the most important problems of theoretical physics. If one prepares a list of the most important problems in quantum theory addressed over the years, this subject will undoubtedly be on it. The purpose of this communication is to spell out in a modern, comprehensive and rigorous way by invoking the Pauli exclusion principle, the underlying mathematics involved in the quantum mechanics that establishes matter in bulk is stable, and, for completeness, elaborate, rigorously as well, on the unusually large volume matter occupies. The latter was clearly emphasized in *words* by Ehrenfest to Pauli in 1931 on the occasion of the Lorentz medal (Ehrenfest, 1595) to this effect: "We take a piece of metal, or a stone. When we think about it, we are astonished that this quantity of matter should occupy so large a volume." He went on by stating that the Pauli exclusion principle is the reason: "Answer: only the Pauli Principle, no two electrons in the same state." On the other hand, if the Pauli exclusion principle is not invoked, it is interesting to quote Dyson (Dyson, 1967) who states: "[such] matter in bulk would collapse into a condensed high-density phase. The assembly of any two macroscopic objects would release energy comparable to that of an atomic bomb. Matter without the exclusion principle is unstable." In the translated version of the book by Tomonaga on spin (Tomonaga, 1997), one reads in the Preface: "The existence of spin, and the statistics associated with it, is the most subtle and ingenious design of Nature - without it the whole universe would collapse."

The drastic difference between matter, with the exclusion principle, and "bosonic matter," i.e., for which the Pauli exclusion principle is not invoked, with Coulomb interactions, is that the ground-state energy for the latter, $E_N \sim -N^{\alpha}$, with $\alpha>1$, where $(N+N)$ denotes the number of the negatively charged particles plus an equal number of positively charged particles. This behavior for "bosonic matter" is unlike that of matter, with the exclusion principle, for which $\alpha=1$ (see Dyson, 1967; Dyson and Lenard, 1967; Lenard and Dyson, 1968; Lieb and Thirring, 1975; Lieb, 1979; Manoukian and Muthaporn 2002; Manoukian and Muthaporn, 2003; Muthaporn and Manoukian, 2004; Manoukian and Sirininlakul, 2004; Manoukian and Muthaporn, 2003; Muthaporn and Manoukion, 2004). A power law behavior with $\alpha>1$, implies instability, as the formation of a single system consisting of $(2N+2N)$ particles is favored over two separate systems brought together each consisting of $(N+N)$ particles, and the energy released upon collapse of the two systems into one, being proportional to $[(2N)^{\alpha}-2(N)^{\alpha}]$, will be overwhelmingly large for realistic large N, e.g., $N\sim10^{23}$. The instability of "bosonic matter" is not a characteristic of the dimensionalityof space (Muthaporn and Manoukian, 2004). We have been particularly interested in recent years on the density limit of matter with (Manoukian and Sirininlakul, 2005) and without (Manoukian *et al.,* 2006) the exclusion principle, and the size of such matter in bulk as more and more matter is put together from the point of view mentioned above by Ehrenfest. For completeness, we thus also elaborate rigorously on the large extent (Manoukian, 2013) of matter and its intimate connection with the exclusion principle. Our findings are summarized in the concluding section, which also pin points the strategy of attack and how the explicit statements of the extension of matter are extracted from the theory. The underlying *technical* details, not just in words, are spelled out and

*Author for correspondence: manoukian_eb@hotmail.com

given right here in the bulk of the paper that lead to our explicit conclusions.

The Hamiltonian of consideration in this work is defined by the well known expression

$$H=\sum_{i=1}^{N}\frac{\mathbf{p}_i^2}{2m}+\sum_{i<j}^{N}\frac{e^2}{|\mathbf{x}_i-\mathbf{x}_j|}-\sum_{i=1}^{N}\sum_{j=1}^{k}\frac{Z_j e^2}{|\mathbf{x}_i-\mathbf{R}_j|}+\sum_{i<j}^{k}\frac{Z_iZ_je^2}{|\mathbf{R}_i-\mathbf{R}_j|}, \quad (1)$$

where $Z_i|e|$ denotes the charge of a j^{th} positively charged particle, $\mathbf{x}_i, \mathbf{R}_j$, corresponding, respectively, to the positions of the negatively and the positively charged particles, and m denotes the mass of the negatively charged particles. We also consider neutral matter, that is, $\sum_{i=1}^{k}Z_j = N$. The solution of the stability problem rests on a basic ine quality derived by the legendary Julian Schwinger (Schwinger, 1961) given in §2, followed by a non-binding theorem attributed to Edward Teller (Teller, 1962 ; Lieb and Thirring, 1975) in §3. The N power law of the ground-state energy of matter and its stability is established in §4. The large volume aspect of matter is elaborated upon next, followed by our conclusions in §5.

2. The Schwinger Bound

Given a Hamiltonian

$$h=\frac{\mathbf{p}^2}{2m}-f(\mathbf{x}), \qquad f(\mathbf{x})\geq 0 \quad (2)$$

in three dimensional space, then the number of eigenvalues less than a parameter $-\xi$, where $\xi>0$, denoted by $N[h,-\xi]$, satisfies the inequality

$$N[h,-\xi]\leq\left(\frac{m}{2\hbar^2}\right)^{3/2}\frac{1}{\pi\sqrt{\xi}}\int d^3\mathbf{x}\, f^2(\mathbf{x}). \quad (3)$$

If we choose

$$-\xi=-\frac{1+\epsilon}{\pi^2}\left(\frac{m}{2\hbar^2}\right)^3\left(\int \mathrm{d}^3\mathbf{x}\, f^2(\mathbf{x})\right)^2, \quad (4)$$

then $N[h,-\xi] < 1$, that is $N[h,-\xi] = 0$, and the spectrum is empty below the value on the right-hand side of (4). Hence the right-hand side of (4) gives the following *lower bound* to the spectrum of h

$$h\geq-\frac{1+\epsilon}{\pi^2}\left(\frac{m}{2\hbar^2}\right)^3\left(\int \mathrm{d}^3\mathbf{x}\, f^2(\mathbf{x})\right)^2, \quad (5)$$

for any $\epsilon>0$.

Another useful formula for obtaining a lower bound to the Hamiltonian in (2) is obtained from (3) by integrating the latter over ξ. This will give us an upper bound to the sum of the negative eigenvalues of the Hamiltonian $h=\left[\mathbf{p}^2/2m-f(\mathbf{x})\right]$ as in (2). To this end, we use the identity

$$N[h_0-f,-\xi;\xi>0]=N[h_0-(f-\frac{\xi}{2});-\frac{\xi}{2};0<\xi\leq 2f(\mathbf{x})],$$
$$h_0=\frac{\mathbf{p}^2}{2m}. \quad (6)$$

That is,

$$\int_0^\infty \mathrm{d}\xi\, N[h_0-f,-\xi]\leq\left(\frac{m}{2\hbar^2}\right)^{3/2}\frac{\sqrt{2}}{\pi}\int \mathrm{d}^3\mathbf{x}\int_0^{2\upsilon(\mathbf{x})}\frac{\mathrm{d}\xi}{\sqrt{\xi}}\left(f(\mathbf{x})-\frac{\xi}{2}\right)^2, \quad (7)$$

which leads to

$$\int_0^\infty \mathrm{d}\xi N[h_0-f,-\xi]\leq\frac{4}{15\pi}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int \mathrm{d}^3\mathbf{x}\left(f(\mathbf{x})\right)^{5/2}, \quad (8)$$

referred to as a Lieb-Thirring bound (Lieb and Thirring, 1975), providing an upper bound for the negative of the sum of the negative eigenvalues (if any), counting degeneracy, of a Hamiltonian h, such as the one in (2). Since the ground-state energy cannot be less than the sum of the negative eigenvalues, this equation provides a lower bound for the ground-state energy with

$$h\geq-\frac{4}{15\pi}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int \mathrm{d}^3\mathbf{x}(f(x))^{(5/2)}. \quad (9)$$

3. A Non-Binding Theorem

We introduce the functional

$$F[\varrho;Z_1,...,Z_k,\mathbf{R}_1,...,\mathbf{R}_k]$$
$$=(3\pi^2)^{5/3}\frac{\hbar^2}{10\pi^2\, m\,\beta}\int \mathrm{d}^3\mathbf{x}\,\varrho^{5/3}(\mathbf{x})-\sum_{j=1}^{k}Z_j\,\mathrm{e}^2\int \mathrm{d}^3\mathbf{x}\frac{\varrho(\mathbf{x})}{|\mathbf{x}-\mathbf{R}_j}$$
$$+\frac{\mathrm{e}^2}{2}\int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{x}'\varrho(\mathbf{x})\frac{1}{|\mathbf{x}-\mathbf{x}'|}\varrho(\mathbf{x}')+\sum_{1\leq i<j\leq k}\frac{Z_i\,Z_j\mathrm{e}^2}{|\mathbf{R}_i-\mathbf{R}_j|},. \quad (10)$$

Here $\varrho(\mathbf{x})$ is an arbitrary positive function, and $\beta>0$ is an arbitrary dimensio nless parameter. Also, $Z_j|\mathrm{e}|$ denotes the charge of a j^{th} positively charged particle and $\mathbf{R}_j$, corresponds to the positions of these positively charged particles - the nuclei. The functional in (10) is minimized for ϱ and taken to be ϱ_0 satisfying the equation

$$(3\pi^2)^{2/3}\frac{\hbar^2}{2m\beta}\,\varrho_0^{2/3}(\mathbf{x};k)=$$
$$\sum_{i=1}^{k}\frac{Z_i\,\mathrm{e}^2}{|\mathbf{x}-\mathbf{R}_i|}-\mathrm{e}^2\int \mathrm{d}^3\mathbf{x}'\frac{1}{|\mathbf{x}-\mathbf{x}'|}\varrho_0(\mathbf{x}';k). \quad (11)$$

as obtained by the functional differentiation of (10) with respect to ϱ, and by setting the result equal to zero as done in Lagrangian mechanics. That is,

$$F[\varrho; Z_1, ..., Z_k, \mathbf{R}_1, ..., \mathbf{R}_k] \geq F[\varrho_0; Z_1, ..., Z_k, \mathbf{R}_1, ..., \mathbf{R}_k]. \quad (12)$$

In particular, let $\varrho^{(i)}_{\mathrm{TF}}$ satisfy the equation

$$(3\pi^2)^{2/3}\frac{\hbar^2}{2m\beta}\left(\varrho^{(i)}_{\mathrm{TF}}(\mathbf{x})\right)^{2/3}(\mathbf{x}) = \frac{Z_i\,\mathrm{e}^2}{|\mathbf{x}-\mathbf{R}_i|} - \mathrm{e}^2\int \mathrm{d}^3\mathbf{x}'\frac{1}{|\mathbf{x}-\mathbf{x}'|}\,\varrho^{(i)}_{\mathrm{TF}}(\mathbf{x}'), \quad (13)$$

where $\varrho^{(i)}_{\mathrm{TF}}$ is the so-called Thomas-Fermi density with $m \to m\beta,\ Z \to Z_i$, and from (12), we have (Lieb and Thirring, 1975; Teller, 1962; Wightman et. al, 1991)

$$F[\varrho_0; Z_1, ..., Z_k, \mathbf{R}_1, ..., \mathbf{R}_k] \geq \sum_{i=1}^{k} F[\varrho^{(i)}_{\mathrm{TF}}; Z_i, \mathbf{R}_i], \quad (14)$$

and finally

$$F[\varrho; Z_1, ..., Z_k, \mathbf{R}_1, ..., \mathbf{R}_k] \geq \beta E_{\mathrm{TF}}(1)\sum_{i=1}^{k} Z_i^{7/3}, \quad (15)$$

where $E_{\mathrm{TF}}(Z)$ is the Thomas-Fermi energy for atoms, and numerically

$$E_{\mathrm{TF}}(1) \simeq -1.5375\left(\frac{m\mathrm{e}^4}{2\hbar^2}\right). \quad (16)$$

This inequality states that a system identified by the parameters $[Z_1, ..., Z_k, \mathbf{R}_1, ..., \mathbf{R}_k]$ cannot have an (optimized) energy functional (10) less than the sum of the (optimized) energy functionals of any two subsystems identified by the parameters $[Z_1, ...Z_\ell, \mathbf{R}_1, ...\mathbf{R}_\ell]$, $[Z_{\ell+1}, ..., Z_k, \mathbf{R}_{\ell+1}, ..., \mathbf{R}_k]$, for $\ell < k$. Due to this, the theorem embodied in the inequality is referred to as a no binding theorem.

4. The N Power Law of the Ground-State Energy of Matter and Stability

In detail (15) reads

$$(3\pi^2)^{5/3}\frac{\hbar^2}{10\pi^2\, m\,\beta}\int \mathrm{d}^3\mathbf{x}\,\varrho^{5/3}(\mathbf{x}) - \sum_{j=1}^{k} Z_j\,\mathrm{e}^2\int \mathrm{d}^3\mathbf{x}\frac{\varrho(\mathbf{x})}{|\mathbf{x}-\mathbf{R}_j|} + \frac{\mathrm{e}^2}{2}\int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{x}'\varrho(\mathbf{x})\frac{1}{|\mathbf{x}-\mathbf{x}'|}\varrho(\mathbf{x}') + \sum_{1\leq i<j\leq k}\frac{Z_i\,Z_j\mathrm{e}^2}{|\mathbf{R}_i-\mathbf{R}_j|} \;\geq\; \beta\,E_{\mathrm{TF}}(1)\sum_{j=1}^{k} Z_j^{7/3}. \quad (17)$$

where we recall that ϱ is an arbitrary positive function. From the above inequality, we may find a lower bound to the (repulsive) potential interaction part between the nuclei to be

$$\sum_{1\leq i<j\leq k}\frac{Z_i\,Z_j\mathrm{e}^2}{|\mathbf{R}_i-\mathbf{R}_j|} \geq \sum_{j=1}^{k} Z_j\,\mathrm{e}^2\int \mathrm{d}^3\mathbf{x}\frac{\varrho(\mathbf{x})}{|\mathbf{x}-\mathbf{R}_j|} - \frac{\mathrm{e}^2}{2}\int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{x}'\varrho(\mathbf{x})\frac{1}{|\mathbf{x}-\mathbf{x}'|}\varrho(\mathbf{x}') - (3\pi^2)^{5/3}\frac{\hbar^2}{10\pi^2\, m\,\beta}\int \mathrm{d}^3\mathbf{x}\,\varrho^{5/3}(\mathbf{x}) + \beta\,E_{\mathrm{TF}}(1)\sum_{j=1}^{k} Z_j^{7/3}. \quad (18)$$

This inequality, in turn, allows us to find a lower bound to the (repulsive) potential interaction part between the electrons, by making the substitutions: $k \to N$, $Z_j \to 1$, $\mathbf{R}_j \to \mathbf{x}_j$, for $j = 1, ..., N$:

$$\sum_{1\leq i<j\leq N}\frac{\mathrm{e}^2}{|\mathbf{x}_i-\mathbf{x}_j|} \geq \sum_{j=1}^{N}\mathrm{e}^2\int \mathrm{d}^3\mathbf{x}\frac{\varrho(\mathbf{x})}{|\mathbf{x}-\mathbf{x}_j|} - \frac{\mathrm{e}^2}{2}\int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{x}'\varrho(\mathbf{x})\frac{1}{|\mathbf{x}-\mathbf{x}'|}\varrho(\mathbf{x}') - (3\pi^2)^{5/3}\frac{\hbar^2}{10\pi^2\, m\,\beta}\int \mathrm{d}^3\mathbf{x}\,\varrho^{5/3}(\mathbf{x}) + \beta\,E_{\mathrm{TF}}(1)\,N. \quad (19)$$

Since ϱ is an arbitrary positive function, we may take it to denote the electron density

$$\varrho(\mathbf{x}) = N\sum_{\sigma_1,\ldots,\sigma_N}\int \mathrm{d}^3\mathbf{x}_2 \ldots \mathrm{d}^3\mathbf{x}_N|\Psi(\mathbf{x}\sigma_1, \mathbf{x}_2\sigma_2, ..., \mathbf{x}_N\sigma_N)|^2, \quad (20)$$

where $\Psi(\mathbf{x}\sigma_1, \mathbf{x}_2\sigma_2, ..., \mathbf{x}_N\sigma_N)$ is a normalized wave function, anti-symmetric under the interchange of any pair $(\mathbf{x}_i\sigma_i) \leftrightarrow (\mathbf{x}_j\sigma_j)$, and the sums are over spins. The total number of particles is obtained by integrating over the number density $\varrho(\mathbf{x})$

$$\int \mathrm{d}^3\mathbf{x}\,\varrho(\mathbf{x}) = N. \quad (21)$$

We also need to derive a lower bound to the expectation value of the kinetic energy:

$$T = \sum_{\sigma_1,\ldots,\sigma_N}\int \mathrm{d}^3\mathbf{x}_1 \ldots \mathrm{d}^3\mathbf{x}_N\Psi^*(\mathbf{x}_1\sigma_1, \mathbf{x}_2\sigma_2, ..., \mathbf{x}_N\sigma_N) \times\Big(\sum_{i=1}^{N}\frac{-\hbar^2\nabla_i^2}{2m}\Big)\Psi(\mathbf{x}_1\sigma_1, \mathbf{x}_2\sigma_2, ..., \mathbf{x}_N\sigma_N). \quad (22)$$

To this end, we use the Schwinger bound in (9) and introduce, in the process, a hypothetical Hamiltonian

$$\sum_{i=1}^{N}\Big(\frac{-\hbar^2\nabla_i^2}{2m} - f(\mathbf{x}_i)\Big), \quad (23)$$

where

$$f(\mathbf{x}) = \frac{5}{3}\frac{\varrho^{2/3}(\mathbf{x})}{\int \mathrm{d}^3\mathbf{x}\,\varrho^{5/3}(\mathbf{x})}T, \qquad T = \langle\Psi|\sum_{j=1}^{N}\frac{\mathbf{p}_j^2}{2m}|\Psi\rangle. \quad (24)$$

It is easily verified that

$$\langle\Psi\,|\sum_{j=1}^{N} f(\mathbf{x}_j)\,|\,\Psi\rangle = \frac{5}{3}T. \quad (25)$$

Allowing multiplicity and spin degeneracy, we can put the N fermions in the lowest energy levels of the hypothetical Hamiltonian $[\sum_{j=1}^{N}\left(\mathbf{p}_j^2/2m - f(\mathbf{x}_j)\right)]$, in conformity with the Pauli exclusion principle, if $N \leq$ the number of such levels. If N is larger than this number of levels, the remaining free fermions may be chosen to have arbitrary small $(\to 0)$ kinetic energies, and be infinitely separated, to define the lowest energy of this Hamiltonian. Hence in all cases, this Hamiltonian is bounded below by 2, for allowing spin orientations, times the sum of the negative energy levels of the Hamiltonian, $(\mathbf{p}^2/2m - f(\mathbf{x};))$, allowing in the sum for multiplicity but not for spin degeneracy. Hence we obtain the bound

$$\langle \Psi | \sum_{j=1}^{N} \Big(\frac{\mathbf{p}_j^2}{2m} - f(\mathbf{x}_j) \Big) | \Psi \rangle \geq -2 \frac{4}{15\pi} \Big(\frac{2m}{\hbar^2} \Big)^{3/2} \int \mathrm{d}^3\mathbf{x} (f(\mathbf{x}))^{5/2}. \tag{26}$$

From (25), (26),

$$-\frac{2}{3} T \geq -2 \frac{4}{15\pi} \Big(\frac{2m}{\hbar^2} \Big)^{3/2} \Big(\frac{5}{3} \Big)^{5/2} T^{5/2} \Big(\int \mathrm{d}^3\mathbf{x}\, \varrho^{5/3}(\mathbf{x}) \Big)^{-3/2}, \tag{27}$$

leading to

$$\frac{3}{5} \Big(\frac{3\pi}{4} \Big)^{2/3} \frac{\hbar^2}{2m} \int \mathrm{d}^3\mathbf{x}\, \varrho^{5/3}(\mathbf{x}) \leq T. \tag{28}$$

Now we have all the ingredients to obtain a lower bound of the Hamiltonian in (1). To this end,

$$\sum_{i=1}^{N} \sum_{j=1}^{k} Z_j \, \mathrm{e}^2 \Big\langle \Psi \Big| \frac{1}{|\mathbf{x}_i - \mathbf{R}_j|} \Big| \Psi \Big\rangle = \sum_{j=1}^{k} Z_j \, \mathrm{e}^2 \int \mathrm{d}^3\mathbf{x} \frac{1}{|\mathbf{x} - \mathbf{R}_j|} \varrho(\mathbf{x}). \tag{29}$$

$$\sum_{i=1}^{N} \mathrm{e}^2 \int \mathrm{d}^3\mathbf{x}\, \varrho(\mathbf{x}) \Big\langle \Psi \Big| \frac{1}{|\mathbf{x} - \mathbf{x}_i|} \Big| \Psi \Big\rangle = \mathrm{e}^2 \int \mathrm{d}^3\mathbf{x}\, \mathrm{d}^3\mathbf{x}' \varrho(\mathbf{x}) \frac{1}{|\mathbf{x} - \mathbf{x}'|} \varrho(\mathbf{x}'), \tag{30}$$

and hence from (19)

$$\sum_{1 \leq i < j \leq N} \mathrm{e}^2 \Big\langle \Psi \Big| \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|} \Big| \Psi \Big\rangle \geq \frac{\mathrm{e}^2}{2} \int \mathrm{d}^3\mathbf{x}\, \mathrm{d}^3\mathbf{x}' \varrho(\mathbf{x}) \frac{1}{|\mathbf{x} - \mathbf{x}'|} \varrho(\mathbf{x}') - (3\pi^2)^{5/3} \frac{\hbar^2}{10\pi^2 m \beta} \int \mathrm{d}^3\mathbf{x}\, \varrho^{5/3}(\mathbf{x}) + \beta N E_{\mathrm{TF}}(1). \tag{31}$$

Moreover from (28) - (31), (1), we have

$$\langle \Psi | H | \Psi \rangle \geq (3\pi^2)^{5/3} \frac{\hbar^2}{10\pi^2 m \beta'} \int \mathrm{d}^3\mathbf{x}\, \varrho^{5/3}(\mathbf{x}) - \sum_{j=1}^{k} Z_j \, \mathrm{e}^2 \int \mathrm{d}^3\mathbf{x} \frac{1}{|\mathbf{x} - \mathbf{R}_j|} \varrho(\mathbf{x}) + \frac{\mathrm{e}^2}{2} \int \mathrm{d}^3\mathbf{x}\, \mathrm{d}^3\mathbf{x}' \varrho(\mathbf{x}) \frac{1}{|\mathbf{x} - \mathbf{x}'|} \varrho(\mathbf{x}') + \sum_{1 \leq i < j \leq k} \frac{Z_i Z_j \mathrm{e}^2}{|\mathbf{R}_i - \mathbf{R}_j|} + \beta N E_{\mathrm{TF}}(1), \tag{32}$$

where

$$\frac{1}{\beta'} = \frac{\frac{3}{5} \big(\frac{3\pi}{4} \big)^{2/3} - \frac{(3\pi^2)^{5/3}}{5\pi^2} \frac{1}{\beta}}{\frac{(3\pi^2)^{5/3}}{5\pi^2}} = \Big(\frac{1}{4\pi} \Big)^{2/3} - \frac{1}{\beta}. \tag{33}$$

For a positive β', we must choose $\beta > (4\pi)^{2/3}$. The sum of the first four terms on the right-hand side of the inequality in (32) coincides with the expression on the left-hand side of the inequality (17) with β in the latter simply replaced by β'. Hence

$$\langle \Psi | H | \Psi \rangle \geq \beta' E_{\mathrm{TF}}(1) \sum_{j=1}^{k} Z_j^{7/3} + \beta N E_{\mathrm{TF}}(1). \tag{34}$$

Optimizing over β leads to the Lieb-Thirring bound (Lieb and Thirring, 1975)

$$\langle \Psi | H | \Psi \rangle \geq E_{\mathrm{TF}}(1) (4\pi)^{2/3} N \Big[1 + \Big(\sum_{j=1}^{k} \frac{Z_j^{7/3}}{N} \Big)^{1/2} \Big]^2, \tag{35}$$

where E_{TF} is given in (16). Setting $Z = \max_j Z_j$, we obtain

$$\langle \Psi | H | \Psi \rangle \geq -8.3104 \Big(\frac{m\mathrm{e}^4}{2\hbar^2} \Big) N \big[1 + Z^{2/3} \big]^2. \tag{36}$$

The numerical value 8.3104 may be further reduced (Hundertmark, 2000; Dolbeault *et al*., 2008), but this will not be important in the subsequent analysis (see also (Federbush, 1975; Graf, 1997). The left-hand side of the inequality in (36) provides a lower bound to the spectrum.

An upper bound to the ground-energy is also readily derived by noting that any trial wave function *cannot* give a lower bound to the ground-state energy, otherwise this would contradict the very definition of the ground-state energy. A trial wave function may be chosen to obtain (Manoukian, 2013; p.779) the following upper bound to the ground-state energy E_N:

$$E_N \leq -0.0450 \Big(\frac{m\mathrm{e}^4}{2\hbar^2} \Big) N, \tag{37}$$

thus establishing the N power law behavior of the ground-state energy $E_N \sim -N$, with a *finite* (negative) numerical coefficient, as discussed in the Introduction.

Note that the negative spectrum of the Hamiltonian in (1) is not empty for matter. Envisage a situation where we have infinitely separated N clusters: k hydrogenic atoms in their ground states, of nuclear charges $Z_1|\mathrm{e}|, \ldots, Z_k|\mathrm{e}|$, each having one negatively charged particle, and there are also $(N-k)$ free negatively charged particles with vanishingly small kinetic energies. The ground-state of such a system is $-\sum_{i=1}^{k} Z_i^2 m\mathrm{e}^4/2\hbar^2$. Let $|\varphi(m)\rangle$ denote a normalized strictly negative energy state of matter. That is,

$$-\varepsilon_N[m] \leq \langle \varphi(m) | H | \varphi(m) \rangle < 0, \tag{38}$$

where $-\varepsilon_N[m] = E_N < 0$ denotes the lower end of the spectrum, and we have emphasized its dependence on the mass m. By definition of the ground-state, the state $|\varphi(m/2)\rangle$ cannot lead for $\langle \varphi(m/2) | H | \varphi(m/2) \rangle$ to a numerical value lower than $-\varepsilon_N[m]$ for the same Hamiltonian with mass m. That is,

$$-\varepsilon_N[m] \leq \langle \varphi(m/2) | H | \varphi(m/2) \rangle, \tag{39}$$

where we note that the interaction part V in the Hamiltonian in (1) is independent of the mass scale m. Accordingly, we may rewrite the above equation in detail as

$$-\varepsilon_N[m]\leq\langle\varphi(m/2)|\Big[\sum_{i=1}^{N}\frac{\mathbf{p}_i^2}{2m}+V\Big]|\varphi(m/2)\rangle. \tag{40}$$

This equation, in turn implies that for $m\to 2m$,

$$-\varepsilon_N[2m]\leq\langle\varphi(m)|\Big[\sum_{i=1}^{N}\frac{\mathbf{p}_i^2}{4m}+V\Big]|\varphi(m)\rangle. \tag{41}$$

Upon simplifying

$$\sum_{i=1}^{N}\frac{\mathbf{p}_i^2}{2m}+V=\Big[\sum_{i=1}^{N}\frac{\mathbf{p}_i^2}{4m}+V\Big]+\sum_{i=1}^{N}\frac{\mathbf{p}_i^2}{4m}\,. \tag{42}$$

Eqs. (40), (41) imply that

$$\langle\varphi(m)|\sum_{i=1}^{N}\frac{\mathbf{p}_i^2}{2m}|\varphi(m)\rangle\leq 2\varepsilon_N[2m], \tag{43}$$

for all states $|\varphi(m)\rangle$ for which (38) is true.

From the bounds to the spectra in (36), together with the lower bound of the expectation value of the kinetic energy part in (28), we then have the following bounds

$$\frac{3}{5}\Big(\frac{3\pi}{4}\Big)^{2/3}\frac{\hbar^2}{2m}\int \mathrm{d}^3\mathbf{x}\,\varrho^{5/3}(\mathbf{x})\;<\;\Big\langle\Psi\Big|\sum_{i=1}^{N}\frac{\mathbf{p}_i^2}{2m}\Big|\Psi\Big\rangle\;<\;16.63\Big(\frac{me^4}{\hbar^2}\Big)N\big[1+Z^{2/3}\big]^2. \tag{44}$$

This, in turn, gives the following key bound for the integral of some power of the particle *density* $\varrho(\mathbf{x})$:

$$\int \mathrm{d}^3\mathbf{x}\,\varrho^{5/3}(\mathbf{x})<32\,\frac{m^2\mathrm{e}^4}{\hbar^4}N\,[1+Z^{2/3}]^2. \tag{45}$$

Now let $\mathbf{x}$ denote the position of an electron relative, for example, to the center of the mass of the nuclei, recalling that the Pauli exclusion was invoked in deriving the bound of the power of the electron number-density in (45). Let

$$\chi_R(\mathbf{x})=1,\text{ if }\ \mathbf{x}\ \text{ lies within a sphere of radius }\ R,\ \text{ and }\ =0,\ \text{ otherwise.} \tag{46}$$

Then, clearly, for the probability to have the electrons within a sphere of radius R, we have

$$\begin{aligned}\mathrm{Prob}\big[\,|\mathbf{x}_1|\leq R,...,|\mathbf{x}_N|\leq R\,\big]&\leq \mathrm{Prob}\big[\,|\mathbf{x}_1|\leq R\,\big]\\ &=\frac{1}{N}\int \mathrm{d}^3\mathbf{x}\,\chi_R(\mathbf{x})\,\varrho(\mathbf{x})\\ &\leq\frac{1}{N}\Big[\int \mathrm{d}^3\mathbf{x}\,\varrho^{5/3}(\mathbf{x})\Big]^{3/5}(v_R)^{2/5},\end{aligned} \tag{47}$$

where in the last inequality, we use Hölder's inequality, the fact that $\chi_R(\mathbf{x})^{2/5}=\chi_R(\mathbf{x})$, and where $v_R=4\pi R^3/3$.

From (45) and (47), we have the fundamental inequality (Manoukian and Sirininlakul, 2005)

$$\mathrm{Prob}\big[|\mathbf{x}_1|\leq R,...,|\mathbf{x}_N|\leq R\big]\Big(\frac{N}{v_R}\Big)^{2/5}<8\Big(\frac{1}{a_0^3}\Big)^{2/5}[1+Z^{2/3}]^{6/5}, \tag{48}$$

where $a_0=\hbar^2/me^2$ is the Bohr radius. One may infer from this equation the *inescapable* fact that *necessarily* for a non-vanishing probability of having the electrons within a sphere of radius R, the corresponding volume v_R grows not any slower than the first power of N for $N\to\infty$, *since otherwise the left-hand side of the inequality would go to infinity and would be in contradiction with the finite upper bound on the its right-hand side*. That is, *necessarily*, the radius R grows not any slower than $N^{1/3}$ for $N\to\infty$. No wonder matter occupies such a large volume!

5. Summary

We summarized the key results in the above analyses. The Pauli exclusion is not only sufficient for establishing that matter in the quantum setting is stable but is also necessary. This is precisely the condition that gives an N power law behavior of the ground-state energy, otherwise by revoking the exclusion principle, the ground-state energy would lead to power law behavior, N^α, $\alpha>1$, (Lieb, 1979; Manonkian and Muthaporn, 2002; Maroukian and Muthaporn, 2003a; Muthaporn and Manoukian, 2004; Maroukian and Siriniolakul, 2004; Monoukian and Muthaporn, 2003b; Manoukian, 2013), implying instability as discussed in the Introduction. The fact that the electron density, as obtained from (44) satisfies the bound

$$\int \mathrm{d}^3\mathbf{x}\,\varrho^{5/3}(\mathbf{x})<32\frac{m^2\mathrm{e}^2}{\hbar^4}N[1+Z^{2/3}]^2, \tag{49}$$

with an upper bound with a single power of N, which allows us to infer that matter with an extension radius R grows not any slower than $N^{1/3}$ for $N\to\infty$ as established in (48). "Bosonic matter" behaves completely differently and we refer the reader to the investigation in (Manoukian *et al*., 2006) for the relevant details involving its collapsing stage.

Acknowledgments
The authors would like to thank their colleagues at the Institute for their enthusiasm and the keen interest they have shown in this presentation. One of the authors (EBM) would also like to thank his colleagues C. Muthaporn and S. Sirininlakul for earlier discussions and collaborations.

References

Dolbeault, J., Laptev, A. and Loss, M. E. 2008. Lieb-Thirring inequalities with improved constants. Journal of the European Mathematical Society10, 1121-1126.

Dyson, F. J. 1967. Ground-state energy of a finite system of charged particles. Journal of Mathematical Physics. 8, 1538-1545.

Dyson, F. J. and Lenard, A. 1967. Stability of matter I. Journal of Mathematical Physics 8, 423-434.

Ehrenfest, P., Ansprache zur Verleihung der Lorentz medaille an ProfessorWolfgang Pauli am 31 Oktober 1931. (Address on award of Lorentz medal to Professor Wolfgang Pauli on 31 October 1931). In Klein, M. J. (Editor), Paul Ehrenfest : Collected scientific papers, North-Holland, Amsterdam (1959) p. 617. [The address appeared originally in P. Ehrenfest, Versl. Akad. Amsterdam 40 (1931) 121.

Federbush, P. 1975. A new approach to the stability of matter. Journal of Mathematical Physics 16, 347-351.

Graf, G. M. 1997. Stability of matter through an electrostatic inequality. Helvetica Physica Acta70, 72-79.

Hundertmark, D., Laptev, A. and Weidl, T. 2000. New bounds on the Lieb-Thirring constants. Inventiones Mathematicae 140, 693-704.

Lenard, A. and Dyson, F. J. 1968. Stability of matter II. Journal of Mathematical Physics 9, 698-709.

Lieb E. H. and Thirring W. E. 1975. Physical Review Letters 35, 687-689; [35 (1975) 1116(E); Thirring, W. E. (Ed.). The Stability of Matter: From Atoms to Stars, Selecta of E. H. Lieb (Springer, Heidelberg, 2005).

Lieb, E. H. 1979. The $N^{5/3}$ law for bosons. Physics Letters 70A, 71-73.

Manoukian, E. B. 2013.Why matter occupies so large a volume?. Communications in Theoretical Physics 60, 677-686.

Manoukian, E. B. and Muthaporn, C. 2002. The collapse of "bosonic matter". Progress of Theoretical Physics 107, 927-939.

Manoukian, E. B., Muthaporn, C. and Sirininlakul, S. 2006. Collapsing stage of "bosonic matter". Physics Letters 352A, 488-490.

Manoukian, E. B. and Muthaporn, C. 2003a. $N^{5/5}$ law for bosons for arbitrary N. Progress of Theoretical Physics110, 385-391.

Manoukian, E. B. and Muthaporn, C. 2003b. Is "bosonic matter" unstable in 2D?. Journal of Physics A: Mathematical & General 36, 653-663.

Manoukian, E. B. and Sirininlakul, S. 2005. High-density limit and inflation of matter. Physical Review Letters 95(190402), 1-3.

Manoukian, E. B. and Sirininlakul, S. 2004. Rigorous lower bounds for the ground state energy of matter. Physics Letters 332A, 54-59; [337A, (2004) 496(E).

Muthaporn, C. and Manoukian, E. B. 2004a. N^2 law for bosons in 2D. Reports on Mathematical Physics 53, 415-424.

Muthaporn, C. and Manoukian, E. B. 2004b. Instability of "bosonic matter" in all dimensions. Physics Letters 321A, 152-154.

Schwinger, J. 1961. On the bound states of a given potential. Proceedings of the National Academy of Sciences U.S.A. 47, 122-129.

Teller, E. 1962.On the stability of molecules in the Thomas-Fermi theory. Reviews of Modern Physics 34, 627-631.

Tomonaga, S.-T. (Translator T. Oka). 1997. The Story of Spin. University of Chicago Press, Chicago. Preface.

Wightman, A. S. et al. 1991. Studies in Mathematical Physics. Princeton University Press, Princeton, New Jersey, PP. 269-303.

Editor-in-Chief's Message

Dear Readers,

Welcome to the first issue of the second volume of Research and Knowledge. Entering the second year of publication of the journal has been a rewarding experience for all the staff involved. We have had the pleasure of reading and selecting some excellent manuscripts to publish and have worked with many inspiring authors. In addition, the staff has been working hard behind the scenes to ensure that the journal has the best IT to help it to grow and develop in the future.

While considering the growth of Research and Knowledge it should be noted that we are up to eleven papers in this issue. The papers cover an interesting range of topics, from which there should be something for nearly everyone.

The issue starts with a physics paper about 'Quantum theory of matter in bulk: modern treatment' which is complemented by a manuscript related to superconductivity. There is then engineering work presented in papers about sawdust briquettes and the effect of materials used to make pellets. Then the medical field is covered with a manuscript that considers the use of pulse waves in a model of an aneurysmal aorta.

There are also a range of manuscripts about biology that range from a study of the genetics of weedy rice to using medaka as a model animal. While there is a zoology paper about a new species of Cipangopaludina.

As usual the staff of Research and Knowledge hope that you enjoy reading this issue and that you will consider us or recommend us when you or your colleagues have work to publish in the future.

Regards

Professor Wichian Magtoon, Ph.D.
Dean, Faculty of Science, Mahasarakham University
Editor-in-Chief, Research and Knowledge

CONTENTS

Instructions for Authors

Aim & Scope

Research & Knowledge is a peer-reviewed international journal with open access that is published by the Research Center of Mahasarakham University. The aim of the journal is to provide a platform for researchers, academics, professionals, practitioners and students to publish and share knowledge in the form of high quality original research and review papers.

The journal publishes in both hardcopy and online versions. It publishes papers in the fields of science and technology such as biology, biotechnology, botany, chemistry, ecology, engineering, environmental science, ichthyology, geology, genetics, mathematics, microbiology, molecular biology, organic chemistry, paleontology, physics, plants and animal science, statistics, axonomy and zoology. All manuscripts can be submitted through online submission or email. For further information, please visit: https://rk.msu.ac.th.

Manuscript Preparation and Submission

The manuscript should be clear and concise. It should use the same font throughout of at least 12 point, double-line spacing with margins of at least 2 cm all round and with page and line numbering. Manuscripts must be in either British or American English, but not a mixture of both, and all measures should be reported in SI units. The manuscript must be saved as a DOC (not DOCX) or RTF file before submission.

1. Cover Letter

A cover letter should be included that states a category for the manuscript being submitted and confirms that it is not being submitted for publication elsewhere. The corresponding author must provide complete contact details (with email address) and confirm that all authors have agreed with the submission. If there is any conflict of interest or any third-party copyrighted material, it must be mentioned in the cover letter.

2. Manuscript and Accompanying Files

Title

The title should be short and simple so that it is easy for the readers to understand. Titles that are too long will not be remembered by the readers. The title should be representative of the whole paper and be an accurate reflection of the contents.

List of Authors

Full names and affiliations of all authors that contributed a significant input into producing the experimental results reported as well as the writing of the paper should be included. All authors who are listed will be required to take public responsibility for the work presented. One of the listed authors should be designated the corresponding author, and it is their responsibility to manage the publication process on behalf of the other authors. The full postal address, phone number and email address of the corresponding author must be included.

Abstract

The abstract must be no longer than 250 words and should clearly present the study's aims, methods, main findings and significant conclusions. It should be written as a single paragraph that has a focus on the novel aspects of the work presented. It should be able to standalone without reference to the rest of the paper and should not contain any citations. Minimize the use of non-standard abbreviations.

Keywords

Following the abstract, 3 to 5 keywords or phrases should be included. These should represent of the paper as they will be used for indexing. (Authors are reminded that any terms that do not appear in the title, abstract or keywords will not be used with reliability by search engines to find the article.)

Manuscript Text

The main body of the text should be divided into sections such as **1. Introduction, 2. Materials and Methods**, **3. Results**, **4. Discussions and Conclusions, Acknowledgements** and **References**. Long articles may need subheadings within some sections (especially the Results and Discussion sections) to clarify their content. All heading must follow:

- Main heading: begins with a number (except for Acknowledgements and References), has the first letter of each significant word capitalized and is in bold type, such as **1. Introduction**, **2. Materials and Methods** and **References**.
- Subheading: in bold type with numbering as ".1", ".2", etc., such as **"4.1 Sedimentation rate and sea-level fluctuation".**
- Sub-subheading: typed in italics and numbered ".1", ".2", etc. such as "*4.1.1 Sedimentation rate in the peritidal environment*".

Results

All data and results should be presented in a clear and logical order. The tables and the figures should each be numbered sequentially in the order that they are first referred to in the text. In the text do not restate all the information from the tables and figures, just highlight the most important observations.

Discussions and Conclusions

This should focus on presenting the new and important aspects of the work. The conclusions should be presented clearly at the end of this section. This section should only contain new information related to the interpretation of the results and not be a restating of the information form the Introduction or Results sections. Possible areas for future related work should be considered in this section.

Acknowledgements

This section should include the names of anyone who contributed to the work presented but does not qualify to be an author. In addition, material and financial (with grant number) support should be identified.

Tables

Tables should be included in the text of the manuscript where they are first cited. They should be numbered in the order that they appear in the text, and each table should have a legend that gives a brief title. Each column should have a heading. Do not include tables that are not cited in the text. If any data is used that has been published previously it is the Author's responsibility to get permission to use it and fully acknowledge the source.

Figures

All figures must be saved as TIFF or EPS files (with resolution at least 300 dpi for color or gray scale, more than 1,000 dpi for line drawings and more than 600 dpi for combination figures). Figures should be professionally drawn, photographed or digitized. Letters, numbers and symbols should be clear and even throughout and of sufficient size. Shading and hatches should be used with care and consideration of the final size of the image being made. All figures should be numbered consecutively according to the order in which they have been first cited in the main text. (Do not include figures that are not cited in the text.)
If any data is used that has been published previously it is the Author's responsibility to get permission to use it and fully acknowledge the source. Data presented in graphs should be in an appropriate format and error bars should be included. All lines and data points should be of a suitable size so that they will be easily identifiable in the final version. In histograms, the use of pattern or gradient fills should be avoided. Do not include figure numbers nor captions of the figure within the figure.

Captions

The figure captions must be listed at the end of the main text such as "Figure 1. Photograph of the study locality in Saraburi, Thailand"or "Figure 2. (a) Photomicrograph of microfossil observed in the thin-section. (b) Magnification of the microfossil in (a)".

References

Should be arranged alphabetically with 'and' before the last author. Authors' names should have the first letter capitalized. All words should be listed in full and begin with an initial capital for the first word, with all other words that are not proper nouns being in lower case.

Journal

Kofukuda, D., Isozaki, Y. and Igo, H. 2014. A remarkable sea-level drop and relevant biotic responses across the Guadalupian-Lopingian (Permian) boundary in lowlatitude mid-Panthalassa: Irreversible changes recorded in accreted paleo-atoll limestones in Akasaka and Ishiyama, Japan. Journal of Asian Earth Science 82, 47–65.

Book

Flügel, E. 2004. Microfacies of carbonate rocks: Analysis, interpretation and application. Springer-Verlag, Berlin, pp. 976.

Aigner, T. 1982. Calcareous tempestites, storm-dominated stratification in Upper Muschelkalk Limestones (Middle Triassic, SW-Germany). In: Einsele, G. and Seilacher, A. (Eds.), Cyclic and Event Stratification. Springer-Verlag, Berlin, pp. 180–198.

PhD thesis

Thambunya, S. 2005. Lithofacies and diagenesis of the Khao Khad Formation in the vicinity of Changwat Saraburi, Central Thailand. Ph.D. dissertation, Department of Geology, Chulalongkorn University, Thailand.