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Rarita–Schwinger massless field in covariant and Coulomb-like gauges

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The massless Rarita–Schwinger field propagator used in perturbative interacting field theories involving this field is derived in covariant and the Coulomb-like gauges for the first time by explicitly invoking the gauge constraints as it was for a massless vector field over the years.

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1. Introduction

There has been much interest in the Rarita–Schwinger field[23,24] in recent years, e.g. Refs. 1–22. Apart from the obvious role that a Rarita–Schwinger field plays in supergravity, e.g. Ref. 25, as the superpartner of the graviton, it has been important in describing spin-$\frac{3}{2}$ baryon resonances, e.g. Refs. 3, 17 and 19. Spin-$\frac{3}{2}$ particles may also play an important role in cancelation of gauge anomalies[21] in self consistent constructions of grand unified theories. Recent investigations[6,13–16] suggest that spin-$\frac{3}{2}$ may have an important role in the evolution of the universe. Some solutions to the wave equations of Rarita–Schwinger particles have been obtained, e.g. Refs. 1, 3 and 26, and an investigation[18] of the so-called heat kernel for the Rarita–Schwinger field has been carried out. Here, we recall that the heat kernel and its Schwinger-DeWitt expansion[27–29] have been useful in analyzing loop divergences in effective actions.[28,29] Interesting work has been also done in a study[30] of "Loop Quantum Supergravity". Investigations of consistency problems in the theoretical descriptions of massive spin-$\frac{3}{2}$ field theories go back to studies in the classic Refs. 31 and 32, see also Refs. 5, 9–12, 21, 22 and 33. In the light of modern field theory, massive Rarita–Schwinger fields may be expected to be generated, for example, by a Higgs mechanism.

The above interesting considerations invite us to consider further the role of the Rarita–Schwinger field in quantum field theory. The purpose of this communication is to develop the corresponding essential analysis for the massless Rarita–Schwinger field in covariant and the Coulomb-like gauges to be used in perturbative interacting theories involving this field as it was for a massless vector field over the years. We do this by *explicitly* invoking the gauge conditions in deriving field equations and the underlying propagator in such general gauges. Three points must be emphasized at the outset in any analysis of this type:

(i) A quantum theory may be defined only after gauge fixing.

(ii) If, say, $\bar{\psi}^\mu_a$, denotes the (adjoint of the) Rarita–Schwinger field, where μ denotes a Lorentz index, while a denotes a spinor index, then the variation of the action, with respect to $\bar{\psi}^\mu_a$, for example, will have the general form:

$$\delta\mathcal{A} = \int (\mathrm{d}x)\delta\bar{\psi}^\mu_a(x)G_{a\mu}(x)\,, \qquad (\mathrm{d}x) = \mathrm{d}x^0\,\mathrm{d}x^1\,\mathrm{d}x^2\,\mathrm{d}x^3 \tag{1}$$

and due to a gauge constraint imposed on the fields, *not all the components* of the Rarita–Schwinger field may be varied independently.[28,34] That is, one may not conclude, in general, that $G_{a\mu}(x) = 0$, a point that is not sufficiently emphasized in the literature.

(iii) In order to derive the explicit expression of the propagator, in the above arbitrary gauges, one may couple the field ψ^μ_a, to an external source $\bar{K}_{a\mu}$, and similarly for the adjoint of the field, and one, *a priori*, *cannot* impose any constraints on the source $\bar{K}_{a\mu}(x)$ since the need arises to vary its components *independently* to generate arbitrary field interactions as well as of generating Green functions. As we will see, this gives rise to additional terms in the propagator which, in general, may not be neglected.

2. Covariant Gauges

We consider the Lagrangian density of the Rarita-Schwinger field in the form given by Schwinger (see Ref. 35, p. 191):

$$\mathcal{L} = -\frac{1}{2\,\mathrm{i}}\bar{\psi}_\mu[\gamma\cdot\overleftrightarrow{\partial}\eta^{\mu\nu} - (\gamma^\mu\overleftrightarrow{\partial}^\nu + \gamma^\nu\overleftrightarrow{\partial}^\mu) + \gamma^\mu(-\gamma\cdot\overleftrightarrow{\partial})\gamma^\nu]\psi_\nu\,, \tag{2}$$

$\overleftrightarrow{\partial}^\mu = \overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu$, suppressing the spinor indices, where, with our convention,

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}\,, \qquad [\eta^{\mu\nu}] = \mathrm{diag}[-1, 1, 1, 1] \tag{3}$$

and hence, in particular $\gamma_\mu\gamma^\mu = -4$.

With the action defined by $\mathcal{A} = \int(\mathrm{d}x)\,\mathcal{L}(x)$, a variation of $\bar{\psi}_\mu(x)$, gives

$$\delta\mathcal{A} = \int(\mathrm{d}x)\delta\bar{\psi}_\mu(x)G^\mu(x)\,, \tag{4}$$

$$G^\mu(x) = -\left[\eta^{\mu\nu}\frac{\gamma\cdot\partial}{\mathrm{i}} - \frac{\gamma^\mu\partial^\nu}{\mathrm{i}} - \frac{\gamma^\nu\partial^\mu}{\mathrm{i}} - \gamma^\mu\frac{\gamma\cdot\partial}{\mathrm{i}}\gamma^\nu\right]\psi_\nu(x)\,, \qquad \partial^\mu G^\mu(x) = 0\,. \tag{5}$$

The second equality in (5) implies that the action is invariant under gauge transformations $\delta\bar{\psi}_\mu(x) = \partial_\mu\bar{\Lambda}(x)$, and similarly for $\psi_\mu(x)$, where Λ is an arbitrary spinor.

We consider arbitrary covariant gauges defined by

$$\partial^\mu\psi_\mu(x) = \mathrm{i}\lambda\gamma\cdot\partial\chi(x)\,, \tag{6}$$

where λ is an arbitrary parameter and $\chi(x)$ is a spinor field. In order to be able to vary the components of $\bar{\psi}_\mu$ independently, the gauge constraint in (6) must be derived from a modified action integral. To this end, we modify the Lagrangian density as follows:

$$\mathcal{L}(x) \to \mathcal{L}(x) - \partial_\mu\bar{\psi}^\mu(x)\chi(x) - \bar{\chi}(x)\partial_\mu\psi^\mu(x) - \lambda\bar{\chi}\frac{\gamma\cdot\partial}{\mathrm{i}}\chi$$
$$+ \bar{K}^\mu(x)\psi_\mu(x) + \bar{\psi}_\mu(x)K^\mu(x)\,, \tag{7}$$

where in order to derive the explicit expression for the propagator we have introduced a coupling to an external source $K^\mu(x)$ with, *a priori*, no constraints imposed on it. Now, we may vary all the field components $\bar{\psi}^\mu(x)$, $\bar{\chi}(x)$, for example, in the Lagrangian density on the right-hand side of (7), independently leading, respectively, to

$$[\eta^{\mu\nu}\gamma\cdot p - \gamma^\mu p^\nu - \gamma^\nu p^\mu - \gamma^\mu\gamma\cdot p\gamma^\nu]\psi_\nu = K^\mu + \mathrm{i}p^\mu\chi\,, \tag{8}$$

$$p^\nu\psi_\nu = \mathrm{i}\lambda\gamma\cdot p\chi\,, \tag{9}$$

where $p^\mu = \partial^\mu/\mathrm{i}$, and we recognize (9) as the gauge constraint (6) which is now derived. Upon comparing the equation obtained by multiplying (8) by p_μ with the one obtained by multiplying (8) by γ_μ, and by eliminating in the process of the spinor field χ, which now satisfies the equation

$$p^2\chi = \mathrm{i}p\cdot K\,, \tag{10}$$

a tedious analysis leads to the following equations:

$$p^\nu\psi_\nu = -\lambda\frac{\gamma\cdot pp\cdot K}{p^2}\,, \tag{11}$$

$$\gamma^\nu\psi_\nu = \left(\lambda - \frac{1}{2}\right)\frac{p\cdot K}{p^2} - \frac{\gamma\cdot p\gamma\cdot K}{2p^2}\,. \tag{12}$$

From (10)–(12), we may rewrite (8) as

$$\gamma\cdot p\psi^\mu = \left[\eta^{\mu\nu} + \left(\lambda - \frac{3}{2}\right)\frac{p^\mu p^\nu}{p^2} - \frac{p^\mu\gamma\cdot p\gamma^\nu}{2p^2} - \frac{p^\nu\gamma\cdot p\gamma^\mu}{2p^2} + \frac{\gamma^\mu\gamma^\nu}{2}\right]K_\nu\,. \tag{13}$$

Multiplying the above equation by $(-\gamma\cdot p)$ gives

$$p^2\psi^\mu = \left[\left(\eta^{\mu\nu} + \left(\lambda - \frac{1}{2}\right)\frac{p^\mu p^\nu}{p^2}\right)(-\gamma\cdot p) + \frac{1}{2}(p^\mu\gamma^\nu - p^\nu\gamma^\mu) + \frac{\gamma^\mu\gamma\cdot p\gamma^\nu}{2}\right]K_\nu\,. \tag{14}$$

Elimination of an auxiliary field in this direct formalism is equivalent to integrating over it in the path-integral one. This should give the same result for the propagator as ours, but the underlying (continual) integrations are unnecessarily more tedious. An independent path-integral derivation by the reader would be welcome.

From the above equation, the following explicit expression for the propagator emerges:

$$D_+^{\mu\nu}(x-x') = \int \frac{(\mathrm{d}k)}{(2\pi)^4} D_+^{\mu\nu}(k)\, \mathrm{e}^{\mathrm{i}k(x-x')}\,, \tag{15}$$

where

$$D_+^{\mu\nu}(k) = \left[\left(\eta^{\mu\nu} + \left(\lambda - \frac{1}{2}\right)\frac{k^\mu k^\nu}{k^2}\right)(-\gamma\cdot k) + \frac{1}{2}(k^\mu\gamma^\nu - k^\nu\gamma^\mu) + \frac{\gamma^\mu\gamma\cdot k\gamma^\nu}{2}\right]\frac{1}{(k^2-\mathrm{i}\epsilon)}\,. \tag{16}$$

Taking the vacuum expectation value $\langle 0_+|\cdot|0_-\rangle$ of (14), and using the fact that no constraint was imposed on K_μ, to write $(-\mathrm{i})\delta\langle 0_+|0_-\rangle/\delta\bar{K}_\mu(x) = \langle 0_+|\psi^\mu(x)|0_-\rangle$, and a functional integration with respect to $K_\mu(x)$, gives the expression

$$\langle 0_+|0_-\rangle = \exp\left[\mathrm{i}\int(\mathrm{d}x)(\mathrm{d}x')\,\bar{K}_\mu(x)\,D_+^{\mu\nu}(x-x')K_\nu(x')\right]. \tag{17}$$

Only if the conservation law $\partial^\mu K_\mu(x) = 0$ is now imposed, one may effectively obtain the following expression for the propagator *in* (17) in the momentum description

$$D_+^{\mu\nu}(k) \to \left[-\gamma\cdot k\eta^{\mu\nu} + \frac{\gamma^\mu\gamma\cdot k\gamma^\nu}{2}\right]\frac{1}{(k^2-\mathrm{i}\epsilon)}\,, \tag{18}$$

which is independent of the gauge parameter and describes a pure (massless) spin-$\frac{3}{2}$ particle. For a careful treatment of the latter for a *conserved* source $\partial^\mu K_\mu(x) = 0$ (see Ref. 35, pp. 129 and 130), as well of the derivation of the propagator in (18). Equation (17) as a generating functional for generating interactions in a dynamical theory, however, in a perturbative setting, *independent* variations with respect to the components of the source K_μ (and of its adjoint) are necessary to generate Green functions and describe dynamics, in general, and hence no constraints may *a priori* be imposed on it in such general cases. In dealing with the latter general *extensions* as just mentioned, if a constraint, *a priori*, is set on $K_\mu(x)$, one may not be able to carry independent variations of its components, as variation of one component might imply variation of another of its components.

In short, the Rarita-Schwinger propagator to be used in perturbative *dynamical* interacting theories, involving this field, in covariant gauges, is given by the expression in (16) and *not* by the one on the right-hand side of (18) as one may naïvely expect.

3. Coulomb-Like Gauge

The Coulomb-like gauge reads

$$\partial_i \psi_a^i(x) = 0 \tag{19}$$

with a sum over $i = 1, 2, 3$. Again due to this constraint, the field components $\psi_a^i(x)$ may not be varied independently. With the aim of describing a spin-$\frac{3}{2}$ particle, we may, however, express the above field in terms of fields $U_b^i(x)$, $\rho_b(x)$ as follows:

$$\psi_a^i(x) = \pi_{ab}^{ij} U_b^j(x) - \frac{1}{2}\left(\gamma^i - \frac{\boldsymbol{\gamma}\cdot\boldsymbol{\partial}\partial^i}{\boldsymbol{\partial}^2}\right)_{ab} \rho_b(x)\,, \tag{20}$$

where

$$\pi^{ij} = \left(\delta^{ij} - \frac{\partial^i\partial^j}{\boldsymbol{\partial}^2}\right) + \frac{1}{2}\left(\gamma^i - \frac{\boldsymbol{\gamma}\cdot\boldsymbol{\partial}\partial^i}{\boldsymbol{\partial}^2}\right)\left(\gamma^j - \frac{\boldsymbol{\gamma}\cdot\boldsymbol{\partial}\partial^j}{\boldsymbol{\partial}^2}\right), \tag{21}$$

satisfying

$$\begin{gathered} \partial_i \pi^{ij} = 0\,, \qquad \partial_j \pi^{ij} = 0\,, \qquad \gamma^i \pi^{ij} = 0\,, \\ \pi^{ij}\gamma^j = 0\,, \qquad \pi^{ij}\pi^{jk} = \pi^{ik}\,. \end{gathered} \tag{22}$$

Now, we may vary the components $(\bar{\psi}^0, \bar{U}^j, \bar{\rho})$ to obtain

$$\gamma^i\psi^i = -\mathrm{i}\frac{\boldsymbol{\gamma}\cdot\boldsymbol{\partial}}{\boldsymbol{\partial}^2}\gamma^0 K_0\,, \tag{23}$$

$$\psi^0 = \frac{1}{2\boldsymbol{\partial}^2}\frac{\gamma\cdot\partial}{\mathrm{i}}K^0 - \frac{1}{2\boldsymbol{\partial}^2}\left[\frac{\partial^i}{\mathrm{i}} + \frac{\boldsymbol{\gamma}\cdot\boldsymbol{\partial}\gamma^i}{\mathrm{i}}\right]\gamma^0 K_i\,, \tag{24}$$

$$-\Box\psi^i = -\frac{\gamma\cdot\partial}{\mathrm{i}}\alpha^{ij}K^j - \frac{\Box}{2\boldsymbol{\partial}^2}\left[\frac{\partial^i}{\mathrm{i}} + \frac{\boldsymbol{\gamma}\cdot\boldsymbol{\partial}\gamma^i}{\mathrm{i}}\right]\gamma^0 K_0\,, \tag{25}$$

$$\alpha^{ij} = -\frac{1}{2}\left(\delta^{ik} - \frac{\partial^i\partial^k}{\boldsymbol{\partial}^2}\right)\gamma^\ell\gamma^k\left(\delta^{\ell j} - \frac{\partial^\ell\partial^j}{\boldsymbol{\partial}^2}\right). \tag{26}$$

The propagator may be readily obtained from (24) and (25) to be

$$D_{\mathrm{C}}^{\mu\nu}(x-x') = \int \frac{(\mathrm{d}k)}{(2\pi)^4} D_{\mathrm{C}}^{\mu\nu}(k)\mathrm{e}^{\mathrm{i}k(x-x')}\,, \tag{27}$$

$$\begin{aligned} D_{\mathrm{C}}^{ij}(k) &= \frac{(-\gamma\cdot k)}{k^2 - \mathrm{i}\epsilon}\alpha^{ij}(k) \\ &= \frac{1}{k^2 - \mathrm{i}\epsilon}\left(\delta^{is} - \frac{k^i k^s}{\mathbf{k}^2}\right)\gamma^\ell\left(-\frac{\gamma\cdot k}{2}\right)\gamma^s\left(\delta^{\ell j} - \frac{k^\ell k^j}{\mathbf{k}^2}\right), \end{aligned} \tag{28}$$

$$\alpha^{ij}(k) = -\frac{1}{2}\left(\delta^{is} - \frac{k^i k^s}{\mathbf{k}^2}\right)\gamma^\ell \gamma^s \left(\delta^{\ell j} - \frac{k^\ell k^j}{\mathbf{k}^2}\right), \tag{29}$$

$$\begin{gathered} D_{\mathrm{C}}^{00}(k) = \frac{\gamma \cdot k}{2\mathbf{k}^2}, \qquad D_{\mathrm{C}}^{0i}(k) = \frac{(k^i + \boldsymbol{\gamma}\cdot\mathbf{k}\gamma^i)\gamma^0,}{2\mathbf{k}^2}, \\ D_{\mathrm{C}}^{i0}(k) = -\frac{(k^i + \boldsymbol{\gamma}\cdot\mathbf{k}\gamma^i)\gamma^0}{2\mathbf{k}^2}. \end{gathered} \tag{30}$$

The generating functional $\langle 0_+ | 0_+ \rangle$ is then given by

$$\langle 0_+ | 0_- \rangle = \exp\left[\mathrm{i}\int (\mathrm{d}x)(\mathrm{d}x')\bar{K}_\mu(x) D_{\mathrm{C}}^{\mu\nu}(x - x') K_\nu(x')\right]. \tag{31}$$

We note that D_{C}^{00}, D_{C}^{0i}, D_{C}^{i0} provide phase factors in (31), and only D_{C}^{ij} propagates.

The spin content of the propagator is derived from the numerator of the propagator given on the extreme right-hand side in Eq. (28). For example for the electron, the propagator is given by $(-\gamma k + m)/(k^2 + m^2 - \mathrm{i}\epsilon)$. The numerator is given and expanded in terms of Dirac spinors as $(-\gamma k + m) = 2m \sum_\sigma u(\mathbf{k},\sigma)\bar{u}(\mathbf{k},\sigma)$, with spin projections $\sigma = \pm 1/2$, and hence the spin is $1/2$. In the present case, the numerator is explicitly given by

$$P^{ij}(k) = \left(\delta^{is} - \frac{k^i k^s}{\mathbf{k}^2}\right)\gamma^\ell \left(-\frac{\gamma \cdot k}{2}\right)\gamma^s \left(\delta^{\ell j} - \frac{k^\ell k^j}{\mathbf{k}^2}\right). \tag{32}$$

By working in the chiral representation of the gamma matrices, most convenient for massless particles, we may define spin-$\frac{1}{2}$ Dirac spinors

$$\begin{gathered} u(\mathbf{k}, +1/2) = \sqrt{|\mathbf{k}|}\begin{pmatrix}\xi_+ \\ 0\end{pmatrix}, \qquad u\left(\mathbf{k}, -\frac{1}{2}\right) = \sqrt{|\mathbf{k}|}\begin{pmatrix}0 \\ \xi_-\end{pmatrix}, \\ \sum_{\sigma=\pm 1/2} u(\mathbf{k},\sigma)\bar{u}(\mathbf{k},\sigma) = -\frac{\gamma \cdot k}{2}, \end{gathered} \tag{33}$$

with $\mathbf{k} = |\mathbf{k}|(0,0,1)$, $\xi_+^\top = (1,0)$, $\xi_-^\top = (0,1)$, and polarization vectors $\mathbf{e}_{+1} = (1, -\mathrm{i}, 0)/\sqrt{2}$, $\mathbf{e}_{-1} = (-1, -\mathrm{i}, 0)/\sqrt{2}$, satisfying the completeness relation

$$\delta^{ij} = \frac{k^i k^j}{\mathbf{k}^2} + \sum_{\kappa=\pm 1} \mathrm{e}_\kappa^i(\mathbf{k}) \mathrm{e}{*}_\kappa^j(\mathbf{k}), \tag{34}$$

to rewrite (32) as

$$\begin{aligned} P^{ij}(k) = \sum_{\kappa,\kappa'=\pm 1} & \mathrm{e}_\kappa^i(\mathbf{k})\, \mathrm{e}{*}_\kappa^s(\mathbf{k})\gamma^\ell (u(\mathbf{k}, +1/2)\bar{u}(\mathbf{k}, +1/2) \\ & + u(\mathbf{k}, -1/2)\bar{u}(\mathbf{k}, -1/2))\gamma^s \mathrm{e}_{\kappa'}^\ell(\mathbf{k}) \mathrm{e}{*}_{\kappa'}^j(\mathbf{k}). \end{aligned} \tag{35}$$

The following identities readily follow:

$$\sum_{\kappa'} \mathrm{e}^{*j}_{\kappa'} \mathrm{e}^{i}_{\kappa'} \gamma^i = \delta^{j\,1}\gamma^1 + \delta^{j2}\gamma^2\,, \tag{36}$$

$$\begin{aligned} \sum_{\kappa'} \mathrm{e}^{j}_{\kappa'} \mathrm{e}^{*i}_{\kappa'} \gamma^i u(\mathbf{k}, +1/2) &= \sqrt{2}\mathrm{e}^{j}_{-} u(\mathbf{k}, -1/2)\,, \\ \bar{u}(\mathbf{k}, +1/2) \sum_{\kappa'} \mathrm{e}^{*j}_{\kappa'} \mathrm{e}^{i}_{\kappa'} \gamma^i &= -\sqrt{2}\mathrm{e}^{j}_{-} \bar{u}(\mathbf{k}, -1/2) \end{aligned} \tag{37}$$

and similarly, with $(+1/2) \leftrightarrow (-1/2)$. From (35)–(37), we obtain

$$P^{ij}(k) = \sum_{\zeta=\pm 3/2} V^i(\mathbf{k}, \zeta) \bar{V}^j(\mathbf{k}, \zeta)\,, \tag{38}$$

$$V^i(\mathbf{k}, \pm 3/2) = \mathrm{e}^{i}_{\pm 1}(\mathbf{k}) u(\mathbf{k}, \pm 1/2)\,, \tag{39}$$

giving rise to a cancelation of the two terms $\mathrm{e}^{i}_{\pm 1}(\mathbf{k}) u(\mathbf{k}, \mp 1/2)$, associated with spin-$\frac{1}{2}$ projections, thus describing two helicity states of a (massless) particle of spin-$\frac{3}{2}$. The same conclusion is obviously reached from the covariant gauge one with polarization vectors $e^{\mu}_{\kappa} = (0, \mathbf{e}_{\kappa})$.

Again, the propagator of the massless Rarita–Schwinger field in the Coulomb-like gauge in describing perturbative interacting theories has to be taken as given in (28)/(30) with no deletions in it, in general, may be possible.

The propagator of the massless Rarita–Schwinger field in covariant and Coulomb-like gauges, have been derived and are spelled out, respectively, in (16), and (28)/(30) to be used in perturbative dynamical interacting theories. In general, no terms in them may be omitted without further justification.

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