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Abstract. The options are contracts that give a holder the right to buy or sell an underlying asset at the specific price before or on the expiration date. In return, the holder must pay the premium, the price of an option, to the seller in order to make contracts. As the financial problem occurs with many Thai farmers because of the fluctuation of agricultural product price. In this work, we try to price the American put option by using Feynman path integral in order to create the rice price insurance. The numerical procedures are separated into 2 parts. Firstly, the Atlantic path is determined by quasi-European method. Secondly, hybrid lattice Monte-Carlo method is used to simulate the underlying asset price path in order to evaluate the expectation profit of the option, or the premium. Moreover, the volatility rate of rice price is determined. As a result, the premium of American put option for rice is determined, and there is a deviation less than 8% comparing with Monte Carlo valuation with LSM algorithm in these examples. However, the result suggests that in this work's assumption, the option pricing by using Feynman path integral is not effective to be used.

1. Introduction

There are many people in Thailand who are farmers. However, Thai farmers have to take many risks, especially a risk from the fluctuation of agricultural product price, which make many Thai farmers face the financial problems. The risk from the fluctuation of agricultural product price can be neutralize by using the option which allows farmers to sell their product at the predetermined price.

Option is a one of the financial instruments that is able to neutralize the risk of holder. It is a contract that give the right, not the obligation, to the holder to sell or buy an underlying asset at the specific price before the maturity date. In return, the holder must pay the premium for the right to sell or buy an underlying asset. Option has many types. If the option allows holder to buy an underlying asset, the option is called call option. In contrast, if the option gives holder the right to sell an underlying asset, the option is called put option. Moreover, if the holder is allowed to exercise only on the maturity date, the option is called European option. However, if the holder is allowed to exercise at any time before and on the maturity date, the option is called American option. Also, if the payout of the option is fixed and will pay only when the price of underlying asset is more or less than the strike price, the option is called digital option.

So, if the insurance is created by using the American put option as a model, the Thai farmers will be able to guarantee their product price. The reasons for choosing American put option are, first, the farmers must have right to sell, not to buy, their product (put option) and, second, the farmers should be able to trade at any date because if the agricultural products are harvested before the maturity date on the option, it might be degraded by the increase of humidity or insects which made us choose American option instead of the European option.

In this work, rice will be used as an example of the agricultural product to determine the premium of the American put option. In the next section, the necessary knowledge for pricing an American put option will be provided. Follow by the result, discussion, and conclusion in section 3, 4 and 5, respectively.

2. Materials and methods

2.1. Black-Scholes equation and formula

In 1973, Black and Scholes invented the partial differential equation (PDE) that can price the European option premium. The equation is constructed by hedging the portfolio of the option holder. After that, the result came in the form of the Black-Scholes equation

$$\frac{\partial C}{\partial t} = rC - rS\frac{\partial C}{\partial S} - \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}, \tag{1}$$

where C is the option price (premium) of the call option; S is the underlying asset price; t is time; r is short-term interest rate; and σ is volatility. It should be noted that the Black-Scholes equation has two interesting points. First, the expectation value of the underlying asset is vanished from the equation, meaning that the option price does not depend on the opinion of the investor about the return of the underlying asset. Second, the equation is perfectly hedged; in other words, the randomness of both option price and underlying asset price are vanished [1]. Also, the Black-Scholes equation can be solved for the exact solution (Black-Scholes formula), which is

$$C = S(t_0)N(d_+) - \exp(-r\tau)EN(d_-), \tag{2}$$

where $S(t_0)$ is the underlying asset price at the starting date; E is strike price; τ is time to maturity date; function d is

$$d_- = \{\ln[S(t_0)E^{-1}] + (r - 0.5\sigma^2)\tau\}(\sigma^2\tau)^{\frac{-1}{2}};$$
$$d_+ = d_- + \sigma\sqrt{\tau}$$

and $N(\zeta)$ is the standard normal cumulative distribution function which is

$$N(\zeta) = \int_{-\infty}^{\zeta} \frac{\mathrm{d}u}{\sqrt{2\pi}} \exp\left(\frac{-u^2}{2}\right).$$

Moreover, the put option price can be written as

$$P_{\mathrm{Euro}} = \exp(-r\tau)EN(-d_-) - S(t_0)N(-d_+) \tag{3}$$

where P_{Euro} is the European put option price [2].

2.2. Feynman path integral

In quantum mechanics, Edward Schrodinger introduced an equation to explain the nature of a quantum particle as follows.

$$\mathrm{i}\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V(x)\Psi=\widehat{\mathcal{H}}\Psi, \tag{4}$$

where Ψ is a wave function of the particle; m is the particle mass; x is the particle position; t is time; $V(x)$ is a potential of the particle; and $\widehat{\mathcal{H}}$ is a Hamiltonian operator. Then, the unitary operator, that is

$$\widehat{U}(t'-t)=\exp[-\frac{\mathrm{i}}{\hbar}\widehat{\mathcal{H}}\,(t'-t)], \tag{5}$$

was constructed. The unitary operator is used to map wave function from t to t', which Richard Feynman used to demonstrate the quantum path

$$\Psi(x',t')=\int_{-\infty}^{+\infty}\mathrm{d}x\ p(x',t';x,t)\ \Psi(x,t), \tag{6}$$

and $p(x',t';x,t)$ is called propagator. The structure of a propagator is

$$p(x',t';x,t)\ =\langle x'|\widehat{U}(\varepsilon)|x\rangle=(\frac{m}{2\mathrm{i}\pi\hbar\varepsilon})^{\frac{1}{2}}\exp[\frac{\mathrm{i}\varepsilon}{\hbar}\mathcal{L}(x',x)]=\int_{x}^{x'}\mathcal{D}[x(t)]\exp\{\frac{\mathrm{i}}{\hbar}S[x(t)]\}, \tag{7}$$

where ε is $(t'-t)$, $\mathcal{L}(x',x)$ is a Lagrangian; $S[x(t)]$ is a classical path; and

$$\int_{x}^{x'}\mathcal{D}[x(t)]=\lim_{\substack{N\to\infty\\ \varepsilon\to 0}}(\frac{m}{2\mathrm{i}\pi\hbar\varepsilon})^{\frac{N}{2}}(\prod_{\mathrm{k}=1}^{N-1}\int_{-\infty}^{+\infty}\mathrm{d}x_{\mathrm{k}}).$$

It can be seen that the propagator can represent the path of the particle that moves from (x,t) to (x',t') [3,4].

2.3. Black-Scholes propagator

Substitute $x=\ln(S)$ in equation (1) and compare it with Schrodinger's wave equation (4). It can be seen that the option price is similar to the wave function in equation (4), and the Hamiltonian operator is

$$\mathcal{H}_{\mathrm{BS}}=-\frac{\sigma^2}{2}\frac{\partial^2}{\partial x^2}+(\frac{\sigma^2}{2}-r)\frac{\partial}{\partial x}+r. \tag{8}$$

Therefore, the Black-Scholes propagator is

$$p_{\mathrm{BS}}=(2\pi\sigma^2\tau)^{\frac{-1}{2}}\exp\{-r\tau-[x-x'-(r-0.5\sigma^2)\tau]^2(\sigma^2\tau)^{\frac{-1}{2}}\}. \tag{9}$$

The reason that quantum mechanics can be adapted for option pricing is that the option price is a stochastic process like the wave function of a quantum particle. However, unlike the wave function in quantum mechanics, the option price cannot be a complex number [1,5].

2.4. Digital option price

The digital option is a type of option that will pay the holder at the fixed amount if the underlying asset price is higher, for a call option, or lower, for a put option than the strike price. In this work, the cash-or-nothing put option, which is a subtype of digital option, will be used. So, the cash-or-nothing put option pricing can be calculated by

$$P_{\text{Digital}} = \exp(-r\tau)\, N(-d_-) \tag{10}$$

where P_{Digital} is price of cash-or-nothing put option [2].

2.5. Quasi-European method

The quasi-European method is used to simulate the Atlantic path defined as the price path that the return from exercising the option is equal to the value of the option if it was kept holding. At the maturity date (T), the value of holding the option is equal to 0. Therefore, the Atlantic path on the maturity date ($\widehat{P}_T$) is the exercise price E.

On the day before the expiration date, the value of the Atlantic path for the American put option is lower than that of the maturity date because if the holder holds the options longer for one day, it is the same as holding a European option. So, the Atlantic path value on the day before the expiration date ($\widehat{P}_{T-1}$) is equal to $E - P_{\text{Euro}}(\widehat{P}_{T-1}, t_{T-1})$.

Moreover, at the other previous date before the maturity date, the Atlantic path always has a lower price than that of the day after because if the underlying asset price is lower or equal to the Atlantic path, for the put option case, the holder will exercise and will get the profit from the difference of Atlantic path and the exercise price. However, if the price is higher than the Atlantic path, the holder will not exercise and will gain nothing from the option, like the cash-or-nothing option. Therefore, the Atlantic path value at any day before the next-to-last day ($\widehat{P}_\lambda$) is equal to $E - P_{\text{Euro}}(\widehat{P}_{T-1}, t_{T-1}) - (\widehat{P}_{\lambda+1} - E)\, P_{\text{Digital}}(\widehat{P}_\lambda, t_\lambda)$ [5].

2.6. Hybrid Monte Carlo with lattice approximation

In order to evaluate the price path of the underlying asset, it is possible to use the propagator for doing Monte Carlo simulation because the propagator represents the probability of the price path, which is the Black-Scholes propagator in this work.

The simulation is started by setting up the first point at t_0 as $S(t_0)$. Then, simulate the lattices at t_1 by using the propagator to generate the value of each lattice. After that, scale the lattices into the range from 0 to 1. Finally, uniformly choosing the random number from 0 to 1, the price of the underlying asset at the time t_1 is equal to the mean value of the lattice before and after the random number. Do the simulation until the underlying asset price lower or equal to the Atlantic path or until the time expires [5].

3. Results

According to the rice price data (2010-2019) from the Office of Agricultural Economics, Ministry of Agriculture and Cooperatives, Thailand. The volatility (σ) of rice price is about 1.1% per day. In this work, it is assumed that the short-term interest rate and volatility are constant, which means the Black-Scholes propagator is possible to be used. Assume that the short-term interest rate (r) is 1% per year (0.00274% per day); time to maturity is 100 business days (5 months) which is approximately 150 calendar days used for planting and harvesting the rice; and the precision of lattice is 4 digits. The results are shown in table 1.

Table 1. Result of the option price determined by using Feynman path integral comparing with Monte Carlo valuation with LSM method [6], time used to process in both method by using CPU Intel Core i7 1.8 GHz with 16GB RAM, and deviation of Feynman path integral results from Monte Carlo with LSM results which is used as our benchmark.

Starting price (Thai baht)	Exercise price (Thai baht)	Feynman path integral result (Thai baht)	Feynman path integral time used	Monte Carlo with LSM result (Thai baht)	Monte Carlo with LSM time used (s)	Deviation (%)
22,000	20,000	190 ± 4	$3^h15^m \pm 11^m$	176.464	0.16 ± 0.01	7.67
18,000	20,000	2,170 ± 7	$2^h57^m \pm 05^m$	2,117.775	0.154 ± 0.001	2.50
18,000	22,000	4,050 ± 20	$3^h01^m \pm 07^m$	4,008.085	0.139 ± 0.002	1.01

4. Discussion

In the quasi-European method, the Atlantic path cannot be found by the ordinary iteration. The reason might because the low volatility of rice price could affect the equation in the quasi-European method. In this work, the problem was solved by using the particular condition of the put option, which is the Atlantic path at the day t must lower than the Atlantic path at the day before, $t + 1$, and greater than 0. Therefore, the Atlantic path is determined by finding the value in the range from 0 to the Atlantic path price on the day before that minimizes the difference between the Atlantic path price and the function of the Atlantic path.

However, the path integral method still has an advantage that is the case when the condition of the option or the nature of the underlying asset price is changed. For example, the interest rate or volatility is not constant, or the underlying asset price is the function of interest rate, e.g., bond. This method can be used easily by changed the propagator into that condition; then, the premium can be calculated with the same procedure.

5. Conclusion

After calculating the rice American put option by using the Feynman path integral, the accuracy of the premium is acceptable. However, under the assumption that the short-term interest rate and volatility are constant, the efficiency of this method is quite low comparing to Monte Carlo valuation with LSM algorithm.

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