COSMOLOGICAL DYNAMICS OF NON-MINIMAL DERIVATIVE COUPLING TO GRAVITY IN PALATINI FORMALISM

SOMPHOACH SAICHAEMCHAN

A Thesis Submitted to Graduate School of Naresuan University
in Partial Fulfillment of the Requirements
of the Master of Science Degree in Theoretical Physics
July 2018

Thesis entitled "Cosmological Dynamics of Non-Minimal Derivative Coupling to Gravity in Palatini Formalism"
by Somphoach Saichaemchan
has been approved by the Graduate School as partial fulfillment of the requirements for the Master of Science in Theoretical Physics of Naresuan University.

Oral Defense Committee

.... Chair
(Assistant Professor Seckson Sukhasena, Ph.D.)

.... Advisor
(Associate Professor Burin Gumjudpai, Ph.D.)

.... Co-Advisor
(Assistant Professor Pichet Vanichchapongjaroen, Ph.D.)

.... External Examiner
(Assistant Professor Sikarin Yoo-Kong, Ph.D.)

Approved

....
(Associate Professor Paisarn Muneesawang, Ph.D.)
Dean of the Graduate School

ACKNOWLEDGMENTS

I would like to thank my advisor Assoc. Prof. Dr. Burin Gumjudpai, my co-advisor Dr. Pichet Vanichchapongjarareon, Narakorn Kaewkhao, and Yuttana Jawralee, my labmates in the Collegium Fisica Studiorum. I thank IF faculty members and present master's students. At last, I thank all my supporters, my uncle and my family.

Somphoach Saichaemchan

LIST OF CONTENTS

LIST OF CONTENTS (CONT.)

LIST OF FIGURES

Title	COSMOLOGICAL DYNAMICS OF NON-MINIMAL DERIVATIVE COUPLING TO GRAVITY IN PALATINI FORMALISM
Author	Somphoach Saichaemchan
Advisor	Associate Professor Burin Gumjudpai, Ph.D.
Academic Paper	Thesis M.S. in Theoretical Physics,
	Naresuan University, 2017
Keywords	Non-Minimal Derivative Coupling, Inflation, Scalar-Tensor Theory

ABSTRACT

We study Non-Minimal Derivative Coupling to Einstein tensor (NMDC) model of scalar field. We considered it in Palatini formalisms. This study considers early universe during chaotic inflation driven by a scalar field. We analyse acceleration condition and dynamical phase portrait. We compare our results to standard general relativity (GR) case ($\kappa = 0$), metric formalism case. We present acceleration condition in the three cases. In Palatini-formalism, phase portrait indicates new saddle point and enlarged acceleration region.

CHAPTER I

INTRODUCTION

1.1 Background and motivation

Current evidences from astrophysical observations convince accelerating expansion of universe, for instance, redshift of Supernovae Type Ia measurement [1, 2, 3]. The SN Ia observation shows that the universe comprises of dark energy about 70% causing late-time cosmic acceleration. In very early universe, inflation supported by present observations, e.g. cosmic microwave background (CMB) anisotropies [4], WMAP [5], was introduced [6] to solve standard big bang cosmology problems such as flatness problem and horizon problem. Contemporary cosmology focuses on either the early inflationary or late time accelerating expansion hypothesized as effects of dynamical scalar field such as inflaton or quintessence [7], k-essence [8] or by effect of modified gravity such as scalar-tensor theories [9, 10, 11].

One can extend scalar-tensor theories to the coupling of scalar field to gravity sector such as $f(\phi, \phi_{,\mu}, \phi_{,\mu\nu}, ...)$ which is motivated by Brans-Dicke theory [12] such that it includes Non-Minimal Coupling (NMC) or Non-Minimal Derivative Coupling (NMDC) terms. The NMC and NMDC are found to be a spacial case of Horndeskis theory which is generalization of gravitational theory with at most second-order derivatives in the equations of motion, making the Horndeski action the most general scalar-tensor theory [13]. NMDC model can be found in lower-energy limits of extra-dimension theories [14] and Weyl anomaly of $\mathcal{N} = 4$ conformal supergravity [15, 16]. It is possible to have coupling terms between derivative of the scalar field and gravity in form of $\kappa_1 R\phi_{,\mu}\phi^{,\mu}$ and $\kappa_2 R_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$ which as well result in acceleration without loss of generality [14, 17]. Consider a special case of $\kappa_1 R\phi_{,\mu}\phi^{,\mu}$ and $\kappa_2 R_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$ term, we can set $\kappa \equiv \kappa_2 = -2\kappa_1$ and combination of these two terms gives the Einstein tensor which couples to kinetic part of scalar field as $G_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$ [18, 19, 20, 21, 22, 23, 24, 25].

1.2 Objectives

- Derive acceleration condition of NMDC model both in metric and Palatini formalism.
- Introduce autonomous systems of the models.
- Present dynamical phase portraits of the models with acceleration region.

1.3 Framework

- FRW universe
- NMDC model
- metric and Palatini formalism
- Phase portraits

1.4 Unit and notation

Units with $c = \hbar = k_B = 1$

$[G] = [M_{rmP}^{-2}]$

$[\mathrm{length}] = [\mathrm{time}] = [\mathrm{L}]$

$[\mathrm{mass}] = [\mathrm{energy}] = [\mathrm{M}]$

$[\text{energy density: } \rho] = [\text{pressure: } P] = [\mathrm{ML}^{-3}] = [\mathrm{L}^{-4}]$

$[g_{\mu\nu}] = [\mathrm{dimensionless}]$

$[\Gamma^{\mu}_{\nu\kappa}] = [\mathrm{L}^{-1}]$

$[R^{\mu}_{\nu\kappa\sigma}] = [R_{\mu\nu}] = [R] = [\mathrm{L}^{-2}]$

$[\phi] = [\mathrm{M}]$

$[\kappa] = [\mathrm{M}^{-2}]$

$[V(\phi)] = [\mathrm{M}^4]$

$[H] = [\mathrm{M}]$

$[G] = [\mathrm{M}^{-2}]$

Notation

- G : Newton's gravitational constant ($G = 6.67 \times 10^{-8}\mathrm{cm}^3\mathrm{g}^{-1}\mathrm{sec}^{-2}$)
- M_{rmP} : Reduced Plank mass ($M_{rmP} = (8\pi G)^{-1/2} = 2.4357 \times 10^{18}\mathrm{GeV}$)
- a : Scale factor of the universe (where $a_0 = 1$ at the present time)
- t : The cosmic time
- $\dot{x}$: Derivative of x variable with respect to t
- x' : Derivative of x variable with respect to $N \equiv \ln a$

- H : Hubble parameter $\left(H \equiv \frac{\dot{a}}{a}\right)$
- ρ : Energy density
- P : Pressure
- w : Equation of state parameter $\left(w = \frac{P}{\rho}\right)$
- $w_{\rm eff}$: Effective equation of state parameter
- R : Ricci scalar
- GR : Einstein's general relativity theory
- $g_{\mu\nu}$: Metric tensor
- $\Gamma^{\mu}_{\nu\kappa}$: Connection field
- $G_{\mu\nu}(g)$: Einstein tensor in metric formalism
- $G_{\mu\nu}(\Gamma)$: Einstein tensor in Palatini formalism
- ϕ : Scalar field
- $V(\phi)$: Scalar field potential

CHAPTER II

STANDARD COSMOLOGY

2.1 Cosmological principle

The Standard cosmology assumes cosmological principle obeying large scale observational data. These require that the universe is homogeneous and isotropic. Homogeneity implies that property of space is independent of the position and isotropy means that universe looks the same from all directions. Hence the metric that describes homogeneous and isotropy of space-time is Friedmann-Roberson-Walker (FRW) metric given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left[\frac{\mathrm{d}r^2}{1 - Kr^2} + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2 \right) \right], \tag{2.1}$$

where t is cosmic time. The coordinates r, θ, ϕ are co-moving coordinates, a is scale factor and the constant K is spatial curvature. Flat, closed, open geometries correspond to zero, positive, negative values of k respectively.

2.2 Hubble's law expansion

Hubble's observation showed that galaxies are moving away from us, therefore the universe is not static but expanding. A relation between velocity and physical distance of objects from us is

$$\vec{v} = H_0 \vec{r}, \tag{2.2}$$

where $\vec{r}$ is physical distance, $\vec{v}$ is velocity of moving object and H_0 is Hubble constant at present time t_0. Considering physical distance $\vec{r}$ and co-moving distance $\vec{x}$ can be written as

$$\vec{r} = a(t)\vec{x}, \tag{2.3}$$

where $a(t)$ is scale factor. As a result, recession velocity definition is

$$\vec{v} = \frac{\dot{a}(t)}{a(t)}\vec{r}, \tag{2.4}$$

and the Hubble parameter is defined as

$$H \equiv \frac{\dot{a}(t)}{a(t)}. \tag{2.5}$$

2.3 Cosmological equations

Einstein field equation of general relativity is the following equation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}, \tag{2.6}$$

where $G_{\mu\nu}, R_{\mu\nu}, R, G$, and $T_{\mu\nu}$ are Einstein tensor, Ricci tensor, Ricci scalar, gravitational constant and the energy-momentum tensor, respectively. We give $c = 1$ and solve the Einstein field equation for FRW universe, and we obtain Friedmann and acceleration equation:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}, \tag{2.7}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho - 3P\right), \tag{2.8}$$

where $\rho(t)$ and $P(t)$ are the energy density and the pressure. Differentiate Eq. (2.7) with respect to time,

$$\ddot{a} = \frac{8\pi G}{3\dot{a}}\left(2\rho a\dot{a} + a^2\dot{\rho}\right), \tag{2.9}$$

Eq. (2.9) is substituted in Eq. (2.8) hence

$$\dot{\rho} + 3H\left(\rho + P\right). \tag{2.10}$$

This equation is called the continuity equation.

The density parameter which is ratio of the energy density and critical density is explained as

$$\Omega \equiv \frac{\rho}{\rho_c} = \frac{8\pi G}{3H^2}\rho, \tag{2.11}$$

where critical density is

$$\rho_c = \frac{3H^2}{8\pi G}. \tag{2.12}$$

The Friedmann can be rewritten as

$$\Omega - 1 = \frac{K}{H^2 a^2}, \tag{2.13}$$

2.4 Inflation

We believe that inflation occurred in the early universe, thus it solves several problems such as flatness and horizon problems [6]. The existence of inflation [26] is when,

$$\ddot{a} > 0, \tag{2.14}$$

which corresponds to

$$\rho + 3P < 0. \tag{2.15}$$

Considering a scalar field, ϕ, the energy density and the pressure of the scalar field are

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{2.16}$$

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{2.17}$$

where $V(\phi)$ is potential energy of scalar field and the condition of inflation is $\dot{\phi}^2 < V(\phi)$. We substitute the energy density Eq. (2.16) and the pressure Eq. (2.17) into Eq. (2.7) and Eq. (2.10) giving

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \tag{2.18}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0, \tag{2.19}$$

where $K = 0$, $c = 1$ and $V_{,\phi}(\phi) \equiv \mathrm{d}V/\mathrm{d}\phi$. The slow-roll condition is introduced $\dot{\phi}^2 \ll V(\phi)$ and $\ddot{\phi} \ll 3H\dot{\phi}$. Eq. 2.18 and Eq. (2.19) are approximately given as

$$H^2 \simeq \frac{8\pi G}{3}V(\phi), \tag{2.20}$$

$$\dot{\phi} \simeq -\frac{V_{,\phi}}{3H}. \tag{2.21}$$

Slow-roll parameter are defined as

$$\epsilon \equiv \frac{1}{16\pi G}\left(\frac{V_{,\phi}}{V}\right)^2, \quad \eta \equiv \frac{V_{,\phi\phi}}{8\pi G V}. \tag{2.22}$$

Slow-roll approximation is satisfied when $0 < \epsilon \ll 1$ and $\eta \ll 1$. The inflationary phase ends when $\dot{\phi}^2 \simeq V(\phi)$.

CHAPTER III

NON-MINIMAL DERIVATIVE COUPLING WITH SCALAR FIELD IN COSMOLOGY

3.1 Non-Minimal Derivative Coupling to gravity

Since inflation is driven by scalar field called *"the inflaton"* [6], scalar field models are allowed to find dynamic of inflaton. Recent scalar field models can describe accelerating expansion or inflation of the universe. One of the scalar-tensor theories is Non-Minimal Derivatives Coupling (NMDC) which is extended version of Non-Minimal Coupling model (NMC) that is motivated by Brans-Dicke models [12]. The NMC and NMDC are found to be a spacial case of Horndeskis theory generalization of gravitational theory with at most second-order derivatives in the equations of motion, making the Horndeski action the most general scalar-tensor theory [13].

Lagrangian densities of NMDC like $\kappa_1 R\phi_{,\mu}\,\phi^{,\mu}$ and $\kappa_2 R_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$ [14] is possible to have coupling term between derivative of scalar filed and gravity without loss of generality. For spacial case of the coupling as $\kappa \equiv -2\kappa_1 = \kappa_2$, combination of the two terms give the Einstein tensor [18, 19, 20, 21, 22, 23, 24, 25].

We consider NMDC model which take the action with this case

$$S = \int \mathrm{d}x^4 \sqrt{-g}\left[\frac{R}{8\pi G_{\mathrm{N}}} - \left(\varepsilon g_{\mu\nu} + \kappa G_{\mu\nu}\right)\phi^{,\mu}\phi^{,\nu} - 2V(\phi)\right]. \tag{3.1}$$

The model is a subclass of the Horndeski action (with $G_5 = \phi\kappa/2$) which is generalized action that is Ostrogradski instability free [13].

3.2 NMDC-Palatini action

The gravitational action of NMDC in Palatini formalism is proposed [25] as

$$S_{\text{Palatini}} = \frac{M_{\text{P}}^2}{2} \int \mathrm{d}x^4 \sqrt{-g} \left[\tilde{R}(\Gamma) - \left(\varepsilon g_{\mu\nu} + \kappa \tilde{G}_{\mu\nu}(\Gamma) \right) \phi^{;\mu} \phi^{;\nu} - V(\phi) \right], \tag{3.2}$$

where $G_{\mu\nu}(\Gamma) = R_{\mu\nu}(\Gamma) - \frac{1}{2}R(\Gamma), \tilde{R}(\Gamma) = g^{\mu\nu}\tilde{R}_{\mu\nu}(\Gamma)$ and Ricci tensor is defined by connection field $\tilde{R}_{\mu\nu}(\Gamma) = \tilde{R}^{\lambda}_{\mu\lambda\nu}(\Gamma) = \partial_\lambda \Gamma^{\lambda}_{\mu\nu} - \partial_\nu \Gamma^{\lambda}_{\mu\lambda} + \Gamma^{\lambda}_{\sigma\lambda}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\lambda}_{\sigma\nu}\Gamma^{\sigma}_{\mu\lambda}$. Tilde symbol represents the variables which depend on connection field (Γ)

This action Eq. (3.2) is derived in Palatini formalism which allows two dynamical fields, metric field g and connection field Γ, to vary.

3.3 Field equations of slow-roll NMDC in Palatini formalism

The field equation of NMDC in Palatini formalism was derived and slow-roll approximated [25]. With slow-roll condition, $0 < |\dot{\phi}| \ll 1$, it gives modified Friedmann equation,

$$H^2 \simeq \frac{\rho_{\text{tot}}}{3M_{\text{P}}^2} \left[1 + \frac{3\kappa\dot{\phi}^2}{2M_{\text{P}}^2} (1 + w_{\text{eff}}) \right], \tag{3.3}$$

where $\rho_{\text{tot}} = \rho_\phi + \rho_{\text{m}}$ is total density and w_{eff} is the effective equation of state parameter. Klein-Gordon equation was derived as [25]

$$\begin{aligned} \ddot{\phi} \left[\varepsilon - \frac{9\kappa\dot{H}}{2} \left(1 - \frac{\kappa\dot{\phi}^2}{M_{\text{P}}^2} \right) - \frac{3\kappa H^2}{2} \left(5 - \frac{6\kappa\dot{\phi}^2}{M_{\text{P}}^2} \right) \right] \\ + 3H\dot{\phi} \left[\varepsilon - \left(\frac{\ddot{H}}{H} + 4\dot{H} \right) \kappa \left(1 - \frac{\kappa\dot{\phi}^2}{2M_{\text{P}}^2} \right) \right] + V_{,\phi} \simeq 0. \end{aligned} \tag{3.4}$$

The NMDC-Palatini effect plays role in chaotic inflationary model, the tensor-to-scalar ratio and spectral index could pass the Planck 2015 constraint for a range of κ.

CHAPTER IV

ACCELERATION CONDITION

We consider NMDC model of which the coupling would affect inflation. First, we derive acceleration condition of NMDC model in metric formalism using Klein-Gordon equation. We investigate the effect of connection field in Palatini formalism in comparison to the metric formalism.

4.1 Acceleration condition of NMDC in metric formalism

NMDC Klein-Gordon equation in metric formalism [24] is

$$\varepsilon\left(\ddot{\phi}+3H\dot{\phi}\right)-3\kappa\left(H^2\ddot{\phi}+2H\dot{H}\dot{\phi}+3H^3\dot{\phi}\right)=-V_{,\phi}\,. \tag{4.1}$$

Under slow-roll condition $|\dot{H}|\ll|H^2|$, the Eq. (4.1) is rewritten as

$$\begin{aligned}\ddot{\phi}\left(\varepsilon-3\kappa H^2\right)+3H\dot{\phi}\left(\varepsilon-2\kappa\dot{H}-3\kappa H^2\right)+V_{,\phi} &= 0\\ \ddot{\phi}\left(\varepsilon-3\kappa H^2\right)+3H\dot{\phi}\left(\varepsilon-3\kappa H^2\right)+V_{,\phi} &= 0.\end{aligned} \tag{4.2}$$

Approximating $\ddot{\phi}\approx 0$ at late time, the trajectory is

$$\dot{\phi}\simeq-\frac{V_{,\phi}}{3H\left(\varepsilon-3\kappa H^2\right)}. \tag{4.3}$$

Friedmann equation is expressed in [24],

$$H^2\simeq\frac{8\pi G}{3}\left[\frac{\dot{\phi}^2}{2}\left(\varepsilon-9\kappa H^2\right)+V\right]. \tag{4.4}$$

Acceleration condition is defined $\ddot{a}/a\equiv\dot{H}+H^2$, differentiating Eq. (4.4) with respect to time t

$$
\begin{aligned}
H^2 &\simeq \frac{1}{3M_{\mathrm{P}}^2}\left[\frac{\varepsilon\dot{\phi}^2}{2} - \frac{9\kappa\dot{\phi}^2H^2}{2} + V\right] \\
2H\dot{H} &\simeq \frac{1}{3M_{\mathrm{P}}^2}\left[\varepsilon\dot{\phi}\ddot{\phi} - \frac{9\kappa}{2}\left(2\dot{\phi}\ddot{\phi}H^2 + \dot{\phi}^2(2H\dot{H})\right) + V_{,\phi}\,\dot{\phi}\right] \\
&\simeq \frac{1}{3M_{\mathrm{P}}^2}\left[\varepsilon\dot{\phi}\ddot{\phi} - 9\kappa\dot{\phi}\ddot{\phi}H^2 + 9\kappa\dot{\phi}^2H\dot{H} + V_{,\phi}\,\dot{\phi}\right] \\
\dot{H} &\simeq \frac{1}{6M_{\mathrm{P}}^2}\left[\frac{\varepsilon\dot{\phi}\ddot{\phi}}{H} - 9\kappa\dot{\phi}\ddot{\phi}H - 9\kappa\dot{\phi}^2\dot{H} + \frac{V_{,\phi}\,\dot{\phi}}{H}\right],
\end{aligned}
\tag{4.5}
$$

so acceleration condition is expressed as

$$
\begin{aligned}
\dot{H} + H^2 &\simeq \frac{\varepsilon\ddot{\phi}\dot{\phi}}{6HM_{\mathrm{P}}^2} - \frac{9\kappa\dot{\phi}\ddot{\phi}H}{6M_{\mathrm{P}}^2} - \frac{9\kappa\dot{\phi}^2\dot{H}}{6M_{\mathrm{P}}^2} + \frac{V_{,\phi}\,\dot{\phi}}{6HM_{\mathrm{P}}^2} + \frac{\varepsilon\dot{\phi}}{6M_{\mathrm{P}}^2} - \frac{9\kappa\dot{\phi}^2H^2}{6M_{\mathrm{P}}^2} + \frac{V}{3M_{\mathrm{P}}^2} \\
&\simeq \frac{\dot{\phi}^2}{6M_{\mathrm{P}}^2}(\varepsilon - 9\kappa H^2 - 9\kappa\dot{H}) + \frac{V}{3M_{\mathrm{P}}^2} + \frac{\dot{\phi}\ddot{\phi}}{6M_{\mathrm{P}}^2}\left(\frac{\varepsilon}{H} - 9\kappa H\right) + \frac{V_{,\phi}\,\dot{\phi}}{6HM_{\mathrm{P}}^2} \\
&\simeq \frac{\dot{\phi}^2}{6M_{\mathrm{P}}^2}(\varepsilon - 9\kappa H^2 - 9\kappa\dot{H}) + \frac{V}{3M_{\mathrm{P}}^2} + \frac{\dot{\phi}\ddot{\phi}}{6M_{\mathrm{P}}^2H}(\varepsilon - 9\kappa H^2) + \frac{V_{,\phi}\,\dot{\phi}}{6HM_{\mathrm{P}}^2}.
\end{aligned}
\tag{4.6}
$$

Slow-roll condition, $|\dot{H}| \ll |H^2|$, is valid hence Eq. (4.6) is

$$
\frac{\ddot{a}}{a} \equiv \dot{H} + H^2 \approx \frac{\dot{\phi}^2}{6M_{\mathrm{P}}^2}(\varepsilon - 9\kappa H^2) + \frac{V}{3M_{\mathrm{P}}^2} + \frac{\dot{\phi}\ddot{\phi}}{6M_{\mathrm{P}}^2H}(\varepsilon - 9\kappa H^2) + \frac{V_{,\phi}\,\dot{\phi}}{6HM_{\mathrm{P}}^2}. \tag{4.7}
$$

Using Eq. (4.4) and Friedmannn equation that is approximated by slow-roll field, $\dot{\phi}^2 \ll V$,

$$
H^2 \approx \frac{V}{3M_{\mathrm{P}}^2} \tag{4.8}
$$

in Eq. (4.7),

$$
\begin{aligned}
\frac{\ddot{a}}{a} &\approx \frac{\dot{\phi}^2}{6M_{\rm P}^2}(\varepsilon - 9\kappa H^2) + \frac{V}{3M_{\rm P}^2} && (4.9)\\
&\quad + \frac{\dot{\phi}}{6M_{\rm P}^2 H}\left(\frac{-V_{,\phi} - 3H\dot{\phi}(\varepsilon - 3\kappa H^2)}{\varepsilon - 3\kappa H^2}\right)(\varepsilon - 9\kappa H^2) + \frac{V_{,\phi}\,\dot{\phi}}{6M_{\rm P}^2 H} \\
&= \frac{\dot{\phi}^2}{6M_{\rm P}^2}(\varepsilon - 9\kappa H^2) + \frac{V}{3M_{\rm P}^2} && (4.10)\\
&\quad + \frac{\dot{\phi}}{6M_{\rm P}^2 H}\left(\frac{-V_{,\phi}}{\varepsilon - 3\kappa H^2} - 3H\dot{\phi}\right)(\varepsilon - 9\kappa H^2) + \frac{V_{,\phi}\,\dot{\phi}}{6M_{\rm P}^2 H} \\
&= \frac{\dot{\phi}^2}{6M_{\rm P}^2}(\varepsilon - 9\kappa H^2) + \frac{V}{3M_{\rm P}^2} - \frac{\dot{\phi}V_{,\phi}\,(\varepsilon - 9\kappa H^2)}{6M_{\rm P}^2 H(\varepsilon - 3\kappa H^2)} \\
&\quad - \frac{\dot{\phi}^2}{2M_{\rm P}^2}(\varepsilon - 9\kappa H^2) + \frac{V_{,\phi}\,\dot{\phi}}{6M_{\rm P}^2 H} \\
&= -\frac{\dot{\phi}^2}{3M_{\rm P}^2}(\varepsilon - 9\kappa H^2) + \frac{V}{3M_{\rm P}^2} + \frac{V_{,\phi}\,\dot{\phi}}{6M_{\rm P}^2 H}\left[1 - \frac{(\varepsilon - 9\kappa H^2)}{(\varepsilon - 3\kappa H^2)}\right] \\
&= -\frac{1}{3M_{\rm P}^2}\left[\dot{\phi}^2\left(\varepsilon - 9\kappa\frac{V}{3M_{\rm P}^2}\right) - V\right] + \frac{V_{,\phi}\,\dot{\phi}}{6M_{\rm P}^2\frac{\sqrt{V}}{\sqrt{3}M_{\rm P}}}\left[1 - \frac{\left(\varepsilon - 9\kappa\frac{V}{3M_{\rm P}^2}\right)}{\left(\varepsilon - 3\kappa\frac{V}{3M_{\rm P}^2}\right)}\right] \\
&= -\frac{1}{3M_{\rm P}^2}\left[\dot{\phi}^2\left(\varepsilon - 3\kappa\frac{V}{M_{\rm P}^2}\right) - V\right] + \frac{\sqrt{3}V_{,\phi}\,\dot{\phi}M_{\rm P}}{6M_{\rm P}^2\sqrt{V}}\left[1 - \frac{\left(\varepsilon - 3\kappa\frac{V}{M_{\rm P}^2}\right)}{\left(\varepsilon - \kappa\frac{V}{M_{\rm P}^2}\right)}\right] && (4.11)
\end{aligned}
$$

For $|\kappa V| \ll |M_{\rm P}^2$ to avoid super-Planckian regime, binomial approximation, $(1+x)^a \approx 1 + ax$, is valid hence Eq. (4.11) is approximated as

$$
\frac{\ddot{a}}{a} \approx -\frac{1}{3M_{\rm P}^2}\left[\dot{\phi}^2\left(\varepsilon - 3\kappa\frac{V}{M_{\rm P}^2}\right) - V\right] + \frac{\sqrt{3}V_{,\phi}\,\dot{\phi}M_{\rm P}}{6M_{\rm P}^2\sqrt{V}}\left[1 - \left(\varepsilon - 3\kappa\frac{V}{M_{\rm P}^2}\right)\left(\varepsilon + \kappa\frac{V}{M_{\rm P}^2}\right)\right]. \tag{4.12}
$$

Hence,

$$
\begin{aligned}
\frac{\ddot{a}}{a} &\approx -\frac{1}{3M_{\rm P}^2}\left[\dot{\phi}^2\left(\varepsilon - 3\kappa\frac{V}{M_{\rm P}^2}\right) - V\right] + \frac{\sqrt{3}V_{,\phi}\,\dot{\phi}M_{\rm P}}{6M_{\rm P}^2\sqrt{V}}\left[1 - \left(\varepsilon^2 - \frac{2\kappa\varepsilon V}{M_{\rm P}^2} - \frac{3\kappa^2 V^2}{M_{\rm P}^4}\right)\right] \\
&\approx -\frac{1}{3M_{\rm P}^2}\left[\dot{\phi}^2\left(\varepsilon - 3\kappa\frac{V}{M_{\rm P}^2}\right) - V\right] + \frac{\sqrt{3}V_{,\phi}\,\dot{\phi}M_{\rm P}}{6M_{\rm P}^2\sqrt{V}}\left[1 - 1 + \frac{2\kappa\varepsilon V}{M_{\rm P}^2} + \cancelto{\approx 0}{\frac{3\kappa^2 V^2}{M_{\rm P}^4}}\right] \\
&= -\frac{1}{3M_{\rm P}^2}\left[\dot{\phi}^2\left(\varepsilon - 3\kappa\frac{V}{M_{\rm P}^2}\right) - V\right] + \frac{\sqrt{3}V_{,\phi}\,\dot{\phi}M_{\rm P}}{6M_{\rm P}^2\sqrt{V}}\left(\frac{2\kappa\varepsilon V}{M_{\rm P}^2}\right) \\
&= -\frac{1}{3M_{\rm P}^2}\left[\dot{\phi}^2\left(\varepsilon - 3\kappa\frac{V}{M_{\rm P}^2}\right) - V\right] + \frac{\kappa\varepsilon\dot{\phi}V_{,\phi}\sqrt{V}}{\sqrt{3}M_{\rm P}^3}, && (4.13)
\end{aligned}
$$

where $\varepsilon^2 = 1$. For $\ddot{a}/a > 0$, we use late time trajectory Eq. (4.3) and rewrite Eq. (4.13) as

$$
\begin{aligned}
\frac{\dot{\phi}^2\left(\varepsilon - 3\kappa\frac{V}{M_{\rm P}^2}\right)}{3M_{\rm P}^2} &< \frac{V}{3M_{\rm P}^2} + \frac{\kappa\varepsilon\dot{\phi}V_{,\phi}\sqrt{V}}{\sqrt{3}M_{\rm P}^3} \\
\dot{\phi}^2\left(\varepsilon - 3\kappa\frac{V}{M_{\rm P}^2}\right) &< V\left(1 + \frac{\sqrt{3}\kappa\varepsilon\dot{\phi}V_{,\phi}}{\sqrt{V}M_{\rm P}}\right) \\
\dot{\phi}^2 &< \frac{V\left(1 + \frac{\sqrt{3}\kappa\varepsilon\dot{\phi}V_{,\phi}}{\sqrt{V}M_{\rm P}}\right)}{\left(\varepsilon - 3\kappa\frac{V}{M_{\rm P}^2}\right)} \\
\dot{\phi}^2 &< \frac{V\left\{1 + \sqrt{3}\kappa\varepsilon V_{,\phi}\left[-\frac{M_{\rm P}V_{,\phi}}{\sqrt{3V}\left(\varepsilon - \frac{\kappa V}{M_{\rm P}^2}\right)}\right]\left(\sqrt{V}M_{\rm P}\right)^{-1}\right\}}{\left[\varepsilon - 3\kappa\frac{V}{M_{\rm P}^2}\right]} \\
\dot{\phi}^2 &< V\left[1 - \left(\frac{\kappa\varepsilon(V_{,\phi})^2}{V\left(\varepsilon - \frac{\kappa V}{M_{\rm P}^2}\right)}\right)\right]\left[\varepsilon - \frac{9\kappa V}{3M_{\rm P}^2}\right]^{-1}. \qquad (4.14)
\end{aligned}
$$

We apply chaotic inflation potential, $V(\phi) = V_0\phi^2$, to Eq. (4.14) and obtain

$$
\dot{\phi}^2 < V_0\phi^2\left[1 - \frac{4\kappa\varepsilon V_0}{\left(\varepsilon - \frac{\kappa V_0\phi^2}{M_{\rm P}^2}\right)}\right]\left[\varepsilon - \frac{3\kappa V_0\phi^2}{M_{\rm P}^2}\right]^{-1}, \qquad (4.15)
$$

where $V_0 = \frac{1}{2}m^2$, and m is mass of scalar field.

4.2 Acceleration condition of NMDC in Palatini formalism

Klein-Gordon equation of NMDC modified gravity in Palatini formalism is shown in [25] under slow-roll approximation, it is

$$
\begin{aligned}
&\ddot{\phi}\left[\varepsilon - \frac{9\kappa\dot{H}}{2}\left(1 - \frac{\kappa\dot{\phi}^2}{M_{\rm P}^2}\right) - \frac{3\kappa H^2}{2}\left(5 - \frac{6\kappa\dot{\phi}^2}{M_{\rm P}^2}\right)\right] \\
&\qquad + 3H\dot{\phi}\left[\varepsilon - \left(\frac{\ddot{H}}{H} + 4\dot{H}\right)\kappa\left(1 - \frac{\kappa\dot{\phi}^2}{2M_{\rm P}^2}\right)\right] + V_{,\phi} \simeq 0. \qquad (4.16)
\end{aligned}
$$

During inflationary era, equation of motion (Eq. (4.16)) is approximately rewritten as

$$
\begin{aligned}
\ddot{\phi} &\simeq \frac{-V_{,\phi} - 3H\dot{\phi}\left[\varepsilon - \left(\cancelto{\simeq 0}{\frac{\ddot{H}}{H}} + 4\dot{H}\right)\kappa\left(1 - \frac{\kappa\dot{\phi}^2}{2M_{\mathrm{P}}^2}\right)\right]}{\varepsilon - \cancelto{\simeq 0}{\frac{9\kappa\dot{H}}{2}\left(1 - \frac{\kappa\dot{\phi}^2}{M_{\mathrm{P}}^2}\right)} - \frac{3\kappa H^2}{2}\left(5 - \frac{6\kappa\dot{\phi}^2}{M_{\mathrm{P}}^2}\right)} \\
\ddot{\phi} &\simeq \frac{-V_{,\phi} - 3H\dot{\phi}\left[\varepsilon - 4\dot{H}\kappa\left(1 - \frac{\kappa\dot{\phi}^2}{2M_{\mathrm{P}}^2}\right)\right]}{\varepsilon - \frac{3\kappa H^2}{2}\left(5 - \frac{6\kappa\dot{\phi}^2}{M_{\mathrm{P}}^2}\right)} \\
\ddot{\phi} &\simeq \frac{-V_{,\phi} - 3H\dot{\phi}\left(\varepsilon - 4\dot{H}\kappa\right)}{\varepsilon - \frac{15\kappa H^2}{2}} \qquad (4.17) \\
\ddot{\phi} &\simeq \frac{-V_{,\phi} - 3H\dot{\phi}\varepsilon}{\varepsilon - \frac{15\kappa H^2}{2}} \qquad (4.18)
\end{aligned}
$$

where $\left|\ddot{H}/H\right| \ll |\dot{H}| \ll |H^2|$ and $0 < |\dot{\phi}| \ll 1$. Eq.(4.17) is a second order approximation for it is used to derive very late time trajectory. $4\dot{H}\kappa$ is negligible in Eq.(4.18) as a result of ε domination.

Modified Friedman equation is derived in [25] with slow-rolling field,

$$H^2 \simeq \frac{\rho_{\mathrm{tot}}}{3M_{\mathrm{P}}^2}\left[1 + \frac{3\kappa\dot{\phi}^2}{2M_{\mathrm{P}}^2}(1 + w_{\mathrm{eff}})\right], \qquad (4.19)$$

where $w_{\mathrm{eff}} = w_\phi = P_\phi/\rho_\phi$. We differentiate Eq. (4.19) with respect to time and fluid equation, $\dot{\rho} = -3H(\rho + P)$, is substituted

$$\begin{aligned}
H^2 \simeq &\quad \frac{1}{3M_{\rm P}^2}\left[\rho_\phi + \frac{3\kappa\dot\phi^2}{2M_{\rm P}^2}(\rho_\phi + P_\phi)\right] \\
2H\dot H \simeq &\quad \frac{1}{3M_{\rm P}^2}\left[\dot\rho_\phi + \frac{3\kappa}{2M_{\rm P}^2}(2\dot\phi\ddot\phi)(\rho_\phi + P_\phi) + \frac{3\kappa\dot\phi^2}{2M_{\rm P}^2}(\dot\rho_\phi + \dot P_\phi)\right] \\
\dot H \simeq &\quad \frac{1}{2M_{\rm P}^2}\left[\frac{\dot\rho_\phi}{3H} + \frac{\kappa\dot\phi\ddot\phi}{HM_{\rm P}^2}(\rho_\phi + P_\phi) + \frac{3\kappa\dot\phi^2}{2M_{\rm P}^2}\left(\frac{\dot\rho_\phi}{3H} + \frac{\dot P_\phi}{3H}\right)\right] \\
\simeq &\quad \frac{1}{2M_{\rm P}^2}\left\{-(\rho_\phi + P_\phi) + \frac{\kappa\dot\phi\ddot\phi}{HM_{\rm P}^2}(\rho_\phi + P_\phi) + \frac{3\kappa\dot\phi^2}{2M_{\rm P}^2}\left[-(\rho_\phi + P_\phi) + \frac{\dot P_\phi}{3H}\right]\right\} \\
\simeq &\quad \frac{1}{2M_{\rm P}^2}\left[-\varepsilon\dot\phi^2 + \frac{\kappa\dot\phi\ddot\phi}{HM_{\rm P}^2}\varepsilon\dot\phi^2 + \frac{3\kappa\dot\phi^2}{2M_{\rm P}^2}\left(-\varepsilon\dot\phi^2 + \frac{\varepsilon\dot\phi\ddot\phi - V,_\phi\dot\phi}{3H}\right)\right] \\
\simeq &\quad \frac{1}{2M_{\rm P}^2}\left(-\varepsilon\dot\phi^2 + \frac{\kappa\varepsilon\dot\phi^3\ddot\phi}{HM_{\rm P}^2} - \frac{3\varepsilon\kappa\dot\phi^4}{2M_{\rm P}^2} + \frac{\kappa\varepsilon\dot\phi^3\ddot\phi}{2HM_{\rm P}^2} - \frac{\kappa\dot\phi^3V,_\phi}{2HM_{\rm P}^2}\right) \\
\dot H \simeq &\quad -\frac{\varepsilon\dot\phi^2}{2M_{\rm P}^2} + \frac{3\kappa\varepsilon\dot\phi^3\ddot\phi}{4HM_{\rm P}^4} - \frac{3\varepsilon\kappa\dot\phi^4}{4M_{\rm P}^4} - \frac{\kappa\dot\phi^3V,_\phi}{4HM_{\rm P}^4}. \qquad (4.20)
\end{aligned}$$

$\dot H$ can be derived in another way. Using a density and pressure of scalar field as $\rho_\phi = \varepsilon\dot\phi^2/2 + V$ and $P_\phi = \varepsilon\dot\phi^2/2 - V$ respectively,

$$\begin{aligned}
H^2 &\simeq \frac{\rho_\phi}{3M_{\rm P}^2}\left[1 + \frac{3\kappa\dot\phi^2}{2M_{\rm P}^2}\left(1 + \frac{P_\phi}{\rho_\phi}\right)\right] \\
&\simeq \frac{1}{3M_{\rm P}^2}\left[\rho_\phi + \frac{3\kappa\dot\phi^2}{2M_{\rm P}^2}(\rho_\phi + P_\phi)\right] \qquad (4.21)\\
&\simeq \frac{1}{3M_{\rm P}^2}\left[\frac{\varepsilon\dot\phi^2}{2} + V(\phi) + \frac{3\kappa\dot\phi^2}{2M_{\rm P}^2}\left(\frac{\varepsilon\dot\phi^2}{2} + V(\phi) + \frac{\varepsilon\dot\phi^2}{2} - V(\phi)\right)\right] \\
&\simeq \frac{1}{3M_{\rm P}^2}\left(\frac{\varepsilon\dot\phi^2}{2} + V(\phi) + \frac{3\kappa\dot\phi^2}{2M_{\rm P}^2}\varepsilon\dot\phi^2\right) \qquad (4.22)\\
&\simeq \frac{1}{3M_{\rm P}^2}\left(\frac{\varepsilon\dot\phi^2}{2} + V(\phi) + \frac{3\kappa\varepsilon\dot\phi^4}{2M_{\rm P}^2}\right) \qquad (4.23)
\end{aligned}$$

where $w_{\rm eff} = P_{\rm tot}/\rho_{\rm tot} \simeq P_\phi/\rho_\phi$. We assumed here that the scalar field is dominant. Differentiating Eq. (4.23) with respect to time we obtain slow-rolling of $\dot H$,

$$
\begin{aligned}
2H\dot{H} &\approx \frac{1}{3M_{\mathrm{P}}^2}\left(\varepsilon\dot{\phi}\ddot{\phi} + V_{,\phi}\dot{\phi} + \frac{6\kappa\varepsilon\dot{\phi}^3\ddot{\phi}}{M_{\mathrm{P}}^2}\right) \\
\dot{H} &\approx \frac{1}{6HM_{\mathrm{P}}^2}\left(\varepsilon\dot{\phi}\ddot{\phi} + V_{,\phi}\dot{\phi} + \frac{6\kappa\varepsilon\dot{\phi}^3\ddot{\phi}}{M_{\mathrm{P}}^2}\right).
\end{aligned} \tag{4.24}
$$

Eq. (4.24) can be transformed to Eq. (4.20) by substituting Klein-Godon equation Eq. (4.16) into $\ddot{\phi}$ approximately.

The acceleration, which is defined by $\ddot{a}/a \equiv \dot{H} + H^2$, is found from Eq. (4.22) and Eq. (4.20),

$$
\begin{aligned}
\frac{\ddot{a}}{a} \simeq & -\frac{\varepsilon\dot{\phi}^2}{2M_{\mathrm{P}}^2} + \frac{3\kappa\varepsilon\dot{\phi}^3\ddot{\phi}}{4HM_{\mathrm{P}}^4} - \frac{3\varepsilon\kappa\dot{\phi}^4}{4M_{\mathrm{P}}^4} - \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4HM_{\mathrm{P}}^4} \\
& + \frac{\varepsilon\dot{\phi}^2}{6M_{\mathrm{P}}^2} + \frac{V(\phi)}{3M_{\mathrm{P}}^2} + \frac{3\kappa\varepsilon\dot{\phi}^4}{6M_{\mathrm{P}}^4} \\
\simeq & \frac{-\varepsilon\dot{\phi}^2}{3M_{\mathrm{P}}^2} + \frac{3\kappa\varepsilon\dot{\phi}^3\ddot{\phi}}{4HM_{\mathrm{P}}^4} - \frac{\kappa\varepsilon\dot{\phi}^4}{4M_{\mathrm{P}}^4} - \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4HM_{\mathrm{P}}^4} + \frac{V(\phi)}{3M_{\mathrm{P}}^2} \\
\simeq & \frac{-\varepsilon\dot{\phi}^2}{3M_{\mathrm{P}}^2}\left(1 + \frac{3\kappa\dot{\phi}^2}{4M_{\mathrm{P}}^2}\right) + \frac{V(\phi)}{3M_{\mathrm{P}}^2} + \frac{3\kappa\varepsilon\dot{\phi}^3}{4HM_{\mathrm{P}}^4}\left(\ddot{\phi} - \frac{V_{,\phi}}{3\varepsilon}\right).
\end{aligned} \tag{4.25}
$$

Substituting Eq. (4.18) in Eq. (4.25),

$$
\begin{aligned}
\frac{\ddot{a}}{a} \simeq & \frac{-\varepsilon\dot{\phi}^2}{3M_{\mathrm{P}}^2} + \frac{V(\phi)}{3M_{\mathrm{P}}^2} + \frac{3\kappa\varepsilon\dot{\phi}^3}{4HM_{\mathrm{P}}^4}\left[\left(\frac{-V_{,\phi} - 3H\dot{\phi}\varepsilon}{\varepsilon - \frac{15\kappa H^2}{2}}\right) - \frac{V_{,\phi}}{3\varepsilon}\right] \\
\simeq & \frac{-\varepsilon\dot{\phi}^2}{3M_{\mathrm{P}}^2} + \frac{V(\phi)}{3M_{\mathrm{P}}^2} + \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4HM_{\mathrm{P}}^4}\left[\left(\frac{-3\varepsilon - 9H\varepsilon^2\dot{\phi}/V_{,\phi}}{\varepsilon - \frac{15\kappa H^2}{2}}\right) - 1\right] \\
\simeq & \frac{-\varepsilon\dot{\phi}^2}{3M_{\mathrm{P}}^2} + \frac{V(\phi)}{3M_{\mathrm{P}}^2} + \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4HM_{\mathrm{P}}^4}\left(\frac{-3\varepsilon - 9H\varepsilon^2\dot{\phi}/V_{,\phi} - \varepsilon + \frac{15\kappa H^2}{2}}{\varepsilon - \frac{15\kappa H^2}{2}}\right) \\
\simeq & \frac{-\varepsilon\dot{\phi}^2}{3M_{\mathrm{P}}^2} + \frac{V(\phi)}{3M_{\mathrm{P}}^2} - \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4HM_{\mathrm{P}}^4}\left(\frac{4\varepsilon + 9H\varepsilon^2\dot{\phi}/V_{,\phi} - \frac{15\kappa H^2}{2}}{\varepsilon - \frac{15\kappa H^2}{2}}\right) \\
\simeq & \frac{-\varepsilon\dot{\phi}^2}{3M_{\mathrm{P}}^2} + \frac{V(\phi)}{3M_{\mathrm{P}}^2} - \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4HM_{\mathrm{P}}^4}\left(\frac{4\varepsilon - \frac{15\kappa H^2}{2}}{\varepsilon - \frac{15\kappa H^2}{2}}\right),
\end{aligned} \tag{4.26}
$$

the $\dot{\phi}^4$ terms can be neglected in slow-roll regime where non-coupling part is $(-\varepsilon\dot{\phi}^2+$

$V)/3M_{\rm P}^2$. Friedmann equation,Eq. (4.23), is substituted to Eq. (4.26)

$$\frac{\ddot{a}}{a} \simeq \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V(\phi)}{3M_{\rm P}^2} - \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4HM_{\rm P}^4}\left[\frac{4\varepsilon - \frac{15\kappa\left(\frac{\varepsilon\dot{\phi}^2}{6M_{\rm P}^2}+\frac{1}{3M_{\rm P}^2}V(\phi)+\frac{\kappa\varepsilon\dot{\phi}^4}{2M_{\rm P}^4}\right)}{2}}{\varepsilon - \frac{15\kappa\left(\frac{\varepsilon\dot{\phi}^2}{6M_{\rm P}^2}+\frac{1}{3M_{\rm P}^2}V(\phi)+\frac{\kappa\varepsilon\dot{\phi}^4}{2M_{\rm P}^4}\right)}{2}}\right] \tag{4.27}$$

Considering chaotic inflation potential $V(\phi) = V_0\phi^n$ which $n = 2$ and in the range $|\kappa| \ll |M_{\rm P}^2/V|$. It should be of sub-Plankian regime[25] that allows binomial approximation, therefore

$$\begin{aligned}
\dot{H} + H^2 \simeq\ & \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V(\phi)}{3M_{\rm P}^2} - \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4HM_{\rm P}^4}\left(4\varepsilon - \frac{15\kappa H^2}{2}\right)\left(\varepsilon + \frac{15\kappa H^2}{2}\right) \\
\simeq\ & \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V(\phi)}{3M_{\rm P}^2} - \frac{\kappa\dot{\phi}^3 V_{,\phi}}{4\left(\frac{\sqrt{V(\phi)}}{\sqrt{3}M_{\rm P}}\right)M_{\rm P}^4}\left[4\varepsilon - \frac{15\kappa\left(\frac{V(\phi)}{3M_{\rm P}^2}\right)}{2}\right]\left[\varepsilon + \frac{15\kappa\left(\frac{V(\phi)}{3M_{\rm P}^2}\right)}{2}\right] \\
\simeq\ & \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V(\phi)}{3M_{\rm P}^2} - \frac{\sqrt{3}\kappa\dot{\phi}^3 V_{,\phi}}{4\sqrt{V(\phi)}M_{\rm P}^3}\left(4\varepsilon - \frac{5\kappa V(\phi)}{2M_{\rm P}^2}\right)\left(\varepsilon + \frac{5\kappa V(\phi)}{2M_{\rm P}^2}\right) \\
\simeq\ & \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V(\phi)}{3M_{\rm P}^2} - \frac{\sqrt{3}\kappa\dot{\phi}^3 V_{,\phi}}{4\sqrt{V(\phi)}M_{\rm P}^3}\left(4\varepsilon^2 - \frac{5\kappa\varepsilon V(\phi)}{2M_{\rm P}^2} + \frac{10\kappa\varepsilon V(\phi)}{M_{\rm P}^2} - \frac{25\kappa^2\varepsilon^2 V(\phi)^2}{4M_{\rm P}^4}\right) \\
\simeq\ & \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V(\phi)}{3M_{\rm P}^2} - \frac{\sqrt{3}\kappa\dot{\phi}^3 V_{,\phi}}{4\sqrt{V(\phi)}M_{\rm P}^3}\left(4\varepsilon^2 + \frac{15\kappa\varepsilon V(\phi)}{2M_{\rm P}^2} - \cancelto{\simeq 0}{\frac{25\kappa^2\varepsilon^2 V(\phi)^2}{4M_{\rm P}^4}}\right) \\
\simeq\ & \frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V(\phi)}{3M_{\rm P}^2} - \frac{\sqrt{3}\kappa\dot{\phi}^3 V_{,\phi}}{\sqrt{V(\phi)}M_{\rm P}^3}\left(1 + \frac{15\kappa\varepsilon V(\phi)}{8M_{\rm P}^2}\right).
\end{aligned} \tag{4.28}$$

The acceleration condition reads, $\ddot{a}/a > 0$ and

$$\frac{-\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} + \frac{V(\phi)}{3M_{\rm P}^2} - \frac{\sqrt{3}\kappa\dot{\phi}^3 V_{,\phi}}{\sqrt{V(\phi)}M_{\rm P}^3}\left(1 + \frac{15\kappa\varepsilon V(\phi)}{8M_{\rm P}^2}\right) > 0. \tag{4.29}$$

The condition is computed where acceleration expansion of universe is, for chaotic inflation potential is applied, how much magnitude of kinetic energy of scalar field are the expansion.

$$\begin{aligned}
\frac{\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} &< \frac{V(\phi)}{3M_{\rm P}^2} - \frac{\sqrt{3}\kappa\dot{\phi}^3 V_{,\phi}}{\sqrt{V(\phi)}M_{\rm P}^3}\left(1 + \frac{15\kappa\varepsilon V(\phi)}{8M_{\rm P}^2}\right) \\
\frac{\varepsilon\dot{\phi}^2}{3M_{\rm P}^2} &< \frac{V(\phi)}{3M_{\rm P}^2}\left[1 - \frac{3\sqrt{3}\kappa\dot{\phi}^3 V_{,\phi}}{V(\phi)\sqrt{V(\phi)}M_{\rm P}}\left(1 + \frac{15\kappa\varepsilon V(\phi)}{8M_{\rm P}^2}\right)\right] \\
\varepsilon\dot{\phi}^2 &< V(\phi)\left[1 - \frac{3\sqrt{3}\kappa}{M_{\rm P}}\left(\frac{V_{,\phi}}{V(\phi)\sqrt{V(\phi)}}\right)\dot{\phi}^3\left(1 + \frac{15\kappa\varepsilon V(\phi)}{8M_{\rm P}^2}\right)\right]
\end{aligned} \tag{4.30}$$

The Eq. (4.18) is approximated to $\dot{\phi} \approx -V_{,\phi}/(3H\varepsilon)$ by very late-time, $\ddot{\phi} \approx 0$, and $H \approx \sqrt{V}/\sqrt{3}M_{\rm P}$ such that

$$\dot{\phi} \approx -\frac{V_{,\phi}\, M_{\rm P}}{\sqrt{3V}\varepsilon},$$

therefore

$$\begin{aligned}
\varepsilon\dot{\phi}^2 &< V(\phi)\left[1 - \frac{3\sqrt{3}\kappa}{M_{\rm P}}\left(\frac{V_{,\phi}}{V(\phi)\sqrt{V(\phi)}}\right)\left(-\frac{M_{\rm P}V_{,\phi}}{\sqrt{3}\sqrt{V(\phi)\varepsilon}}\right)^3\left(1 + \frac{15\kappa\varepsilon V(\phi)}{8M_{\rm P}^2}\right)\right]\\
\varepsilon\dot{\phi}^2 &< V(\phi)\left[1 + \frac{\kappa M_{\rm P}^2}{\varepsilon}\left(\frac{V_{,\phi}^4}{V(\phi)^3}\right)\left(1 + \frac{15\kappa\varepsilon V(\phi)}{8M_{\rm P}^2}\right)\right]\\
\varepsilon\dot{\phi}^2 &< V(\phi)\left[1 + \frac{\kappa M_{\rm P}^2}{\varepsilon}\left(\frac{\sqrt{2\epsilon_{V,{\rm GR}}}^3}{M_{\rm P}^3}\right)V_{,\phi}\left(1 + \frac{15\kappa\varepsilon V(\phi)}{8M_{\rm P}^2}\right)\right]\\
\varepsilon\dot{\phi}^2 &< V(\phi)\left[1 + \frac{\kappa M_{\rm P}^2}{\varepsilon}\left(\frac{\sqrt{2\epsilon_{V,{\rm GR}}}^3}{M_{\rm P}^3}\right)V_{,\phi}\left(1 + \frac{15\kappa\varepsilon V(\phi)}{8M_{\rm P}^2}\right)\right]\\
\varepsilon\dot{\phi}^2 &< V(\phi)\left[1 + \frac{\kappa}{\varepsilon M_{\rm P}}\sqrt{2\epsilon_{V,{\rm GR}}}^3 V_{,\phi}\left(1 + \frac{15\kappa\varepsilon V(\phi)}{8M_{\rm P}^2}\right)\right]. \qquad (4.31)
\end{aligned}$$

$\epsilon_{V,{\rm GR}} \equiv M_{\rm P}^2(V_{,\phi}/V)^2/2$ is slow-roll parameter in which chaotic inflation potential, $V(\phi) = V_0\phi^2$, is considered so they gives

$$\begin{aligned}
\epsilon_{V,{\rm GR}} &= \frac{M_{\rm P}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2\\
&= \frac{M_{\rm P}^2}{2}\left(\frac{2V_0\phi}{V_0\phi^2}\right)^2\\
&= \frac{2M_{\rm P}^2}{\phi^2}. \qquad (4.32)
\end{aligned}$$

The acceleration condition is hence,

$$\begin{aligned}
\varepsilon\dot{\phi}^2 &< V_0\phi^2\left[1 + \frac{\kappa}{\varepsilon M_{\rm P}}\sqrt{2\left(\frac{2M_{\rm P}^2}{\phi^2}\right)}^3(2V_0\phi)\left(1 + \frac{15\kappa\varepsilon V_0\phi^2}{8M_{\rm P}^2}\right)\right]\\
\varepsilon\dot{\phi}^2 &< V_0\phi^2\left[1 + \frac{\kappa}{\varepsilon M_{\rm P}}\left(\frac{2M_{\rm P}}{\phi}\right)^3(2V_0\phi)\left(1 + \frac{15\kappa\varepsilon V_0\phi^2}{8M_{\rm P}^2}\right)\right]\\
\varepsilon\dot{\phi}^2 &< V_0\phi^2\left[1 + \frac{\kappa}{\varepsilon}\left(\frac{8M_{\rm P}^2}{\phi^2}\right)(2V_0)\left(1 + \frac{15\kappa\varepsilon V_0\phi^2}{8M_{\rm P}^2}\right)\right]\\
\varepsilon\dot{\phi}^2 &< V_0\phi^2\left[1 + \left(\frac{16V_0\kappa M_{\rm P}^2}{\varepsilon\phi^2}\right) + 30\kappa^2V_0^2\right]. \qquad (4.33)
\end{aligned}$$

Equations of motion of the model in metric and Palatini formalism are applied in slow-roll regime that is derived by [24, 25]. $\dot{H}$ are derived in slow-roll regime to use in acceleration condition $\ddot{a}/a \equiv \dot{H} + H^2$. We show mathematical representations of these two conditions ,Eq. (4.15) and Eq. (4.33), where power law chaotic potential ,$V(\phi) = V_0\phi^2$, is applied to the conditions.

CHAPTER V

DYNAMICAL PHASE PORTRAITS OF NMDC MODEL

5.1 Acceleration region

The condition for accelerating expansion of standard general relativity is

$$\varepsilon\dot{\phi}^2 < V_0\phi^2. \tag{5.1}$$

In NMDC model, the acceleration conditions are modified in Section. (4.1) and Section. (4.2) with slow-roll approximation.

5.1.1 Metric formalism case of the NMDC model

Acceleration condition of NMDC in metric formalism is expressed by Eq.(4.15)

$$\dot{\phi}^2 < V_0\phi^2\left[1-\left(\frac{4\kappa V_0}{M_{\rm P}\left(\varepsilon-\frac{\kappa V_0\phi^2}{M_{\rm P}^2}\right)}\right)\right]\Big/\left[\varepsilon-\frac{3\kappa V_0\phi^2}{M_{\rm P}^2}\right].$$

Let $\kappa < 0$, acceleration occurs for kinetic energy of scalar field that is considered needs to be less than potential part. In metric formalism case, acceleration can be attained easier than GR.

5.1.2 Palatini formalsim case of the NMDC model

For NMDC in Palatini formalism, there is Eq.(4.33)

$$\varepsilon\dot{\phi}^2 < V_0\phi^2\left[1+\left(\frac{16V_0\kappa M_{\rm P}^2}{\varepsilon\phi^2}\right)\right].$$

The acceleration is harder to be achieved since the field must move even slower than GR.

5.2 Autonomous system

Autonomous system is differential equation system which do not explicitly depend on time. Suppose that there exist n differential equations in a system.

$$\begin{aligned}
\tfrac{\mathrm{d}x_1}{\mathrm{d}t} = \dot{x}_1 \quad &= f_1(x_1, x_2, x_3, ..., x_n) \\
\tfrac{\mathrm{d}x_2}{\mathrm{d}t} = \dot{x}_2 \quad &= f_2(x_1, x_2, x_3, ..., x_n) \\
\tfrac{\mathrm{d}x_3}{\mathrm{d}t} = \dot{x}_3 \quad &= f_3(x_1, x_2, x_3, ..., x_n) \\
\vdots \qquad\quad &\vdots \qquad \vdots \\
\tfrac{\mathrm{d}x_n}{\mathrm{d}t} = \dot{x}_n \quad &= f_n(x_1, x_2, x_3, ..., x_n),
\end{aligned}$$

where $\mathrm{d}x_1/\mathrm{d}t = \dot{x}_1, \mathrm{d}x_2/\mathrm{d}t = \dot{x}_2, \mathrm{d}x_3/\mathrm{d}t = \dot{x}_3 ..., \mathrm{d}x_n/\mathrm{d}t = \dot{x}_n$ are time derivative of variable x and $f_1(x_1, x_2, x_3, ..., x_n), ..., f_n(x_1, x_2, x_3, ..., x_n)$ are flow or velocity function of x that are not explicitly time-dependent. Therefore dynamical system can be given in an autonomous form [27, 28].

5.3 Second-order differential equation

In some case, a second-order differential equation, Eq. (5.2) there exits derivative of the flow function. One can introduce new variable such that the second-order differential equation can be reduced to first-order differential equation,

$$\ddot{x} = f(x, \dot{x}). \tag{5.2}$$

In doing this, we propose new variable y that is defined by

$$y \equiv \dot{x}. \tag{5.3}$$

From Eq. (5.2) and Eq. (5.3), autonomous equations are

$$\dot{x} = y \tag{5.4}$$

$$\dot{y} = f(x, y). \tag{5.5}$$

The differential equation system of Eq. (5.4) and Eq. (5.5) is called two dimensional closed autonomous system.

5.4 NMDC to gravity of scalar field autonomous and phase portrait

The autonomous systems are combination of the Klein-Gordon and Friedmann equations in slow-roll regime. Three cases are GR, NMDC model in metric and Palatini formalisms. We approximated Hubble variable to $H = \frac{\sqrt{V}}{\sqrt{3}M_{\rm P}}$ so that dimensions of equation can be reduced to two dimensions.

5.4.1 Phase portrait and acceleration region of scalar field in GR limit ($\kappa = 0$)

When $\kappa = 0$ is set, the GR limit is achieved from both metric and Palatini cases,

$$\ddot{\phi} = \frac{-V_{,\phi} - 3H\dot{\phi}\varepsilon}{\varepsilon}. \tag{5.6}$$

Define $\dot{\phi} \equiv \psi$, together with Eq. (5.6), the system is of,

$$\begin{aligned} \dot{\phi} &= \psi \\ \dot{\psi} &= -V_{,\phi} - 3H\psi, \end{aligned}$$

where we set $\varepsilon = 1$ (for canonical scalar field). Phase portrait of this autonomous system and acceleration condition Eq. (5.1) of GR case are shown in Fig. 1

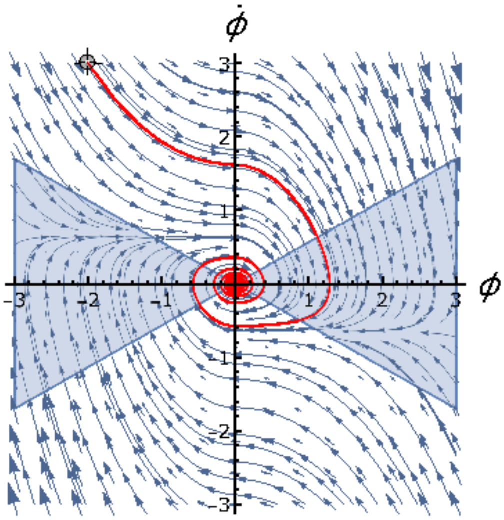

Figure 1: Phase portrait with acceleration region of standard general relativity case for canonical field ($\varepsilon = 1$), $m = 0.8$ and $M_{\rm P} = 1.0$

5.4.2 Phase portrait of NMDC model in metric formalism

Equation of motion of NMDC model in metric formalism is Eq. (4.2) but it is second-order differential equation as,

$$\ddot{\phi} = -\frac{V_{,\phi} - 3H\dot{\phi}(\varepsilon - 3\kappa H^2)}{(\varepsilon - 3\kappa H^2)}. \tag{5.7}$$

New variable is introduced by $\dot{\phi} = \psi$ as same as GR case hence

$$\begin{aligned}
\dot{\phi} &= \psi \\
\dot{\psi} &= -\frac{V_{,\phi} - 3H\psi(\varepsilon - 3\kappa H^2)}{\varepsilon - 3\kappa H^2},
\end{aligned}$$

phase portrait of this system is shown as well as the acceleration region in Fig. 2.

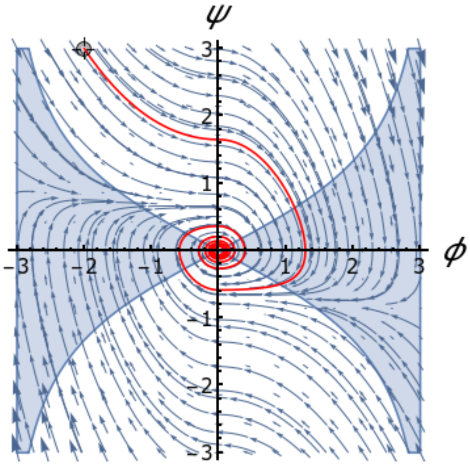

Figure 2: Phase portrait with acceleration region of NMDC to gravity in metric formalism by setting $\kappa = 0.1$, $m = 0.8$ and $M_{\rm P} = 1.0$

5.4.3 Phase portrait of NMDC model in Palatini formalism

NMDC in Palatini formalism, autonomous system is derived from equation of motion Eq. (4.18)

$$\ddot{\phi} \simeq \frac{-V_{,\phi} - 3H\dot{\phi}\varepsilon}{\left(\varepsilon - \frac{15\kappa H^2}{2}\right)}$$

hence

$$\begin{aligned} \dot{\phi} &= \psi \\ \dot{\psi} &= -\frac{-V_{,\phi} - 3H\psi\varepsilon}{\left(\varepsilon - \frac{15\kappa H^2}{2}\right)}, \end{aligned}$$

and phase portrait is shown in Fig. 3.

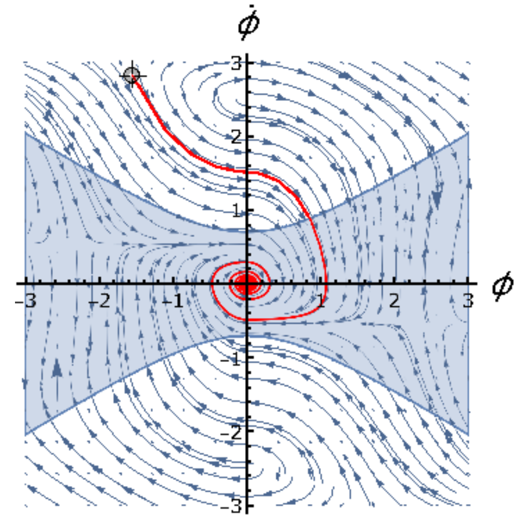

Figure 3: Phase portrait with acceleration region of NMDC to gravity in Palatini formalism by setting $\kappa = 0.3$, $m = 0.8$ **and** $M_{\mathrm{P}} = 1.0$

CHAPTER VI

CONCLUSIONS

In Non-Minimal derivative coupling (NMDC) to gravity model, we study dynamical behavior of canonical scalar field which the derivative term is coupling to Einstein tensor in special case as $\kappa_1 R\phi_{,\mu}\phi^{,\nu}$ and $\kappa_2 R^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ where $\kappa \equiv \kappa_2 = -2\kappa_1$. In flat FLRW universe, quasi-de Sitter phase can occur when $\kappa > 0$ and there is initial singularity for $\kappa < 0$ at the inflation age. In Palatini formalism, Ricci scalar and Ricci tensor are function of dynamical connection field (Γ) hence the Einstein tensor is a function of connection field, ($\tilde{G}_{\mu\nu}(\Gamma)$).

Cosmological equations of NMDC model are derived and approximated in slow-roll condition both in metric and Palatini formalism [25]. We derived acceleration conditions of NMDC-Palatini model and see that the accelerations can be increased for $\kappa > 0$ Eq. (4.33) at inflation. For power law potential of chaotic inflation, $V \propto \phi^2$, we have presented the slow-roll autonomous systems and phase portraits of the NMDC model both in metric and Palatini formalism with slow-roll regime in 2 dimensional system. In NMDC-metric model, comparison between the NMDC model and GR show that NMDC effect enlarges the acceleration at large field. However NMDC-Palatini model affects to enhance acceleration at small field in comparison with GR case, there are new saddle point in phase portrait. Although $\kappa > 0$ of NMDC model can enhance acceleration at inflation, the model allows superluminal sound speed of scalar field traveling. Future work should change the potential and $\kappa < 0$ to avoid superluminal traveling of scalar field.

REFERENCES

REFERENCES

[1] Riess, A. G., Filippenko, A. V., Challis, P., Clocchiatti, A., Diercks, A., Garnavich, P. M., ..., Kirshner, R. P., et al. (1998). Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal, 116*(3), 1009.

[2] Perlmutter, S., Aldering, G., Della Valle, M., Deustua, S., Ellis, R. S., Fabbro, S., ..., Hook, I. M., et al. (1998). Discovery of a supernova explosion at half the age of the universe. *Nature, 391*(6662), 51–54.

[3] Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R. A., Nugent, P., Castro, P. G., ..., Groom, D. E., et al. (1999). Measurements of ω and λ from 42 high-redshift supernovae. *The Astrophysical Journal, 517*(2), 565.

[4] Komatsu, E., Smith, K. M., Dunkley, J., Bennett, C. L., Gold, B., Hinshaw, G., ..., Page, L., et al. (2011). Seven-year wilkinson microwave anisotropy probe (WMAP*) observations: cosmological interpretation. *The Astrophysical Journal Supplement Series, 192*(2), 18.

[5] Tegmark, M., Strauss, M. A., Blanton, M. R., Abazajian, K., Dodelson, S., Sandvik, H., ..., Bahcall, N. A., et al. (2004). Cosmological parameters from sdss and wmap. *Physical Review D, 69*(10), 103501.

[6] Guth, A. H. (1981). The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems. *Physical Review D, 23*, 347–356.

[7] Ratra, B. & Peebles, P. J. E. (1988). Cosmological consequences of a rolling homogeneous scalar field. *Physical Review D, 37*(12), 3406.

[8] Chiba, T., Okabe, T., & Yamaguchi, M. (2000). Kinetically driven quintessence. *Physical Review D, 62*(2), 023511.

[9] De Felice, A., & Tsujikawa, S. (2010). f(R) Theories. *Living Reviews in Relativity, 13*(1), 3.

[10] Copeland, E. J., Sami, M., & Tsujikawa, S. (2006). Dynamics of dark energy. *International Journal of Modern Physics D, 15*(11), 1753–1935.

[11] Nojiri, S., & Odintsov, S. D. (2011). Unified cosmic history in modified gravity: from f (R) theory to lorentz non-invariant models. *Physics Reports, 505*(2), 59–144.

[12] Brans, C., & Dicke, R. H. (1961) Mach's principle and a relativistic theory of gravitation. *Physical Review, 124*(3), 925.

[13] Horndeski, G. W. (1974). Second-order scalar-tensor field equations in a four-dimensional space. *International Journal of Theoretical Physics, 10*(6), 363–384.

[14] Capozziello, S., & Lambiase, G. (1999). Nonminimal derivative coupling and the recovering of cosmological constant. *General Relativity and Gravitation, 31*(7), 1005–1014.

[15] Liu, H., & Tseytlin, A. A. (1998). D= 4 super yang-mills, d= 5 gauged supergravity and d= 4 conformal supergravity. *Nuclear Physics B, 533*(1-3), 88–108.

[16] Nojiri, S., & Odintsov, S. D. (1998). Conformal anomaly for dilaton coupled theories from AdS/CFT correspondence. *Physics Letters B, 444*(1-2), 92–97.

[17] Granda, L. N., & Cardona, W. (2010). General non-minimal kinetic coupling to gravity. *Journal of Cosmology and Astroparticle Physics, 2010*(07), 021.

[18] Sushkov, S. V. (2009). Exact cosmological solutions with nonminimal derivative coupling. *Physical Review D, 80*(10), 103505.

[19] Saridakis, E. N., & Sushkov, S. V. (2010). Quintessence and phantom cosmology with nonminimal derivative coupling. *Physical Review D, 81*(8), 083510.

[20] Germani, C., & Kehagias, A. (2010). New model of inflation with nonminimal derivative coupling of standard model higgs boson to gravity. *Physical review letters, 105*(1), 011302.

[21] Sushkov, S. V. (2012). Realistic cosmological scenario with nonminimal kinetic coupling. *Physical Review D, 85*(12), 123520.

[22] Tsujikawa, S. (2012). Observational tests of inflation with a field derivative coupling to gravity. *Physical Review D, 85*(8), 083518.

[23] Sadjadi, H. M. (2011). Super-acceleration in a nonminimal derivative coupling model. *Physical Review D, 83*(10), 107301.

[24] Gumjudpai, B., & Rangdee, P. (2015). Non-minimal derivative coupling gravity in cosmology. *General Relativity and Gravitation, 47*(11), 140.

[25] Kaewkhao, N., & Gumjudpai, B. (2018). Cosmology of non-minimal derivative coupling to gravity in palatini formalism and its chaotic inflation. *Physics of the Dark Universe, 20*, 20 – 27.

[26] Bassett, B. A., Tsujikawa, S., & Wands, D. (2006). Inflation dynamics and reheating. *Reviews of Modern Physics, 78*(2), 537.

[27] Hirsch, M. W., Smale, S., & Devaney, R. L. (2012). *Differential equations, dynamical systems, and an introduction to chaos.* Cambridge: Academic press.

[28] Jordan, D. W., & Smith, P. (1999). *Nonlinear ordinary differential equations: an introduction to dynamical systems*, volume 2. Oxford: Oxford University Press.

BIOGRAPHY

BIOGRAPHY

Name-Surname Somphoach Saichaemchan

Date of Birth May 19, 1999

Place of Birth Bangkok Province, Thailand

Address 108/1 Moo 5 Tambon Huaypai, Amphoe Saweangha, Angtong Province, Thailand 14150

Education Background

2014 B.S. (Physics), University of Phayao, Phayao, Thailand

Publications

Saichaemchan, S., & Gumjudpai, B., (2017). Non-minimal derivative coupling in Palatini cosmology: acceleration in chaotic inflation potential. *Journal of Physics: Conference Series, 901*(1), 012010.