FEATURES OF THE PRIMORDIAL COSMOLOGICAL PERTURBATIONS FROM MULTI-FIELD INFLATION

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ABSTRACT

In this thesis, we study some features of the non-gaussianity in the multi-field inflationary model with the curvature in the field space. Using the ΔN formalism, we compute bispectrum and also non-gaussianity parameter in the squeezed limit. For this inflationary model, we have found that the curvature in the field space can affect both the shape and amplitude of the non-gaussianity parameter. The amplitude of the backward non-gaussianity parameter can be enhanced by the magnitude of the curvature in the field space. Furthermore, shape of the non-gaussianity parameter can be largely effected by the curvature in field space for some ranges of parameters. Hence, the future observations of the squeezed bispectrum can put tight constrain on the curvature in the field space of multi-field inflationary models.

CHAPTER I

INTRODUCTIONS

1.1 Background and motivation

Looking as a perspective, nowadays, our universe is very smooth globally, i.e. homogeneous and isotropic on the scales larger than a few hundred megaparsecs. However, on smaller scales, some inhomogeneities and anisotropies, for example, the solar system, the galactic structures, etc, are observed. In the standard cosmology, the non-smoothness in the observable universe is a consequence of the small inhomogeneity and anisotropy in the early universe which can be viewed as perturbations around the smoothness. According to the Hubble law, the present universe is expanding. Inversely, as the further and further time away from the present, the universe in the history has to be smaller and smaller than the size observed nowadays. Based on this idea, some interesting troubles come out. Because, to obtain the homogeneous and isotropic universe as observed today, we need some mechanisms for providing suitable initial conditions. Even if one computes the dynamics of the universe by adding up every recognized medium, i.e. matter and radiation, into the Einstein field equation, it is not enough to explain why the observed universe is spatially flat and why the observed universe homogeneous as well as isotropic on large scales. these two problems are called, respectively, flatness and horizon problems implying the insolvency of the hot big bang model. To solved these problems, inflation, the epoch in which the universe rapidly expand with the acceleration, is proposed. Not only solving these two problems, but inhomogeneities and anisotropies observed at the present can be explained as an effect of the quantum fluctuation of the inflaton field. The brief scenario is that, when the wavelength becomes longer than the Hubble radius, the quantum fluctuation of the inflaton will induce the classical perturbations, e.g. metric perturbations and density perturbations, in the universe on large scales during inflation. The classical perturbations will then be the seed of inhomogeneities and anisotropies in the late-time universe when its wavelength is again smaller than the Hubble radius after reheating.

The data from the observations indicates that the hot big bang model with the inflation is the most reliable model at this moment. Then, one can study the primordial universe and its consequent effect on the observable universe via the quantities being able to indicate physical properties of the whole observable universe, i.e. smooth and perturbed parts. These quantities are n-point correlation functions which are ensemble averages of the n times product of considered quantities. In cosmology, the mean average which is ensemble average of a single quantity can be used to describe the smooth part of the observable universe, while the variance can be indicated as the two-point correlation function of the perturbations around the mean. In Gaussian case, it is enough to use only just mean and variance to describe the statistical properties of the perturbations, because every odd-point correlation functions vanish, and every even-point correlation function can be written in terms of the two-point correlation function.

By definition, the ensemble average is the average over numerous samples, however, practically, there is only one universe which we can observe, hence, to measure the n-point correlation function of any quantities in the observed universe, one has to use the ergodic hypothesis yielding the equivalence between ensemble and spatial averages. Since the n-point correlation function in the early universe of any physical quantity can be related to the present physical quantity. Furthermore, different inflationary models yield different predicted n-point correlation functions, then, we can use this fact to constrain inflationary models using the observation. For instance, so far, one knows that the universe is nearly smooth on large scales, and the evidence from observations indicates that odd-point correlation functions used to measure the non-Gaussianity approximately vanish. Hence, via this fact, one is allowed to rule out some models which yield high non-Gaussianities. Instead of n-point correlation functions, the prediction from a given inflationary model is more convenient to be presented in form of spectra which are Fourier transformations of n-point correlation functions, i.e. power spectrum and bispectrum correspond to two point and three-point correlation functions respectively. Using the cosmological perturbation theory, one can compute primordial perturbations for a particular inflationary model, and also one can relate the primordial perturbations with homogeneity and isotropy observed nowadays. However, in practice, for some complicated inflationary models, one may meet troubles from solving the evolution equation of perturbations directly. Fortunately, to avoid the difficulty, one is able to use an easier method which is called ΔN formalism. Using this method, one can compute evolutions of perturbations on large scales during inflation without solving full evolution equations for perturbations, and we will discuss the essential idea of the ΔN formalism later more in reviews of the literature.

Recently, theoretical investigations on the squeezed bispectrum have got a lot of attention, because it is possible to obtain more accurate data about the highly squeezed bispectrum in near future. The squeezed bispectrum is the spectrum in the case where one of the wave numbers of the perturbation is smaller than others. Nevertheless, it is hard to compute the highly squeezed mode of the bispectrum for general models, e.g. multi-field inflationary models, using the standard perturbation theory. However, it is easier to compute it via the ΔN formalism.

The technique for computing the highly squeezed bispectrum for multi-field inflation via the ΔN have been recently proposed[1, 2]. The investigations in those works are restricted for the flat field space. For curved field space, the equilateral bispectrum can also be computed using the ΔN formalism[3]. Definitions of the squeezed and equilateral bispectrum will be stated clearly in the review of the literature. In our research, the main target is to compute the squeezed bispectrum in the inflationary model with arbitrary curvature in the field space.

1.2 Objectives

- To compute the highly squeezed bispectrum for the inflationary model with the curvature in the field space.
- To investigate the influences of the curvature in the field space on the features of the squeezed bispectrum.

1.3 Frameworks

- Chapter 1

  We mention about introductions including the background and motivation which give the brief overview of this thesis, objectives, and frameworks.

- Chapter 2

  We will explain why we do need inflation. Also, the dynamics of each physical variable during inflation as well as some interesting models of inflation will be introduced in this chapter.

- Chapter 3

  The quantities called spectra and a useful method called ΔN formalism which are used both for study the dynamics of the perturbation in inflatons on large scales during inflation, will be defined.

- Chapter 4

  We will show the effects of the curvature in the field space on the non-gaussianity parameter, and the ways to compute them.

- Chapter 5

  The conclusions of this thesis will be mentioned.

CHAPTER II

INFLATION

2.1 Standard big bang model

Based on the cosmological principle which states that the universe is homogeneous and isotropic on large scales, the metric for the standard big-bang cosmology takes the form of the Friedmann-Robertson-Walker (FLRW) metric:

$$g_{\mu\nu} = a^2(t) \begin{pmatrix} -a^{-2}(t) & 0 & 0 & 0 \\ 0 & (1-Kr^2)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin^2(\theta) \end{pmatrix}, \tag{2.1}$$

where coordinates for this metric are t, r, θ, and ϕ, t is the cosmic time, and $a(t)$ is the scale factor. The factor K, which can be -1, 0 and 1, is the factor yielding the open, flat, and closed universes respectively. In the standard big-bang cosmology, the Friedmann and continuity equations used for describing the cosmic dynamics are given by, respectively

$$H^2 = \frac{8\pi}{3m_p^2}\rho - \frac{K}{a^2(t)}, \tag{2.2}$$

$$\dot{\rho_\alpha} + 3H(\rho_\alpha + p) = 0, \tag{2.3}$$

where a dot denotes the derivative with respect to time, $m_p \equiv 1/\sqrt{G}$ is the Planck mass, ρ is the total energy density of the universe, p is the total pressure, and the subscript α runs over r,m, and$d$ representing radition, matter, and dark energy respectively. The Hubble parameter is defined as $H \equiv \dot{a}/a$. The combination of

2.2 and 2.3 yields the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3m_p^2}(\rho + 3p). \tag{2.4}$$

To quantify the contribution of the matter to dynamics of the universe, the new quantity called density parameter, Ω, is defined via 2.2 as

$$\Omega = 1 + \frac{K}{H^2a^2}, \tag{2.5}$$

where $\Omega \equiv 8\pi\rho/3H^2m_p^2$. Also from the standard big-bang cosmology, dynamics of the universe is governed by the contained energy and matter. The role of each matter or energy component on dynamics of the universe can be determined for the epoch which each of them is dominant. For instance, during the radiation dominated epoch, main contributions to the total energy density come from the radition so that ρ on the right-hand side of the equation (2.2) is approximately the radiation energy density ρ_r. For radiation, the pressure p_r is related to ρ_r by $p_r = \rho_r/3$. Then, the equation (2.3) can be integrated and yield $\rho_r \propto a^{-4}$. Substituting the relation between ρ_r and a into the equation (2.2), one obtains $a \propto t^{1/2}$. Applying the previous method to the matter dominated universe and using $p_m = 0$ for matter, $a \propto t^{2/3}$ is obtained.

2.2 Problems and solutions

2.2.1 Flatness problem

The density parameter can be calculated back to the past until the Planck era is reached via the relation

$$\frac{\Omega_p - 1}{\Omega_0 - 1} = \frac{H_0^2 a_0^2}{H_p^2 a_p^2} \tag{2.6}$$

where the subscript p indicates the Planck time, and the subscript 0 refers to the present time. This equation is derived from 2.5. The ratio of scale factors is computed by using the relation $a \propto T^{-1}$ where T is the cosmic temperature or, equivalently, the temperature of the radiation in the universe. Using $T_p \approx 10^{19}\ GeV$ and $T_0 \approx 2.7\ K \simeq 10^{-13}\ GeV$, $a_0/a_p =$. The ratio of Hubble parameters is approximated to be $H_0^2/H_p^2 \approx \rho_0/\rho_p$ because of the spatially-flat property. Since energy densities of the universe at the present and the Planck time are $10^{-47}\ GeV^4$ and $10^{76}\ GeV^4$ respectively, $\Omega_p - 1$ is of the order of 10^{-59} compared with $\Omega_0 - 1$. In order to have the spatially-flat universe at the present, i.e., $\Omega - 1 \approx 0$, the energy density of the early universe is almost, very nearly, but not quite 1. Because of this unnaturally fine-tuned initial value of the energy parameter of the matter to have the flat universe, this is so called the flatness problem.

2.2.2 Horizon problem

In cosmology, the size of the observed universe at the present is of the order of the physical horizon scale l_0, which can be estimated as $l_0 \sim ct_0$, where c is the speed of light, and t_0 is the present time. Since the physical length scales expand via the increase of the scale factor, the magnitude of l_0 can be computed backward to an arbitrary time t_i as follows: multiplying l_0 by the ratio a_i/a_0, the magnitude of l_0 at t_i becomes

$$l_i \sim ct_0 \frac{a_i}{a_0}. \tag{2.7}$$

On the other hand, Computing the magnitude of the size of the causal region at t_i by relation $l = ct_i$, and comparing this magnitude with l_i, one obtains

$$\frac{l_i}{l} = \frac{a_i}{a_0} ct_0 \frac{1}{ct_i} = \frac{a_i}{a_0}\frac{t_0}{t_i} \approx \frac{a_i}{a_0}\frac{H_i}{H_0} \approx \frac{T_0}{T_i}\left(\frac{\rho_i}{\rho_0}\right)^{1/2}. \tag{2.8}$$

To derive the above equation, we use the following: the condition $\rho + p \geqslant 0$ which gives $H \sim t^{-1}$, the relation $a = T_r^{-1}$, and the approximation $H_i^2/H_0^2 \approx \rho_i/\rho_0$. Let t_i be the Planck time, t_p, the above ratio becomes

$$\frac{l_p}{l} \approx \frac{T_0}{T_p}\left(\frac{\rho_p}{\rho_0}\right)^{1/2}. \tag{2.9}$$

Using the same numerical values as 2.2.1, this ratio finally becomes

$$\frac{l_p}{l} \approx 10^{28}. \tag{2.10}$$

This indicates that the size of the observable universe is extreemly larger than the causal scale in the early universe, leading to the question how our universe which consists of a numerous causally disconnected regions can be so smooth, homogeneous and isotropic, on large scales nowadays.

2.2.3 Initial perturbation problem

Even though the observed universe is homogeneous and isotropic on large scales, there are some inhomogeneities and anisotropies, e.g., galaxies, clusters of galaxies, and etc., contained inside as well. To explain the existence of these inhomogeneities and anisotropies, small initial inhomogeneities and anisotropies in the early universe are required. We need some mechanisms to generate small initial inhomogeneities and anisotropies in the early universe. Nevertheless, a natural mechanism cannot be constructed in the standard big bang model. One of the problem is that the big bang can create a huge amout of inhomogeneities and anisotropies which are inconsistent with the observed inhomogeneities and anisotropies at the present.

2.2.4 Solutions

In previous sections, disadvantages of the standard big bang model have been mentioned. Now, let us mention about the way to solve those problems. The ratio on the right-hand side of 2.9 will be changed as well as a_iH_i/a_0H_0, i.e., the comoving Hubble radius ratio. Nevertheless, the order of this ratio can be replaced by $\dot{a}_i/\dot{a}_0$ because of the definition of the Hubble parameter. Using the idea of this epoch with this ratio, not only just more or much more than unity, to have this ratio in the order of unity is possible, because, applying the acceleration from the repulsive force, i.e., $\ddot{a} > 0$, it is possible to have $\dot{a}_i/\dot{a}_0 = 1$. Obviously, a_i and a_0 have been assumed to be used in and out of the special epoch respectively. Physically, based on this idea, the size of the observed universe at t_i and the particle horizon at t_i can be in the same order. It means that the smooth universe can be described as a small smooth region having rapidly expanded enough in the special epoch before becoming the observed universe today. In addition, this small region is actually contained inside the bigger one whose smoothness is ignored since it is larger than our observed universe by the way. Now, the problem 2.2.2 has been solved. For the problem 2.2.1, the solution can be recovered as well by recalling the equation

(2.5) and using $H = \dot{a}/a$. Hence, we obtain

$$(\Omega - 1)\dot{a}^2 = K. \tag{2.11}$$

Using the above equation, since K is constant, the dynamic of Ω is contributed by the dynamic of $\dot{a}$. Namely, since $\ddot{a} > 0$ in the special epoch, $\dot{a}$ increases as well. Then, in this epoch, $\Omega \to 1$ while t is increasing. This solves the problem 2.2.1 because, in this epoch, the density parameter is naturally fine tuned by the increasing of $\dot{a}$ automatically. Finally, for the solution of the problem 2.2.3, the key point is the decreasing of the comoving Hubble radius, $(aH)^{-1}$, and the concentration of radiation and matter of the universe. Because of the concentration of radiation and matter in this epoch, physics can be described by quantum field theory. Since, in this epoch, the quantum fluctuation plays an important role, then let us consider its behavior. Considering the wavelength of a fluctuation, λ, inside the small region whose size is L in the acceleration epoch, it is found that there is no any substance, e.g., radiation, matter, or etc, occurs because of the propagation of the fluctuation. Hence, the universe is still completely smooth. Nevertheless, via the evolution of $\dot{a}$, it can be shown that the universe is not always smooth. Since $\lambda \propto a$, the comoving wavelength, $\lambda_c = \lambda/a$, remains constant while $L \sim (aH)^{-1} = \dot{a}^{-1}$ rapidly decreases, then, after the smooth observed universe evolves for a while, there is a period that $\lambda_c \gg L$ exists. In this period, the fluctuation cannot propagate anymore. On the other hand, it is frozen inside and outside the smooth observed universe and becomes a classical substance. In addition, there are various quantities of the substance outside the observable universe, but, for the inside, the quantity is the same. The quantity of the substance inside evolves via a as the observable universe evolves. Hence, the observable universe is still smooth. However, by the way, this period is going to disappear because the universe evolves eventually into the deceleration epoch, and, finally, the wavelength of the fluctuation is again smaller than the observable universe. After it gets into the observable universe, there are so many little different quantities of the classical substance get into the observable universe as well. By small differences between each quantities of the classical substance, initial homogeneity and isotropy of the observable universe are caused. Hence, the existence of the acceleration epoch can yield the solution for the problem 2.2.3. Since the universe rapidly expands in this epoch, It is called inflation. However, to have inflation is not enough to have our observable universe at the present. Because, in inflation, the expansion is supposed to be contributed by the inflatons as long as possible until the universe reaches the initial state of our observable universe at the end of inflation. Then, $-\dot{H}/H$ is required to be so smaller than unity to keep the universe expands throughout inflation, and, to stop inflation at the end to let the deceleration epoch starts, $-\dot{H}/H$ is required to be of the order of unity as well. This condition is called the slow-roll condition, and $-\dot{H}/H$ is defined to be the slow-roll parameter.

2.3 Inflationary dynamics

As discussed in the introduction, one can start with an action which describes the dynamics of many inflatons,

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R + P(X, \phi^I)\right) \tag{2.12}$$

with

$$X = -\frac{1}{2} G_{IJ} \nabla_\mu \phi^I \nabla^\mu \phi^J \,, \tag{2.13}$$

where $8\pi G$ has been set to be 1 for simplicity. Throughout this paper, the summation convention will be used on the Greek and Latin indices both. The energy-momentum tensor derived from (2.12), takes the form

$$T^{\mu\nu} = P\, g^{\mu\nu} + P_{,X}\, G_{IJ}\, \nabla^\mu \phi^I \nabla^\nu \phi^J \,, \tag{2.14}$$

where $P_{,X}$ is the partial derivative of P with respect to X. The dynamics of the inflatons are governed by the equation of motion which can be obtained by varying this action with respect to the inflatons, and it takes the form

$$\nabla_\mu (P_{,X} G_{IJ} \nabla^\mu \phi^J) - \frac{1}{2} P_{,X} (\nabla_\mu \phi^K)(\nabla^\mu \phi^L) \partial_I G_{KL} + P_{,I} = 0 \,, \tag{2.15}$$

where G_{IJ} is the field space metric, and $P_{,I} \equiv dP/d\phi^I$.

2.3.1 Background evolution

In a spatially flat FLRW (Friedmann-Lemaitre-Robertson-Walker) space-time, the line element takes the form

$$ds^2 = -dt^2 + a^2 \delta_{ij} dx^i dx^j. \tag{2.16}$$

The energy-momentum tensor reduces to that of a perfect fluid with energy density

$$\rho = 2XP_{,X} - P \,, \tag{2.17}$$

P the pressure, and the inflatons are homogeneous. Hence, the Friedmann equations can be written in the form

$$H^2 = \frac{1}{3}(2XP_{,X} - P)\,, \tag{2.18}$$

and

$$\dot{H} = -XP_{,X}. \tag{2.19}$$

Actually, the second equation is the other form of the equation (2.4). The equations of motion (2.15) for the scalar fields reduce to

$$\ddot{\phi}^I + \Gamma^I_{JK}\dot{\phi}^J\dot{\phi}^K + \left(3H + \frac{\dot{P_{,X}}}{P_{,X}}\right)\dot{\phi}^I - \frac{1}{P_{,X}}G^{IJ}P_{,J} = 0\,, \tag{2.20}$$

where Γ^I_{JK} denotes the Christoffel symbol associated with G_{IJ}. The equation (2.20) can also be written in the shorter form as

$$\mathcal{D}_t\dot{\phi}^I + \left(3H + \frac{\dot{P_{,X}}}{P_{,X}}\right)\dot{\phi}^I - \frac{1}{P_{,X}}G^{IJ}P_{,J} = 0\,, \tag{2.21}$$

where

$$\mathcal{D}_t\dot{\phi}^I \equiv \ddot{\phi}^I + \Gamma^I_{JK}\dot{\phi}^J\dot{\phi}^K\,. \tag{2.22}$$

2.3.2 Perturbed evolution

Referring to the theory of linear cosmological perturbations, the scalarly perturbed FLRW metric is written as

$$ds^2 = a^2\left(-(1+2A)d\eta^2 + 2\partial_i B dx^i d\eta + \left[(1-2\psi)\delta_{ij} + 2\partial_{ij}E\right]dx^i dx^j\right)\,, \tag{2.23}$$

where $\eta \equiv \int dt/a$. Since, in the equation (2.23), there are two gauge degrees of freedom, these metric perturbations can be combined to yield the gauge-invariant potentials which are the Bardeen's potentials, defined by

$$\Phi \equiv A - \frac{d}{dt}\left[a^2(\dot{E} - B/a)\right], \tag{2.24}$$

$$\Psi \equiv \psi + a^2 H(\dot{E} - B/a). \tag{2.25}$$

In addition, because of the local covariant property of Einstein theory, the infinitesimal-coordinate transformation, $\tilde{x}^\mu = x^\mu + \xi^\mu$, where x^μ four coordinates, and ξ^μ are small vectors transforming x^μ to $\tilde{x}^\mu$, can be used to get rid of the excessive variables via the tensor transformation, i.e.,

$$\tilde{g}_{\mu\nu}(\tilde{x}^\zeta) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}. \tag{2.26}$$

Since, there are several coordinates used generally, here, some examples of the gauge and its dynamical variables are shown. For the conformal Newtonian gauge,

$$A_n = A - \mathcal{H}(E' - B) - E'' + B',\ \psi_n = \psi + \mathcal{H}(E' - B), \tag{2.27}$$

where $'$ denotes the derivative with respect to η, and $\mathcal{H} \equiv a'/a$, and the subscript, n, denotes that of the conformal Newtonian gauge, but none of any subscript stands for that of an arbitrary gauge. For the uniform curvature gauge,

$$A_c = A + \psi + \left(\frac{\psi}{\mathcal{H}}\right)',\ B_c = B - \frac{\psi}{\mathcal{H}} - E', \tag{2.28}$$

where the subscript, c, denotes that of the uniform curvature gauge. For the uniform density gauge,

$$A_d = A - \mathcal{H}\frac{\delta\rho}{\bar{\rho}'} - \left(\frac{\delta\rho}{\bar{\rho}'}\right)',\ \psi_d = \psi + \mathcal{H}\frac{\delta\rho}{\bar{\rho}'}, \tag{2.29}$$

where $\delta\rho$ and $\bar{\rho}$ are the perturbed and background energy densities, and the subscript, d, denotes that of the uniform density gauge. For the comoving gauge,

$$A_m = A - \mathcal{H}(v + B) + v' + B', \;\; \psi_m = \psi - \mathcal{H}(v + B), \tag{2.30}$$

where v is a scalar whose gradient becomes the curl-free part of the spatial velocity of the perfect fluid, and the subscript, m, denotes that of the comoving gauge [7, 8].

The useful gauge choices for study the cosmological perturbations during inflation are uniform density and comoving gauge The field perturbations at horizon crossing can be computed using the uniform-curvature gauge, and, from the linear cosmological perturbation theory, the curvature perturbations are conserved in the comoving gauge, $\mathbf{v}_{fluids} = 0$, on large scales, if there is no any entropy perturbation. Also, from the linear cosmological perturbation theory, the curvature perturbation in comoving and uniform-density gauges are equivalent on large scales, i.e. $\Psi_{\delta\rho=0} \approx \Psi_{\mathbf{v}_{fluids}=0}$. Then, the constancy of curvature perturbations in comoving and uniform-density gauges on large scales is supported by the vanishing of entropy perturbations suggested by the observation. Hence, the most convenient choices of gauge for studying perturbations during inflation are the comoving gauges, the uniform-curvature gauge, and the uniform-density gauge. In fact, to complete the computation for explanation of perturbations via the standard cosmological perturbation, the initial curvature perturbations at the beginning of radiation, i.e. after inflation, are needed. Hence, the constancy of curvature perturbations on large scales allows us to compute its initial condition during radiation without considering the evolution at the conjunction between inflation and radiation. To get the evolutions of perturbations, one can obtain the equations of motion of perturbations by perturbing the equation (2.21) and replacing the background FLRW metric by the metric 2.23. As mentioned that eventually the gauge degrees of freedom has to be destroyed, It is convenient to destroy them by using the uniform curvature gauge. Via the Fourier transformation, we obtain

$$\frac{D^2\delta\phi^a_{\mathbf{k}}}{dt^2} + 3H\frac{D\delta\phi^a_{\mathbf{k}}}{dt} - R^a{}_{bcd}\dot{\phi}^b\dot{\phi}^c\delta\phi^d_{\mathbf{k}} + q^2\delta\phi^a_{\mathbf{k}} + V^{;a}{}_{;b}\delta\phi^b_{\mathbf{k}} = \frac{1}{a^3}\frac{D}{dt}\left(\frac{a^3}{H}\dot{\phi}^a\dot{\phi}^b\right)h_{bc}\delta\phi^c_{\mathbf{k}}, \tag{2.31}$$

where $\mathbf{k}$ is the comoving wavenumber, and $R^a{}_{bcd} = \Gamma^a{}_{bd,c} - \Gamma^a{}_{bc,d} + \Gamma^a{}_{ce}\Gamma^e{}_{db} - \Gamma^a{}_{de}\Gamma^e{}_{cb}$. Under the slow-roll conditions, the equation (2.31) approximately becomes

$$3H\frac{D\delta\phi^a_{\mathbf{k}}}{dt} - R^a{}_{bcd}\dot{\phi}^b\dot{\phi}^c\delta\phi^d_{\mathbf{k}} + V^{;a}{}_{;b}\delta\phi^b_{\mathbf{k}} = 3\dot{\phi}^a\dot{\phi}^b h_{bc}\delta\phi^c_{\mathbf{k}} \tag{2.32}$$

[5].

2.3.3 Inflationary models

2.3.3.1 Quadratic model.

Here is one of the standard models of the inflaton, called the quadratic model. The potential of this model takes the form

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2, \tag{2.33}$$

where m_ϕ is the mass of ϕ, and $P = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$. Applying this potential to equation (2.12)-(2.22) under the slow-roll conditions, one obtains evolution equation for the background universe as follows

$$H^2 \simeq \frac{1}{3}V(\phi) = \frac{1}{6}m_\phi \phi^2, \tag{2.34}$$

$$\dot{\phi} \simeq -\frac{V_{,\phi}}{3H} = \frac{2}{\phi}. \tag{2.35}$$

Actually, equation (2.34) and (2.35) are inflationary Friedmann and continuity equations respectively. The slow-roll parameter, $\epsilon \equiv -\dot{H}/H$, can also approximately take the other form as

$$\epsilon = \frac{1}{2}\frac{V_{,\phi}}{V}. \tag{2.36}$$

By using this potential, the extra slow-roll parameter can also be defined as

$$\eta \equiv \left|\frac{V,\phi\phi}{V}\right| \ll 1. \tag{2.37}$$

This new parameter is used to maintain the condition that $\epsilon \ll 1$ during inflation.

2.3.3.2 Double quadratic model.

Let us, now, consider the other inflationary model. In stead of having only one scalar field in the model as the previous, this model is modified to drive the universe by two scalar fields, ϕ and χ. For this model, using $V(\phi, \chi) = \frac{1}{2}m_\phi \phi^2 + \frac{1}{2}m_\chi \chi^2$, $P = X(\dot{\phi}, \dot{\chi}) - V(\phi, \chi)$, and the slow-roll conditions, we can also have the Friedmann and continuity equations respectively as

$$H^2 \simeq \frac{1}{3}V(\phi, \chi) = \frac{1}{6}\left(m_\phi \phi^2 + m_\chi \chi^2\right), \tag{2.38}$$

$$\dot{\phi} \simeq -\frac{V_{,\phi}}{V} = \frac{2}{\phi}, \tag{2.39}$$

$$\dot{\chi} \simeq -\frac{V_{,\chi}}{V} = \frac{2}{\chi}. \tag{2.40}$$

For the slow-roll parameters, they both are computed to be $\epsilon = \epsilon_\phi + \epsilon_\chi$, and

$$\epsilon_I = \frac{1}{2}\frac{V_{,I}}{V}, \tag{2.41}$$

$$\eta_I = \left|\frac{V_{,II}}{V}\right|, \tag{2.42}$$

where the index, I, runs for ϕ and χ respectively.

CHAPTER III

SPECTRA OF THE PERTURBATION & ΔN FORMALISM

3.1 Power spectrum & bispectrum

The theoretical prediction of the cosmological model can be connected to observational data by using the n-point correlation functions. The n-point correlation function of the quantity ξ in real spaces can be written as $\langle \prod_{i=1}^{n} \xi_{\mathbf{x}_i} \rangle$ where $\xi_{\mathbf{x}_i}$ is the value of ξ at the position $\mathbf{x}_i$, and $\langle \Box \rangle$ is the ensemble averages of $\Box$. Using the definition of the Fourier transformations of the two-point correlation function and the homogeneity and isotropy of statistic, the power spectrum, P_k, is defined as

$$\begin{aligned}
\left\langle \widetilde{\xi}_{\mathbf{k}_1} \widetilde{\xi}_{\mathbf{k}_2} \right\rangle &= \int_{-\infty}^{\infty} d^3x_1 d^3x_2 \exp(-i(\mathbf{k}_1 \cdot \mathbf{x}_1 + \mathbf{k}_2 \cdot \mathbf{x}_2)) \langle \xi_{\mathbf{x_1}} \xi_{\mathbf{x_2}} \rangle \\
&= \int_{-\infty}^{\infty} d^3x_1 d^3R \exp(-i(\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{x_1}) \exp(-i\mathbf{k}_2 \cdot \mathbf{R}) \langle \xi_{\mathbf{0}} \xi_R \rangle \\
&= (\int_{-\infty}^{\infty} d^3x_1 \exp(-i(\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{x}_1)(\int_{-\infty}^{\infty} d^3R \exp(-i\mathbf{k}_2 \cdot \mathbf{R}) \langle \xi_{\mathbf{0}} \xi_R \rangle) \\
&= (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \int_{-\infty}^{\infty} d^3R \exp(-i\mathbf{k}_2 \cdot \mathbf{R}) \langle \xi_{\mathbf{0}} \xi_R \rangle \\
&= (2\pi)^3 \delta^{(3)} (\mathbf{k}_1 + \mathbf{k}_2) P_{k_2} \qquad (3.1)
\end{aligned}$$

where R is the distant between two points, i.e. the magnitude of $\mathbf{R}$. Due to statistical homogeneity and isotropy, one can set $\mathbf{x}_1$ in two-point correlation function to be $\mathbf{0}$ without lost of generality. In the above equation, we have defined

$$P_{k_2} \equiv \int_{-\infty}^{\infty} d^3R \exp(-i\mathbf{k}_2 \cdot \mathbf{R}) \langle \xi_0 \xi_R \rangle . \qquad (3.2)$$

Hence, P_k can be transformed back to be $\langle \xi_0 \xi_R \rangle$ by

$$\langle \xi_0 \xi_R \rangle = (2\pi)^{-3} \int_{-\infty}^{\infty} d^3 k e^{i\mathbf{k}\cdot\mathbf{R}} P_k = (2\pi)^{-3} \int_{0}^{\infty} d\omega dk k^2 e^{i\mathbf{k}\cdot\mathbf{R}} P_k \tag{3.3}$$

where k and R is the magnitude of $\mathbf{k}$ and $\mathbf{R}$ respectively, and ω is the solid angle around $\mathbf{R}$. From equation (3.3), the dimensionless power spectrum, $\mathcal{P}_k$, can be defined by setting $\mathbf{R} = 0$ and using $\int d\omega = 4\pi$ as

$$\mathcal{P}_k \equiv (2\pi^2)^{-1} k^3 P_k. \tag{3.4}$$

Likewise, the bispectrum, B, or a spectrum of an arbitrary order, S_n where n is the order, can be defined as

$$\left\langle \prod_{i=1}^{n} \xi_{(\mathbf{k}_i)} \right\rangle \equiv (2\pi)^3 \, \delta^{(3)} \left(\sum_{i=1}^{n} \mathbf{k}_i \right) S_n, \tag{3.5}$$

hence, by definition, one can see that $P = S_2$ and $B = S_3$. The property of delta function, $\delta^{(3)} \left(\sum_{i=1}^{n} \mathbf{k}_i \right)$, confirms that the summation of wave vectors, $\mathbf{k}_i$, have to vanish, i.e. the combination of wave vectors forms a polygon. In the case of the power spectrum, the configuration of wave vectors is forced to be one pattern which is the configuration of two opposite vectors with the same magnitude, i.e. $P_{k_1} = P_{k_2}$. However, in the case of the bispectrum, there are infinite possible configurations, but important features of non-Gaussianity can be represented by the following configurations: the first is called equilateral bispectrum with the property $k_1 = k_2 = k_3$, the second is the squeezed bispectrum which is $k_1 = k_2 \gg k_3$. One can consider this type as an isosceles.

As it was mentioned, non-Gaussianity of the curvature perturbations can be investigated by computing the three-point correlation function. For the local shape, the bispectrum can be computed using the local ansatz, i.e.

$$\Psi_x = G_x + \frac{3}{5} f_{NL}^{local} (G_x^2 - \langle G^2 \rangle) \tag{3.6}$$

where Ψ_x is the curvature perturbations, and G_x is a Gaussian field. For the Gaussian case, f_{NL}^{local} vanishes, i.e. $\Psi_x = G_x$. One is leaded to the fact that, for the Gaussian curvature perturbations, three-point correlation functions vanish, i.e. $\langle \Psi_{x_1} \Psi_{x_2} \Psi_{x_3} \rangle = 0$. Hence, also for Gaussian curvature perturbations, $B_\Psi(k_1, k_2, k_3)$ vanishes. Using the local ansatz and Wick's theorem, f_{NL}^{local}, which is called the non-Gaussianity parameter, can be computed as

$$\frac{3}{5} f_{NL}^{local} = \frac{B_\Psi(k_1, k_2, k_3)}{2(P_\Psi(k_1)P_\Psi(k_2) + P_\Psi(k_2)P_\Psi(k_3) + P_\Psi(k_3)P_\Psi(k_1))}. \tag{3.7}$$

It can be seen that f_{NL}^{local} quantifies a size of non-Gaussianity. For squeezed modes, the above equation can be reduced to

$$\frac{3}{5} f_{NL}^{local} \simeq \frac{B_\Psi(k_l, k_s)}{4P_\Psi(k_l)P_\Psi(k_s)}, \quad k_s \gg k_l \tag{3.8}$$

where $k_1 \simeq k_2 = k_s$, $k_3 = k_l$ and k_l and k_s denote wave vectors of long and short wavelengths respectively, so that $k_s \gg k_l$. From the above equation, the coupling term, $P_\Psi(k_s)P_\Psi(k_s)$, in equation (3.7), is ignored because it is proportional to k_s^{-6} while others are proportional to $k_s^{-3}k_l^{-3}$. During inflation, the dimensionless power spectrum is found to near be scale-invariant. The deviation from being scale-invariant is characterized by the spectral index, n_s, as

$$n_s = 1 + \frac{d \log(\mathcal{P}_\Psi)}{d \log k}. \tag{3.9}$$

Using the local ansatz with the squeezed mode, i.e. $G_x = G_l + G_s(x)$ where subscripts, l and s, directly mean long and short wavelengths respectively, the curvature perturbation can be written as

$$\begin{aligned}\Psi_x &= (G_l + G_s(x)) + \frac{3}{5} f_{NL}^{local}((G_l + G_s(x))^2 - \langle (G_l + G_s(x))^2 \rangle) \\ &= G_l + \frac{3}{5} f_{NL}^{local}(G_l^2 - \langle G_l^2 \rangle) + G_s(x) + \frac{3}{5} f_{NL}^{local}(G_s^2(x) + 2G_l G_s(x)). \end{aligned} \quad (3.10)$$

Neglecting the second order, curvature perturbations for the short-wavelength mode, Ψ_s, is then of the order

$$\Psi_s(x) \sim \left(1 + \frac{6}{5} f_{NL}^{local} G_l\right) G_s(x). \quad (3.11)$$

As one can see from the above equation, G_l on the R.H.S shows the modulation of the long-wavelength on short wavelengths when short wavelengths are crossing the horizon. This modulation causes the non-Gaussianity in curvature perturbations. However, the detail about the modulation will be explained further in section (4.2).

3.2 ΔN formalism

To describe the dynamics of the scales factor, one can use numbers of e-folding , N. By definition, the numbers of e-folding is $N \equiv \log\left(a(t_2)/a(t_1)\right)$. From this definition the numbers of e-folding can also takes the form as $N = \int_{t_1}^{t_2} dtH$. In uniform-density gauge, $\delta\rho = 0$, or uniform-curvature gauge, $\Psi = 0$, in which vanishing of $E_{\delta\rho=0}$ or $E_{\Psi=0}$ is required,

the leftover spatial part of ds^2 given in equation (2.23) is $a^2(t)\delta_{ij}2\Psi_{\delta\rho=0}$ and $a^2(t)\delta_{ij}$ consecutively. This motivates us to describe curvature perturbations in these two gauges by introducing a new scale factor, $\widetilde{a}$, which depend on both spatial coordinates and time coordinate via this definition

$$\widetilde{a}(x,t) \equiv \exp(\Psi(x,t))a(t) \approx (1+\Psi(x,t))a(t). \tag{3.12}$$

Likewise, as the background metric, one can also define a new Hubble parameter, $\widetilde{H}$, as $\widetilde{H} \equiv \dot{\widetilde{a}}/\widetilde{a}$. Since $\Psi = 0$ in the uniform-curvature gauge, one can easily see that

$$\widetilde{H}_{\delta\rho=0} = \frac{\dot{\widetilde{a}}_{\delta\rho=0}}{\widetilde{a}_{\delta\rho=0}} = H + \dot{\Psi}_{\delta\rho=0}, \quad \widetilde{H}_{\Psi=0} = \frac{\dot{\widetilde{a}}_{\Psi=0}}{\widetilde{a}_{\Psi=0}} = H \tag{3.13}$$

where H is the background Hubble parameter. Here, one can define the numbers of e-folding in both to gauges by using the same definition from the background as $\widetilde{N}(x,t_0,t) \equiv \log\left(\widetilde{a}(x,t)/\widetilde{a}(x,t_0)\right)$. Because one can independently fix the gauge, it is possible to choose uniform-curvature and uniform-density gauges at t_1 and t_2 respectively. Then, one can prove that

$$\Delta N = \widetilde{N}_{\delta\rho=0}(x,t_2) - N(t_2) = \log\left(\frac{\widetilde{a}(x,t_2)}{a(t_1)}\right) - \log\left(\frac{a(t_2)}{a(t_1)}\right) = \Psi_{\delta\rho=0}\,(x,t_2) \tag{3.14}$$

where $N(t_1,t_2) \equiv \log(a(t_2)/a(t_1))$. This is the fundamental equation of the ΔN approach. In practice, the ΔN formalism can be used to compute curvature perturbations via the separate universe approach. The elementary ideas of the separate universe approach will be shown as follows: on scales larger than the Hubble radius, the universe can be viewed as many separate universes, i.e. many small separate disconnected regions which separately evolve as the FLRW universe. The FLRW universe evolves via the evolution of $a(t)$ while the small region of the universe on large scales evolves via $\widetilde{a}(x,t)$. However, the evolution of each separate region depends on the initial value of $\widetilde{a}$ that coincides with the local area of the region. Using this idea, the inhomogeneity and anisotropy can be viewed as the effect of the difference between the evolution of each local region. This approach is called the separate universe approach. During inflation, dynamics of the universe is govern by the inflaton, ϕ. For a scalar field inflaton, ϕ can be split as $\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t)$ where $\phi(t)$ is the homogeneous and isotropic background field, and $\delta\phi(x,t)$ are very small field perturbations. Using the idea of the separate universe, the Friedmann equation for the local region will take this form

$$\widetilde{H}^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2(x,t) + V(\phi(x,t))\right) \tag{3.15}$$

However, the slow-roll condition during inflation, i.e. $\dot{\phi}^2 \ll V(\phi)$, forces this equation to be

$$\widetilde{H} = \widetilde{H}(\phi(x,t)) \approx \sqrt{\frac{1}{3}V(\phi(x,t))}. \tag{3.16}$$

Using the above equation, one can consider the evolution of the universe via ϕ in each small region. Applying this idea to the ΔN approach, curvature perturbations, $\Psi_{\delta\rho=0}(x,t)$, become

$$\begin{aligned}\Psi_{\delta\rho=0}(x,t) &= \Delta N(\phi(x,t)) = \Delta N(\bar{\phi}(t) + \delta\phi(x,t)) \\ &= \widetilde{N}(\bar{\phi}(t) + \delta\phi(x,t)) - N(\bar{\phi}(t)) \\ &= N_{\bar{\phi}}\delta\phi + \frac{1}{2}N_{\bar{\phi}\bar{\phi}}\delta\phi^2 + \frac{1}{6}N_{\bar{\phi}\bar{\phi}\bar{\phi}}\delta\phi^3 + \cdots \end{aligned} \tag{3.17}$$

where the subscript ϕ denotes the derivative with respect to ϕ. Using the last above equation, the n-point correlation function of curvature perturbations and its spectrum can be computed if we know $\delta\phi$ in uniform-curvature gauge at horizon crossing and the evolution of the background without considering the nonlinear evolution of curvature perturbations, i.e., in order to compute the n-point correlation function of curvature perturbations and its spectrum via the cosmological perturbation theory, one has to consider the nonlinear evolution of curvature perturbations also.

CHAPTER IV

EFFECT OF CURVATURE IN FIELD SPACE ON SQUEEZED BISPECTRUM

4.1 Standard approach

For the model of multi scalar fields minimally coupling with the physical curvature, its actions can be written as shown

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(R + G_{ab}\partial_\mu\phi^a\partial^\mu\phi^b - 2W(\phi^a)\right) \tag{4.1}$$

where the reduced Planck mass is set to be unity, and ϕ^a denote each fields. Because the field space metric tensor, G_{ab}, is assumed to be none trivial, and the slow roll conditions is used, the evolution of multi fields can be written as

$$\frac{d\phi^a}{dN} = -G^{ab}\frac{V_b}{V}. \tag{4.2}$$

Using this model with the ΔN formalism, i.e. equation (3.17), for the leading order, the dimensionless power spectrum takes the form as

$$\mathcal{P}_\psi = \frac{k^3}{2\pi^2}|\Psi|^2 = \frac{k^3}{2\pi^2}|N_a\delta\phi^a N_b\delta\phi^b|^2 = N_aN_b\mathcal{P}^{ab}_{\delta\phi} = N_aN^a\left(\frac{H}{2\pi}\right)^2, \tag{4.3}$$

where $\mathcal{P}^{ab}_{\delta\phi}$ is the dimensionless power spectrum of fields perturbations at horizon crossing, t_c. Similarly, the bispectrum of curvature perturbations can be yielded as

$$B_\Psi(k_1,k_2,k_3) =$$
$$N_aN_bN_cB_{\delta\phi}(k_1,k_2,k_3) + \left(N_aN^aN_{cd}P_{\delta\phi}(k_1)P_{\delta\phi}(k_2) + 2perms\right). \tag{4.4}$$

For the equilateral mode, since the magnitude of each wave vector of the field perturbation is equal, all field perturbations cross the horizon at the same time. Therefore, the first term on the R.H.S represents the non-Gaussianity of field perturbations at horizon crossing. Then, this term is called intrinsic non-Gaussianity. In the standard cosmological perturbation theory, this term can be computed from the action of the third order perturbations. Regularly, this term is proportional to the slow-roll parameter of inflation which is smaller than unity. The second term is influenced by nonlinear evolutions on large scales. This term can be computed by the standard cosmological perturbation theory or, alternatively, the ΔN approach.

For the squeezed mode, field perturbations do not cross the horizon at the same time. Therefore, the first term has to be computed when short wavelengths perturbations cross the horizon. However, at that time, the longer wavelength had already evolved on large scales. Its evolution affects the time at which the short wavelengths exist the horizon leading to the correlation between short and long wavelengths. In this case, instead of the intrinsic non-Gaussianity, the first term on the R.H.S represents the correlation between short and long wavelengths. In order to compute this term, one can use the standard cosmological perturbation theory or the ΔN approach. From the action of the third order perturbations for a single field, one can compute the non-Gaussianity parameter of squeezed mode in term of spectral index as

$$\frac{3}{5}f_{NL} = \frac{1-n_s}{4} \tag{4.5}$$

which is called the consistency relation. This relation is true for a single field inflaton only. To avoid the complexity of computing the third order action for multi fields, the ΔN approach used to compute the non-Gaussianity parameter for the squeezed mode has been proposed recently[1, 2]. This approach will be reviewed in the next section.

4.2 New technique to compute the highly squeezed mode

According to the previous section, non-Gaussianity in the squeezed mode is caused by the correlation between short and long wavelength modes of perturbations. Hence, bispectrum for squeezed mode can be viewed as the power spectrum of short-wavelength mode modulated by the long-wavelength mode, i.e. $\langle \Psi_l \Psi_s \Psi_s \rangle \simeq \langle \Psi_l \langle \Psi_s \Psi_s \rangle \rangle$. $\Psi_l \equiv \Psi(\widetilde{\mathbf{x}})$ can be defined from $\Psi(x)$ by averaging $\Psi(x)$ over the region which is larger than k_l^{-1} using the window function $W(k_l|\widetilde{\mathbf{x}} - \mathbf{x}|)$ as

$$\Psi_l \equiv \Psi(\widetilde{\mathbf{x}}) = \int d^3x W(k_l|\widetilde{\mathbf{x}} - \mathbf{x}|)\Psi(\mathbf{x}). \tag{4.6}$$

From this definition, one can get

$$\begin{aligned} \langle \Psi_l \mathcal{P}_{\Psi_s}(k_s) \rangle &= \frac{k_s^3}{2\pi^2} \int d^3x d^3\widetilde{x} e^{-i\mathbf{k}_s \cdot \widetilde{\mathbf{x}}} W(k_l x) \left\langle \Psi(\mathbf{x}) \Psi(-\frac{\widetilde{\mathbf{x}}}{2}) \Psi(\frac{\widetilde{\mathbf{x}}}{2}) \right\rangle \\ &= \frac{k_s^3}{(2\pi^2)(2\pi)^6} \int d^3x d^3\widetilde{x} d^3p d^3q e^{i(\mathbf{p}\cdot(\mathbf{x}+\frac{\widetilde{\mathbf{x}}}{2}) + \mathbf{q}\cdot\widetilde{\mathbf{x}} - \mathbf{k}_s\cdot\widetilde{\mathbf{x}})} W(k_l x) B_\Psi(p, q, |\mathbf{p}+\mathbf{q}|) \\ &= \frac{k_s^3}{(2\pi^2)(2\pi)^3} \int d^3p d^3q \delta^{(3)} \left(\frac{\mathbf{p}}{2} + \mathbf{q} - \mathbf{k}_s \right) \widetilde{W} \left(\frac{p}{k_l} \right) B_\Psi(p, q, |\mathbf{p}+\mathbf{q}|). \end{aligned} \tag{4.7}$$

We choose the Fourier transformation of the window function, $\widetilde{W}(K)$ to be the delta function, $\widetilde{W}(p/k_l) = \delta(\log p - \log k_l)$, for selecting out the mode $p = k_l$ in the above integration. Hence the above equation is reduced to

$$\langle \Psi_l \mathcal{P}_{\Psi_s} \rangle \simeq \frac{(k_s k_l)^3}{(2\pi^2)^2} B_\Psi(k_l, k_s, k_s) \tag{4.8}$$

where we have used the approximation that $k_l \ll k_s$. Using the above equation with equation (3.8), one can show that the local nonlinear parameter of the squeezed mode is given by

$$\frac{3}{5} f_{NL}^{local}(k_l, k_s) = \frac{\langle \Psi_l \mathcal{P}_{\Psi_s} \rangle}{4\mathcal{P}_{\Psi_l} \mathcal{P}_{\Psi_s}}. \tag{4.9}$$

Using this relation, one can compute the bispectrum via the ΔN formalism. Let A^* and B^* be points in the field space when the long and short-wavelength modes are crossing the horizon, and B be a point in the field space at which the short wavelength perturbations cross the horizon without the effect of the long wavelength. Making an expansion around background field at the point B yields

$$\begin{aligned}
\langle \Psi_l \mathcal{P}_{\Psi_s}|_{B^*} \rangle \simeq& \langle \Psi_l \mathcal{P}_{\Psi_s}|_B \rangle + \langle \Psi_l \mathcal{P}_\Psi(k)_{,b} \left(\phi(B^*) - \phi(B)\right) \rangle \\
=& \underbrace{N_a \langle \delta\phi_l^a \mathcal{P}_\Psi(k)|_B \rangle}_{=0} + N_a \mathcal{P}_\Psi(k)_{,b} \left\langle \delta\phi_l^a (\phi_l^b(B^*) - \phi_l^b(B)) \right\rangle \\
=& N(k_l^*)_a \mathcal{P}_\Psi(k)_{,b} \left\langle \delta\phi_l^a \delta\widetilde{\phi}_l^b \right\rangle
\end{aligned} \tag{4.10}$$

where the perturbed field, $\delta\widetilde{\phi}_l^b$, describes the shifting from B to B^*, i.e. deviating from one to another slow-roll path, and $\mathcal{P}_{,a}$ denotes the derivative of $\mathcal{P}$ with respect to the scalar field ϕ^a. At leading order, this perturbed field is given by

$$\begin{aligned}
\delta\widetilde{\phi}_l^a =& \phi^a(\phi^b(A^*) + \delta\phi^b(A^*)) - \phi^a(\phi^b(A^*)) \\
\simeq& \phi^a(\phi^b(A^*)) + \frac{\partial \phi_B^a}{\partial \phi^b} \delta\phi^a(A^*) - \phi^a(\phi^b(A^*)) \\
=& \left. \frac{\partial \phi_B^a}{\partial \phi^b} \right|_{A^*} \delta\phi^b(A^*)
\end{aligned} \tag{4.11}$$

where we have already used separate universe approach. Hence, finally, the non-linear parameter becomes

$$\frac{3}{5} f_{NL}^{local}(k_l, k_s) = \frac{\langle \Psi_l \mathcal{P}_{\Psi_s} \rangle}{4\mathcal{P}_{\Psi_l}|_{A^*}\mathcal{P}_{\Psi_s}|_B} = \frac{N_a|_{A^*}\mathcal{P}_{\Psi_s,b}|_B \phi^b_a|_{A^*}\mathcal{P}_{\delta\phi}|_{A^*}}{4\mathcal{P}_{\Psi_l}|_{A^*}\mathcal{P}_{\Psi_s}|_B}. \tag{4.12}$$

For a single scalar field, the above equation will be reduced to

$$\begin{aligned}\frac{3}{5} f_{NL}^{local}(k_l, k_s) =& \frac{N_\phi|_{A^*}\mathcal{P}_{\Psi_s,\phi}|_B \frac{\partial}{\partial\phi}\phi(B)|_{A^*}\mathcal{P}_{\delta\phi}|_{A^*}}{4\mathcal{P}_{\Psi_l}|_{A^*}\mathcal{P}_{\Psi_s}|_B} \\ =& \frac{1}{4}\frac{N_\phi|_{A^*}N_\phi|_B \frac{\partial}{\partial\phi}\phi(B)|_{A^*}(\frac{\partial}{\partial N}(\log \mathcal{P}_{\Psi_s})|_B)(\mathcal{P}_{\delta\phi}|_{A^*})}{\mathcal{P}_{\Psi_l}|_{A^*}} \\ =& \frac{1}{4}\frac{\partial}{\partial N}\log \mathcal{P}_{\Psi_s}\bigg|_B = -\frac{1}{4}\frac{\partial}{\partial \log k}\log \mathcal{P}_{\Psi_s}\bigg|_B \frac{\partial \log k}{\partial \log a} \\ \simeq& \frac{1-n_s}{4}\end{aligned} \tag{4.13}$$

where $k = aH$, and we have used the same method as we did in equation (4.3) and equation (3.9). It yields again the consistency relation we have mentioned.

4.3 Squeezed bispectrum in multi-inflaton with curved field space

We now consider the multi-field inflation with the action

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}G_{IJ}\partial_\mu\phi^I\partial^\mu\phi^J - W(\phi^I),\right], \tag{4.14}$$

where $W(\phi^I)$ is the potential of inflatons.

In the following consideration, We concentrate on two-inflaton field with the additive separable potential

$$W(\phi, \chi) = U(\phi) + V(\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2\,, \tag{4.15}$$

where we have set $\phi^I \equiv (\phi, \chi)$. We choose to work with the metric

$$G_{IJ} = \begin{pmatrix} 1 & 0 \\ 0 & G(\phi) \end{pmatrix}, \qquad \text{where} \qquad G(\phi) \equiv \lambda_1 + \lambda_2 \phi^p \,. \tag{4.16}$$

Here, λ_1, λ_2 and p are the constant parameters.

4.3.1 Background evolution

Varying the action (4.14) with respect to metric tensor of spacetime, and inserting the FLRW line element from equation (2.16) into the result, we obtain Friedmann equation which can be written as

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{2W}{6 - (\phi')^2 - G(\phi)(\chi')^2}\,, \tag{4.17}$$

where a prime denotes derivative with respect to number of e-folding of the background universe. The evolution equation for the background field can be obtained by varying the action (4.14) with respect to the field, which yields

$$\phi'' - \frac{1}{2} G_\phi (\chi')^2 = -\frac{W}{H^2}\left(\phi' + \frac{U_\phi}{W}\right), \tag{4.18}$$

$$\chi'' + \frac{G_\phi}{G}\phi'\chi' = -\frac{W}{H^2}\left(\chi' + \frac{V_\chi}{GW}\right), \tag{4.19}$$

where subscripts $_\phi$ and $_\chi$ denote derivative with respect to ϕ and χ respectively. At the lowest order in slow roll approximation, Eq. (4.17) gives $3H^2 = W$, and the above equations become

$$\phi' = -\frac{U_\phi}{U}\,, \qquad \chi' = -\frac{V_\chi}{GV}\,. \tag{4.20}$$

These equations give the following constrained equation:

$$\int_{\phi_1}^{\phi_2} \frac{d\phi}{G(\phi) U_\phi(\phi)} = \int_{\chi_1}^{\chi_2} \frac{d\chi}{V_\chi(\chi)}\,, \tag{4.21}$$

where (ϕ_1, χ_1) and (ϕ_2, χ_2) are any points in field space. Substituting the expression for $G(\phi)$ in Eq. (4.16) into the above equation, the relation between the fields ϕ and χ along trajectory which passes a point (ϕ_1, χ_1) in the field space is

$$\phi^p = \frac{\lambda_1 f_\phi(\phi_1)\,(\chi/\chi_1)^{rp}}{1 - \lambda_2 f_\phi(\phi_1)\,(\chi/\chi_1)^{rp}}\,, \tag{4.22}$$

where $r \equiv \lambda_1 m_\phi^2/m_\chi^2$ and

$$f_\phi(\phi_1) \equiv \frac{\phi_1^p}{\lambda_1 + \lambda_2 \phi_1^p} = \frac{\phi_1^p}{G(\phi_1)}\,. \tag{4.23}$$

Using Eq. (4.20), the number of e-folding for the background universe between times t_1 and t_2 or equivalently between points (ϕ_1, χ_1) and (ϕ_2, χ_2) in the field space can be computed as

$$N \equiv \int_{t_1}^{t_2} H dt = -\int_{\phi_1}^{\phi_2} \frac{W(\phi,\chi)}{U_\phi(\phi)} d\phi = \int_{\phi_2}^{\phi_1} \frac{U(\phi)}{U_\phi(\phi)} d\phi + \int_{\chi_2}^{\chi_1} G(\phi(\chi)) \frac{V(\chi)}{V_\chi(\chi)} d\chi\,. \tag{4.24}$$

The above integration cannot be evaluated analytically. Hence, we perform numerical integration by inserting Eq. (4.22) into the above equation and evaluating the integration from $N = 85$ to $N = 0$ at the end of inflation. The results from the numerical integration are plotted in figure (1). From the plot, we see that $\phi \to 0$ at the end of inflation, which follows from Eq. (4.22) that if $r \gg 1$, the value of the field ϕ at the end of inflation can be extremely smaller than the value at the initial stage of inflation. From Eq. (4.24), we see that λ_1 can enhance the number of e-folding for a given value of χ, so that the initial value of ϕ reduces when λ_1 increases. Since ϕ drops towards zero quickly when χ starts to dominate dynamics of inflation, the parameters λ_2 and p have no effect on dynamics of inflation when the number of e-folding is close to zero.

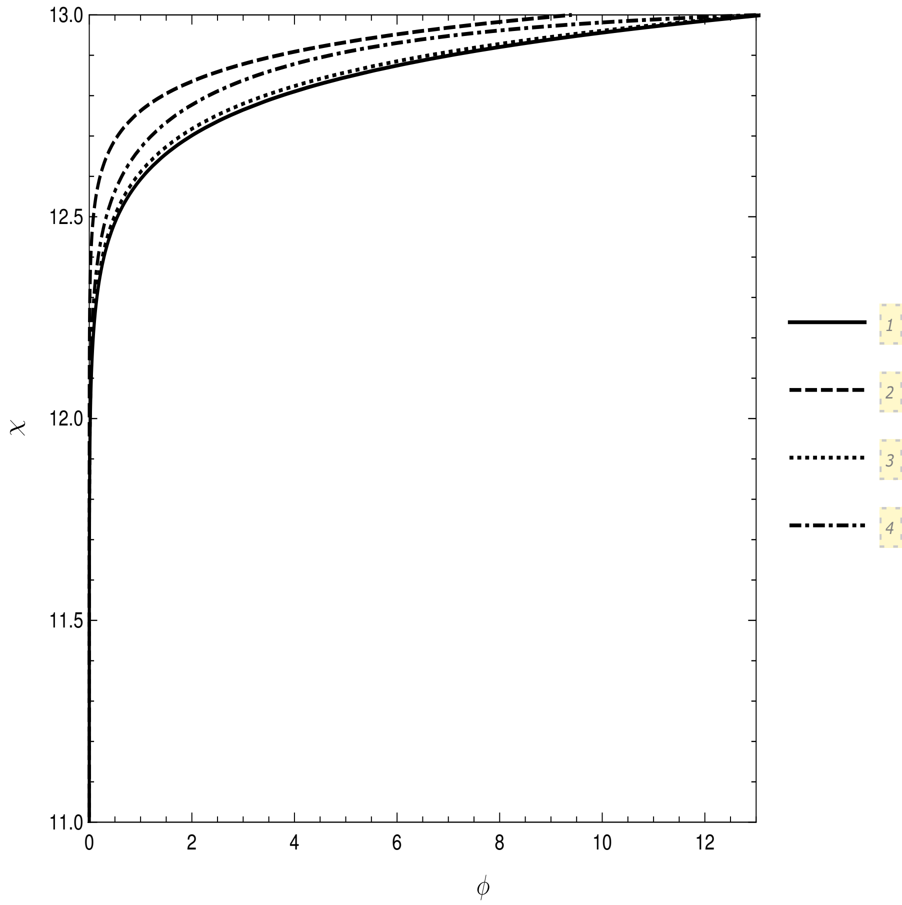

Figure 1: Trajectories in the field space for various values of λ_1, λ_2 and p. Lines 1, 2, 3 and 4 correspond to the cases where $(\lambda_1, \lambda_2, p) = (1, 0, 1), (1.5, 0, 1), (1, 0.01, 1)$, and $(1, 0.01, 2)$ respectively. In the plot, $m_\phi/m_\chi = 9$, $\chi = \sqrt{2/\lambda_1}$ at the end of inflation and $\chi = 13$ at the initial stage of inflation for all cases while the initial value of ϕ is chosen such that the total number of e-folding of inflation is 85.

4.3.2 Squeezed bispectrum

Inserting equation (4.3) into equation (4.12), we get

$$\frac{3}{5} f_{\rm NL}^{(s)}(N_A, N_B) = \frac{N_I|_A G^{IK}(\phi_A)}{4(N_I N^I)|_A} \left(\frac{2N_{K'K}N^{K'} + N_{J'}N_{K'}G^{J'K'}_{,K}}{N_I N^I} + \frac{W_{,K}}{W} \right)_B \frac{\partial \varphi^J_B}{\partial \varphi^K}\bigg|_A . \tag{4.25}$$

where a subscript $_{,K}$ denotes derivative with respect to φ^K. In order to compute $f_{\rm NL}^{(s)}$ for model of interests, we insert the expressions for the potential and metric given in equation (4.15) and (4.16) into the above equation. We compute the expressions for N_I, N_{IJ} and $\partial\phi_B^J/\partial\phi^K|_A$ in appendix (A). Since we are interested in $f_{\rm NL}$ at the end of inflation at which $\phi(t = t_u) \to 0$, we compute $f_{\rm NL}^{(s)}$ for the forward formulation by substituting equation (A.17), (A.18) and (A.26) into equation (4.25) while equation (A.34) is used instead of equation (A.26) for the backward $f_{\rm NL}^{(s)}$. Plots of $f_{\rm NL}^{(s)}$ for various values of parameters p, λ_1, λ_2 are shown in figures (2) - (4). For all plots, we set $m_\phi = 9 \times 10^6$ GeV, $m_\phi/m_\chi = 9$, and denote number of e-folding at which the long wavelength and short wavelength perturbations exist the horizon by N_L and N_S respectively. From the plots, we see that the curvature of the field space can significantly affects both amplitude and shape of $f_{\rm NL}^{(s)}$. The amplitude of the backward $f_{\rm NL}$ increases and the peak is shifted to larger N_L when λ_2 and p increase, while the shape of $f_{\rm NL}^s z$ can be largely altered when $p = 3$. Unlike the change of the curvature in field space, the increase of λ_1 just shift the peaks of $f_{\rm NL}$.

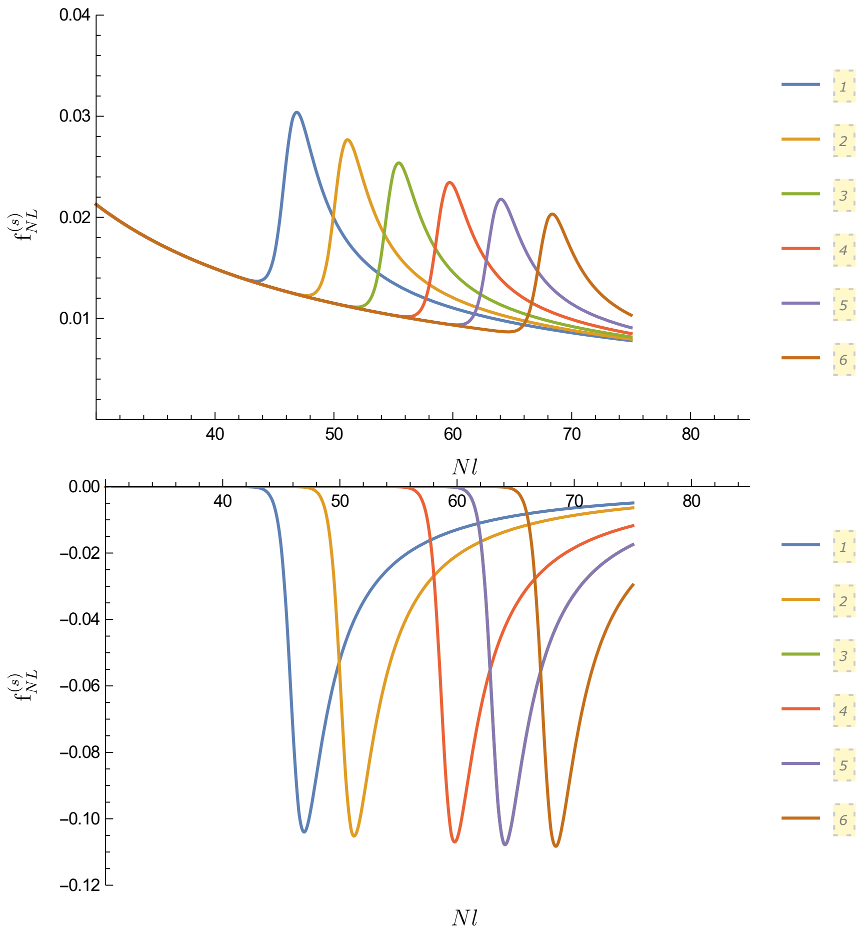

Figure 2: Nongaussianity parameter for forward(upper panel) and backward(lower panel) formulations. Lines 1, 2, 3, 4, 5, and 6 correspond to the cases $\lambda_1 = 1, 1.1, 1.2, 1.3, 1.4,$ and 1.5 respectively. For all lines, $\lambda_2 = 0, p = 1$.

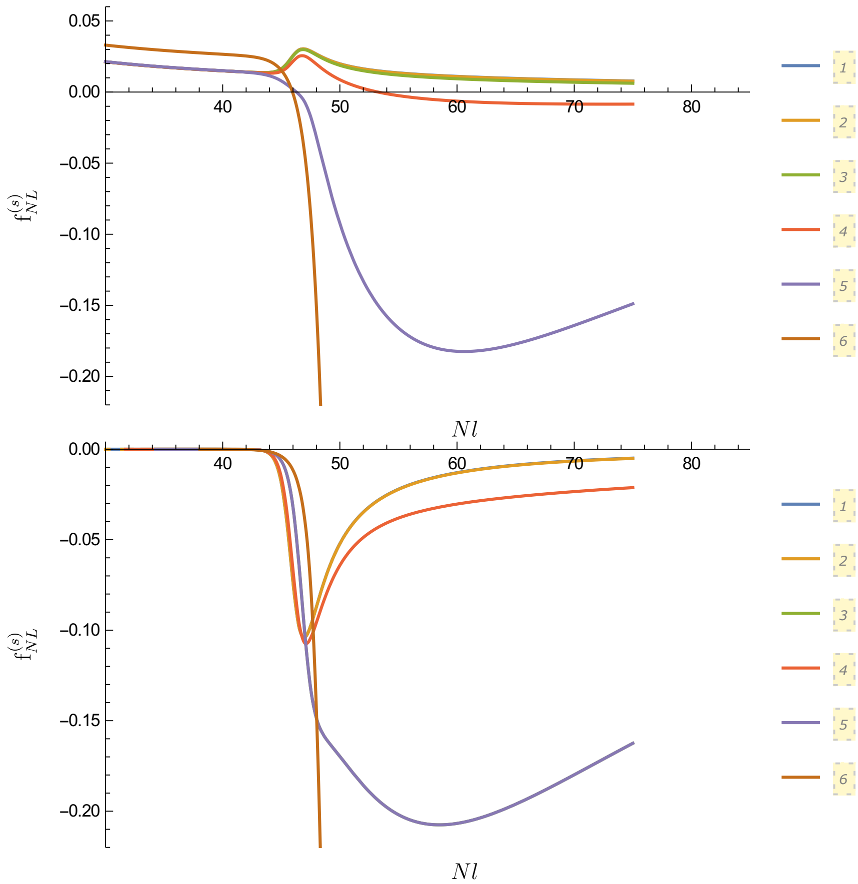

Figure 3: Nongaussianity parameter for forward(upper panel) and backward(lower panel) formulations. Lines 1, 2, 3, 4, 5, and 6 correspond to the cases $\lambda_2 = 0, 0.0001, 0.001, 0.01, 0.1$, and 1 respectively. For all lines, $\lambda_1 = 0, p = 1$.

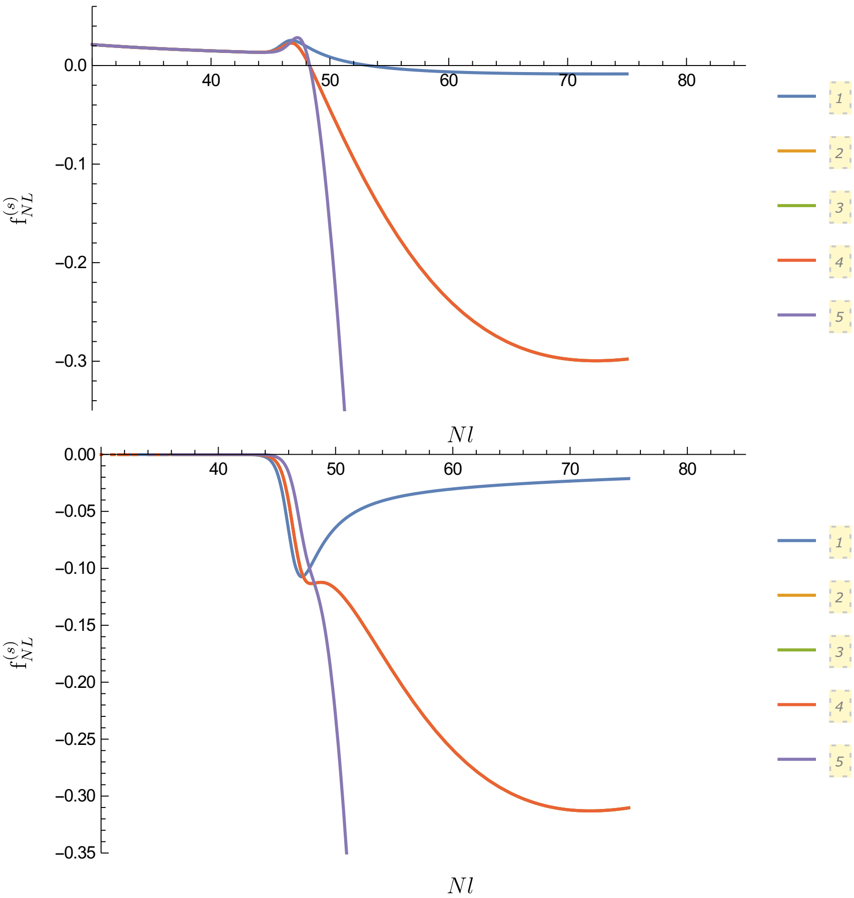

Figure 4: Nongaussianity parameter for forward(upper panel) and backward(lower panel) formulations. Lines 1, 2, 3, 4, 5, and 6 correspond to the cases that $p = 1, 2,$ and 3 respectively. For all lines, $\lambda_1 = 1, \lambda_2 = 0.01$.

CHAPTER V

CONCLUSIONS

In this thesis, the effects of curvature in field space of multi fields inflationary models on the squeezed bispectrum are investigated. Using ΔN formalism, we compute squeezed bispectrum both in the forward and backward cases. We have found that the effects of curvature in field space on the squeezed bispectrum can be quantified by the slopes of the field-space metric and the differences of number of e-folding between flat and curved models, arising from moving between two specific points in field space. According to our analysis, The number of e-folding for moving between two particular points in field space of curved field-space model is less than that of flat field-space model. From our concrete form of the metric in field space, amplitudes and shapes of f_{NL} can be affected by the curvature in field space. When λ_2 and p increase, amplitudes of backward f_{NL} increase, while amplitudes of forward f_{NL} decrease, and peaks of both cases shift to larger NL. Furthermore, shapes of both forward and backward f_{NL} are largely modified when $p = 3$. This indicates that future detections of squeezed bispectrum can put tight constrains on the curvature in field space of multi-field inflationary model.

APPENDIX

APPENDIX A N_I, N_{IJ} AND $\partial\varphi_B^J/\partial\varphi_A^K$

A.1 N_I

In order to compute N_I, we set t_1 and t_2 in Eq. (4.24) to be times at which the specific perturbations modes are on the spatially flat and uniform density hypersurfaces respectively. Thus we have

$$N = \int_{\phi_u}^{\phi_*} \frac{U(\phi)}{U_\phi(\phi)} d\phi + \int_{\chi_u}^{\chi_*} G(\phi(\chi)) \frac{V(\chi)}{V_\chi(\chi)} d\chi\,, \tag{A.1}$$

where subscripts $_*$ and $_u$ denote evaluation at the time when perturbations are on the spatially flat and uniform density hypersurfaces respectively. Differentiating the above equation with respect to ϕ_* and χ_*, we get

$$N_\phi \equiv \frac{\partial N}{\partial \phi_*} = \frac{U_*}{U_{\phi_*}} - \frac{\partial \phi_u}{\partial \phi_*}\frac{U_u}{U_{\phi_u}} - \frac{\partial \chi_u}{\partial \phi_*} G(\phi_*) \frac{V_u}{V_{\chi_u}} + \int_{\chi_u}^{\chi_*} \frac{\partial G(\phi(\chi))}{\partial \phi_*}\frac{V(\chi)}{V_\chi(\chi)} d\chi\,, \tag{A.2}$$

$$N_\chi \equiv \frac{\partial N}{\partial \chi_*} = G(\phi_*) \frac{V_*}{V_{\chi_*}} - \frac{\partial \chi_u}{\partial \chi_*} G(\phi_u) \frac{V_u}{V_{\chi_u}} - \frac{\partial \phi_u}{\partial \chi_*}\frac{U_u}{U_{\phi_u}} + \int_{\chi_u}^{\chi_*} \frac{\partial G(\phi(\chi))}{\partial \chi_*}\frac{V(\chi)}{V_\chi(\chi)} d\chi\,. \tag{A.3}$$

The derivative of fields at $t = t_u$ with respect to fields at $t = t_*$ can be computed from Eq. (4.21) and the condition $\delta\rho = 0$ on uniform density hypersurfaces. Setting (ϕ_1, χ_1) and (ϕ_2, χ_2) in Eq. (4.21) to be (ϕ_*, χ_*) and (ϕ_u, χ_u) respectively, and differentiating the result with respect to ϕ_* and χ_*, we get

$$0 = \frac{1}{U_{\phi_*} G(\phi_*)} - \frac{\partial \phi_u}{\partial \phi_*} \frac{1}{U_{\phi_u} G(\phi_u)} + \frac{\partial \chi_u}{\partial \phi_*}\frac{1}{V_{\chi_u}} \tag{A.4}$$

$$0 = \frac{\partial \phi_u}{\partial \chi_*} \frac{1}{U_{\phi_u} G(\phi_u)} + \frac{1}{V_{\chi_*}} - \frac{\partial \chi_u}{\partial \chi_*}\frac{1}{V_{\chi_u}}\,. \tag{A.5}$$

On the uniform density hypersurfaces, there are no perturbations in energy density $\delta\rho = 0$, so that if the slow roll approximation is assumed, we have $\delta\rho \simeq \delta W =$

$U_{\phi u}\delta\phi + U_{\chi u}\delta\chi$ and therefore

$$
\begin{aligned}
0 &= U_{\phi_u}\frac{\partial\phi_u}{\partial\phi_*} + V_{\chi_u}\frac{\partial\chi_u}{\partial\phi_*}\,, \\
0 &= U_{\phi_u}\frac{\partial\phi_u}{\partial\chi_*} + V_{\chi_u}\frac{\partial\chi_u}{\partial\chi_*}\,. \qquad \text{(A.6)}
\end{aligned}
$$

Solving the above four equations, we obtain

$$\frac{\partial\phi_u}{\partial\phi_*} = \frac{G(\phi_u)\,U_{\phi_u}V^2_{\chi_u}}{G(\phi_*)\,U_{\phi_*}V^2_{\chi_u} + G(\phi_*)\,U_{\phi_*}G(\phi_u)\,U^2_{\phi_u}}\,, \qquad \text{(A.7)}$$

$$\frac{\partial\chi_u}{\partial\phi_*} = -\frac{G(\phi_u)\,U^2_{\phi_u}V_{\chi_u}}{G(\phi_*)\,U_{\phi_*}V^2_{\chi_u} + G(\phi_*)\,U_{\phi_*}G(\phi_u)\,U^2_{\phi_u}}\,, \qquad \text{(A.8)}$$

$$\frac{\partial\phi_u}{\partial\chi_*} = -\frac{G(\phi_u)\,U_{\phi_u}V^2_{\chi_u}}{V_{\chi_*}G(\phi_u)\,U^2_{\phi_u} + V_{\chi_*}V^2_{\chi_u}}\,, \qquad \text{(A.9)}$$

$$\frac{\partial\chi_u}{\partial\chi_*} = \frac{G(\phi_u)\,U^2_{\phi_u}V_{\chi_u}}{V_{\chi_*}G(\phi_u)\,U^2_{\phi_u} + V_{\chi_*}V^2_{\chi_u}}\,. \qquad \text{(A.10)}$$

In order to evaluate the integration terms in Eqs. (A.2) and (A.3), we express the field space metric G in terms of χ by inserting Eq. (4.22) into Eq. (4.16) as

$$G(\phi(\chi)) = \frac{\lambda_1}{1-\lambda_2 f_\phi(\phi_1)(\chi/\chi_1)^{rp}}\,. \qquad \text{(A.11)}$$

Setting $(\phi_1, \chi_1) = (\phi_*, \chi_*)$, and differentiating this equation with respect to ϕ_* and χ_* , we respectively get

$$\frac{\partial G(\phi(\chi))}{\partial\phi_*} = \frac{\partial f_\phi(\phi_*)}{\partial\phi_*}\frac{\lambda_1\lambda_2 f_\phi(\phi_*)(\chi/\chi_*)^{rp}}{(1-\lambda_2 f_\phi(\phi_*)(\chi/\chi_*)^{rp})^2} = \frac{1}{\lambda_1 f_\phi(\phi_*)}\frac{\partial f_\phi(\phi_*)}{\partial\phi_*}G(\phi)\phi^p\,, \qquad \text{(A.12)}$$

$$\frac{\partial G(\phi(\chi))}{\partial\chi_*} = -\frac{1}{\chi_*}\frac{\lambda_1 rp\lambda_2 f_\phi(\phi_*)(\chi/\chi_*)^{rp}}{(1-\lambda_2 f_\phi(\phi_*)(\chi/\chi_*)^{rp})^2} = -\frac{rp\lambda_2}{\lambda_1\chi_*}G(\phi)\phi^p\,, \qquad \text{(A.13)}$$

Inserting the above relation into the integration terms in Eqs. (A.2) and (A.3) and performing suitable integration by parts, we can write the parts that cannot be integrated analytically in terms of the number of e-folding given Eq. (A.1) and obtain

$$N_\phi = \frac{G_* U_* V_{\chi_u}^2 + G_* U_* G_u U_{\phi_u}^2 - G_u U_u V_{\chi_u}^2 + G_u^2 V_u U_{\phi_u}^2}{G_* U_{\phi_*} V_{\chi_u}^2 + G_* G_u U_{\phi_*} U_{\phi_u}^2} + \frac{\lambda_1 \delta_N}{2 r_2 G_* \phi_*}\,, \tag{A.14}$$

$$N_\chi = \frac{G_* V_* + \frac{G_u \left(U_u V_{\chi_u}^2 - G_u V_u U_{\phi_u}^2\right)}{G_u U_{\phi_u}^2 + V_{\chi_u}^2}}{V_{\chi_*}} - \frac{\delta_N}{2\chi_*}\,, \tag{A.15}$$

where we have used $df_\phi/d\phi_s = p\lambda_1 f_\phi/(\phi_* G_*)$ and

$$\delta_N \equiv -4N + \phi_*^2 - \phi_u^2 + G\left(\phi_*\right)\chi_*^2 - G\left(\phi_u\right)\chi_u^2\,. \tag{A.16}$$

The number of e-folding N in the above equation is given by Eq. (A.1). It is clear that $\delta_N = 0$ when the curvature in field space disappears, i.e., $\lambda_2 = 0$. From figure (1), we see that $\phi \to 0$ at the end of inflation. Since we are interested to evaluate $f_{\rm NL}$ at the end of inflation, we set t_u to be a time at the end of inflation, so that we have $\phi_u \sim 0$ and therefore the above equations become

$$N_\phi = \frac{\phi_*}{2} + \frac{\lambda_1 \delta_N}{2 r_2 G_* \phi_*}\,, \quad N_\chi = \frac{G_* \chi_*}{2} - \frac{\delta_N}{2\chi_*}\,. \tag{A.17}$$

where we have insert the expressions for U and V from Eq. (4.15) into the above equation.

A.2 N_{IJ}

Since we are interested in the case $\phi_u \sim 0$, N_{IJ} can be computed by differentiating Eq. (A.17) with respect to ϕ_* and χ_* and the results are

$$
\begin{aligned}
N_{IJ} &= \begin{pmatrix} N_{\phi\phi} & N_{\phi\chi} \\ N_{\chi\phi} & N_{\chi\chi} \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{G_*}{2} \end{pmatrix} + \begin{pmatrix} -\lambda_1 \frac{\delta_N (G_* r_2 + 2\lambda_1) + G_{\phi_*} r_2 \phi_* \left(\delta_N - G_* \chi_*^2\right)}{2 G_*^2 r_2^2 \phi_*^2} & \frac{\delta_N \lambda_1}{G_* r_2 \phi_* \chi_*} \\ \frac{\delta_N \lambda_1}{G_* r_2 \phi_* \chi_*} & -\frac{\delta_N}{2\chi_*^2} \end{pmatrix}, \quad \text{(A.18)}
\end{aligned}
$$

where $G_{\phi_*} \equiv dG(\phi_*)/d\phi_*$.

A.3 $\partial\varphi^J_B/\partial\varphi^K_A$

We first set (ϕ_1, χ_1) and (ϕ_2, χ_2) in Eq. (4.21) to (ϕ_A, χ_A) and (ϕ_B, χ_B) respectively. Differentiating the result with respect to ϕ_A and χ_A, we respectively get

$$0 = \frac{1}{U_{\phi_A} G(\phi_A)} - \frac{\partial\phi_B}{\partial\phi_A}\frac{1}{U_{\phi_B} G(\phi_B)} + \frac{\partial\chi_B}{\partial\phi_A}\frac{1}{V_{\chi_B}} \qquad (A.19)$$

$$0 = \frac{\partial\phi_B}{\partial\chi_A}\frac{1}{U_{\phi_B} G(\phi_B)} + \frac{1}{V_{\chi_A}} - \frac{\partial\chi_B}{\partial\chi_A}\frac{1}{V_{\chi_B}} . \qquad (A.20)$$

In order to solve the above equations for $\partial\varphi^J_B/\partial\varphi^K_A$, we need two more equations of $\partial\varphi^J_B/\partial\varphi^K_A$. The required equations can be obtained by differentiate the equation for the number of e-folding with respect to ϕ_A and χ_A. However, there are two choices of the specification of a point B at which the short wavelength perturbation mode exits the horizon. We consider each specification separately in the following sections.

A.3.1 Forward formulation

For the forward formulation, a point B in field space is specified from a point A such that $N_A - N_B$ is constant where N_A and N_B are the number of e-folding realised backwords in time from the end of inflation to times at which the long and short wavelength exit horizon respectively. Hence, we have

$$N_A - N_B = \int_{\phi_B}^{\phi_A} \frac{U(\phi)}{U_\phi(\phi)} d\phi + \int_{\chi_B}^{\chi_A} G(\phi(\chi)) \frac{V(\chi)}{V_\chi(\chi)} d\chi = \text{constant} \,. \tag{A.21}$$

Differentiating the above equations with respect to ϕ_A and χ_A, we get

$$0 = \frac{U_A}{U_{\phi_A}} - \frac{\partial \phi_B}{\partial \phi_A}\frac{U_B}{U_{\phi_B}} - \frac{\partial \chi_B}{\partial \phi_A} G_A \frac{V_B}{V_{\chi_B}} + \int_{\chi_B}^{\chi_A} \frac{\partial G(\phi(\chi))}{\partial \phi_*}\frac{V(\chi)}{V_\chi(\chi)} d\chi \,, \tag{A.22}$$

$$0 = G_A \frac{V_A}{V_{\chi_A}} - \frac{\partial \chi_B}{\partial \chi_A} G_B \frac{V_B}{V_{\chi_B}} - \frac{\partial \phi_B}{\partial \chi_A}\frac{U_B}{U_{\phi_B}} + \int_{\chi_B}^{\chi_A} \frac{\partial G(\phi(\chi))}{\partial \chi_*}\frac{V(\chi)}{V_\chi(\chi)} d\chi \,. \tag{A.23}$$

Expressing the integrations in the above equations in terms of the number of e-folding and solving Eq. (A.19), (A.20), (A.22) and (A.23), we obtain

$$\Gamma^f_{BA} \equiv \begin{pmatrix} \frac{\partial \phi_B}{\partial \phi_A} & \frac{\partial \phi_B}{\partial \chi_A} \\ \frac{\partial \chi_B}{\partial \phi_A} & \frac{\partial \chi_B}{\partial \chi_A} \end{pmatrix} \tag{A.24}$$

$$= \begin{pmatrix} \frac{U_{\phi_B}(G_A U_A + G_B V_B)}{G_A U_{\phi_A} W_B} & \frac{U_{\phi_B}(G_A V_A - G_B V_B)}{V_{\chi_A} W_B} \\ \frac{V_{\chi_B}(G_A U_A - G_B U_B)}{G_A G_B U_{\phi_A} W_B} & \frac{V_{\chi_B}(G_B U_B + G_A V_A)}{G_B V_{\chi_A} W_B} \end{pmatrix} + \begin{pmatrix} \frac{U_{\phi_B}}{W_B} I_1 & \frac{U_{\phi_B}}{W_B} + I_2 \\ \frac{V_{\chi_B}}{G_B W_B} I_1 & \frac{V_{\chi_B}}{G_B W_B} + I_2 \end{pmatrix} ,$$

where

$$I_1 \equiv \frac{\lambda_1 \delta_{N_{BA}}}{2 r_2 G_A \phi_A} \,,$$

$$I_2 \equiv -\frac{\delta_{N_{BA}}}{2\chi_A} \,,$$

$$\delta_{N_{BA}} \equiv -4N_{BA} + \phi_A^2 - \phi_B^2 + G(\phi_A)\chi_A^2 - G(\phi_B)\chi_B^2 \,, \tag{A.25}$$

where $N_{BA} \equiv N_B - N_A$ which is given in Eq. (A.21). Substituting the expressions for the potentials from Eq. (4.15) into Eq. (A.24), we get

$$\Gamma^{f}_{BA} = \begin{pmatrix} \frac{1}{2W_B}\left(\phi_A\phi_B m_\phi^2 + \frac{G_B m_\chi^2 \phi_B \chi_B^2}{G_A\phi_A} + \frac{m_\chi^2\delta_N}{G_B}\right) & -\frac{m_\phi^2\phi_B}{2\chi_A\chi_B W_B}\left(G_B\chi_B^3 - G_A\chi_A^2\chi_B + \delta_N\chi_A\right) \\ \frac{1}{2W_B} m_\chi^2\chi_B\left(\frac{\delta_N m_\chi^2}{m_\phi^2 G_B^2\phi_B} + \frac{\phi_A}{G_B} - \frac{\phi_B^2}{G_A\phi_A}\right) & \frac{-\delta_N\chi_A m_\chi^2 + G_A\chi_A^2\chi_B m_\chi^2 + G_B m_\phi^2\phi_B^2\chi_B}{2G_B\chi_A W_B} \end{pmatrix}. \tag{A.26}$$

A.3.2 Backward formulation

In the backward formulation, points B and A in field space are specified by N_B and N_A which are the number of e-folding realised backwords in time from the end of inflation to times at which the short and long wevlength perturbations exit horizon respectively. For this case, the additional equations for $\partial\varphi_B^J/\partial\varphi_A^K$ can be obtained by differentiating equation

$$N_B = \int_{\phi_u}^{\phi_B} \frac{U(\phi)}{U_\phi(\phi)} d\phi + \int_{\chi_B}^{\chi_u} G(\phi(\chi)) \frac{V(\chi)}{V_\chi(\chi)} d\chi \,, \tag{A.27}$$

with respect to ϕ_A and χ_A. The differentiation gives two equations describing relations among $\partial\varphi_B^I/\partial\varphi_A^J$ and $\partial\varphi_u^I/\partial\varphi_A^J$. The expressions for $\partial\varphi_u^I/\partial\varphi_A^J$ can be computed using the same approach as for Eq. (A.10), and the results take similar form as in Eq. (A.10) with the replacement of evaluation at point * by evaluation at point A in field space. Inserting these results into the relations among $\partial\varphi_B^I/\partial\varphi_A^J$ and $\partial\varphi_u^I/\partial\varphi_A^J$ obtained from the differentiate of Eq. (A.27) with respect to ϕ_A and χ_A, we get two relations for $\partial\varphi_B^I/\partial\varphi_A^J$. Solving these two relations together with

Eqs. (A.19) and (A.20), we obtain

$$\begin{aligned}\frac{\partial\phi_B}{\partial\phi_A} &= \frac{U_{\phi_B}\left(G_u\left(G_BV_B-G_uV_u\right)U_{\phi_u}^2+\left(G_uU_u+G_BV_B\right)V_{\chi_u}^2\right)}{G_AU_{\phi_A}W_B\left(G_uU_{\phi_u}^2+V_{\chi_u}^2\right)}\\ &\quad -\frac{U_{\phi_B}}{W_B}\frac{\lambda_1\delta_{N_B}}{2r_2G_A\phi_A}, \qquad (A.28)\\ \frac{\partial\phi_B}{\partial\chi_A} &= -\frac{U_{\phi_B}\left(G_u\left(G_BV_B-G_uV_u\right)U_{\phi_u}^2+\left(G_uU_u+G_BV_B\right)V_{\chi_u}^2\right)}{V_{\chi_A}W_B\left(G_uU_{\phi_u}^2+V_{\chi_u}^2\right)}\\ &\quad \frac{U_{\phi_B}}{W_B}\frac{\delta_{N_B}}{2\chi_A}, \qquad (A.29)\\ \frac{\partial\chi_B}{\partial\phi_A} &= -\frac{V_{\chi_B}\left(G_BU_B\left(G_uU_{\phi_u}^2+V_{\chi_u}^2\right)+G_u\left(G_uU_{\phi_u}^2V_u-U_uV_{\chi_u}^2\right)\right)}{G_AG_BU_{\phi_A}W_B\left(G_uU_{\phi_u}^2+V_{\chi_u}^2\right)}\\ &\quad -\frac{V_{\chi_B}}{G_BW_B}\frac{\lambda_1\delta_{N_B}}{2r_2G_A\phi_A}, \qquad (A.30)\\ \frac{\partial\chi_B}{\partial\chi_A} &= \frac{V_{\chi_B}\left(G_BU_B\left(G_uU_{\phi_u}^2+V_{\chi_u}^2\right)+G_u\left(G_uU_{\phi_u}^2V_u-U_uV_{\chi_u}^2\right)\right)}{G_BW_BV_{\chi_A}\left(G_uU_{\phi_u}^2+V_{\chi_u}^2\right)}\\ &\quad -\frac{V_{\chi_B}}{G_BW_B}\frac{\delta_{N_B}}{2\chi_A}, \qquad (A.31)\end{aligned}$$

where

$$\delta_{N_B}\equiv -4N_B+\phi_B^2-\phi_u^2+G\left(\phi_B\right)\chi_B^2-G\left(\phi_u\right)\chi_u^2\,, \qquad (A.32)$$

and N_B is given by Eq. (A.27). Using $\phi_u\sim 0$ and the expressions for the potentials from Eq. (4.15), we get $U_u\sim U_{\phi_u}\sim 0$ and consequently the above equations give

$$\begin{aligned}\Gamma^b_{BA} &\equiv \begin{pmatrix}\frac{\partial\phi_B}{\partial\phi_A} & \frac{\partial\phi_B}{\partial\chi_A}\\ \frac{\partial\chi_B}{\partial\phi_A} & \frac{\partial\chi_B}{\partial\chi_A}\end{pmatrix} \qquad (A.33)\\ &= \begin{pmatrix}\frac{G_BU_{\phi_B}V_B}{G_AU_{\phi_A}W_B} & -\frac{G_BU_{\phi_B}V_B}{V_{\chi_A}W_B}\\ -\frac{U_BV_{\chi_B}}{G_AU_{\phi_A}W_B} & \frac{U_BV_{\chi_B}}{V_{\chi_A}W_B}\end{pmatrix}+\begin{pmatrix}-\frac{U_{\phi_B}}{W_B}\frac{\lambda_1\delta_{N_B}}{2r_2G_A\phi_A} & \frac{U_{\phi_B}}{W_B}\frac{\delta_{N_B}}{2\chi_A}\\ -\frac{V_{\chi_B}}{G_BW_B}\frac{\lambda_1\delta_{N_B}}{2r_2G_A\phi_A} & \frac{V_{\chi_B}}{G_BW_B}\frac{\delta_{N_B}}{2\chi_A}\end{pmatrix}.\end{aligned}$$

Substituting the expressions for the potentials from Eq. (4.15) into the above equation, we obtain

$$\Gamma^b_{BA} = \begin{pmatrix}\frac{\phi_Bm_\chi^2}{2G_A\phi_AW_B}\left(G_B\chi_B^2-\delta_{N_B}\right) & -\frac{\phi_B}{2\chi_AW_B}\left(G_Bm_\chi^2\chi_B^2-m_\phi^2\delta_{N_B}\right)\\ -\frac{\chi_Bm_\chi^2}{2m_\phi^2\phi_AG_AW_B}\left(m_\phi^2\phi_B^2+\frac{m_\chi^2}{G_B}\delta_{N_B}\right) & \frac{\chi_B}{2\chi_AW_B}\left(m_\phi^2\phi_B^2+\frac{m_\chi^2}{G_B}\delta_{N_B}\right)\end{pmatrix}. \qquad (A.34)$$

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