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Time-dependent Lagrangian Perturbation Theory with The Dynamical Dark Energy

Teeraparb Chantavat[1]

[1] ThEP's Laboratory of Cosmology and Gravity, The Institute for Fundamental Study "The Tah Poe Academia Institute", Naresuan University, Phitsanulok, 65000, Thailand

E-mail: `teeraparbc@nu.ac.th`

Abstract. This article gives a brief overview of the time-dependent Lagrangian perturbation theory as a theoretical tool to predict the behavior of galaxy clustering in quasi-linear regimes. While most of the Lagrangian perturbation theories are based on an Einstein-De Sitter space where the growth function is time independent, more accurate theoretical predictions could be achieved by time-dependent growth functions in a more general cosmology. This is directly applicable to the dynamical dark-energy where the clustering behavior can be recognized with time-dependent growth functions.

1. Introduction

One of the main goals of cosmology is to understand the large-scale structure of the Universe. The structures that we see in the galaxy surveys today are derived from the widely accepted gravitational instability with primordial matter density fluctuations [1, 2]. A quantitative understanding of the dynamics of the structure formation requires a theoretical modeling that can be applied with statistical tools to test the theory against observations. Predicting the behavior of the formation of the structures is crucially important as a direct way to validate the current dynamical theory of gravity with the observations such as the Baryonic Acoustic Oscillations (BAOs) [3]. As a simple, but accurate, model for numerical predictions, an irrotational and pressureless fluid of cold dark matter physics is normally considered in the non-relativistic regime [4]. The observable could, in principle, be predicted. When higher-order terms are considered, one would expect that the numerical predictions should be more accurate; however, with the cost of higher complexities and computational time.

There are two main classes of non-relativistic cosmological perturbation theory. One is expressed observable quantities in terms of the Eulerian frame, called Standard Perturbation Theory (SPT) [5, 6, 7]. Other one is given in the Lagrangian frame, call Lagrangian Perturbation Theory (LPT) [8, 9]. This *article* I shall give a brief overview of the time dependent Lagrangian perturbation theory up to the third order with is sufficiently high enough for accurate numerical predictions especially the applications of the dynamical dark energy where the equation of state is a function of time.

2. Lagrangian Perturbation Theory

In Lagrangian perturbation theory, the dynamical variable is the displacement field $\boldsymbol{x}(t) = \boldsymbol{q}(\boldsymbol{x}) + \boldsymbol{\Psi}(\boldsymbol{q}, t)$, where $\boldsymbol{\Psi}$ is the displacement field, $\boldsymbol{q}$ is the position of the particle at the initial

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time. The dynamical equation is given by

$$J(\boldsymbol{q},t)\nabla\cdot\left[\frac{\mathrm{d}^2}{\mathrm{d}t^2}\boldsymbol{\Psi}+2H(t)\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\Psi}\right]=\frac{3}{2}\Omega_M(t)H^2(t)(J-1), \tag{1}$$

where $J(\boldsymbol{q},t)$ is the Jacobian of the transformation between $\boldsymbol{x}$ and $\boldsymbol{q}$. The Hubble parameter, $H(t)$, composes of the component of the universe mainly cold dark matter $\Omega_M(t)$ and dynamical dark energy $\Omega_{\rm DE}(t)$ with the constant curvature parameter k.

$$\frac{H^2(t)}{H_0^2}=\Omega_M(t)+\Omega_{\rm DE}(t)-\frac{kc^2}{a^2(t)}. \tag{2}$$

The dark energy depends on the equation of state parameter as

$$w(t)=w_0+w_1\frac{z(t)}{1+z(t)}, \tag{3}$$

where $z(t)$ is the redshift w_0 and w_1 are constants [10, 11].

3. The Evolution Equations for Perturbations

In order to describe the large scale structure that we see in the universe, we shall expand the density field and the velocity field as

$$\delta(\boldsymbol{x},t) = \delta_1(\boldsymbol{x},t)+\delta_2(\boldsymbol{x},t)+\ldots, \tag{4}$$
$$\boldsymbol{v}(\boldsymbol{x},t) = \boldsymbol{v}_1(\boldsymbol{x},t)+\boldsymbol{v}_2(\boldsymbol{x},t)+\ldots, \tag{5}$$

where δ_n and $\boldsymbol{v}_n$ is the corresponding nth order perturbative overdensity and velocity field respectively.

3.1. First Order

To the first order, we can write the overdensity as

$$\delta_1(\boldsymbol{x},t)=D_1(t)\epsilon, \tag{6}$$

where $D_1(t)$ is the linear growth function. $\epsilon(\boldsymbol{x})$ is the initial background overdensity. The linear growth function can be found by solving the second-order differential equation with appropriated initial conditions;

$$\ddot{D}_1+2H\dot{D}_1-\frac{3}{2}\Omega_M H^2 D_1=0. \tag{7}$$

3.2. Second Order

In order to calculate the second order term, the velocity potential, $\varphi(\boldsymbol{x})$, is needed,

$$\boldsymbol{v}_1(\boldsymbol{x},t)=-\dot{D}_1\boldsymbol{\nabla}\varphi(\boldsymbol{x}). \tag{8}$$

It turns out that the time-dependent growth function for the second-order perturbation $D_2(t)$ is given by

$$\ddot{D}_2+2H\dot{D}_2-\frac{3}{2}\Omega_M H^2 D_2=\frac{3}{2}\Omega_M H^2 D_1^2. \tag{9}$$

As one can see the second order growth function is proportional to D_1^2.

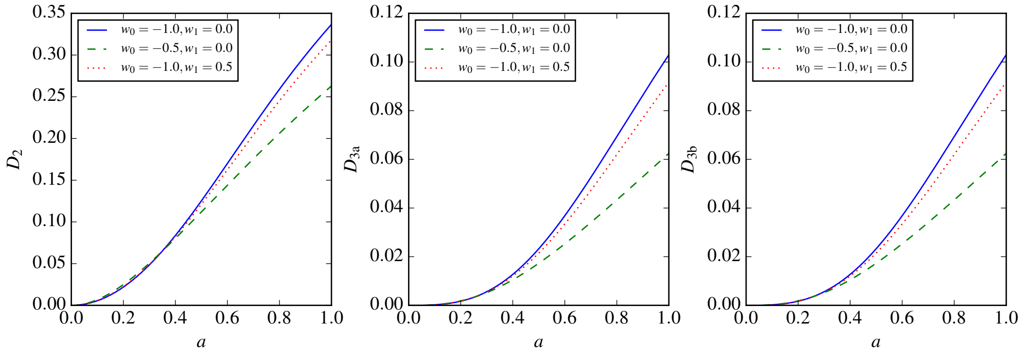

Figure 1. The time evolution functions for third order terms $\Omega_{M,0} = 0.3$, $\Omega_{\mathrm{DE},0} = 0.7$ for different values of w_0 and w_1 with the normalization condition $D_1 = 1$ at $a = 1$.

3.3. Third Order

There are two time dependent growth functions for the third order terms $D_{3\mathrm{a}}(t)$ and $D_{3\mathrm{b}}(t)$ which are given by

$$\ddot{D}_{3\mathrm{a}} + 2H\dot{D}_{3\mathrm{a}} - \frac{3}{2}\Omega_M H^2 D_{3\mathrm{a}} = \frac{3}{2}\Omega_M H^2 \left(D_1^3 + 3D_1 D_2\right), \tag{10}$$

$$\ddot{D}_{3\mathrm{b}} + 2H\dot{D}_{3\mathrm{b}} - \frac{3}{2}\Omega_M H^2 D_{3\mathrm{b}} = \frac{3}{2}\Omega_M H^2 D_1^3. \tag{11}$$

Similar, to the second-order growth function, the third-order growth functions are proportional to D_1^3.

4. Results

In this article, we shall assume a flat cosmology with dark energy component i.e. $\Omega_{\mathrm{M},0} = 0.3, \Omega_{\mathrm{DE},0} = 0.7$ and $k = 0$. The time evolution equations are solved using fourth order Runge-Kutta method with the condition $D_1(a = 1) = 1$ i.e. the linear growth function is equal to unity at the present epoch. The second and third-order growth functions are shown in FIG. 1 for different cosmological models with different value of w_0 and w_1. The growth functions are sensitive to the equation of state parameter of the dynamical dark energy. The difference in growth rate is more pronounced at the present epoch.

5. Conclusions

We study the third order Lagrangian perturbation theory and derive the time dependent evolution functions. We find that there are different growth functions for different perturbative orders; however, there are two independent evolution functions for the third-order quantities which describe different terms in the expansion. The evolution function is sensitive to the equation of state parameter of the dynamical dark energy w which is helpful for large-scale structure surveys, where the effect of time-dependent w would be discernible. While the effect of w on large-scale structure would look marginally different from the time-independent one's, it is possible to stack the differences in many redshift slides to amplify the difference. This can be achieved with the the Alcock-Paczyński (AP) test [12] where the change in the angular diameter distance can be recognized. Our approach is advantageous to the standard perturbation theory (SPT) approach since we only need to specify the initial conditions for the first order quantities in the density and velocity field.

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