



Qualifying Examination



Round 1/2564 (1/2021)

30 August 2021

Name:.....

Student ID:.....

Part 2: Subjects Relating to student's Research Field : General Relativity and Cosmology

Instructions and cautions

1. Choose 3 out of the following 6 problem relating student's field.
2. Write down detailed and clear solution to problems of your choice.
3. Complete answer are preferred to fragments.
4. Write on one side of the paper only and begin each answer on a space provided.
5. Write legibly, otherwise you place yourself at a grave disadvantage.

Question No.	Checked Question to be marked
1	
2	
3	
4	
5	
6	

Name:.....

1 **Manifold, Curvature, Gravitation, Spherically symmetric solution, Black holes**

Question 1. One of the most popular solutions of Einstein field equation is a static and spherically-symmetric solution.

1.1) For 4-dimensional spacetime, write down a general form of the metric solution which is respect to the static condition and spherical symmetry as well as explain how to obtain it.

[3 points]

1.2) The vacuum solution can be written as

$$ds^2 = - \left(1 - \frac{2\mu}{r}\right) c^2 dt^2 + \left(1 - \frac{2\mu}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

where μ is a constant. Indentify the singularity points and then explain how they are different.

[3 points]

1.3) In case of spherically symmetric solution (does not need to be static), explain how the solution should be by comparing to the static one.

[4 points]

Name:.....

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2 Field theory of gravity

Question 2. The Einstein field equation can be obtained in terms of classical field theory associated with the Einstein-Hilbert action as follows

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} + L_m \right),$$

2.1) Find the form of the energy momentum tensor to satisfy the Einstein field equation according to the variational principle. [4 points]

2.2) By requiring that the action must be invariant under general coordinate transformation, show that the energy momentum tensor will be covariantly conserved. [6 points]

Name:.....

Name:.....

3 Linearized gravity, Gravitational wave

Question 3. It is well-known that the solution for the linearized gravitation is a wave solution called gravitational wave.

3.1) Identify the number of propagating degrees of freedom of the gravitational wave and then explain how to obtain such a number according to the symmetry of the theory. [5 points]

3.2) One of the crucial differences between the electromagnetic wave and gravitational wave is that it is not possible to generate gravitational wave by using a dipole source while it is possible for electromagnetic wave. Explain how/why they are different. [5 points]

Name:.....

Name:.....

4 Inflationary cosmology: Based on the book of V. Mukhanov

Question 4. One of the important keys of the inflationary model in cosmology which make it differs from other accelerated expansion of the universe is that the "graceful exit".

4.1) Explain what is the "graceful exit" in this context then give the simple potential form for a scalar field to characterize the model which is graceful exit and is not graceful exit. [4 points]

4.2) Inflationary model can be classified into three classes; Old inflation, New inflation and Chaotic inflation. Explain how these three classes are different. [6 points]

Name:.....

5 Physical Foundations of Cosmology, chapter 5, and lecture on inflation universe for Cosmology I course, Acceleration equation

Question 5. One of generalized versions of scalar field inflation is the scalar field with non-canonical kinetic terms called k -inflation. The action of such a model can be written as

$$S = \int d^4x \sqrt{-g} (P(X, \phi)), \quad X = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi.$$

5.1) Find the energy momentum tensor of the scalar field. [4 points]

5.2) Identify the energy density and pressure of the scalar field by comparing to the perfect fluid one and then find the condition on the form of P to obtain the acceleration phase of the universe. [3 points]

5.3) Find the conditions on the form of P to obtain the graceful exit. [3 points]

Name:.....

6 Problems of Newtonian cosmology, shortcomings of the Big Bang model

Question 6. In 1927, Lemaitre proposed a model of the evolution of the universe. This model provides the evolution of the universe classified into 3 stages; Cosmic expansion, very slow expansion and fast expansion. Later on, this model is known as "hot Big Bang" model.

6.1) Explain in details how the universe evolves in each stage and what is the dominant content in the universe at each stage. [6 points]

6.2) Explain how this model cannot be used to explain the universe nowadays and then find the proper solutions to this shortcomings. [4 points]

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