



Qualifying Examination



Round 1/2564 (1/2021)

30 August 2021

Name:.....

Student ID:.....

Part 2: Subjects Relating to student's Research Field : Complex system and Network Theory

Instructions and cautions

1. Choose 3 out of the following 6 problem relating student's field.
2. Write down detailed and clear solution to problems of your choice.
3. Complete answer are preferred to fragments.
4. Write on one side of the paper only and begin each answer on a space provided.
5. Write legibly, otherwise you place yourself at a grave disadvantage.

Question No.	Checked Question to be marked
1	
2	
3	
4	
5	
6	

1 Network basics

Question 1. For a undirected network with $N = \{1, 2, 3, 4, 5, 6, 7\}$ as set of nodes and edges set $E = \{(1,2), (1,3), (1,4), (2,3), (2,5), (3,5), (3,1), (3,2), (4,5), (5,6), (2,7), (4,7)\}$. Find the

1.1) Adjacency matrix [2 marks]

1.2) Network density [2 marks]

1.3) Degree of each node, degree distribution and average degree of the network. [4 marks]

1.4) Distance matrix. [2 marks]

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2 Network connectivity and walks

Question 2 Consider an undirected network $G(N, E)$ with 5 nodes ($N = \{1, 2, 3, 4, 5\}$) and set of edges as

$$E = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 5), (3, 5), (3, 1), (3, 2), (4, 5)\}.$$

2.1) Draw the above network. Also give the adjacency matrix. [2 marks]

2.2) How do you estimate the number of total possible walks of length K between any two given nodes in a network from the adjacency matrix ? Find number of all possible walks of length 4 between the following pairs of nodes $(1, 2)$ and $(1, 5)$. [3 marks]

2.3) Find all the closed paths of length 3 from each node. [2 marks]

2.4) What is a complete network ? Calculate the number of edges in a complete network (directed and undirected) having N vertices. How many more edges are needed to make it $G(N, E)$ as a complete network ? [1 marks]

2.5) Find the number of edges in network H which is the Complement of G . Find adjacency matrix of network H . [2 marks]

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3 Random networks

Question 3.1) Derive the expression for the degree distribution of Erdos Renyi graph $ER(N, p)$, where N is the number of nodes in the network and p is the probability with which any two nodes in the network are connected. [5 marks]

3.2) Find the average degree of a random graph (Erdos Renyi) $ER(N, p)$, where N is the number of nodes in the network and p is the probability with which any two nodes in the network are connected. [5 marks]

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4 Spectral properties of Network

Question 4.1) Define the energy of a network. [2 marks]

4.2) How to evaluate the robustness and the flow of information in a network ? [3 marks]

4.3) What is the Fiedler Eigenvalue and Eigenvector?

Give their physical significance. [3 marks]

4.4) Define spectral gap of a network. [2 marks]

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5 Network Laplacian and centrality

Question 5.1) Define the Laplacian matrix of a network. What are the bounds for the eigen values of the Laplacian matrix? [2 marks]

5.2) Define the Subgraph centrality of a node in a network. How can we model the effect of external parameter say β on the sub graph centrality. Using subgraph centrality, discuss the Estrada index of a network. [5 marks]

5.3) How can we represent a network in the form of Density matrix? Define the Von-Neumann Entropy for a network? [3 marks]

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6 Minimum spanning Tree

Question 6.1) What is the difference between a spanning tree and minimum spanning tree?

[3 marks]

6.2) Given the weighted network in Figure 1, find the minimum spanning tree (MST) of the network.

[4 marks]

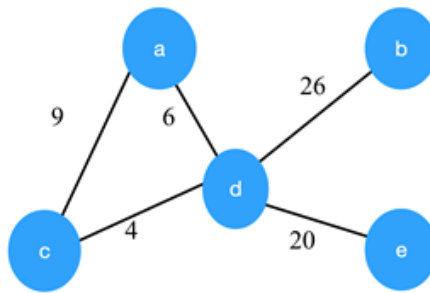


Figure 1: Weighted network

6.3) Consider a network of 3 vertices with adjacency matrix as

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad (1)$$

How many different minimum spanning trees are possible? What is the weight cost of each MST ?

[3 marks]

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