



Qualifying Examination



Academic Year 2026 (2569) : PART 2 : ROUND 1

10 August 2026

Name:.....

Student ID:.....

Part 2: Subjects Relating to student's Research Field : General Relativity and Black Hole Physics

Instructions and cautions

1. Choose 3 out of the following 6 questions relating student's field.
2. Write down detailed and clear solution to problems of your choice.
3. Complete answer are preferred to fragments.
4. Write on one side of the paper only and begin each answer on a space provided.
5. Write legibly, otherwise you place yourself at a grave disadvantage.

Question No.	Checked Question to be marked
1	
2	
3	
4	
5	
6	

Name:.....

1 Manifold, Curvature, Gravitation, Einstein equation

(CHC)

Question 1.

1.1) [10 points.] Assuming Torsion free and $\nabla_\mu A^\nu = \partial_\mu A^\nu + \Gamma_{\alpha\mu}^\nu A^\alpha$, show that the Riemann curvature tensor

$$R^\alpha_{\beta\mu\nu} = \partial_\mu \Gamma_{\beta\nu}^\alpha - \partial_\nu \Gamma_{\beta\mu}^\alpha + \Gamma_{\beta\nu}^\rho \Gamma_{\rho\mu}^\alpha - \Gamma_{\beta\mu}^\rho \Gamma_{\rho\nu}^\alpha.$$

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2 Field theory of gravity

(CHC)

Question 2. The Dirac gamma matrices are a set of matrices satisfying the Clifford algebra $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$, where $g^{\mu\nu}$ denotes the symmetric spacetime metric. The antisymmetrized products of Dirac gamma matrices are defined by $\gamma^{\mu_1\mu_2\cdots\mu_p} = \gamma^{[\mu_1} \gamma^{\mu_2} \cdots \gamma^{\mu_p]}$, show that

2.1) [4 points.]

$$\gamma^{\mu\nu} = \gamma^{[\mu} \gamma^{\nu]} = \gamma^\mu \gamma^\nu - g^{\mu\nu}.$$

2.2) [6 points.]

$$\gamma^{\mu\nu\alpha} = \gamma^{[\mu} \gamma^\nu \gamma^{\alpha]} = \gamma^\mu \gamma^\nu \gamma^\alpha - \gamma^\mu g^{\nu\alpha} + \gamma^\nu g^{\alpha\mu} - \gamma^\alpha g^{\mu\nu}.$$

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3 Hamiltonian formalism for General Relativity

Question 3.

3.1) [2 points.] Write down the line element for the ADM metric.

3.2) [2 points.] Are the lapse function and shift vector dynamical variables in General Relativity? Why?

3.3) [2 points.] What is the difference between extrinsic and intrinsic curvatures?

3.4) [4 points.] Compute the momentum conjugate to a scalar field described by the following Lagrangian in the 3+1 formalism:

$$L = \int d^3x \left[\frac{1}{2N^2} \dot{\phi}^2 - \frac{N^i}{N^2} \partial_i \phi \dot{\phi} - \frac{1}{2} \left(h^{ij} - \frac{N^i N^j}{N^2} \right) \partial_i \phi \partial_j \phi - V(\phi) \right], \quad (1)$$

where N , N^i , and h_{ij} are the lapse function, shift vector, and induced metric on the three-dimensional hypersurfaces. Here, a dot denotes the derivative with respect to time, while $\partial_i \phi \equiv \partial \phi / \partial x^i$ denotes the derivative with respect to the spatial coordinate x^i .

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4 Quantum Cosmology

Question 4.

4.1) [3 points.] What is superspace in quantum cosmology?

4.2) [3 points.] What is minisuperspace in quantum cosmology?

4.3) [4 points.] Write down the quantum operator in the canonical quantization formalism for the dynamical variables in General Relativity.

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5 Black hole solutions

Question 5.

5.1) [3 points.] What is the Birkhoff's theorem [1], and explain how this theorem relates to the source of gravitational waves [2].

5.2) [3 points.] For the static spacetime with spherical symmetry, the general form of the metric can be written as

$$ds^2 = -e^{2\alpha(r)} dt^2 + e^{2\beta(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

and the energy-momentum tensor with the form of $T_\nu^\mu = \text{diag}(-\rho(r), -\rho(r), P(r), P(r))$ where $\rho(r) = \rho_0(l^3/r^3)$. By using the conservation of the energy-momentum tensor $\nabla_\mu T_\nu^\mu$, find the solution for $P(r)$

5.3) [4 points.] The non-zero independent components of the Ricci tensor can be written as

$$R_{00} = -e^{2(\alpha-\beta)} \left(\alpha'^2 - \beta' \alpha' + \alpha'' + \frac{2\alpha'}{r} \right), \quad (2)$$

$$R_{11} = \alpha'^2 - \beta' \alpha' + \alpha'' - \frac{2\beta'}{r}, \quad (3)$$

$$R_{22} = -e^{-2\beta} (-r\alpha' + e^{2\beta} + r\beta' - 1). \quad (4)$$

By using the Einstein field equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, find the solution [2] and explain the possibility of linking this solution to quantum effects [2].

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6 Black hole thermodynamics

Question 6.

For the Schwarzschild black hole solution with generalized entropy $S = \frac{A_h}{4} \frac{1}{1+\alpha r_h^2}$, where α is a parameter characterizing the modification of the entropy.

6.1) [5 points.] Find the first law of the black hole thermodynamics.

6.2) [5 points.] Find the heat capacity and then analyze the local stability of the black hole

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