



Qualifying Examination



Academic Year 2023 (2566) : PART 2 : ROUND 2

26 JAN 2024

Name:.....

Student ID:.....

Part 2: Subjects Relating to student's Research Field : General Relativity and Cosmology

Instructions and cautions

1. Choose 3 out of the following 6 questions relating student's field.
2. Write down detailed and clear solution to problems of your choice.
3. Complete answer are preferred to fragments.
4. Write on one side of the paper only and begin each answer on a space provided.
5. Write legibly, otherwise you place yourself at a grave disadvantage.

Question No.	Checked Question to be marked
1	
2	
3	
4	
5	
6	

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1 Manifold, Curvature, Gravitation, Spherically symmetric solution, Black holes

Question 1.

1.1). Prove the following relation and find S

$$\nabla_X(\nabla_Y Z^\alpha) - \nabla_Y(\nabla_X Z^\alpha) = S Z^\alpha + R^\alpha_{\rho\mu\nu} X^\mu Y^\nu Z^\rho,$$

where $\nabla_B = B^\mu \nabla_\mu$ and $R^\alpha_{\rho\mu\nu}$ is the curvature tensor.

[6 points]

1.2). Show the degrees of freedom for Riemann curvature tensor $R_{\mu\nu\alpha\beta}$ and Ricci tensor $R_{\mu\nu}$ in the five-dimensional spacetimes. One needs to declare the reductions with the symmetry properties.

[4 points]

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2 Field theory of gravity

Question 2.

For the gravitational theory with Palatini formalism, the connection is promoted to be the dynamical field.

2.1) What are the violated assumptions in General relativity of Einstein [2 points]

2.2) For the Einstein-Hilbert action, find the equations of motion (You can use Palatini equation directly: $\delta R_{\mu\nu} = \nabla_\rho \delta \Gamma_{\mu\nu}^\rho - \nabla_\nu \delta \Gamma_{\mu\rho}^\rho$). [4 points]

2.3) Explain the advantages and disadvantages of this formalism compare to General Relativity of Einstein. [4 points]

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3 Linearized gravity, Gravitational wave

Question 3.

In the linearized Einstein field equations, they can be written in terms of wave equations and gauge conditions as follows

$$\begin{aligned}\partial_\rho \partial^\rho h^{\mu\nu} &= 0, \\ \partial_\mu h^{\mu\nu} &= 0, \quad \partial_\rho \partial^\rho \xi^\mu = 0.\end{aligned}$$

- 3.1) What is the number of degrees of freedom and explain how to count it. [1 point]
- 3.2) Show that the solution is a wave function which is transverse and propagates with speed of light. [2 point]
- 3.3) From the formula of the quadrupole moment of the source $I^{ij} = \int T^{00} y^i y^j d^3y$, find the exact form of the equal-mass (M) circular binary star with radius (a) and constant angular frequency (Ω). [3 points]
- 3.4) Explain the logical steps in order to obtain the expression for the energy loss due to the gravitational wave from this binary star. [4 points]

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4 Physical Foundations of Cosmology: cosmological principle, perfect fluid, Λ CDM model

Question 4.

- 4.1) List the important matter and energy components in the Λ CDM model, and explain briefly what are roles of each components in the universe. [4 points]
- 4.2) Compute the energy density ρ_d of dark energy in terms of redshift z for the case where its equation of state parameter $w_d(z)$ depends on z . [6 points]

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5 Inflationary cosmology: models of cosmic inflation, necessary conditions for cosmic inflation

Question 5.

- 5.1) Compute the slow-roll parameter $\epsilon \equiv -\frac{\dot{H}}{H^2}$ in terms of the potential V of inflaton in the slow-roll limit, where H is the Hubble parameter. [4 points]
- 5.2) What is the graceful exit in inflationary scenario? What are conditions on evolution of ϵ and H required to have graceful exit? [3 points]
- 5.3) Show in detail whether an inflationary model has graceful exit if an inflaton is a phantom field. [3 points]

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6 Late-time cosmology: models of accelerating universe, problem of those models, basic of modified theories of gravity

Question 6.

- 6.1) What is the cosmic coincidence problem? How can this problem be alleviated?[3 points]
- 6.2) What is the chameleon mechanism? How can this mechanism be realized? [3 points]
- 6.3) Compute the redshift at which the universe starts to accelerate for the spatially flat Friedmann universe containing dark matter and dark energy with constant equation of state parameter w_d . Note that you can express your result in terms of w_d and the present values of the densities parameter of dark energy and dark matter. [4 points]

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