



Qualifying Examination



Academic Year 2025 (2568) : PART 2 : ROUND 3

15 MAY 2026

Name:.....

Student ID:.....

Part 2: Subjects Relating to student's Research Field : General Relativity and Black Hole Physics

Instructions and cautions

1. Choose 3 out of the following 6 questions relating student's field.
2. Write down detailed and clear solution to problems of your choice.
3. Complete answer are preferred to fragments.
4. Write on one side of the paper only and begin each answer on a space provided.
5. Write legibly, otherwise you place yourself at a grave disadvantage.

Question No.	Checked Question to be marked
1	
2	
3	
4	
5	
6	

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1 Manifold, Curvature, Gravitation, Einstein equation

Question 1. Consider a vector A^μ in a curved spacetime,

1.1) [10 points.] Assume $\nabla_\mu A^\mu = 0$ and Torsion free, show that

$$\nabla_\mu \nabla^\nu A^\mu = R_\mu{}^\nu A^\mu,$$

where ∇_μ is the covariant derivative, $g_{\mu\nu}$ is the metric tensor, and $R_{\mu\nu}$ is the Ricci tensor.

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2 Field theory of gravity

Question 2. Consider a vector field in a spherically symmetric spacetime. The field equation and the background spacetime are given by

$$\nabla_\mu \nabla^\mu A^\nu = 0,$$

$$ds^2 = -f(r) dt^2 + f(r)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,$$

where $A^\nu = (A^t, A^r, A^\theta, A^\phi)$ and each A^ν is a function of $\{t, r, \theta, \phi\}$.

2.1) [4 points.] Compute the Christoffel symbols $\Gamma_{tt}^r, \Gamma_{r\theta}^\theta, \Gamma_{r\phi}^\phi, \Gamma_{\theta\phi}^\phi, \Gamma_{rr}^r, \Gamma_{\theta\theta}^r, \Gamma_{\phi\phi}^r, \Gamma_{\phi\phi}^\theta$.

2.2) [6 points.] Derive the equation of motion for $\nabla_\mu \nabla^\mu A^\nu = 0$ with the free index $\nu = \theta$ assigned. (Note that $\Gamma_{\mu\nu}^\rho = \Gamma_{\nu\mu}^\rho$ and no need to separate for the equation of motion.)

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3 Hamiltonian formalism for General Relativity

Question 3.

Let us consider the total Hamiltonian for the General Relativity:

$$H = \int d^3x \left(N H_0 + N^i H_i + \lambda \pi_N + \lambda^i \pi_i \right), \quad (1)$$

where λ and λ^i are the Lagrange multipliers, N and N^i are a lapse function and a shift vector, while π_N and π_i are their momentum conjugate. The Hamiltonian and momentum constraints are given by

$$H_0 \equiv \frac{1}{\sqrt{h}} (2\Pi_{ij}\Pi^{ij} - \Pi^2) - \frac{1}{2}\sqrt{h}\mathcal{R}, \quad (2)$$

$$H_i \equiv -2D_j(h_{ik}\Pi^{jk}), \quad (3)$$

where we have set $8\pi G = 1$, Π_{ij} is the conjugate momentum of the induced metric h_{ij} on spacelike hypersurfaces, and $\mathcal{R}$ is the Ricci scalar on the hypersurfaces.

3.1) [4 points.] Determine all the constraints in this theory.

3.2) [3 points.] Show how many propagating degrees of freedom are eliminated by the constraints in the previous problem.

3.3) [3 points.] Describe the procedure for deriving the Lagrangian from the Hamiltonian in Eq. (1).

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4 Quantum Cosmology

Question 4.

4.1) [5 points.] Give at least two reasons why canonical quantization fails in gravity theories.

4.2) [5 points.] Derive the Wheeler-DeWitt equation by applying the canonical quantization to Eq. (2).

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5 Black hole solutions

Question 5.

5.1) [5 points.] What are the differences between static and stationary spacetime [1], and explain how these conditions influence the generic form of the metric tensor [4].

5.2) [5 points.] For the static spacetime with spherical symmetry, the general form of the metric can be written as

$$ds^2 = -e^{2\alpha(r)} dt^2 + e^{2\beta(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

and the non-zero independent components of the Ricci tensor can be found by

$$R_{00} = -e^{2(\alpha-\beta)} \left(\alpha'^2 - \beta' \alpha' + \alpha'' + \frac{2\alpha'}{r} \right), \quad (4)$$

$$R_{11} = \alpha'^2 - \beta' \alpha' + \alpha'' - \frac{2\beta'}{r}, \quad (5)$$

$$R_{22} = -e^{-2\beta} (-r\alpha' + e^{2\beta} + r\beta' - 1). \quad (6)$$

Solving solution for AdS/dS spacetime [3] and explaining the key features of such the solutions [2].

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6 Black hole thermodynamics

Question 6.

6.1) [3 points.] Write down the statement for the zeroth law of black hole mechanics and explain how it is different/similar to one for thermodynamics.

6.2) [3 points.] Explain the physical meaning of the surface gravity and describe how to obtain the formula of the surface gravity in terms of the Killing horizon.

6.3) [4 points.] By using the formula in 6.2), compute the explicit form of the surface gravity for the AdS/dS black hole.

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